第4章 微积分问题的MATLAB求解

山东理工大学数学学院 周世祥 20200320

Contents

极限计算

函数计算

```
sin(pi/4)
% 结果如下:
x=[pi/6,pi/4,pi/3];cos(x) %说明可同时计算三角函数在多点的值
```

```
ans =

    0.7071


ans =

    0.8660    0.7071    0.5000
```

一元函数的极限

limit()函数

$\lim_{x\to 0}\frac{2x-1}{x^2+3}$

```
clear
 syms x;
 limit(((2.*x-1)./(x.^2+3)),x,0)
```

```
ans =

-1/3
```

```
 clear
syms x;
f=sin(x)/x;
limit(f)
```

```
ans =

1
```

```
clear
syms n
limit((1+1/n)^n,inf)
```

```
ans =

exp(1)
```

右极限

```
syms x
limit(log(1+x)/x,x,0,'right')
```

```
ans =

1
```

多重极限

```
 syms x y
 f=((exp(x)+exp(y))/(cos(x)-sin(y)));
 limit(limit(f,x,0),y,0)
```

```
ans =

2
```

导数和微分

```
 clear
syms x
f=2^x+x^(1/2)*log(x);
diff(f)
```

```
ans =

         x                1                                  log(x)
        2  log(2) + ------------- + 0.5000000000000000000 -------------
                     0.5000000000                          0.5000000000
                    x                                     x
```

```
clear
syms x
f=sin(2*x+3);
diff(f,3) %三阶导数
```

```
ans =

                                -8 cos(2 x + 3)
```

偏导

```
clear
syms x y
f=log(exp(2*(x+y^2))+(x^2+y)+sin(1+x^2));
fx=diff(f,x)
fy=diff(f,y)
fxy=diff(fx,y)
fyx=diff(fy,x)
fxx=diff(fx,x)
fyy=diff(fy,y)
fxx=diff(f,x,2)
fyy=diff(f,y,2)
```

```
fx =

                           2                         2
                  2 exp(2 y  + 2 x) + 2 x + 2 x cos(x  + 1)
                  -----------------------------------------
                          2            2             2
                   exp(2 y  + 2 x) + x  + y + sin(x  + 1)

fy =

                                      2
                          4 y exp(2 y  + 2 x) + 1
                   --------------------------------------
                          2            2             2
                   exp(2 y  + 2 x) + x  + y + sin(x  + 1)

fxy =

                     2
```

```
             y exp(2 y  + 2 x)
  8 -------------------------------------- -
           2           2            2
    exp(2 y  + 2 x) + x  + y + sin(x  + 1)

                2                         2                   2
        (2 exp(2 y  + 2 x) + 2 x + 2 x cos(x  + 1)) (4 y exp(2 y  + 2 x) + 1)

           /         2           2            2     2
          /  (exp(2 y  + 2 x) + x  + y + sin(x  + 1))
         /

fyx =

                      2
             y exp(2 y  + 2 x)
  8 -------------------------------------- -
           2           2            2
    exp(2 y  + 2 x) + x  + y + sin(x  + 1)

                2                         2                   2
        (2 exp(2 y  + 2 x) + 2 x + 2 x cos(x  + 1)) (4 y exp(2 y  + 2 x) + 1)

           /         2           2            2     2
          /  (exp(2 y  + 2 x) + x  + y + sin(x  + 1))
         /

fxx =

           2                     2           2     2
  4 exp(2 y  + 2 x) + 2 + 2 cos(x  + 1) - 4 x  sin(x  + 1)
  --------------------------------------------------------
                  2           2            2
           exp(2 y  + 2 x) + x  + y + sin(x  + 1)

                   2                         2     2
           (2 exp(2 y  + 2 x) + 2 x + 2 x cos(x  + 1))
         - --------------------------------------------
                     2           2            2     2
             (exp(2 y  + 2 x) + x  + y + sin(x  + 1))

fyy =

           2              2        2
  4 exp(2 y  + 2 x) + 16 y  exp(2 y  + 2 x)
  -----------------------------------------
          2           2            2
   exp(2 y  + 2 x) + x  + y + sin(x  + 1)

                              2            2
                 (4 y exp(2 y  + 2 x) + 1)
         - ------------------------------------------
                    2           2            2     2
            (exp(2 y  + 2 x) + x  + y + sin(x  + 1))

fxx =

           2                     2           2     2
  4 exp(2 y  + 2 x) + 2 + 2 cos(x  + 1) - 4 x  sin(x  + 1)
  --------------------------------------------------------
                  2           2            2
           exp(2 y  + 2 x) + x  + y + sin(x  + 1)

                   2                         2     2
           (2 exp(2 y  + 2 x) + 2 x + 2 x cos(x  + 1))
```

```
             - --------------------------------------------
                          2           2             2      2
                 (exp(2 y   + 2 x) + x   + y + sin(x   + 1))

fyy =

            2               2         2
   4 exp(2 y   + 2 x) + 16 y   exp(2 y   + 2 x)
   -----------------------------------------
           2           2             2
    exp(2 y   + 2 x) + x   + y + sin(x   + 1)

                                2             2
                      (4 y exp(2 y   + 2 x) + 1)
             - -----------------------------------------
                          2           2             2      2
                 (exp(2 y   + 2 x) + x   + y + sin(x   + 1))
```

积分

```
syms x;
v= int(sin(x)/x,0,1)
vpa(v)
```

```
v =

                                   sinint(1)

ans =

                                 0.9460830704
```

```
clear
syms x
v=int(exp(-2*x),0,1)
vpa(v)
```

```
v =

                               1/2 - 1/2 exp(-2)

ans =

                                 0.4323323584
```

```
clear
syms x;
int(1/x,1,inf) %广义积分

v= int(1/(1+x^2),1,inf)
vpa(v)
```

```
ans =
```

```
                    infinity

v =

                    1/4 pi

ans =

                    0.7853981635
```

热辐射中的反常积分

```
syms x
f=x^3/(exp(x)-1);
int(f,0,inf)
```

```
ans =

                          4
                    1/15 pi
```

```
syms x;
f=1/(x^2+2*x+3); %有理分式积分
v= int(f,-inf,inf)
vpa(v)
```

```
v =

                            1/2
                    1/2 pi 2

ans =

                    2.221441469
```

不定积分

```
syms x y z
f=sin(x*y+z+1);
int(f)
```

```
ans =

                      cos(x y + z + 1)
                    - ----------------
                             y
```

对z的不定积分

```
clear
syms x y z
int(sin(x*y+z+1),z)
```

```
ans =

                          -cos(x y + z + 1)
```

级数求和

```
syms a b n
s=a^n+b*n;
symsum(s)
```

```
ans =

                           n
                          a
                         ----- + (1/2 n - 1/2) n b
                         a - 1
```

```
syms n x
s=sin(n*x);
symsum(s,n)
```

```
ans =

                       sin(x) cos(x n)
                  1/2 --------------- - 1/2 sin(x n)
                         cos(x) - 1
```

求级数 $s = 2^{sin(n*x)}$ 的前 $n-1$ 项，并求它前10项和。

```
syms n
s=2*sin(2*n)+4*cos(4*n)+2^n;
sum_n=symsum(s)
sum10=symsum(s,0,10)
vpa(sum10)
```

```
sum_n =

                       3       2                    4       3
  (-16 sin(n) cos(n)  cos(1)  sin(1) - 16 cos(n)  cos(1)

                                   3                          3
         + 8 sin(1) sin(n) cos(n)  - 2 sin(n) cos(n) cos(1)

                                   2                 4
         + 8 sin(n) cos(n) cos(1)  sin(1) + 16 cos(n)  cos(1)
```

```
                2        3           2               2    n       3
     + 16 cos(n)  cos(1)  + 2 cos(1)  sin(1) cos(n)  + 2  cos(1)

                                                                  2
     + 2 sin(n) cos(n) cos(1) - 4 sin(1) sin(n) cos(n) - 16 cos(n)  cos(1)

        n          /                2
     - 2  cos(1))  /  (cos(1) (cos(1)  - 1))
                  /
```

```
sum10 =

  2051 + 2 sin(2) + 4 cos(4) + 2 sin(4) + 4 cos(8) + 2 sin(6) + 4 cos(12)

     + 2 sin(8) + 4 cos(16) + 2 sin(10) + 4 cos(20) + 2 sin(12)

     + 4 cos(24) + 2 sin(14) + 4 cos(28) + 2 sin(16) + 4 cos(32)

     + 2 sin(18) + 4 cos(36) + 2 sin(20) + 4 cos(40)

ans =

                                2048.277123
```

无穷级数

```
syms n
s1=1/n;
v1=symsum(s1,1,inf)
clear
syms n
s2=1/n^3;
v2=symsum(s2,1,inf)  % zeta(3)
vpa(v2)
```

```
v1 =

                                infinity

v2 =

                                zeta(3)

ans =

                                1.202056903
```

泰勒展开

```
syms x
f=exp(-x);
f6=taylor(f)
```

```
f6 =
```

```
                2       3        4         5
1 - x + 1/2 x  - 1/6 x  + 1/24 x  - 1/120 x
```

```
syms a b x
f=a*sin(x)+b*cos(x);
f1=taylor(f,10) %10阶麦克劳林近似展开
f2=taylor(f,10,pi/2)
```

```
f1 =

                  2          3           4            5            6
  b + a x - 1/2 b x  - 1/6 a x  + 1/24 b x  + 1/120 a x  - 1/720 b x

                   7              8               9
         - 1/5040 a x  + 1/40320 b x  + 1/362880 a x

f2 =

                             -9                      -9
  1. a - 0.2051033808 10    b + (-0.2051033808 10    a - 1. b) %1

                                                 -9       2
         + (-0.5000000000 a + 0.1025516904 10    b) %1

                                -10                      3
         + (0.3418389680 10     a + 0.1666666667 b) %1

                                                 -11       4
         + (0.04166666667 a - 0.8545974200 10     b) %1

                                 -11                       5
         + (-0.1709194840 10     a - 0.008333333333 b) %1

                                                   -12       6
         + (-0.001388888889 a + 0.2848658067 10     b) %1

                                -13                        7
         + (0.4069511524 10     a + 0.0001984126984 b) %1

                                                    -14       8
         + (0.00002480158730 a - 0.5086889405 10     b) %1

                                 -15                  -5       9
         + (-0.5652099339 10     a - 0.2755731922 10    b) %1

  %1 := x - 1.57079632679489656
```

傅里叶展开

function [a0,an,bn] =Fourierzpi(f) % 自定义傅里叶变换

syms x n

a0=int(f,0,2*pi)/pi;

an=int(f*cos(n*x),0,2*pi)/pi;

bn=int(f*sin(n*x),0,2*pi)/pi;

end

```
clear all
syms x
f=x^2;
[a0,an,bn]=Fourierzpi(f)
```

```
a0 =

                                  26.31894507

an =

                -8                   11   2
  0.1161988018 10    (0.1081454405 10    n   sin(6.283185306 n)

                              10
         + 0.3442376286 10    n cos(6.283185306 n)

                             9                       /  3
         - 0.547871202 10   sin(6.283185306 n))   /   n
                                                     /

bn =

                -8                    11                     2
  0.1161988018 10    (-0.1081454405 10    cos(6.283185306 n) n

                              10
         + 0.3442376286 10    n sin(6.283185306 n)

                             9                                    9     /  3
         + 0.547871202 10   cos(6.283185306 n) - 0.547871202 10 )  /   n
                                                                      /
```

傅里叶积分变换

```
% clear
% syms x
% f = exp(-x^2);
% fourier(f)  matlab2016
```

```
% clear
% syms  w
% f = exp(-abs(w));
% fourier(f)
```

傅里叶反变换

```
% clear
% syms a w real
% f=exp(-w^2/(4*a^2));
% F = ifourier(f)
% exp(-a^2*x^2)/(2*pi^(1/2)*(1/(4*a^2))^(1/2)) 用matlab2016
```

拉普拉斯变换

```
clear
syms s
g=1/sqrt(s);
laplace(g)
```

```
ans =

/ pi \1/2
|----|
\ t  /
```

逆变换

```
clear
syms s
f=1/(s^2);
ilaplace(f)
```

```
ans =

t
```

多元函数分析

偏导

```
clear
syms x y z
f=[x*y*z;y;x+z];
v=[x,y,z];
jacobian(f,v) %雅克比矩阵
```

```
ans =

[z y    z x    y x]
[                 ]
[ 0      1      0 ]
[                 ]
[ 1      0      1 ]
```

```
clear
syms x y z
f=x^2+81*(y+1)^2+sin(z);
v=[x,y,z];
jacobian(f,v) %偏导
```

```
ans =

[2 x    162 y + 162    cos(z)]
```

梯度大小--在点(0,0,0)和(1,3,4)处的梯度

```
clear
syms x y z
f=x^2+2*y^2+3*z^2+x*y;
v=[x,y,z];
j=jacobian(f,v);
j1=subs(subs(subs(j,x,0),y,0),z,0);
j2= subs(subs(subs(j,x,1),y,3),z,4)
```

```
j2 =

                                [5    13    24]
```

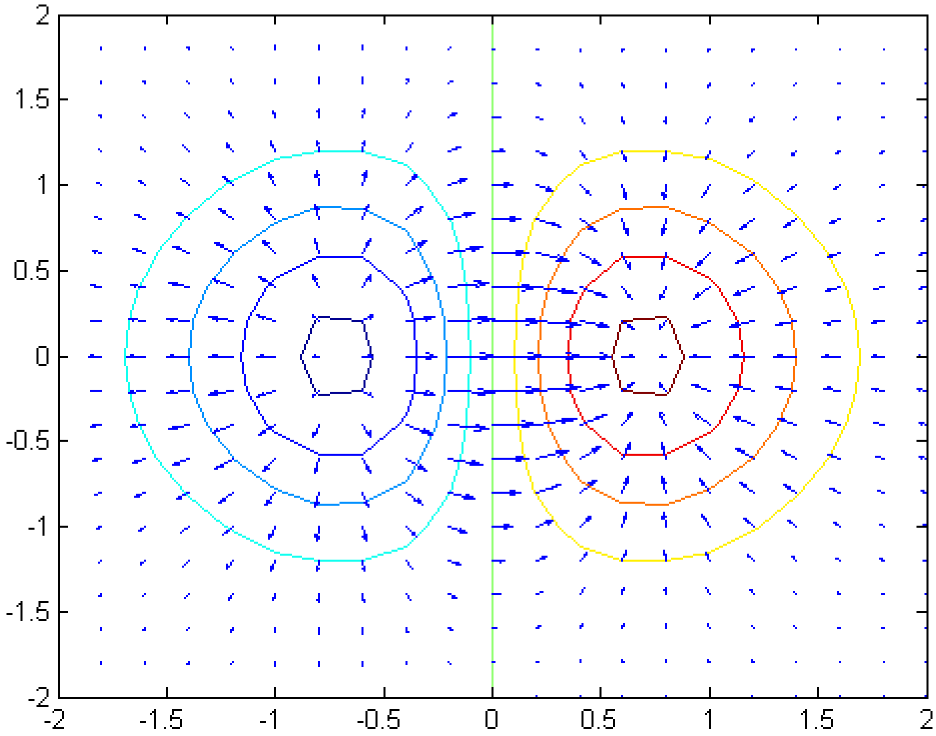
沿v=(1 ,2 ,3)的方向导数

```
clear
syms x y z
f=x^2+2*y^2+3*z^2+x*y;
v=[x,y,z];
j=jacobian(f,v);
v1=[1,2,3];
j.*v1
```

```
ans =

                        [2 x + y    2 x + 8 y    18 z]
```

多元函数的梯度

```
clear
v = -2:0.2:2;
[x,y] = meshgrid(v);
z = x .* exp(-x.^2 - y.^2);
[px,py] = gradient(z,0.2,0.2); %数值梯度
contour(v,v,z), hold on, quiver(v,v,px,py), hold off
```



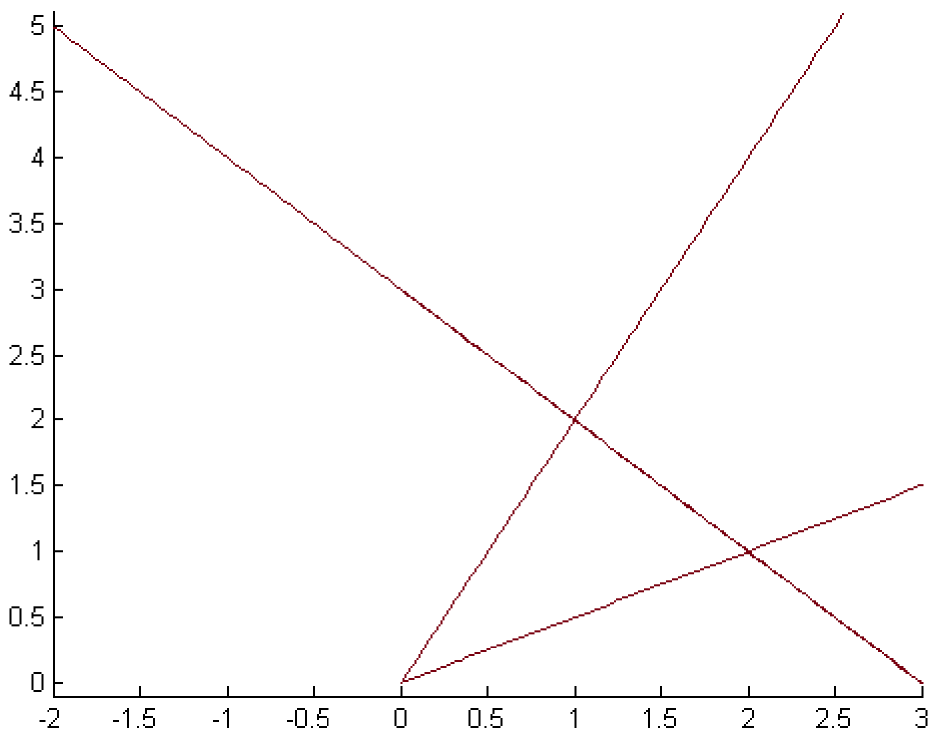
计算二重积分

$\int_D xdxdy$,其中D是由直线$y=2x, y=0.5x, y=3-x$所围成的平面区域。 先划定积分区域

```
clear
syms x y
f=x;
f1=2*x;
f2=0.5*x;
f3=3-x;
ezplot(f1); % 画函数图
hold on
ezplot(f2);
hold on
ezplot(f3);
hold on
ezplot(f3,[-2,3])
A=fzero('2*x-0.5*x',0) %确定积分限
B=fzero('3-x-0.5*x',8)
C=fzero('2*x-(3-x)',4)
ff1=int(f,0.5*x,2*x)  % 计算积分
ff11=int(ff1,0,1)
ff2=int(f,0.5*x,3-x)
ff22=int(ff2,1,2)
 ff11+ff22
```

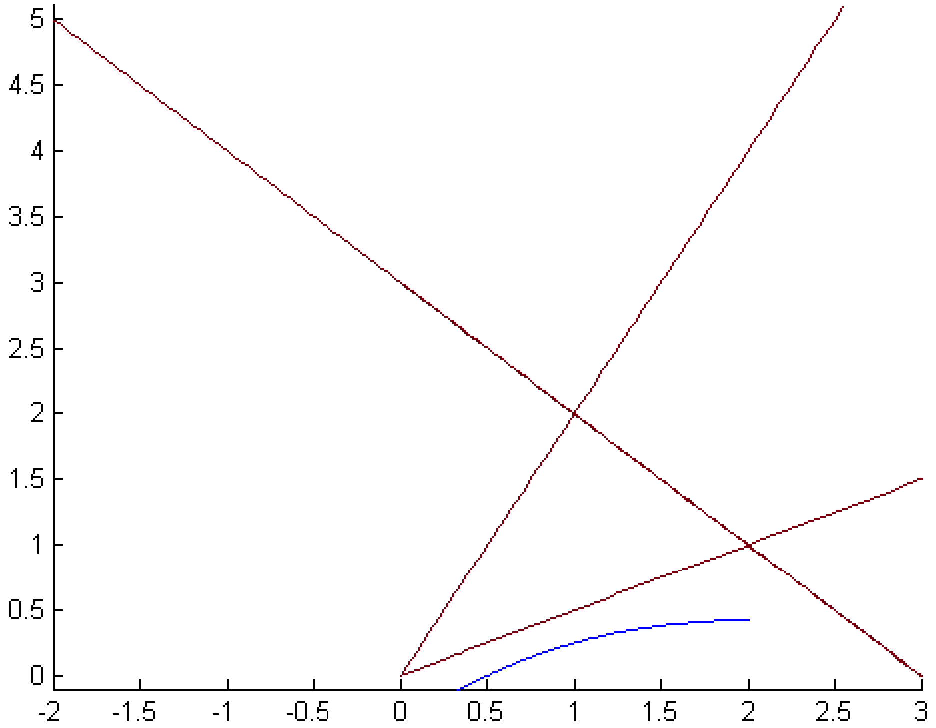
```
A =

     0


B =

     2
```

```
C =

     1


ff1 =

                                                  2
                                     1.875000000 x

ff11 =

                                      0.6250000000

ff2 =

                                          2                  2
                             1/2 (3 - x)  - 0.1250000000 x

ff22 =

                                      0.8750000000

ans =

                                       1.500000000
```



画函数曲线

```
x=-2:0.01:2;
y=((2.*x-1)./(x.^2+3));
plot(x,y,'-',[0],[-1/3],'o')
```



Published with MATLAB® R2014a