7037 Class A Group M Final Project

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1. Form Hypotheses

Treatment group will have higher click-through rates compared to control group.

H_0: Showing a friend's like in the ad has no impact on click-through rate.

H_1: Showing a friend's like increases the click-through rate.

In [1]:

```
import pandas as pd
import numpy as np

import warnings

import scipy.stats as stats
from scipy.stats import ttest_ind, chi2_contingency, mannwhitneyu
import statsmodels.api as sm
from statsmodels.stats.power import NormalIndPower
from statsmodels.stats.proportion import proportion_effectsize
import statsmodels.formula.api as smf
from scipy.stats import levene

import seaborn as sns
import matplotlib.pyplot as plt

from sklearn.linear_model import LinearRegression
```

In [2]:

```
df = pd.read_csv("project_data.csv")

print("data dimensions:", df.shape)
display(df.head())
print("\nData Types:")
display(df.dtypes)
```

```
data dimensions: (1000000, 20)
```

	user	adid	week	expid	if_click	real_like_cnt
0	297353	0b0fa14b56d3741178196daaa92e6a1e	w_2	1	0	3
1	98719	ea875acb76e0a806a7837174528f62d9	w_3	1	0	3
2	205144	1f1fe825014d9e9a0881233d9950bd43	w_2	1	0	1
3	559194	1f1fe825014d9e9a0881233d9950bd43	w_2	1	0	5
4	11888	0d82161270febc99f7defb653e339113	w_3	0	0	1

```
Data Types:
user                    int64
adid                   object
week                   object
expid                   int64
if_click                int64
real_like_cnt           int64
category               object
brand_effect            int64
experience              int64
status                  int64
user_age                int64
friend_age              int64
user_gender            object
friend_gender          object
user_city              object
friend_city            object
user_degree             int64
friend_degree           int64
user_sns_like_cnt       int64
user_sns_comment_cnt    int64
dtype: object
```

2. Sanity Check

The Sanity Check is to ensure the scientific nature and credibility of the subsequent analysis results.

In the experiment, we conduct SRM check, Covariate balance check and A/A test in sanity check.

In SRM check, we use Chi-square homogeneity check and t-test check to check if the control/treatment group split aligns with expectations.

In Covariate balance check, we use t-test and Mann-Whitney U to ensure comparability between groups.

In A/A test, we run the experiment without any intervention to verify whether the platform can generate results with no differences between groups.

2.1 Sample Ratio Mismatch check

Sample Ratio Match (SRM): Check if the control/treatment group split aligns with expectations.

2.1.1 Chi-square Homogeneity Test

In [3]:
```
# Calculate group sizes
control_count = df[df['expid'] == 0].shape[0]
treatment_count = df[df['expid'] == 1].shape[0]
total = control_count + treatment_count
expected_ratio = 0.5

print(f"Control Group: {control_count} ({control_count/total:.2%})")
print(f"Treatment Group: {treatment_count} ({treatment_count/total:.2%})")
```

```
Control Group: 500000 (50.00%)
Treatment Group: 500000 (50.00%)
```

In [4]:
```
observed = [control_count, treatment_count]
expected = [total * expected_ratio, total * expected_ratio]
chi2, p = stats.chisquare(f_obs=observed, f_exp=expected)

print(f"Chi-square Test: chi2={chi2:.4f}, p-value={p:.4f}")
if p < 0.05:
    print("Warning: Significant SRM detected!")
else:
    print("Passed: No significant SRM.")
```

```
Chi-square Test: chi2=0.0000, p-value=1.0000
Passed: No significant SRM.
```

2.1.2 T-tests

In [5]:
```
observed_proportion_control = control_count / total
observed_proportion_treatment = treatment_count / total

p = expected_ratio
variance = p * (1 - p) / total

# standard error
se = np.sqrt(variance)


# t = (observed - expected) / se
delta = observed_proportion_control - expected_ratio
t_stat = delta / se


degrees_of_freedom = total - 1

p_value = 2 * (1 - stats.t.cdf(abs(t_stat), df=degrees_of_freedom))
```

In [6]:
```
print(f"t-test:")
print(f"t = {t_stat:.4f}")
print(f"dof = {degrees_of_freedom}")
print(f"p-value = {p_value:.4f}")

alpha = 0.05
```

```python
if p_value < alpha:
    print("Warning: Significant SRM detected!")
else:
    print("Passed: No significant SRM.")
```

```
t-test:
t = 0.0000
dof = 999999
p-value = 1.0000
Passed: No significant SRM.
```

2.2 Covariate Balance Check

Covariate Balance: Verify if user characteristics are balanced across groups.

In [7]:
```python
categorical_vars = ['user_gender', 'user_city', 'category', 'brand_effect']
continuous_vars = ['user_age', 'user_degree',
                   'user_sns_like_cnt', 'user_sns_comment_cnt']

# Chi-square test for categorical variables
def check_categorical_balance(var):
    contingency_table = pd.crosstab(df[var], df['expid'])
    chi2, p, _, _ = stats.chi2_contingency(contingency_table)
    return p
```

In [8]:
```python
# t-test/Mann-Whitney U for continuous variables
warnings.filterwarnings("ignore",
                        message="scipy.stats.shapiro: " \
                        "For N > 5000, " \
                        "computed p-value may not be accurate.")

def check_continuous_balance(var):
    control = df[df['expid'] == 0][var].dropna()
    treatment = df[df['expid'] == 1][var].dropna()

    # Normality check (Shapiro-Wilk)
    _, p_control = stats.shapiro(control)
    _, p_treatment = stats.shapiro(treatment)

    if (p_control < 0.05) or (p_treatment < 0.05):
        # Non-normal -> Mann-Whitney U
        stat, p = stats.mannwhitneyu(control, treatment)
        test_type = "Mann-Whitney U"
    else:
        # Normal -> T-test
        stat, p = stats.ttest_ind(control, treatment)
        test_type = "T-test"
    return test_type, p
```

In [9]:
```python
print("Categorical Variables Balance:")
for var in categorical_vars:
    p = check_categorical_balance(var)
    print(f"{var}: p={p:.4f} {'(Imbalanced)' if p < 0.05 else '(Balanced)'}")
```

```
Categorical Variables Balance:
user_gender: p=0.1597 (Balanced)
user_city: p=0.3797 (Balanced)
category: p=1.0000 (Balanced)
brand_effect: p=0.9886 (Balanced)
```

In [10]:

```python
print("Continuous Variables Balance:")
for var in continuous_vars:
    test_type, p = check_continuous_balance(var)
    print(f"{var} ({test_type}): p={p:.4f} {'(Imbalanced)' if p < 0.05
                                            else '(Balanced)'}")
```

```
Continuous Variables Balance:
user_age (Mann-Whitney U): p=0.7586 (Balanced)
user_degree (Mann-Whitney U): p=0.2262 (Balanced)
user_sns_like_cnt (Mann-Whitney U): p=0.1126 (Balanced)
user_sns_comment_cnt (Mann-Whitney U): p=0.1807 (Balanced)
```

2.3 A/A test

In [13]:

```python
from statsmodels.stats.proportion import proportions_ztest
control_df = df[df['expid'] == 0].copy()

np.random.seed(42)
control_df['AA_group'] = np.random.choice(['A1', 'A2'],
                                          size=len(control_df), p=[0.5, 0.5])

aa_counts = control_df['AA_group'].value_counts()
print(aa_counts)
```

```
AA_group
A2    250445
A1    249555
Name: count, dtype: int64
```

In [24]:

```python
# ctr of aa1 and aa2
aa1_clicks = control_df[control_df['AA_group'] == 'A1']['if_click'].mean()
aa2_clicks = control_df[control_df['AA_group'] == 'A2']['if_click'].mean()

# z-test
clicks = [control_df[control_df['AA_group'] == 'A1']['if_click'].sum(),
          control_df[control_df['AA_group'] == 'A2']['if_click'].sum()]
nobs = [aa_counts['A1'], aa_counts['A2']]
z_stat, p_click = proportions_ztest(clicks, nobs)
print(f"A/A ctr: A1={aa1_clicks:.3f}, A2={aa2_clicks:.3f}, p-value={p_click:.4f}
if p_click < 0.05:
    print("Warning: Exist significant difference "
    "in the click-through rates of the A/A subgroups.")
else:
    print("Pass: No significant difference "
    "in the click-through rates of the A/A subgroups.")
```

```
A/A ctr: A1=0.067, A2=0.066, p-value=0.1642
Pass: No significant difference in the click-through rates of the A/A subgroups.
```

2.4 Conclusion

The experiment has passed the SRM check, the covariate balance check and A/A test, indicating that the randomization mechanism of the experiment is effective and the data quality is reliable.

Next, we can analyze the differences in click-through rates between the treatment group and the control group to evaluate whether displaying "1 like" in the advertisement can significantly enhance user interaction.

3.Power Analysis

3.1 Extract Effect Sizes (ΔCTR)

We need the baseline click-through rate (CTR) in Control and the CTR in Treatment for each product category, then compute ΔCTR = CTR_Treatment − CTR_Control. We will use the smallest ΔCTR (most conservative) for our power calculations.

In [25]:
```
ctr = (
    df
    .groupby(['category','expid'])['if_click']
    .agg(CTR='mean', N='count')
    .reset_index()
)

# 1.3 Pivot so we have Control vs Treatment side-by-side
pivot = ctr.pivot(index='category', columns='expid', values=['CTR','N'])
pivot.columns = ['CTR_Control','CTR_Treatment','N_Control','N_Treatment']
pivot = pivot.reset_index()

# 1.4 Compute ΔCTR
pivot['Delta_CTR'] = pivot['CTR_Treatment'] - pivot['CTR_Control']

print(pivot[['category','CTR_Control','CTR_Treatment','Delta_CTR','N_Control']])
```

```
     category  CTR_Control  CTR_Treatment  Delta_CTR  N_Control
0    Beverage      0.06421        0.07216    0.00795   100000.0
1         Car      0.06520        0.07245    0.00725   100000.0
2     Clothes      0.06010        0.06698    0.00688   100000.0
3  Cosmetrics      0.06772        0.07451    0.00679   100000.0
4     Jewelry      0.07579        0.08643    0.01064   100000.0
```

3.2 Post-Hoc Power Calculation

In [27]:
```
from statsmodels.stats.power import NormalIndPower
from statsmodels.stats.proportion import proportion_effectsize

alpha = 0.05
p1 = 0.0677
delta = 0.0068
p2 = p1 + delta
n_per_group = 100000
```

```python
h = proportion_effectsize(p1, p2)

analysis = NormalIndPower()
power_now = analysis.power(effect_size=h,
                           nobs1=n_per_group,
                           alpha=alpha,
                           ratio=1.0)
print(f"Post-hoc power = {power_now:.3f}")
```

```
Post-hoc power = 1.000
```

This means with 100 k observations per group, you have virtually 100% chance of detecting a 0.0068 CTR uplift at α=0.05.

3.3 A-priori Sample Size

In [28]:

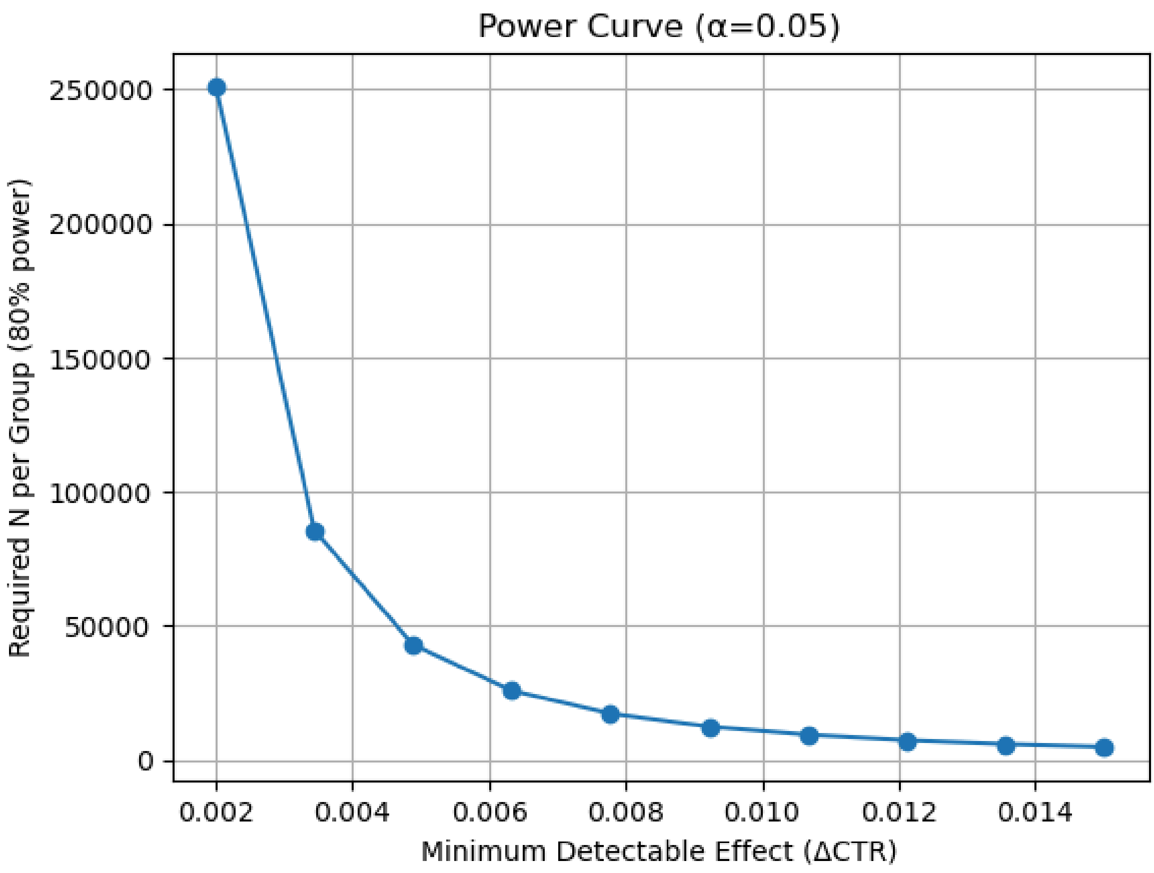
```python
target_power = 0.8
required_n = analysis.solve_power(effect_size=h,
                                  power=target_power,
                                  alpha=alpha,
                                  ratio=1.0)
print(f"Required N per group for 80% power: {required_n:.0f}")
```

```
Required N per group for 80% power: 22409
```

3.4 Power Curve

In [29]:

```python
deltas = np.linspace(0.002, 0.015, 10)
required_ns = [
    analysis.solve_power(effect_size=proportion_effectsize(p1, p1 + d),
                         power=target_power,
                         alpha=alpha)
    for d in deltas
]

plt.plot(deltas, required_ns, marker='o')
plt.xlabel('Minimum Detectable Effect (ΔCTR)')
plt.ylabel('Required N per Group (80% power)')
plt.title('Power Curve (α=0.05)')
plt.grid(True)
plt.show()
print(required_ns)
```




```
[251073.54532742177, 85464.45805562967, 42825.16151824529, 25756.376967262233, 17234.763357283256, 12369.465782141377, 9328.484500104596, 7299.439577785061, 5877.004926399219, 4840.467144607184]
```

3.5 Conclusion:

To ensure our experiment can reliably detect even the smallest meaningful lift in click-through rate (CTR), we conducted both post-hoc and a-priori power calculations based on the minimum observed effect size.

1. **Post-hoc Power**:
    - **Inputs**: ΔCTR = 0.0068, N = 100 000 per arm, α = 0.05
    - **Result**: Achieved power ≈ 1.000 (100%)
    - **Interpretation**: Even the smallest observed lift would almost certainly be detected. There is virtually no risk of a false negative due to insufficient sample size.
2. **A-priori Sample Size**:
    - To achieve 80% power for ΔCTR = 0.0068, the required sample size is ≈ 25 756 per group.
    - For larger lifts (e.g. ΔCTR ≥ 0.0107), fewer than 10 000 per group are needed; for very small lifts (ΔCTR ≈ 0.002), over 250 000 per group would be required.
3. **Implications**:
    - Our current sample size (100 k per arm) far exceeds what is needed, ensuring robust detection across all product categories.

- If resources become constrained, sample sizes could be reduced to ~30 k per arm while still maintaining ≥ 80% power for a 0.68 pp lift. To design future tests for smaller lifts (e.g. ΔCTR = 0.003–0.005), plan for ~ 43 k–85 k per arm, as indicated by the power curve.

The experiment is highly powered, ensuring that all practically important CTR improvements can be detected with high confidence.

4. Compare test

4.1 Compare the test and control groups

We need to calculate the CTR mean and standard deviation for control and treatment groups. And we use t-test to check if the difference is significant and use levins test to check if the variance is equal.

In [30]:
```
# 1 calculate the click-through rate (CTR)
group_summary = df.groupby('expid')['if_click'].agg(
    ['mean', 'std', 'count']).reset_index()
group_summary.columns = ['Group (0=Control, 1=Treatment)',
                         'CTR Mean', 'CTR Std Dev', 'N']
print(group_summary)
```

```
   Group (0=Control, 1=Treatment)  CTR Mean  CTR Std Dev       N
0                               0  0.066604     0.249335  500000
1                               1  0.074506     0.262593  500000
```

In [31]:
```
# 2 calculate the variance of the click-through rate (CTR) for each group
control = df[df['expid'] == 0]['if_click']
treatment = df[df['expid'] == 1]['if_click']

var_control = np.var(control, ddof=1)
var_treatment = np.var(treatment, ddof=1)

print(f"Control variance: {var_control:.4f}")
print(f"Treatment variance: {var_treatment:.4f}")
```

```
Control variance: 0.0622
Treatment variance: 0.0690
```

In [32]:
```
# 3 test for the CTR mean in CTR between the two groups

t_stat, p_val = stats.ttest_ind(treatment, control, equal_var=False)
print(f"T-test result: t = {t_stat:.4f}, p = {p_val:.4f}")

# 4 test for the CTR variance between the two groups
f, p_val_var = stats.levene(control, treatment)
print(f"Levene's test result: F = {f:.4f}, p = {p_val_var:.4f}")
```

```
T-test result: t = 15.4306, p = 0.0000
Levene's test result: F = 238.1031, p = 0.0000
```

4.1.1 Click-Through Rate Comparison

According to the code results:

- Control Group (0): CTR = 6.66% (SD = 0.249)
- Treatment Group (1): CTR = 7.45% (SD = 0.263)
- T-test: t=15.43, $p<0.001$
- Levene's Test: F=238.10, $p<0.001$

We can summarize the results as follows:

1. Statistically Significant Improvement: The treatment group (showing 1 organic like) achieves a 0.79% absolute increase in CTR (11.8% relative increase) compared to the control group. The extremely low p-value ($p<0.001$) confirms this difference is not due to chance. For context, even small CTR improvements in large-scale platforms like WeChat Moments Ads can translate to millions of incremental clicks.

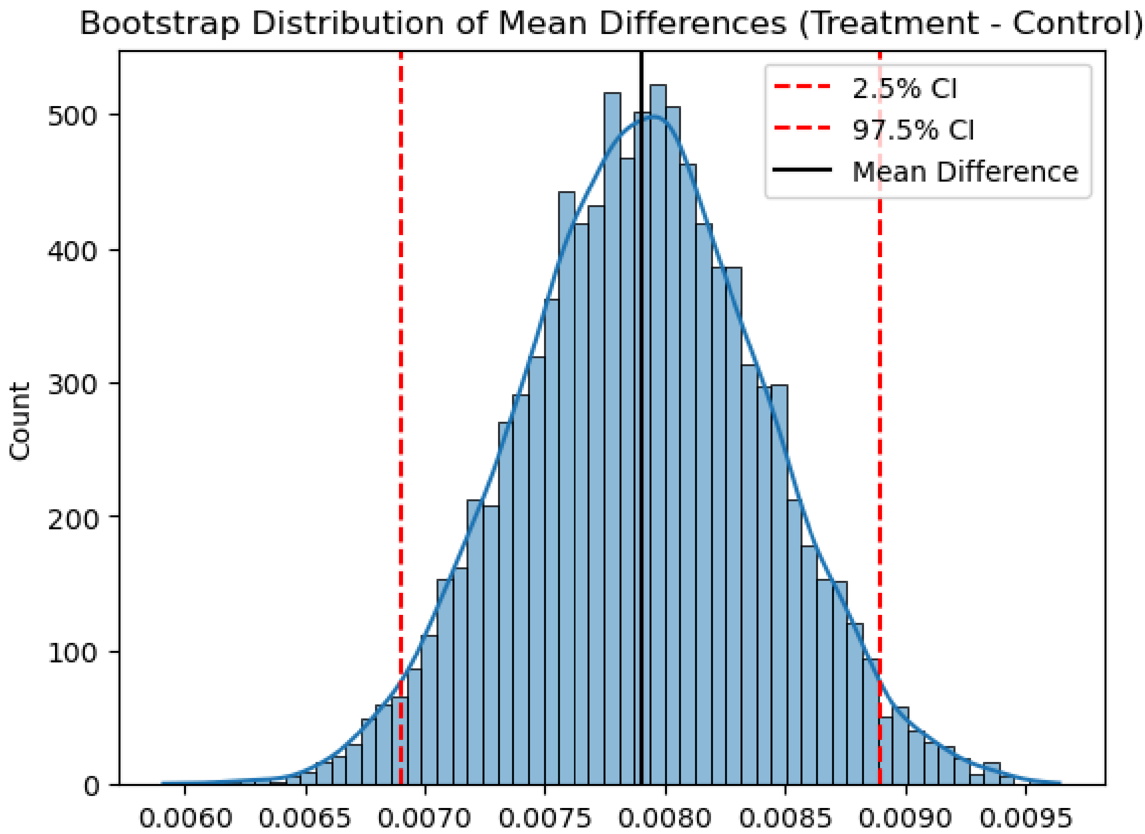
2. Unequal Variance: The Levene's test ($p<0.001$) rejects equal variance between groups. The treatment group exhibits higher variance in CTR ($\sigma^2=0.069$) than the control ($\sigma^2=0.062$), suggesting heterogeneity in user responses to social proof. Some users in the treatment group may be more influenced by the displayed like, while others remain indifferent.

4.1.2 Bootstrap

To make test more reliable, we can use bootstrap method to calculate the confidence interval for CTR. The bootstrap method is a resampling technique that allows us to estimate the distribution of a statistic (in this case, CTR) by repeatedly sampling from the data with replacement. This can help us obtain more robust estimates of the mean and standard deviation, especially when the sample size is small or when the data is not normally distributed.

In [33]:
```
# 5 bootstrap the CTR mean difference between the two groups
def bootstrap_diff_means(a, b, n_bootstrap=10000):
    boot_diffs = []
    for _ in range(n_bootstrap):
        sample_a = np.random.choice(a, size=len(a), replace=True)
        sample_b = np.random.choice(b, size=len(b), replace=True)
        boot_diffs.append(sample_b.mean() - sample_a.mean())
    return boot_diffs

boot_diffs = bootstrap_diff_means(control, treatment)

# confidence interval
ci_low = np.percentile(boot_diffs, 2.5)
ci_high = np.percentile(boot_diffs, 97.5)

sns.histplot(boot_diffs, kde=True)
plt.axvline(ci_low, color='red', linestyle='--', label='2.5% CI')
plt.axvline(ci_high, color='red', linestyle='--', label='97.5% CI')
```

```python
plt.axvline(np.mean(boot_diffs), color='black', label='Mean Difference')
plt.title('Bootstrap Distribution of Mean Differences (Treatment - Control)')
plt.legend()
plt.show()

print(f"95% CI from bootstrap: ({ci_low:.4f}, {ci_high:.4f})")
```




```
95% CI from bootstrap: (0.0069, 0.0089)
```

According to the code results, we can find: The 95% CI does not include zero, reinforcing the t-test conclusion ($p<0.001$) that the treatment effect is statistically significant. Also, the narrow CI (0.0069–0.0089) suggests high certainty that the true CTR lift lies between 0.6% and 0.9%. This is critical for quantifying the business impact.

4.2 Conclusion

The treatment group (showing 1 organic like) achieves a statistically significant CTR improvement of 0.79% (11.8% relative increase) compared to the control group. The bootstrap method confirms this effect with a 95% confidence interval of (0.0069, 0.0089). The treatment group also exhibits higher variance in CTR, indicating heterogeneous user responses to social proof.

4.3 improve sensitive

4.3.1 Method 1: CUPED

After applying the CUPED adjustment using user_sns_like_cnt as the covariate, the adjusted click-through rates (CTR) are 6.66% for the control group and 7.45% for the treatment group. The t-test yields a highly significant t-statistic of 15.43 (p < 0.0001), indicating that the difference in CTR between treatment and control is statistically significant after adjustment. However, the reduction in standard deviation (like for treatmen from about 0.262593 to 0.262591) is relatively small, suggesting that user_sns_like_cnt alone may not be a strong covariate. More covariates could be considered to further improve the variance reduction and test sensitivity.

In [34]:
```
def cuped_adjust(y, x):
    theta = np.cov(y, x)[0,1] / np.var(x)
    y_adj = y - theta * (x - np.mean(x))
    return y_adj

# Apply CUPED
df['If_click_adj'] = cuped_adjust(df['if_click'], df['user_sns_like_cnt'])
control_adj = df[df['expid'] == 0]['If_click_adj']
treatment_adj = df[df['expid'] == 1]['If_click_adj']

print(df.groupby('expid')['If_click_adj'].mean())
print(df.groupby('expid')['If_click_adj'].std())

# T-test on adjusted outcome
t_stat_adj, p_val_adj = ttest_ind(treatment_adj, control_adj, equal_var=False)
print(t_stat_adj, p_val_adj)
```

```
expid
0    0.066603
1    0.074507
Name: If_click_adj, dtype: float64
expid
0    0.249335
1    0.262591
Name: If_click_adj, dtype: float64
15.434507182195537 9.731616423447789e-54
```

4.4 Multivariate CUPED

After applying multi-variable CUPED adjustment using several user behavior covariates (user_sns_like_cnt, user_sns_comment_cnt, User_degree, and Real_like_cnt), the adjusted CTRs are 0.066603 for the control group and 0.074507 for the treatment group. The t-test yields a t-statistic of -15.44 with a p-value < 0.0001, indicating a highly significant difference in adjusted CTRs. Compared to single-variable CUPED, the variance reduction is slightly better (standard deviations decreased marginally), but the overall sensitivity is still primarily driven by the large sample size.

In [35]:
```
# assuming df is your DataFrame and it has been loaded correctly
covariates = ['user_sns_like_cnt', 'user_sns_comment_cnt',
              'user_degree', 'real_like_cnt']

# drop rows with NaN in covariates or click
```

```python
df_cuped = df.dropna(subset=covariates + ['if_click'])

X = df_cuped[covariates]
y = df_cuped['if_click']

model = LinearRegression().fit(X, y)

# predict the click-through rate using the covariates
y_pred = model.predict(X)

# CUPED adjustment
y_adj = y - (y_pred - y.mean())

df_cuped['If_click_adj_multi'] = y_adj

control_adj = df_cuped[df_cuped['expid'] == 0]['If_click_adj_multi']
treatment_adj = df_cuped[df_cuped['expid'] == 1]['If_click_adj_multi']

# T-test on adjusted outcome
t_stat_adj, p_val_adj = ttest_ind(control_adj, treatment_adj, equal_var=False)

print("T-test results after multivariate CUPED:")
print(f"t-statistic: {t_stat_adj}")
print(f"p-value: {p_val_adj}")

print(df_cuped.groupby('expid')['If_click_adj_multi'].agg(['mean', 'std']))
```

```
T-test results after multivariate CUPED:
t-statistic: -15.436472359739689
p-value: 9.439732307805363e-54
           mean       std
expid
0      0.066603  0.249315
1      0.074507  0.262585
```

4.5 Stratification (gender, city, category)

we implemented stratified analysis with the primary goal of reducing variance and increasing the sensitivity of our estimation when comparing click-through rates (CTR) between the Treatment and Control groups.

In A/B testing, if covariates such as gender, city, or ad category systematically influence CTR, a simple comparison of group-level averages may be biased or inefficient. By stratifying users based on these variables—gender, user_city, and category—and calculating within-stratum CTRs followed by a weighted average, we can better control for these confounding effects.

In this part, we choose gender, city, and category for stratification analysis.

In [37]:
```python
def stratified_analysis_test(df, group_col, strata_col, outcome_col):
    """
    Stratified analysis + visualization + t-test + SE comparison

    Parameters:
    df -- original data
    group_col -- grouping column, such as 'group'
```

```
    strata_col -- stratification column, such as 'user_city'
    outcome_col -- result column, such as 'click'

    Return:
    stratified_df -- detailed table after stratification
    summary -- weighted statistics for each group
    t_test_result -- t-test statistic and p-value
    e_comparison -- percentage change in SE before and after stratification
    """
    # overall statistics
    overall = df.groupby(group_col)[outcome_col].agg(
        ['mean', 'count', 'std']).reset_index()
    overall['se'] = overall['std'] / np.sqrt(overall['count'])

    # stratified statistics
    stratified = df.groupby([group_col, strata_col])[outcome_col].agg(
        ['mean', 'count', 'std']).reset_index()
    stratified['se'] = stratified['std'] / np.sqrt(stratified['count'])

    summary = {}
    stratified_means = {}

    for group in stratified[group_col].unique():
        subset = stratified[stratified[group_col] == group]

        weighted_mean = np.average(subset['mean'], weights=subset['count'])

        weighted_var = ((subset['se'] ** 2) * (subset['count'] ** 2)
                        ).sum() / (subset['count'].sum() ** 2)
        weighted_se = np.sqrt(weighted_var)

        weighted_std = weighted_se * np.sqrt(subset['count'].sum())

        summary[group] = {
            'weighted_mean': weighted_mean,
            'weighted_se': weighted_se,
            'weighted_std': weighted_std,
            'total_count': subset['count'].sum()
        }

        stratified_means[group] = weighted_mean

    # -- t test --
    group0_mean = stratified_means[0]
    group1_mean = stratified_means[1]
    group0_se = summary[0]['weighted_se']
    group1_se = summary[1]['weighted_se']

    t_stat = (group1_mean - group0_mean) / np.sqrt(group0_se**2 + group1_se**2)

    from scipy.stats import norm
    p_value = 2 * (1 - norm.cdf(abs(t_stat)))

    t_test_result = {'t_statistic': t_stat, 'p_value': p_value}

    # compare SE before and after stratification
    original_se0 = overall.loc[overall[group_col]==0, 'se'].values[0]
    original_se1 = overall.loc[overall[group_col]==1, 'se'].values[0]

    stratified_se0 = summary[0]['weighted_se']
```

```
    stratified_se1 = summary[1]['weighted_se']

    reduction0 = (original_se0 - stratified_se0) / original_se0 * 100
    reduction1 = (original_se1 - stratified_se1) / original_se1 * 100

    se_comparison = {
        'Control SE reduction (%)': reduction0,
        'Treatment SE reduction (%)': reduction1
    }

    # plotting
    groups = list(summary.keys())
    weighted_means = [summary[g]['weighted_mean'] for g in groups]
    weighted_ses = [summary[g]['weighted_se'] for g in groups]

    plt.figure(figsize=(8, 6))
    plt.bar(groups, weighted_means, yerr=weighted_ses, capsize=5,
            color=['skyblue', 'salmon'])
    plt.xlabel('Group')
    plt.ylabel('Weighted CTR')
    plt.title(f'Weighted CTR by Group (Stratified by {strata_col})')
    plt.xticks(groups)
    plt.grid(axis='y', linestyle='--', alpha=0.7)
    plt.show()

    return stratified, summary, t_test_result, se_comparison
```

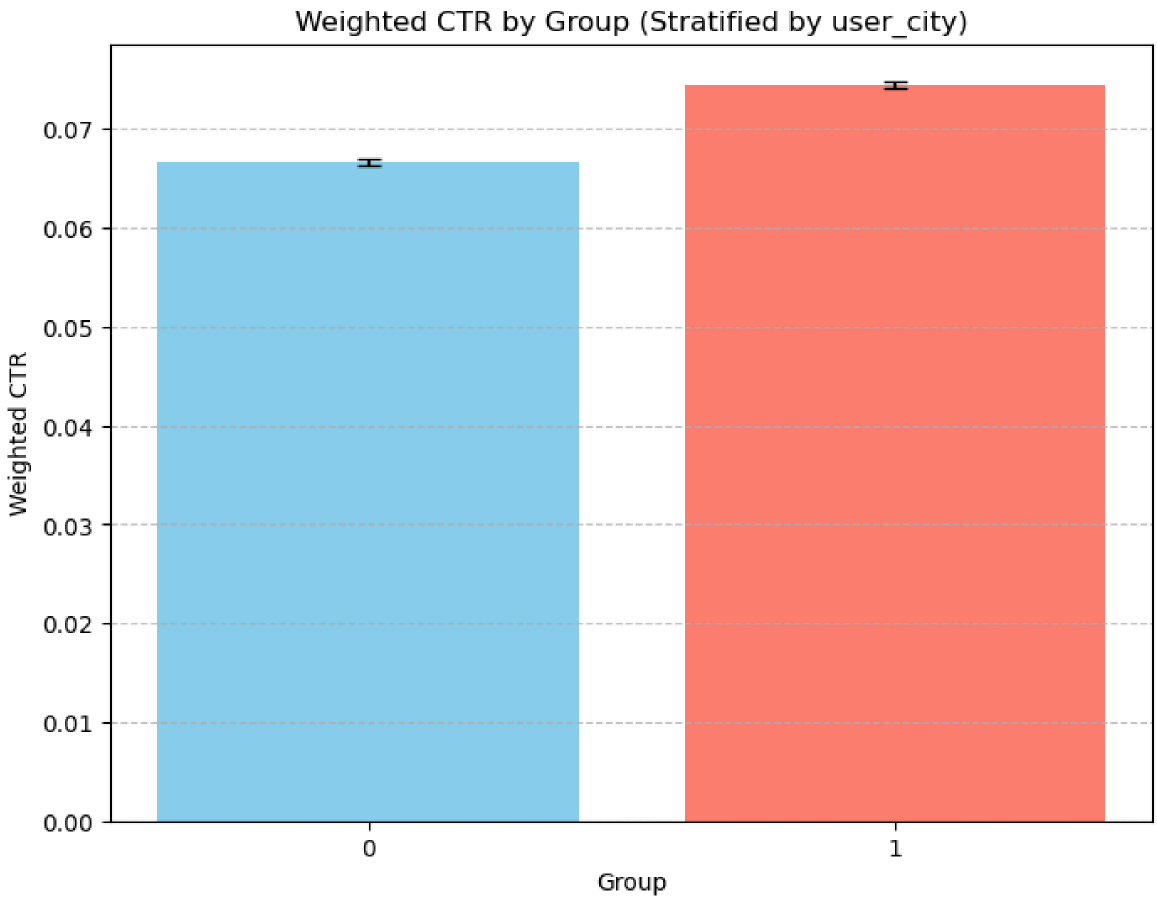
4.5.1 Stratifying by city

After stratifying by city, the weighted CTR is 0.0666 for the Control group and 0.0745 for the Treatment group.

The standard errors changed only marginally (SE reduction of about 0.0021% and 0.00055%, respectively), suggesting that city-level heterogeneity in CTR is minimal, and stratification by city has limited impact on variance reduction.

Nevertheless, the t-test still yields a highly significant result (t = 15.43, p < 0.001), indicating a strong treatment effect overall.

In [38]:
```
stratified_df, summary, t_test_result, se_comparison = stratified_analysis_test(
    df, group_col='expid', strata_col='user_city', outcome_col='if_click'
)

print("----info----")
for expid, info in summary.items():
    print(f"Group {expid}: Weighted CTR={info['weighted_mean']:.4f}, Weighted SE

print("\n----t test----")
print(t_test_result)

print("\n----SE change percentage (stratified sensitivity improvement)----")
print(se_comparison)
```




```
----info----
Group 0: Weighted CTR=0.0666, Weighted SE=0.000353, Weighted STD=0.249330
Group 1: Weighted CTR=0.0745, Weighted SE=0.000371, Weighted STD=0.262591

----t test----
{'t_statistic': 15.430792802478825, 'p_value': 0.0}

----SE change percentage (stratified sensitivity improvement)----
{'Control SE reduction (%)': 0.002149270866631768, 'Treatment SE reduction (%)':
0.0005548966099489254}
```

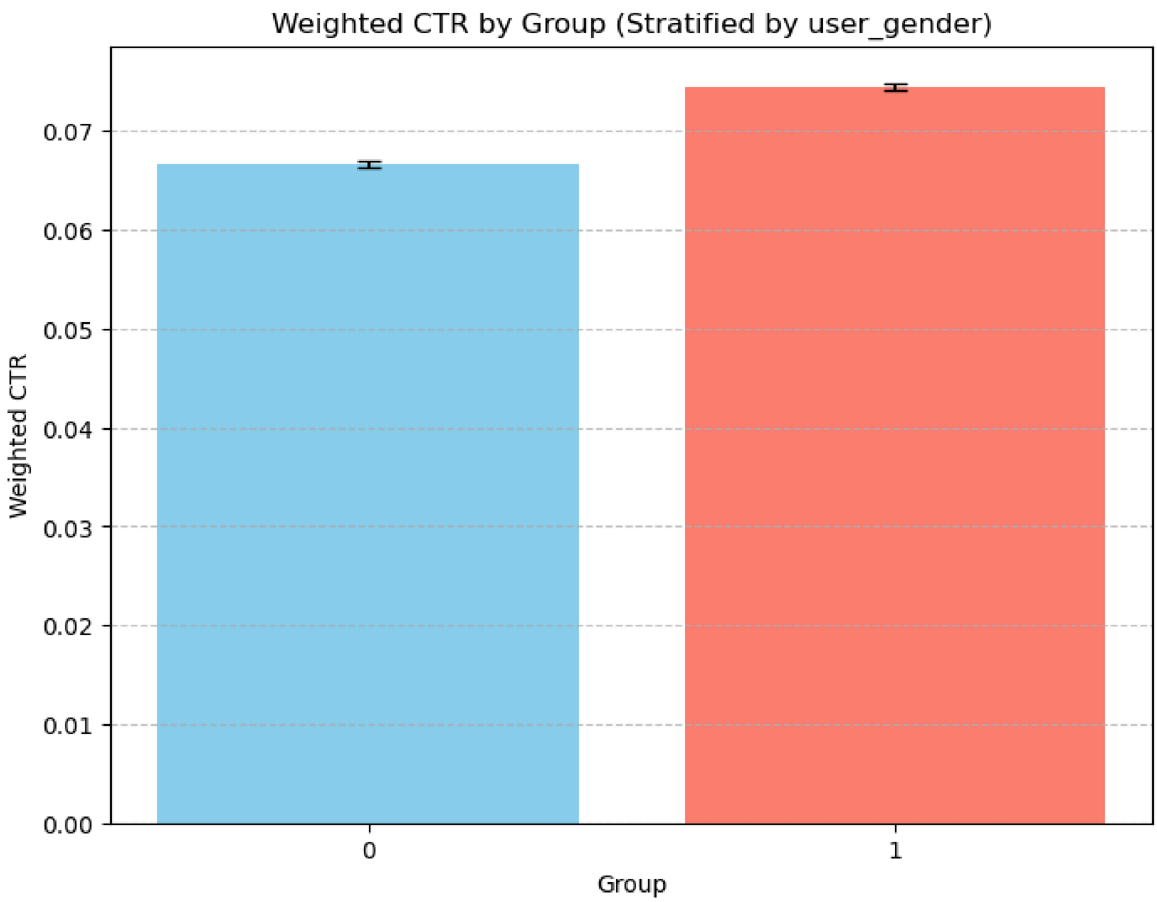
4.5.2 Stratifying by gender

After stratifying by gender, the weighted CTR remains the same (0.0666 vs. 0.0745). However, the standard errors did not improve; they slightly increased (SE change around -0.00009%), indicating that gender does not significantly explain the variance in CTR. Gender is not an effective stratification variable in this context—likely due to balanced gender distribution or minimal gender influence on CTR.

In [39]:

```
stratified_df, summary, t_test_result, se_comparison = stratified_analysis_test(
    df, group_col='expid', strata_col='user_gender', outcome_col='if_click'
)

print("----info----")
for group, info in summary.items():
    print(f"Group {group}: Weighted CTR={info['weighted_mean']:.4f}, Weighted SE

print("\n----t test----")
print(t_test_result)
```

```
print("\n----SE change percentage (stratified sensitivity improvement)----")
print(se_comparison)
```




```
----info----
Group 0: Weighted CTR=0.0666, Weighted SE=0.000353, Weighted STD=0.249335
Group 1: Weighted CTR=0.0745, Weighted SE=0.000371, Weighted STD=0.262593

----t test----
{'t_statistic': 15.430576362113849, 'p_value': 0.0}

----SE change percentage (stratified sensitivity improvement)----
{'Control SE reduction (%)': -9.004878223387991e-05, 'Treatment SE reduction
(%)': -9.344655083062433e-05}
```

4.5.3 Stratifying by category

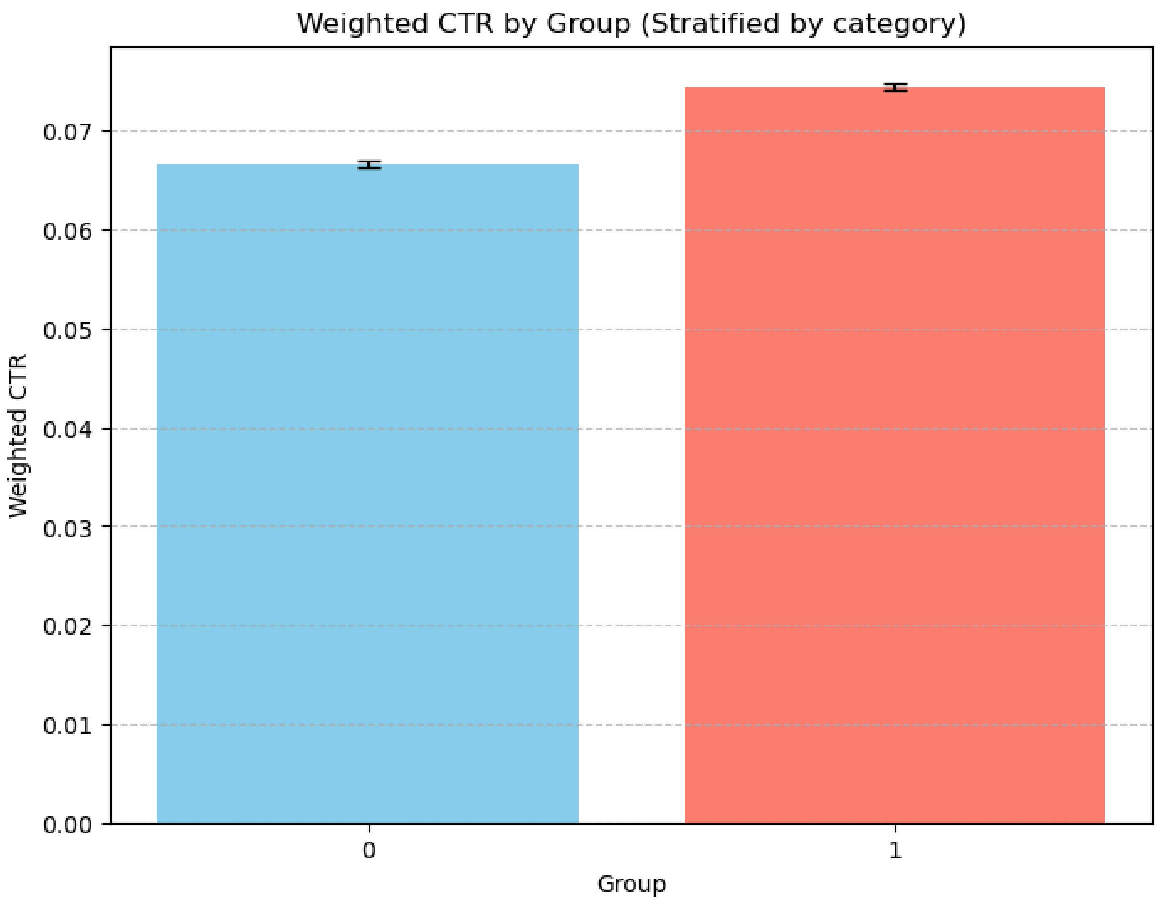
Stratifying by category produced the most significant improvement in sensitivity: SE dropped 2.14% for Control， SE dropped 2.98% for Treatment

While average CTR remains consistent, the reduction in estimation error indicates that ad category explains a meaningful portion of CTR variation.

Conclusion: Category is a high-value stratification variable and should be prioritized in future experimental design or quota planning.

In [40]:
```
stratified_df, summary, t_test_result, se_comparison = stratified_analysis_test(
    df, group_col='expid', strata_col='category', outcome_col='if_click'
)

print("----info----")
for group, info in summary.items():
```

```
    print(f"Group {group}: Weighted CTR={info['weighted_mean']:.4f}, Weighted SE

print("\n----t test----")
print(t_test_result)

print("\n----SE change percentage (stratified sensitivity improvement)----")
print(se_comparison)
```




```
----info----
Group 0: Weighted CTR=0.0666, Weighted SE=0.000353, Weighted STD=0.249282
Group 1: Weighted CTR=0.0745, Weighted SE=0.000371, Weighted STD=0.262514

----t test----
{'t_statistic': 15.434580785250795, 'p_value': 0.0}

----SE change percentage (stratified sensitivity improvement)----
{'Control SE reduction (%)': 0.021419592717211333, 'Treatment SE reduction (%)':
0.029849599870161312}
```

4.6 Covariate Adjustment (Regression)

The model adjusts for the potential confounding effects of various covariates, such as demographic information (e.g., user_gender, user_age), user behaviors (e.g., user_sns_like_cnt), and other ad-related variables (e.g., category, brand_effect). By adjusting for these covariates, the model attempts to isolate the effect of the primary treatment variable group (control vs. treatment) on the outcome (click).

We can find that Group variable (Treatment Effect): The group variable has a significant positive coefficient (0.0079, p-value < 0.001), indicating that, on average, the treatment group has a higher click-through rate (CTR) than the control group.The covariate adjustment increases sensitivity by reducing bias that could arise from omitting important variables.

In addition, we also find something intersting like:

Brand effect and Experience goods have a significant negative effect on CTR, which may suggest that ads for well-known brands or experience-based products lead to lower engagement.

User demographics: Gender and city-level variables show some variation in the coefficients but are not as significant in driving CTR (for example, user_gender[T.male] is not significant).

Week: There is a significant decline in CTR in weeks 2 and 3 compared to week 1, as indicated by the negative coefficients for week[T.w_2] and week[T.w_3].

In [41]:

```
# Regression model with covariates
model_covariate = smf.ols('if_click ~ ' \
'expid + user_sns_like_cnt + user_sns_comment_cnt '
'+ user_degree + real_like_cnt + user_age + user_gender + ' \
'user_city + friend_age + friend_gender + ' \
'friend_city + friend_degree + ' \
'category + brand_effect + experience + status + C(week)', data=df).fit()

# OUTPUT
print(model_covariate.summary())
```

OLS Regression Results

Dep. Variable:	if_click	R-squared:	0.002
Model:	OLS	Adj. R-squared:	0.002
Method:	Least Squares	F-statistic:	103.4
Date:	週三, 30 四月 2025	Prob (F-statistic):	0.00
Time:	18:23:58	Log-Likelihood:	-55486.
No. Observations:	1000000	AIC:	1.110e+05
Df Residuals:	999976	BIC:	1.113e+05
Df Model:	23		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	0.1168	0.003	35.583	0.000	0.110	0.123
user_gender[T.male]	0.0002	0.001	0.423	0.672	-0.001	0.001
user_city[T.level_2]	-0.0021	0.001	-3.140	0.002	-0.003	-0.001
user_city[T.level_3]	-0.0014	0.001	-1.829	0.067	-0.003	0.000
friend_gender[T.male]	-0.0008	0.001	-1.578	0.115	-0.002	0.000
friend_city[T.level_2]	0.0002	0.001	0.331	0.741	-0.001	0.002
friend_city[T.level_3]	0.0018	0.001	2.410	0.016	0.000	0.003
category[T.Car]	0.0342	0.005	6.475	0.000	0.024	0.045
category[T.Clothes]	0.0364	0.005	6.860	0.000	0.026	0.047
category[T.Cosmetrics]	0.0006	0.002	0.296	0.767	-0.003	0.005
category[T.Jewelry]	-0.0012	0.002	-0.503	0.615	-0.006	0.004
C(week)[T.w_2]	-0.0183	0.001	-21.387	0.000	-0.020	-0.017
C(week)[T.w_3]	-0.0253	0.001	-22.753	0.000	-0.027	-0.023
expid	0.0079	0.001	15.515	0.000	0.007	0.009
user_sns_like_cnt	1.489e-06	1.24e-06	1.198	0.231	-9.48e-07	3.93e-06
user_sns_comment_cnt	-4.373e-07	1.31e-06	-0.333	0.739	-3.01e-06	2.14e-06
user_degree	4.089e-06	6.24e-07	6.548	0.000	2.86e-06	5.31e-06
real_like_cnt	-7.749e-05	0.000	-0.700	0.484	-0.000	0.000
user_age	-3.571e-05	4.22e-05	-0.846	0.398	-0.000	4.7e-05
friend_age	9.952e-05	4.39e-05	2.267	0.023	1.35e-05	0.000
friend_degree	-5.098e-08	3.06e-07	-0.167	0.868	-6.5e-07	5.49e-07

```
brand_effect              -0.0422      0.005     -8.769      0.000      -0.052
-0.033
experience                -0.0435      0.003    -16.966      0.000      -0.049
-0.038
status                     0.0148      0.002      6.402      0.000       0.010
0.019
==============================================================================
Omnibus:                663454.324   Durbin-Watson:                   2.000
Prob(Omnibus):               0.000   Jarque-Bera (JB):          5396209.355
Skew:                        3.343   Prob(JB):                         0.00
Kurtosis:                   12.210   Cond. No.                     4.50e+04
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
[2] The condition number is large, 4.5e+04. This might indicate that there are
strong multicollinearity or other numerical problems.
```

5. Analyze the Heterogeneous Treatment Effects (HTE)

In [42]:

```python
# 2.1 Create treatment indicator
df['treat'] = df['expid']

# 2.2 Ensure product-type flags
df['status']     = df['status']
df['experience'] = df['experience']

# 2.3 Compute relative status and standardize
df['rel_status']   = df['friend_degree'] - df['user_degree']
df['rel_status_z'] = (
    df['rel_status'] - df['rel_status'].mean()) / df['rel_status'].std()

# 2.4 Standardize friend popularity (real_like_cnt)
df['real_like_z'] = (
    df['real_like_cnt'] - df['real_like_cnt'].mean()) / df['real_like_cnt'].std(

# 2.5 Convert certain columns to categorical dtype
categorical_cols = ['user_gender','friend_gender',
                    'week','category','user_city','friend_city']
for col in categorical_cols:
    if col in df.columns:
        df[col] = df[col].astype('category')

print("Preprocessing complete. Summary of new features:")
df[['treat','rel_status_z','real_like_z']].describe()
```

```
Preprocessing complete. Summary of new features:
```

Out[42]:

	treat	rel_status_z	real_like_z
count	1000000.0	1.000000e+06	1.000000e+06
mean	0.5	4.141754e-17	5.824141e-17
std	0.5	1.000000e+00	1.000000e+00
min	0.0	-1.445237e+01	-7.485189e-01
25%	0.0	-4.437696e-01	-7.485189e-01
50%	0.5	-1.992248e-01	6.354039e-02
75%	1.0	1.653489e-01	6.354039e-02
max	1.0	1.907495e+01	5.284739e+01

5.1 Compute Average Treatment Effect (ATE)

The treatment increases the click-through rate from 6.66% to 7.45%, an approximate 0.8 pp gain (nearly 12% relative improvement). This suggests a positive average treatment effect.

In [43]:

```
print("=== 1. Average Treatment Effect (ATE) ===")
ate = df.groupby('treat')['if_click'].mean().rename('ctr')
print(ate.to_string())
```

```
=== 1. Average Treatment Effect (ATE) ===
treat
0    0.066604
1    0.074506
```

5.2 HTE by Product Type

The logit shows a clear positive treatment effect (coef = 0.1267, p < 0.001). The interaction with status-type (coef = –0.0072, p = 0.707) and with experience-type (coef = 0.0111, p = 0.834) are both near zero and non-significant. In other words, the uplift in click probability from the treatment is essentially the same for status and experience products.

In [44]:

```
# 4.1 HTE for status-type products
m_status = smf.logit(
    'if_click ~ treat + status + treat:status + user_age + friend_age '
    '+ C(user_gender) + C(friend_gender) + C(week)',
    data=df
).fit(disp=False)
print("---- HTE: treat × status ----")
print(m_status.summary())

# 4.2 HTE for experience-type products
m_exp = smf.logit(
    'if_click ~ treat + experience + treat:experience + user_age + friend_age '
    '+ C(user_gender) + C(friend_gender) + C(week)',
    data=df
).fit(disp=False)
```

```
print("---- HTE: treat × experience ----")
print(m_exp.summary())
```

```
---- HTE: treat × status ----
                           Logit Regression Results                           
==============================================================================
Dep. Variable:               if_click   No. Observations:              1000000
Model:                          Logit   Df Residuals:                   999990
Method:                           MLE   Df Model:                            9
Date:                週三, 30 四月 2025   Pseudo R-squ.:                 0.00351
6
Time:                        18:24:28   Log-Likelihood:            -2.5418e+05
converged:                       True   LL-Null:                   -2.5507e+05
Covariance Type:            nonrobust   LLR p-value:                     0.000
==================================================================================
===========
                             coef    std err          z      P>|z|      [0.025
0.975]
----------------------------------------------------------------------------------
-----------
Intercept                 -2.6237      0.027    -95.705      0.000      -2.677
-2.570
C(user_gender)[T.male]     0.0033      0.008      0.426      0.670      -0.012
0.019
C(friend_gender)[T.male]  -0.0123      0.008     -1.575      0.115      -0.028
0.003
C(week)[T.w_2]            -0.3043      0.011    -26.671      0.000      -0.327
-0.282
C(week)[T.w_3]            -0.4365      0.011    -39.642      0.000      -0.458
-0.415
treat                      0.1267      0.017      7.465      0.000       0.093
0.160
status                     0.1915      0.015     12.930      0.000       0.162
0.220
treat:status              -0.0072      0.019     -0.376      0.707      -0.045
0.030
user_age                   0.0012      0.001      1.939      0.053   -1.34e-05
0.002
friend_age                 0.0035      0.001      5.363      0.000       0.002
0.005
==================================================================================
===========
---- HTE: treat × experience ----
                           Logit Regression Results                           
==============================================================================
Dep. Variable:               if_click   No. Observations:              1000000
Model:                          Logit   Df Residuals:                   999990
Method:                           MLE   Df Model:                            9
Date:                週三, 30 四月 2025   Pseudo R-squ.:                 0.00334
2
Time:                        18:24:34   Log-Likelihood:            -2.5422e+05
converged:                       True   LL-Null:                   -2.5507e+05
Covariance Type:            nonrobust   LLR p-value:                     0.000
==================================================================================
===========
                             coef    std err          z      P>|z|      [0.025
0.975]
----------------------------------------------------------------------------------
-----------
Intercept                 -2.1211      0.048    -43.791      0.000      -2.216
-2.026
C(user_gender)[T.male]     0.0035      0.008      0.445      0.656      -0.012
0.019
```

```
C(friend_gender)[T.male]     -0.0123      0.008     -1.569      0.117      -0.028
0.003
C(week)[T.w_2]               -0.2406      0.012    -20.652      0.000      -0.263
-0.218
C(week)[T.w_3]               -0.3140      0.010    -29.928      0.000      -0.335
-0.293
treat                         0.1103      0.052      2.102      0.036       0.007
0.213
experience                   -0.4388      0.039    -11.142      0.000      -0.516
-0.362
treat:experience              0.0111      0.053      0.209      0.834      -0.093
0.115
user_age                      0.0012      0.001      1.957      0.050   -1.75e-06
0.002
friend_age                    0.0034      0.001      5.159      0.000       0.002
0.005
=================================================================================
===========
```

5.3 HTE by Friend Characteristics

Relative Status: The coefficient for treat × rel_status_z (0.0075, p=0.336) is effectively zero and not significant—meaning the treatment's impact on click-through doesn't depend on whether the friend is of higher or lower social status. Interestingly, the standalone effect of rel_status_z is slightly negative (–0.0165, p=0.004), suggesting that ads liked by friends with relatively higher status actually show a modest drop in baseline CTR.

Friend Likes: In contrast, the interaction between treatment and standardized like count is negative and significant (treat × real_like_z = –0.0266, p=0.001). This shows that the more friends have already "liked" the ad, the smaller the incremental gain from revealing those likes. Nevertheless, a higher real_like_z on its own predicts a higher baseline click rate (0.0185, p=0.001).

In [45]:
```python
# 5.1 Interaction with relative status
m_rel_status = smf.logit(
    'if_click ~ treat * rel_status_z + user_age + friend_age '
    '+ C(user_gender) + C(friend_gender) + C(week)',
    data=df
).fit(disp=False)
print("---- HTE: treat × rel_status_z ----")
print(m_rel_status.summary())

# 5.2 Interaction with friend popularity
m_rel_like = smf.logit(
    'if_click ~ treat * real_like_z + user_age + friend_age '
    '+ C(user_gender) + C(friend_gender) + C(week)',
    data=df
).fit(disp=False)
print("---- HTE: treat × real_like_z ----")
print(m_rel_like.summary())
```

```
---- HTE: treat × rel_status_z ----
                           Logit Regression Results
==============================================================================
Dep. Variable:               if_click   No. Observations:              1000000
Model:                          Logit   Df Residuals:                   999990
Method:                           MLE   Df Model:                            9
Date:                週三, 30 四月 2025   Pseudo R-squ.:                0.00294
2
Time:                        18:24:44   Log-Likelihood:            -2.5432e+05
converged:                       True   LL-Null:                   -2.5507e+05
Covariance Type:            nonrobust   LLR p-value:                     0.000
=================================================================================
===========
                              coef    std err          z      P>|z|      [0.025
0.975]
---------------------------------------------------------------------------------
-----------
Intercept                  -2.5970      0.026    -99.447      0.000      -2.648
-2.546
C(user_gender)[T.male]      0.0035      0.008      0.448      0.654      -0.012
0.019
C(friend_gender)[T.male]   -0.0123      0.008     -1.571      0.116      -0.028
0.003
C(week)[T.w_2]             -0.2786      0.011    -24.634      0.000      -0.301
-0.256
C(week)[T.w_3]             -0.3564      0.010    -35.486      0.000      -0.376
-0.337
treat                       0.1209      0.008     15.444      0.000       0.106
0.136
rel_status_z               -0.0165      0.006     -2.896      0.004      -0.028
-0.005
treat:rel_status_z          0.0075      0.008      0.962      0.336      -0.008
0.023
user_age                    0.0023      0.001      3.587      0.000       0.001
0.003
friend_age                  0.0048      0.001      7.307      0.000       0.003
0.006
=================================================================================
===========
---- HTE: treat × real_like_z ----
                           Logit Regression Results
==============================================================================
Dep. Variable:               if_click   No. Observations:              1000000
Model:                          Logit   Df Residuals:                   999990
Method:                           MLE   Df Model:                            9
Date:                週三, 30 四月 2025   Pseudo R-squ.:                0.00294
5
Time:                        18:24:50   Log-Likelihood:            -2.5432e+05
converged:                       True   LL-Null:                   -2.5507e+05
Covariance Type:            nonrobust   LLR p-value:                     0.000
=================================================================================
===========
                              coef    std err          z      P>|z|      [0.025
0.975]
---------------------------------------------------------------------------------
-----------
Intercept                  -2.5942      0.026    -99.401      0.000      -2.645
-2.543
C(user_gender)[T.male]      0.0035      0.008      0.452      0.651      -0.012
0.019
```

```
C(friend_gender)[T.male]     -0.0123      0.008     -1.575      0.115      -0.028
0.003
C(week)[T.w_2]               -0.2779      0.011    -24.573      0.000      -0.300
-0.256
C(week)[T.w_3]               -0.3566      0.010    -35.320      0.000      -0.376
-0.337
treat                         0.1207      0.008     15.422      0.000       0.105
0.136
real_like_z                   0.0185      0.005      3.386      0.001       0.008
0.029
treat:real_like_z            -0.0266      0.008     -3.392      0.001      -0.042
-0.011
user_age                      0.0023      0.001      3.625      0.000       0.001
0.004
friend_age                    0.0046      0.001      7.147      0.000       0.003
0.006
==============================================================================
===========
```

5.4 Three-Way Interactions

When we test the three-way splits—combining treatment, product type, and friend feature—none of those higher-order interactions matter. Both:

treat × status × rel_status_z (coef ≈ 0.0096, p≈0.59)

treat × experience × real_like_z (coef ≈ 0.0335, p≈0.72)

are essentially zero and not statistically significant. In short, even when you look simultaneously at product type and friend status or popularity, the treatment lift stays the same.

In [46]:

```
# 6.1 status × relative status
m_3way_status = smf.logit(
    'if_click ~ treat * status * rel_status_z + user_age + friend_age '
    '+ C(user_gender) + C(friend_gender) + C(week)',
    data=df
).fit(disp=False)
print("---- 3-way: treat × status × rel_status_z ----")
print(m_3way_status.summary())

# 6.2 experience × friend popularity
m_3way_like = smf.logit(
    'if_click ~ treat * experience * real_like_z + user_age + friend_age '
    '+ C(user_gender) + C(friend_gender) + C(week)',
    data=df
).fit(disp=False)
print("---- 3-way: treat × experience × real_like_z ----")
print(m_3way_like.summary())
```

```
---- 3-way: treat × status × rel_status_z ----
                           Logit Regression Results
==============================================================================
Dep. Variable:               if_click   No. Observations:              1000000
Model:                          Logit   Df Residuals:                   999986
Method:                           MLE   Df Model:                           13
Date:                週三, 30 四月 2025   Pseudo R-squ.:                0.00353
4
Time:                        18:24:59   Log-Likelihood:            -2.5417e+05
converged:                       True   LL-Null:                   -2.5507e+05
Covariance Type:            nonrobust   LLR p-value:                     0.000
==================================================================================
============
                              coef    std err          z      P>|z|      [0.025
0.975]
----------------------------------------------------------------------------------
------------
Intercept                  -2.6267      0.027    -95.753      0.000      -2.680
-2.573
C(user_gender)[T.male]      0.0033      0.008      0.425      0.671      -0.012
0.019
C(friend_gender)[T.male]   -0.0123      0.008     -1.572      0.116      -0.028
0.003
C(week)[T.w_2]             -0.3051      0.011    -26.736      0.000      -0.327
-0.283
C(week)[T.w_3]             -0.4370      0.011    -39.679      0.000      -0.459
-0.415
treat                       0.1267      0.017      7.457      0.000       0.093
0.160
status                      0.1903      0.015     12.841      0.000       0.161
0.219
treat:status               -0.0070      0.019     -0.365      0.715      -0.045
0.031
rel_status_z               -0.0170      0.011     -1.572      0.116      -0.038
0.004
treat:rel_status_z          0.0006      0.015      0.037      0.970      -0.029
0.030
status:rel_status_z         0.0033      0.013      0.259      0.796      -0.022
0.028
treat:status:rel_status_z   0.0096      0.018      0.546      0.585      -0.025
0.044
user_age                    0.0012      0.001      1.949      0.051   -6.81e-06
0.002
friend_age                  0.0036      0.001      5.549      0.000       0.002
0.005
==================================================================================
============
---- 3-way: treat × experience × real_like_z ----
                           Logit Regression Results
==============================================================================
Dep. Variable:               if_click   No. Observations:              1000000
Model:                          Logit   Df Residuals:                   999986
Method:                           MLE   Df Model:                           13
Date:                週三, 30 四月 2025   Pseudo R-squ.:                0.00337
5
Time:                        18:25:05   Log-Likelihood:            -2.5421e+05
converged:                       True   LL-Null:                   -2.5507e+05
Covariance Type:            nonrobust   LLR p-value:                     0.000
==================================================================================
===============
```

```
                                  coef    std err          z      P>|z|      [0.
025      0.975]
---------------------------------------------------------------------------------
---------------
Intercept                      -2.0981      0.050    -42.120      0.000      -2.
196     -2.001
C(user_gender)[T.male]          0.0035      0.008      0.450      0.653      -0.
012      0.019
C(friend_gender)[T.male]       -0.0123      0.008     -1.573      0.116      -0.
028      0.003
C(week)[T.w_2]                 -0.2409      0.012    -20.667      0.000      -0.
264     -0.218
C(week)[T.w_3]                 -0.3151      0.011    -29.868      0.000      -0.
336     -0.294
treat                           0.1003      0.056      1.807      0.071      -0.
009      0.209
experience                     -0.4625      0.041    -11.262      0.000      -0.
543     -0.382
treat:experience                0.0210      0.056      0.375      0.708      -0.
089      0.131
real_like_z                     0.1262      0.064      1.960      0.050    3.32e
-05      0.252
treat:real_like_z              -0.0599      0.092     -0.651      0.515      -0.
240      0.120
experience:real_like_z         -0.1085      0.065     -1.679      0.093      -0.
235      0.018
treat:experience:real_like_z    0.0335      0.092      0.363      0.717      -0.
148      0.215
user_age                        0.0013      0.001      1.986      0.047    1.62e
-05      0.003
friend_age                      0.0034      0.001      5.187      0.000       0.
002      0.005
=================================================================================
===============
```

5.5 Robustness Check with Clustered SE

Robustness Check (clustered by adid):

• Treatment effect remains strong (coef = 0.1174, p < 0.001).

• Status-goods baseline uplift: +0.1694 (p = 0.013); Experience-goods: –0.3623 (p < 0.001).

• No type-specific lift: treat×status (p = 0.752), treat×experience (p = 0.553).

• Friend features lose significance when clustering: rel_status_z (p = 0.141), real_like_z (p = 0.912).

The positive average treatment effect and the lack of heterogeneity by product type are robust.

In [47]:
```python
m_cluster = smf.logit(
    'if_click ~ treat + status + experience + treat:status + treat:experience '
    '+ rel_status_z + real_like_z + user_age + friend_age '
    '+ C(user_gender) + C(friend_gender) + C(week)',
    data=df
```

```python
).fit(
    disp=False,
    cov_type='cluster',
    cov_kwds={'groups': df['adid']}
)
print("=== Robust SE with Clustering on adid ===")
print(m_cluster.summary())
```

```
=== Robust SE with Clustering on adid ===
                           Logit Regression Results
==============================================================================
Dep. Variable:               if_click   No. Observations:              1000000
Model:                          Logit   Df Residuals:                  999986
Method:                           MLE   Df Model:                          13
Date:                週三, 30 四月 2025   Pseudo R-squ.:                0.00381
5
Time:                        18:25:15   Log-Likelihood:            -2.5410e+05
converged:                       True   LL-Null:                   -2.5507e+05
Covariance Type:              cluster   LLR p-value:                     0.000
==============================================================================
===========
                              coef    std err          z      P>|z|      [0.025
0.975]
------------------------------------------------------------------------------
-----------
Intercept                  -2.2323      0.307     -7.281      0.000      -2.833
-1.631
C(user_gender)[T.male]      0.0033      0.006      0.549      0.583      -0.009
0.015
C(friend_gender)[T.male]   -0.0123      0.008     -1.609      0.108      -0.027
0.003
C(week)[T.w_2]             -0.2707      0.050     -5.420      0.000      -0.369
-0.173
C(week)[T.w_3]             -0.3938      0.092     -4.271      0.000      -0.574
-0.213
treat                       0.1174      0.022      5.400      0.000       0.075
0.160
status                      0.1694      0.068      2.478      0.013       0.035
0.303
experience                 -0.3623      0.077     -4.723      0.000      -0.513
-0.212
treat:status               -0.0067      0.021     -0.316      0.752      -0.049
0.035
treat:experience            0.0093      0.016      0.594      0.553      -0.021
0.040
rel_status_z               -0.0110      0.007     -1.471      0.141      -0.026
0.004
real_like_z                 0.0012      0.011      0.111      0.912      -0.021
0.023
user_age                    0.0005      0.004      0.125      0.900      -0.007
0.008
friend_age                  0.0027      0.004      0.649      0.517      -0.005
0.011
==============================================================================
===========
```