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Threshold Functions

FIFI

$$\equiv \frac{T}{k}\sum_n G_f(q, \bar{m}_{fa}^2)\, G_f(q-p, \bar{m}_{fab}^2)$$

$$= \frac{T}{k}\sum_n \frac{1}{(\tilde{q}_0+i\tilde{\mu})^2+1+\bar{m}_{fa}^2}\,\frac{1}{(\tilde{q}_0-\tilde{p}_0+i\tilde{\mu})^2+1+\bar{m}_{fb}^2}$$

$$= \frac{T}{k}\sum_n \frac{1}{\frac{1}{k^2}(q_0+i\mu)^2+1+\bar{m}_{fa}^2}\,\frac{1}{\frac{1}{k^2}(q_0-p_0+i\mu)^2+1+\bar{m}_{fb}^2}$$

$$= \frac{T}{k}k^4\sum_n \frac{1}{(q_0+i\mu)^2+(1+\bar{m}_{fa}^2)k^2}\,\frac{1}{(q_0-p_0+i\mu)^2+(1+\bar{m}_{fb}^2)k^2}$$

$$= \frac{T}{k}k^4\sum_n \frac{1}{-(iq_0-\mu)^2+(1+\bar{m}_{fa}^2)k^2}\,\frac{1}{-(iq_0-ip_0-\mu)^2+(1+\bar{m}_{fb}^2)k^2}$$

$$= \frac{T}{k}k^4\sum_n \frac{1}{(iq_0-\mu)^2-(1+\bar{m}_{fa}^2)k^2}\,\frac{1}{(iq_0-ip_0-\mu)^2-(1+\bar{m}_{fb}^2)k^2}$$

$$iq_0 = \pm k(1+\bar{m}_{fa}^2)^{\frac{1}{2}}+\mu$$

$$iq_0-ip_0-\mu = -ip_0 \pm k(1+\bar{m}_{fa}^2)^{\frac{1}{2}}$$

$$iq_0-ip_0-\mu = \pm k(1+\bar{m}_{fb}^2)^{\frac{1}{2}}$$

$$iq_0-\mu = ip_0 \pm k(1+\bar{m}_{fb}^2)^{\frac{1}{2}}$$

$$iq_0 = ip_0 \pm k(1+\bar{m}_{fb}^2)^{\frac{1}{2}}+\mu$$

$$
\begin{aligned}
= k^3 \Big\{ & n_f\big(k(1+\bar{m}_{fa}^2)^{\frac{1}{2}}+\mu\big) \frac{1}{2k(1+\bar{m}_{fa}^2)^{\frac{1}{2}}} \frac{1}{\big(-ip_0+k(1+\bar{m}_{fa}^2)^{\frac{1}{2}}\big)^2-(1+\bar{m}_{fb}^2)k^2} \\
& + n_f\big(\mu-k(1+\bar{m}_{fa}^2)^{\frac{1}{2}}\big) \frac{1}{-2k(1+\bar{m}_{fa}^2)^{\frac{1}{2}}} \frac{1}{\big(-ip_0-k(1+\bar{m}_{fa}^2)^{\frac{1}{2}}\big)^2-(1+\bar{m}_{fb}^2)k^2} \\
& + n_f\big(ip_0+\mu+k(1+\bar{m}_{fb}^2)^{\frac{1}{2}}\big) \frac{1}{2k(1+\bar{m}_{fb}^2)^{\frac{1}{2}}} \frac{1}{\big(ip_0+k(1+\bar{m}_{fb}^2)^{\frac{1}{2}}\big)^2-(1+\bar{m}_{fa}^2)k^2} + \\
& + n_f\big(ip_0+\mu-k(1+\bar{m}_{fb}^2)^{\frac{1}{2}}\big) \frac{1}{-2k(1+\bar{m}_{fb}^2)^{\frac{1}{2}}} \frac{1}{\big(ip_0-k(1+\bar{m}_{fb}^2)^{\frac{1}{2}}\big)^2-(1+\bar{m}_{fa}^2)k^2} \Big\} + (-
\end{aligned}
$$

$$n_f(ip_0+z)$$

$$p_0 = \frac{2\pi}{\beta} n$$

$$= \frac{1}{e^{\beta(ip_0+z)}+1} = \frac{1}{e^{\beta z}+1}$$

$$= n_f(z)$$

$$n_f(-z) = 1 - n_f(z)$$

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$$\cdots-(1+\bar m_{fb}^2)k^2 = \frac{k^3}{2k}\Big\{ n_f(\mu + k(1+\bar m_{fa}^2)^{\frac{1}{2}})\,\frac{1}{(1+\bar m_{fa}^2)^{\frac{1}{2}}}\,\frac{1}{(-ip_0+k(1+\bar m_{fa}^2)^{\frac{1}{2}})^2-(1+\bar m_{fb}^2)\cdots}$$

$$\cdots+\bar m_{fb}^2)k^2 \quad + (-1+n_f(-\mu+k(1+\bar m_{fa}^2)^{\frac{1}{2}}))\,\frac{1}{(1+\bar m_{fa}^2)^{\frac{1}{2}}}\,\frac{1}{(-ip_0-k(1+\bar m_{fa}^2)^{\frac{1}{2}})^2-(1+\bar m_{fb}^2\cdots}$$

$$\cdots(1+\bar m_{fa}^2)k^2 \quad + n_f(\mu+k(1+\bar m_{fb}^2)^{\frac{1}{2}})\,\frac{1}{(1+\bar m_{fb}^2)^{\frac{1}{2}}}\,\frac{1}{(ip_0+k(1+\bar m_{fb}^2)^{\frac{1}{2}})^2-(1+\bar m_{fa}^2)\cdots}$$

$$\cdots\bar m_{fa}^2)k^2 \quad \Big\} + (-1+n_f(-\mu+k(1+\bar m_{fb}^2)^{\frac{1}{2}}))\,\frac{1}{(1+\bar m_{fb}^2)^{\frac{1}{2}}}\,\frac{1}{(ip_0-k(1+\bar m_{fb}^2)^{\frac{1}{2}})^2-(1+\bar m_{fa}^2)\cdots}$$

Keldysh field theory in the O(N) model

作用量

$$\Gamma_k = \int \left[Z_{\phi,k} (\partial_\mu \phi_{i,q})(\partial^\mu \phi_{i,c}) - U_k(\phi_c, \phi_q) \right]$$

平移介场

$$\begin{cases} \phi_{0,q} = \sigma_q , \quad \phi_{0,c} = \bar{\phi}_{0,c} + \sigma_c & i = 0 \\ \phi_{i,q} = \pi_{i,q} , \quad \phi_{i,c} = \pi_{i,c} & i \neq 0 \end{cases}$$

$$Z_{\phi,k} (\partial_\mu \phi_{i,q})(\partial^\mu \phi_{i,c})$$

$$= Z_{\phi,k} (\partial_\mu \sigma_q)(\partial^\mu \sigma_c) + Z_{\phi,k} (\partial_\mu \pi_{i,q})(\partial^\mu \pi_{i,c})$$

将 $U_k(\phi_c, \phi_q)$ 在

$$\bar{\phi}_q = 0 , \quad \bar{\phi}_c = \begin{cases} \bar{\phi}_{0c} , & i = 0 \\ 0 , & i \neq 0 \end{cases}$$

处展开

$$U_k(\phi_c, \phi_q) = \frac{\partial U_k}{\partial \phi_{i,q}} \Big|_{\bar{\phi}} \phi_{i,q} + \frac{\partial^2 U_k}{\partial \phi_{i,q} \partial \phi_{j,c}} \Big|_{\bar{\phi}} \phi_{i,q} (\phi_{j,c} - \bar{\phi}_{j,c})$$

$$+ \frac{1}{2!} \frac{\partial^3 U_k}{\partial \phi_{i,q} \partial \phi_{j,c} \partial \phi_{k,c}} \Big|_{\bar{\phi}} \phi_{i,q} (\phi_{j,c} - \bar{\phi}_{j,c})(\phi_{k,c} - \bar{\phi}_{k,c})$$

$$+ \frac{1}{3!} \frac{\partial^4 U_k}{\partial \phi_{i,q} \partial \phi_{j,c} \partial \phi_{k,c} \partial \phi_{l,c}} \Big|_{\bar{\phi}} \phi_{i,q} (\phi_{j,c} - \bar{\phi}_{j,c})(\phi_{k,c} - \bar{\phi}_{k,c}) \times (\phi_{l,c} - \bar{\phi}_{l,c})$$

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$$+\frac{1}{3!}\frac{\partial^3 U_k}{\partial\phi_{i,2}\,\partial\phi_{j,2}\,\partial\phi_{k,2}}\Big|_{\bar\phi}\phi_{i,2}\phi_{j,2}\phi_{k,2}$$

$$+\frac{1}{3!}\frac{\partial^4 U_k}{\partial\phi_{i,2}\,\partial\phi_{j,c}\,\partial\phi_{k,2}\,\partial\phi_{l,2}}\Big|_{\bar\phi}\phi_{i,2}\phi_{k,2}\phi_{l,2}(\phi_{j,c}-\bar\phi_{j,c})+\cdots$$

定义

$$\Gamma_{k,2\phi}=\int d^4x\left[Z_{\sigma,k}(\partial_\mu\sigma_2)(\partial^\mu\sigma_c)+Z_{\phi,k}(\partial_\mu\pi_{i,2})(\partial^\mu\pi_{i,c})\right]$$

作 Fourier 变换

$$\sigma_{c/2}(x)=\int\frac{d^4q}{(2\pi)^4}\sigma_{c/2}(q)\,e^{-iq\cdot x}$$

$$\pi_{i,c/2}(x)=\int\frac{d^4q}{(2\pi)^4}\pi_{i,c/2}(q)\,e^{-iq\cdot x}$$

这样有

$$\Gamma_{k,2\phi}\ \int d^4x\ Z_{\phi,k}(\partial_\mu\sigma_2)(\partial^\mu\sigma_c)$$

$$=\int d^4x\ Z_{\phi,k}\ \sigma_2(-\partial^2)\sigma_c$$

$$=\int\frac{d^4q}{(2\pi)^4}\frac{d^4q'}{(2\pi)^4}\int d^4x\ Z_{\phi,k}\ \sigma_2(q')e^{-iq'x}(q^2)\sigma_c(q)e^{-iq\cdot x}$$

$$=\int\frac{d^4q}{(2\pi)^4}\frac{d^4q'}{(2\pi)^4}Z_{\phi,k}\sigma_2(q')q^2\sigma_c(q)(2\pi)^4\delta^4(q'+q)$$

$$=\int\frac{d^4q}{(2\pi)^4}Z_{\phi,k}\ \sigma_2(-q)\,q^2\sigma_c(q)$$

而当 $Z_{\phi,k}$ 不是一个常数而是坐标相关的函数时，则

$$\int d^4x\,d^4y\ \partial_\mu\sigma_2(x)Z_{\phi,k}(x,y)\partial^\mu\sigma_c(y)$$

$$=\int\frac{d^4q}{(2\pi)^4}Z_{\phi,k}(q^2)\sigma_2(-q)\,q^2\sigma_c(q)$$

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其中 $Z_{\phi,k}(x,y) = \int \frac{d^4q}{(2\pi)^4} Z_{\phi,k}(q^2)\, e^{-iq(x-y)}$

$$\Gamma_{k,2\phi} = \int \frac{d^4q}{(2\pi)^4}\Big[Z_{\phi,k}(q^2)\,\sigma_q(-q)\, q^2\, \sigma_c(q) + Z_{\phi,k}(q^2)\,\pi_{i,q}(-q)\, q^2\, \pi_{i,c}(q)\Big]$$

$$\frac{\delta^2\Gamma_{k,2\phi}}{\delta\sigma_q(q')\,\delta\sigma_c(q)} = Z_{\phi,k}(q^2)\, q^2\, (2\pi)^4\delta^4(q+q')$$

$$\frac{\delta^2\Gamma_{k,2\phi}}{\delta\sigma_c(q')\,\delta\sigma_q(q)} = Z_{\phi,k}(q^2)\, q^2 (2\pi)^4\, \delta^4(q+q')$$

$$\frac{\delta^2\Gamma_{k,2\phi}}{\delta\pi_{i,q}(q')\,\delta\pi_{j,c}(q)} = Z_{\phi,k}(q^2)\, q^2\, \delta_{ij}\, (2\pi)^4\delta^4(q+q')$$

$$\frac{\delta^2\Gamma_{k,2\phi}}{\delta\pi_{i,c}(q')\,\delta\pi_{j,q}(q)} = Z_{\phi,k}(q^2)\, q^2\, \delta_{ij}\, (2\pi)^4\delta^4(q+q')$$

设 ~~$\Gamma_{k,U}$~~

$$\Gamma_{k,U} = \int d^4x\,\big(-U_k(\phi_c,\phi_q)\big)$$

$$\Gamma_{k,U}^{qc} = \Gamma$$

$$\big(\Gamma_{k,U}^{\sigma_q\sigma_c}\big)_{qc} = \frac{\delta^2\Gamma_{k,U}}{\delta\sigma_q\,\delta\sigma_c} = \frac{\delta^2\Gamma_{k,U}}{\delta\phi_{0,q}\,\delta\phi_{0,c}} = \big(\Gamma^{\phi\phi}_{k,U}\big)^{qc}_{0q0c}$$

$$= \cancel{\left\{\frac{\partial^2 U}{\partial\phi_{0,q}\,\partial\phi_{0,c}}\Big|_{\phi} + \frac{\partial^3 U}{\partial\phi_{0,q}\,\partial\phi_{0,c}\,\partial\phi_{k,c}}\Big|_{\phi}(\phi_{k,c}\right.}$$

$$\frac{\delta^2 \Gamma_{k,U}}{\delta\phi_{i,2}\,\delta\phi_{j,c}}$$

$$= -\Big\{ \frac{\partial^2 U_k}{\partial\phi_{i,2}\partial\phi_{j,c}}\Big|_{\bar\phi} + \frac{\partial^3 U_k}{\partial\phi_{i,2}\partial\phi_{j,c}\partial\phi_{k,c}}\Big|_{\bar\phi} (\phi_{k,c}-\bar\phi_{k,c})$$

$$+ \frac{1}{2!}\frac{\partial^4 U_k}{\partial\phi_{i,2}\partial\phi_{j,c}\partial\phi_{k,c}\partial\phi_{l,c}}\Big|_{\bar\phi}(\phi_{k,c}-\bar\phi_{k,c})(\phi_{l,c}-\bar\phi_{l,c})$$

$$+ \frac{1}{2!}\frac{\partial^4 U_k}{\partial\phi_{i,2}\partial\phi_{j,c}\partial\phi_{k,2}\partial\phi_{l,2}}\Big|_{\bar\phi}\phi_{k,2}\phi_{l,2} + \cdots \Big\}$$

$$\frac{\partial^2 U_k}{\partial\phi_{0,2}\partial\phi_{0,c}}\Big|_{\bar\phi} = V_k'(\rho_{0,c}) + \frac{1}{2}V_k^{(2)}(\rho_{0,c})\,\bar\phi_{0,c}^2$$

$$= \underbrace{V_k'(\rho_{0,c}) + 2\rho_{0,c}V_k^{(2)}(\rho_{0,c}) = m_\sigma^2}_{\pi介子质量}$$

$$\boxed{\begin{gathered}\rho_c = \frac{\phi_{i,c}^2}{4}\\ \rho_{0,c} = \frac{\bar\phi_{0,c}^2}{4}\\ \bar\phi_{0,c} = 2(\rho_{0,c}\end{gathered}}$$

$$\frac{\partial^3 U_k}{\partial\phi_{0,2}\partial\phi_{0,c}\partial\phi_{k,c}}\Big|_{\bar\phi}(\phi_{k,c}-\bar\phi_{k,c})$$

$$= \Big[\frac{1}{2}V_k^{(2)}(\rho_{0,c})(\bar\phi_{k,c}+2\delta_{0k}\bar\phi_{0,c}) + \frac{1}{4}V_k^{(3)}(\rho_{0,c})\,\bar\phi_{0,c}^2\,\bar\phi_{k,c}\Big](\phi_{k,c}-\bar\phi_{k,c}$$

$$= \frac{1}{2}V_k^{(2)}(\rho_{0,c})(3\bar\phi_{0,c})\sigma_c + \frac{1}{4}V_k^{(3)}(\rho_{0,c})\,\bar\phi_{0,c}^2\,\bar\phi_{0,c}\sigma_c$$

$$= 3(\rho_{0,c})^{\frac{1}{2}}V_k^{(2)}(\rho_{0,c})\,\sigma_c + 2(\rho_{0,c})^{\frac{3}{2}}V_k^{(3)}(\rho_{0,c})\,\sigma_c$$

$$= \Big[3(\rho_{0,c})^{\frac{1}{2}}V_k^{(2)}(\rho_{0,c}) + 2(\rho_{0,c})^{\frac{3}{2}}V_k^{(3)}(\rho_{0,c})\Big]\sigma_c$$

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$$\frac{1}{2!}\frac{\partial^4 U}{\partial\phi_{0,2}\,\partial\phi_{0,c}\,\partial\phi_{k,c}\,\partial\phi_{l,c}}\bigg|_{\bar\phi}(\phi_{k,c}-\bar\phi_{k,c})(\phi_{l,c}-\bar\phi_{l,c})$$

$$= \frac{1}{2}\Big[\frac{1}{2}V_k^{(2)}(\rho_{0,c})(\delta_{kl}+2\delta_{0k}\delta_{0l})$$

$$+\frac{1}{4}V_k^{(3)}(\rho_{0,c})(\bar\phi_{k,c}\bar\phi_{l,c}+\bar\phi_{0,c}\bar\phi_{0,c}\delta_{kl}+2\delta_{0k}\bar\phi_{0,c}\bar\phi_{l,c}+2\delta_{0l}\bar\phi_{0,c}\bar\phi_{k,c})$$

$$+\frac{1}{8}V_k^{(4)}(\rho_{0,c})\bar\phi_{0,c}\bar\phi_{0,c}\bar\phi_{k,c}\bar\phi_{l,c}\Big](\phi_{k,c}-\bar\phi_{k,c})(\phi_{l,c}-\bar\phi_{l,c})$$

$$= \frac{1}{2}\Big[\frac{1}{2}V_k^{(2)}(\rho_{0,c})(\sigma_c^2+\pi_{i,c}^2+2\sigma_c^2)$$

$$+\frac{1}{4}V_k^{(3)}(\rho_{0,c})\big(\bar\phi_{0,c}^2\sigma_c^2+\bar\phi_{0,c}^2(\sigma_c^2+\pi_{i,c}^2)+4\bar\phi_{0,c}^2\sigma_c^2\big)$$

$$+\frac{1}{8}V_k^{(4)}(\rho_{0,c})\bar\phi_{0,c}^4\sigma_c^2\Big]$$

$$= \frac{1}{2}\Big[\frac{1}{2}V_k^{(2)}(\rho_{0,c})(3\sigma_c^2+\pi_{i,c}^2)+\frac{1}{4}V_k^{(3)}(\rho_{0,c})(4\rho_{0,c})(6\sigma_c^2+\pi_{i,c}^2)$$

$$+\frac{1}{8}V_k^{(4)}(\rho_{0,c})(4\rho_{0,c})^2\sigma_c^2\Big]$$

$$= \frac{1}{2}\Big[\frac{1}{2}V_k^{(2)}(\rho_{0,c})(3\sigma_c^2+\pi_{i,c}^2)+\rho_{0,c}V_k^{(3)}(\rho_{0,c})(6\sigma_c^2+\pi_{i,c}^2)$$

$$+2\rho_{0,c}^2V_k^{(4)}(\rho_{0,c})\sigma_c^2\Big]$$

$$= \frac{1}{2}\Big\{\Big[\frac{3}{2}V_k^{(2)}(\rho_{0,c})+6\rho_{0,c}V_k^{(3)}(\rho_{0,c})+2\rho_{0,c}^2V_k^{(4)}(\rho_{0,c})\Big]\sigma_c^2$$

$$+\Big[\frac{1}{2}V_k^{(2)}(\rho_{0,c})+\rho_{0,c}V_k^{(3)}(\rho_{0,c})\Big]\pi_{i,c}^2\Big\}$$

同理

$$\frac{1}{2!}\frac{\partial^4 U_k}{\partial\phi_{0,s}\,\partial\phi_{0,c}\,\partial\phi_{k,s}\,\partial\phi_{l,s}}\Big|_{\bar\phi}\phi_{k,s}\phi_{l,s}$$

$$=\frac{1}{2}\Big\{\Big[\frac{3}{2}V_k^{(2)}(\rho_{0,c})+6\rho_{0,c}V_k^{(3)}(\rho_{0,c})+2\rho_{0,c}^2V_k^{(4)}(\rho_{0,c})\Big]\sigma_s^2$$

$$+\Big[\frac{1}{2}V_k^{(2)}(\rho_{0,c})+\rho_{0,c}V_k^{(3)}(\rho_{0,c})\Big]\pi_{i,s}^2\Big\}$$

$$(\Gamma_{k,0})^{sc}=\frac{\delta^2\Gamma_{k,0}}{\delta\sigma_s\,\delta\sigma_c}=\frac{\delta^2\Gamma_{k,0}}{\delta\phi_{0,s}\,\delta\phi_{0,c}}$$

$$=-\Big\{m_\sigma^2+\Big[3\rho_{0,c}^{\frac{1}{2}}V_k^{(2)}(\rho_{0c})+2\rho_{0,c}^{\frac{3}{2}}V_k^{(3)}(\rho_{0c})\Big]\sigma_c$$

$$+\frac{1}{2}\Big[\frac{3}{2}V_k^{(2)}(\rho_{0,c})+6\rho_{0,c}V_k^{(3)}(\rho_{0,c})+2\rho_{0,c}^2V_k^{(4)}(\rho_{0,c})\Big](\sigma_c^2+\sigma_s^2)$$

$$+\frac{1}{2}\Big[\frac{1}{2}V_k^{(2)}(\rho_{0,c})+\rho_{0,c}V_k^{(3)}(\rho_{0,c})\Big](\pi_{i,c}^2+\pi_{i,s}^2)\Big\}+\cdots$$

$$\frac{\partial^2U_k}{\partial\pi_{i,s}\,\partial\pi_{j,c}}\Big|_{\bar\phi}=V_k'(\rho_{0,c})\,\delta_{ij}=m_\pi^2\,\delta_{ij}$$

$$\frac{\partial^3U_k}{\partial\pi_{i,s}\,\partial\pi_{j,c}\,\partial\phi_{k,c}}\Big|_{\bar\phi}(\phi_{k,c}-\bar\phi_{k,c})$$

$$=\Big[\frac{1}{2}V_k^{(2)}(\rho_{0,c})(\delta_{ij}\bar\phi_{k,c})\Big](\phi_{k,c}-\bar\phi_{k,c})$$

$$=\frac{1}{2}V_k^{(2)}(\rho_{0,c})\,\delta_{ij}\,\bar\phi_{0,c}\,\sigma_c=(\rho_{0,c})^{\frac{1}{2}}V_k^{(2)}(\rho_{0,c})\,\sigma_c\,\delta_{ij}$$

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$$\frac{1}{2!}\frac{\partial^4 U}{\partial\pi_{i,2}\partial\pi_{j,c}\partial\phi_{k,c}\partial\phi_{l,c}}\bigg|_{\bar\phi}(\phi_{k,c}-\bar\phi_{k,c})(\phi_{l,c}-\bar\phi_{l,c})$$

$$=\frac{1}{2}\Big[\frac{1}{2}V_k^{(2)}(\rho_{0,c})(\delta_{ij}\delta_{kl}+\delta_{ik}\delta_{jl}+\delta_{jk}\delta_{il})$$

$$+\frac{1}{4}V_k^{(3)}(\rho_{0,c})(\delta_{ij}\bar\phi_{k,c}\bar\phi_{l,c})\Big](\phi_{k,c}-\bar\phi_{k,c})(\phi_{l,c}-\bar\phi_{l,c})$$

$$=\frac{1}{2}\Big[\frac{1}{2}V_k^{(2)}(\rho_{0,c})(\delta_{ij}(\sigma_c^2+\pi_c^2)+2\pi_{i,c}\pi_{j,c})$$

$$+\frac{1}{4}V_k^{(3)}(\rho_{0,c})\,\delta_{ij}\,\bar\phi_{0,c}\bar\phi_{0,c}\,\sigma_c^2\Big]$$

$$=\frac{1}{2}\Big[\frac{1}{2}V_k^{(2)}(\rho_{0,c})(\sigma_c^2+\pi_c^2)\delta_{ij}+V_k^{(2)}(\rho_{0,c})\pi_{i,c}\pi_{j,c}$$

$$+\rho_{0,c}V_k^{(3)}(\rho_{0,c})\sigma_c^2\delta_{ij}\Big]$$

$$=\frac{1}{2}\Big[\Big(\frac{1}{2}V_k^{(2)}(\rho_{0,c})+\rho_{0,c}V_k^{(3)}(\rho_{0,c})\Big)\sigma_c^2\delta_{ij}$$

$$+\frac{1}{2}V_k^{(2)}(\rho_{0,c})\pi_c^2\delta_{ij}+V_k^{(2)}(\rho_{0,c})\pi_{i,c}\pi_{j,c}\Big]$$

$$\frac{1}{2!}\frac{\partial^4 U}{\partial\pi_{i,2}\partial\pi_{j,c}\partial\phi_{k,2}\partial\phi_{l,2}}\bigg|_{\bar\phi}\phi_{k,2}\phi_{l,2}$$

$$=\frac{1}{2}\Big[\Big(\frac{1}{2}V_k^{(2)}(\rho_{0,c})+\rho_{0,c}V_k^{(3)}(\rho_{0,c})\Big)\sigma_2^2\delta_{ij}$$

$$+\frac{1}{2}V_k^{(2)}(\rho_{0,c})\pi_2^2\delta_{ij}+V_k^{(2)}(\rho_{0,c})\pi_{i,2}\pi_{j,2}\Big]$$

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$$\left(\Gamma^{\pi\pi}_{k,v}\right)^{2c}_{ij} = \frac{\delta^2 \Gamma_{k,v}}{\delta \pi_{i,2}\, \delta \pi_{j,c}}$$

$$= -\Big\{ m_\pi^2 \delta_{ij} + \rho_{0,c}^{\frac{1}{2}} V_k^{(2)}(\rho_{0,c})\, \sigma_c\, \delta_{ij}$$

$$+ \frac{1}{2}\left(\frac{1}{2} V_k^{(2)}(\rho_{0,c}) + \rho_{0,c} V_k^{(3)}(\rho_{0,c})\right)(\sigma_c^2 + \sigma_2^2)\,\delta_{ij}$$

$$+ \frac{1}{4} V_k^{(2)}(\rho_{0,c})(\pi_c^2 + \pi_2^2)\,\delta_{ij} + \frac{1}{2} V_k^{(2)}(\rho_{0,c})(\pi_{i,c}\pi_{j,c} + \pi_{i,2}\pi_{j,2})$$

$$\left(\Gamma^{\sigma\pi}_{k,v}\right)^{2c}_{0j} = \frac{\delta^2 \Gamma_{k,v}}{\delta \sigma_2\, \delta \pi_{j,c}} = \frac{\delta^2 \Gamma_{k,v}}{\delta \phi_{0,2}\, \delta \pi_{j,c}}$$

$$\left.\frac{\partial^2 U_k}{\partial \phi_{0,2}\, \partial \pi_{j,c}}\right|_{\bar{\phi}} = 0$$

$$\left.\frac{\partial^3 U_k}{\partial \phi_{0,2}\, \partial \pi_{j,c}\, \partial \phi_{k,c}}\right|_{\bar{\phi}} (\phi_{k,c} - \bar{\phi}_{k,c})$$

$$= \Big[\frac{1}{2} V_k^{(2)}(\rho_{0,c})\left(\delta_{0j}\cancel{\bar{\phi}_{k,c}} + \delta_{0k}\cancel{\bar{\phi}_{j,c}} + \delta_{jk}\bar{\phi}_{0,c}\right)$$

$$+ \frac{1}{4} V_k^{(3)}(\rho_{0,c})\, \bar{\phi}_{0,c}\, \bar{\phi}_{j,c}\, \bar{\phi}_{k,c} \Big] (\phi_{k,c} - \bar{\phi}_{k,c})$$

$$= \frac{1}{2} V_k^{(2)}(\rho_{0,c})\, \bar{\phi}_{0,c}\, \pi_{j,c}$$

$$= (\rho_{0,c})^{\frac{1}{2}} V_k^{(2)}(\rho_{0,c})\, \pi_{j,c}$$

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$$\frac{1}{2!}\frac{\partial^4 U}{\partial\phi_{0,2}\,\partial\pi_{j,c}\,\partial\phi_{k,c}\,\partial\phi_{l,c}}\bigg|_{\bar\phi}(\phi_{k,c}-\bar\phi_{k,c})(\phi_{l,c}-\bar\phi_{l,c})$$

$$=\frac{1}{2!}\Big[\frac{1}{2}V_k^{(2)}(\rho_{0,c})(\delta_{0k}\delta_{jl}+\delta_{0l}\delta_{jk})$$

$$+\frac{1}{4}V_k^{(3)}(\rho_{0,c})\big(\cancel{\delta_{0j}}\bar\phi_{k,c}\bar\phi_{l,c}+\delta_{kl}\bar\phi_{0,c}\cancel{\bar\phi_{j,c}}+\delta_{0k}\cancel{\bar\phi_{j,c}}\bar\phi_{l,c}$$

$$+\delta_{jl}\bar\phi_{0,c}\bar\phi_{k,c}+\delta_{jk}\bar\phi_{0,c}\bar\phi_{l,c}+\delta_{0l}\bar\phi_{j,c}\bar\phi_{k,c}\big)$$

$$\frac{1}{8}V_k^{(4)}(\rho_{0,c})\bar\phi_{0,c}\bar\phi_{j,c}\bar\phi_{k,c}\bar\phi_{l,c}\Big](\phi_{k,c}-\bar\phi_{k,c})(\phi_{l,c}-\bar\phi_{l,c})$$

$$=\frac{1}{2!}\Big[\frac{1}{2}V_k^{(2)}(\rho_{0,c})\,2\,\sigma_c\pi_{j,c}$$

$$+\frac{1}{4}V_k^{(3)}(\rho_{0,c})\,2\,\bar\phi_{0,c}^2\,\sigma_c\pi_{j,c}\Big]$$

$$=\frac{1}{2}\Big[V_k^{(2)}(\rho_{0,c})\,\sigma_c\pi_{j,c}+2\rho_{0,c}V_k^{(3)}(\rho_{0,c})\,\sigma_c\pi_{j,c}\Big]$$

$$=\frac{1}{2}\left(V_k^{(2)}(\rho_{0,c})+2\rho_{0,c}V_k^{(3)}(\rho_{0,c})\right)\sigma_c\pi_{j,c}$$

$$\frac{1}{2!}\frac{\partial^4 U}{\partial\phi_{0,2}\,\partial\pi_{j,c}\,\partial\phi_{k,2}\,\partial\phi_{l,2}}\bigg|_{\bar\phi}\phi_{k,2}\phi_{l,2}$$

$$=\frac{1}{2}\left(V_k^{(2)}(\rho_{0,c})+2\rho_{0,c}V_k^{(3)}(\rho_{0,c})\right)\sigma_2\pi_{j,2}$$

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所以有

$$\left(\Gamma^{\rho\pi}_{k,U}\right)^{2c}_{0j} = \frac{\delta^2 \Gamma_{k,U}}{\delta\phi_{0,2}\,\delta\pi_{j,c}}$$

$$= -\Big\{ \rho_{0,c}^{\frac{1}{2}} V_k^{(2)}(\rho_{0,c})\,\pi_{j,c}$$

$$+ \frac{1}{2}\left(V_k^{(2)}(\rho_{0,c}) + 2\rho_{0,c} V_k^{(3)}(\rho_{0,c})\right)(\sigma_c \pi_{j,c} + \sigma_2 \pi_j$$

$$\left(\Gamma^{\pi\rho}_{k,U}\right)^{2c}_{i0} = \frac{\delta^2 \Gamma_{k,U}}{\delta\pi_{i,2}\,\delta\phi_{0,c}}$$

$$\left.\frac{\partial^2 U_k}{\partial\pi_{i,2}\,\partial\phi_{0,c}}\right|_{\bar\phi} = V_k'(\rho_{0,c})\,\cancel{\delta_{i0}} + \frac{1}{2} V_k^{(2)}(\rho_{0,c})\,\cancel{\bar\phi_{i,c}}\,\bar\phi_{0,c} = 0$$

$$\left.\frac{\partial^3 U_k}{\partial\pi_{i,2}\,\partial\phi_{0,c}\,\partial\phi_{k,c}}\right|_{\bar\phi} (\phi_{k,c} - \bar\phi_{k,c})$$

$$= \Big[\frac{1}{2} V_k^{(2)}(\rho_{0,c})\left(\cancel{\delta_{i0}}\,\bar\phi_{k,c} + \delta_{ik}\,\bar\phi_{0,c} + \delta_{0k}\,\cancel{\bar\phi_{i,c}}\right)$$

$$+ \frac{1}{4} V_k^{(3)}(\rho_{0,c})\,\cancel{\bar\phi_{i,c}}\,\bar\phi_{0,c}\,\bar\phi_{k,c} \Big](\phi_{k,c} - \bar\phi_{k,c})$$

$$= \frac{1}{2} V_k^{(2)}(\rho_{0,c})\,\bar\phi_{0,c}\,\pi_{i,c}$$

$$= \rho_{0,c}^{\frac{1}{2}} V_k^{(2)}(\rho_{0,c})\,\pi_{i,c}$$

$$\frac{1}{2!}\frac{\partial^4 U}{\partial \pi_{\underline{i},2}\,\partial\phi_{0,c}\,\partial\phi_{k,c}\,\partial\phi_{l,c}}\bigg|_{\bar\phi}(\phi_{k,c}-\bar\phi_{k,c})(\phi_{l,c}-\bar\phi_{l,c})$$

$$=\frac{1}{2}\Big[\frac{1}{2}V_k^{(2)}(\rho_{0,c})(\delta_{\underline{i}0}\delta_{kl}+\delta_{\underline{i}k}\delta_{0l}+\delta_{\underline{i}l}\delta_{0k})$$

$$+\frac{1}{4}V_k^{(3)}(\rho_{0,c})(\cancel{\delta_{\underline{i}0}}\,\bar\phi_{k,c}\bar\phi_{l,c}+\delta_{kl}\,\cancel{\bar\phi_{\underline{i},c}}\,\bar\phi_{0,c}+\delta_{\underline{i}k}\bar\phi_{0,c}\bar\phi_{l,c}$$

$$+\delta_{0l}\,\cancel{\bar\phi_{\underline{i},c}}\,\bar\phi_{k,c}+\delta_{0k}\,\cancel{\bar\phi_{\underline{i},c}}\,\bar\phi_{l,c}+\delta_{\underline{i}l}\bar\phi_{0,c}\bar\phi_{k,c})$$

$$+\frac{1}{8}V_k^{(4)}(\rho_{0,c})\bar\phi_{\underline{i},c}\bar\phi_{0,c}\bar\phi_{k,c}\bar\phi_{l,c}\Big](\phi_{k,c}-\bar\phi_{k,c})(\phi_{l,c}-\bar\phi_{l,c})$$

$$=\frac{1}{2}\Big[\frac{1}{2}V_k^{(2)}(\rho_{0,c})\,2\,\sigma_c\pi_{i,c}+\frac{1}{4}V_k^{(3)}(\rho_{0,c})\,2\,\bar\phi_{0,c}^2\,\sigma_c\pi_{i,c}\Big]$$

$$=\frac{1}{2}\big(V_k^{(2)}(\rho_{0,c})+2\rho_{0,c}V_k^{(3)}(\rho_{0,c})\big)\sigma_c\pi_{i,c}$$

$$\frac{1}{2!}\frac{\partial^4 U}{\partial \pi_{i,2}\,\partial\phi_{0,c}\,\partial\phi_{k,2}\,\partial\phi_{l,2}}\bigg|_{\bar\phi}\phi_{k,2}\phi_{l,2}$$

$$=\frac{1}{2}\big(V_k^{(2)}(\rho_{0,c})+2\rho_{0,c}V_k^{(3)}(\rho_{0,c})\big)\sigma_2\pi_{i,2}$$

따라서

$$(\Gamma_{k,U}^{\pi\sigma})_{i,0}^{2c}=\frac{\delta^2\Gamma_{k,U}}{\delta\pi_{i,2}\,\delta\phi_{0,c}}$$

$$=-\Big\{\rho_{0,c}^{\frac{1}{2}}V_k^{(2)}(\rho_{0,c})\pi_{i,c}$$

$$+\frac{1}{2}\big(V_k^{(2)}(\rho_{0,c})+2\rho_{0,c}V_k^{(3)}(\rho_{0,c})\big)(\sigma_c\pi_{i,c}+\sigma_2\pi_{i,2})\Big\}$$

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$$\bar{\Phi}(q) = \begin{pmatrix} \cancel{\sigma_c(q)} \\ \cancel{\sigma_2(q)} \\ \cancel{\pi_{i,c}(q)} \\ \cancel{\pi_{i,2}(q)} \end{pmatrix} \begin{pmatrix} \sigma_c(q) \\ \pi_{i,c}(q) \\ \sigma_2(q) \\ \pi_{i,2}(q) \end{pmatrix}$$

$$\bar{\Phi}^T(-q) = \left(\sigma_c(-q),\ \pi_{i,c}(-q),\ \sigma_2(-q),\ \pi_{i,2}(-q) \right)$$

$$\frac{\delta^2 \Gamma_k}{\delta\sigma_2(q')\,\delta\sigma_c(q)} = \frac{\delta^2 \Gamma_{k,2\phi}}{\delta\sigma_2(q')\,\delta\sigma_c(q)} + \frac{\delta^2 \Gamma_{k,U}}{\delta\sigma_2(q')\,\delta\sigma_c(q)}$$

$$= \left(Z_{\phi,k}(q^2)\, q^2 - m_\alpha^2\right)(2\pi)^4 \delta^4(q'+q)$$

$$- \Big\{ \left[3\rho_{0,c}^{\frac{1}{2}} V_k^{(2)}(\rho_{0,c}) + 2\rho_{0,c}^{\frac{3}{2}} V_k^{(3)}(\rho_{0,c}) \right] \sigma_c$$

$$+ \frac{1}{2}\left[\frac{3}{2} V_k^{(2)}(\rho_{0,c}) + 6\rho_{0,c} V_k^{(3)}(\rho_{0,c}) + 2\rho_{0,c}^2 V_k^{(4)}(\rho_{0,c}) \right](\sigma_c^2 + \sigma_2^2)$$

$$+ \frac{1}{2}\left[\frac{1}{2} V_k^{(2)}(\rho_{0,c}) + \rho_{0,c} V_k^{(3)}(\rho_{0,c}) \right](\pi_c^2 + \pi_2^2) \Big\} + \cdots$$

$$\frac{\delta^2 \Gamma_k}{\delta\sigma_c(q')\,\delta\sigma_2(q)} = \frac{\delta^2 \Gamma_k}{\delta\sigma_2(q')\,\delta\sigma_c(q)}$$

$$\frac{\delta^2 \Gamma_k}{\delta\pi_{i,2}(q')\,\delta\pi_{j,c}(q)} = \frac{\delta^2 \Gamma_{k,2\phi}}{\delta\pi_{i,2}(q')\,\delta\pi_{j,c}(q)} + \frac{\delta^2 \Gamma_{k,U}}{\delta\pi_{i,2}(q')\,\delta\pi_{j,c}(q)}$$

$$= \left(Z_{\phi,k}(q^2)\, q^2 - m_\pi^2\right)\delta_{ij}\,(2\pi)^4 \delta^4(q+q')$$

$$- \Big\{ \rho_{0,c}^{\frac{1}{2}} V_k^{(2)}(\rho_{0,c})\, \sigma_c\, \delta_{ij}$$

$$+ \frac{1}{2}\left(\frac{1}{2} V_k^{(2)}(\rho_{0,c}) + \rho_{0,c} V_k^{(3)}(\rho_{0,c}) \right)(\sigma_c^2 + \sigma_2^2)\,\delta_{ij}$$

$$+ \frac{1}{4} V_k^{(2)}(\rho_{0,c})(\pi_c^2 + \pi_2^2)\,\delta_{ij} + \frac{1}{2} V_k^{(2)}(\rho_{0,c})(\pi_{i,c}\pi_{j,c} + \pi_{i,2}\pi_{j,2}) \Big\}$$

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$$\frac{\delta^2 \Gamma_k}{\delta \pi_{i,c}(z') \delta \pi_{j,2}(z)} = \frac{\delta^2 \Gamma_k}{\delta \pi_{i,2}(z') \delta \pi_{j,c}(z)}$$

$$\frac{\delta^2 \Gamma_k}{\delta \sigma_2(z') \delta \pi_{j,c}(z)} = \frac{\delta^2 \Gamma_k}{\delta \phi_{0,2}(z') \delta \pi_{j,c}(z)} = \frac{\delta^2 \Gamma_{k,U}}{\delta \phi_{0,2}(z') \delta \pi_{j,c}(z)}$$

$$= -\Big\{ \rho_{0,c}^{\frac{1}{2}} V_k^{(2)}(\rho_{0,c}) \pi_{j,c} + \frac{1}{2} \left(V_k^{(2)}(\rho_{0,c}) + 2 \rho_{0,c} V_k^{(3)}(\rho_{0,c}) \right) \left(\sigma_c \pi_{j,c} + \sigma_2 \pi_{j,2} \right) \Big\}$$

$$\frac{\delta^2 \Gamma_k}{\delta \pi_{i,2}(z') \delta \sigma_c(z)}$$

$$= -\Big\{ \rho_{0,c}^{\frac{1}{2}} V_k^{(2)}(\rho_{0,c}) \pi_{i,c} + \frac{1}{2} \left(V_k^{(2)}(\rho_{0,c}) + 2 \rho_{0,c} V_k^{(3)}(\rho_{0,c}) \right) \left(\sigma_c \pi_{i,c} + \sigma_2 \pi_{i,2} \right) \Big\}$$

$$\frac{\delta^2 \Gamma_k}{\delta \sigma_c(z') \delta \pi_{j,2}(z)} = \frac{\delta^2 \Gamma_k}{\delta \pi_{j,2}(z) \delta \sigma_c(z')}$$

$$\frac{\delta^2 \Gamma_k}{\delta \pi_{i,c}(z') \delta \sigma_2(z)} = \frac{\delta^2 \Gamma_k}{\delta \sigma_2(z) \delta \pi_{i,c}(z')}$$

$$(\Gamma_k^{(2)})_{ab} = \tilde{P}_{ab} + F_{ab} \qquad (\Gamma_k^{(2)})_{ab} = \frac{\overrightarrow{\delta}}{\delta\Phi_a}\Gamma_k\frac{\overleftarrow{\delta}}{\delta\Phi_b}$$

$\tilde{P}$是两点函数，F是两点以上的函数

$$(\Gamma_k^{(2)})_{ab} = \begin{pmatrix} 0 & \begin{matrix} (\Gamma_k^{(2)\sigma\sigma})^{cq} & (\Gamma_k^{(2)\sigma\pi})^{cq}_j \\ (\Gamma_k^{(2)\pi\sigma})^{cq}_i & (\Gamma_k^{(2)\pi\pi})^{cq}_{ij} \end{matrix} \\ \begin{matrix} (\Gamma_k^{(2)\sigma\sigma})^{qc} & (\Gamma_k^{(2)\sigma\pi})^{qc}_j \\ (\Gamma_k^{(2)\pi\sigma})^{qc}_i & (\Gamma_k^{(2)\pi\pi})^{qc}_{ij} \end{matrix} & 0 \end{pmatrix}$$

$$(\Gamma_k^{(2)\sigma\sigma})^{qc} = (\tilde{P}^{\sigma\sigma})^{qc} + (F^{\sigma\sigma})^{qc}$$

$$(\tilde{P}^{\sigma\sigma})^{qc} = (Z_{\phi,k}(q^2)q^2 - m_\sigma^2)(2\pi)^4\delta^4(q'+q)$$

$$(F^{\sigma\sigma})^{qc} = \lambda_{3\sigma}\sigma_c + \frac{1}{2}\lambda_{4\sigma}(\sigma_c^2+\sigma_q^2) + \frac{1}{2}\lambda_{2\sigma2\pi}(\pi_c^2+\pi_q^2)$$

其中

$$\lambda_{3\sigma} = -\left[3\rho_{0,c}^{\frac{1}{2}}V_k^{(2)}(\rho_{0,c}) + 2\rho_{0,c}^{\frac{3}{2}}V_k^{(3)}(\rho_{0,c})\right]$$

$$\lambda_{4\sigma} = -\left[\frac{3}{2}V_k^{(2)}(\rho_{0,c}) + 6\rho_{0,c}V_k^{(3)}(\rho_{0,c}) + 2\rho_{0,c}^2V_k^{(4)}(\rho_{0,c})\right]$$

$$\lambda_{2\sigma2\pi} = -\left[\frac{1}{2}V_k^{(2)}(\rho_{0,c}) + \rho_{0,c}V_k^{(3)}(\rho_{0,c})\right]$$

$$(\Gamma_k^{(2)\sigma\sigma})^{cq} = (\Gamma_k^{(2)\sigma\sigma})^{qc}$$

$$(\Gamma_k^{(2)\pi\pi})^{qc}_{ij} = (\tilde{P}^{\pi\pi})^{qc}_{ij} + (F^{\pi\pi})^{qc}_{ij}$$

$$(\tilde{P}^{\pi\pi})^{qc}_{ij} = (Z_{\phi,k}(q^2)q^2 - m_\pi^2)\delta_{ij}(2\pi)^4\delta^4(q'+q)$$

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$$(F^{\pi\pi})^{2c}_{ij}$$

$$= \lambda_{1\sigma 2\pi}\,\sigma_c\,\delta_{ij} + \frac{1}{2}\lambda_{2\sigma 2\pi}\,\delta_{ij}(\sigma_c^2 + \sigma_2^2)$$

$$+ \frac{1}{2}\lambda_{4\pi}(\pi_c^2 + \pi_2^2)\,\delta_{ij} + \lambda_{4\pi}(\pi_{i,c}\pi_{j,c} + \pi_{i,2}\pi_{j,2})$$

其中 $\lambda_{1\sigma 2\pi} = -\rho_{0,c}^{\frac{1}{2}} V_k^{(2)}(\rho_{0,c})$

$\lambda_{4\pi} = -\frac{1}{2} V_k^{(2)}(\rho_{0,c})$

$$(\Gamma_k^{(2)\pi\pi})^{c2}_{ij} = (\Gamma_k^{(2)\pi\pi})^{2c}_{ij}$$

$$(\Gamma_k^{(2)\sigma\pi})^{2c}_{j} = (F^{\sigma\pi})^{2c}_{j}$$

$$= \lambda_{1\sigma 2\pi}\,\pi_{j,c} + \lambda_{2\sigma 2\pi}(\sigma_c\pi_{j,c} + \sigma_2\pi_{j,2})$$

$= (F_k^{\pi\sigma})^{c2}_i$

$$(\Gamma_k^{(2)\pi\sigma})^{c2}_{i} = (\Gamma_k^{(2)\sigma\pi})^{2c}_{i}$$

$$= \lambda_{1\sigma 2\pi}\,\pi_{i,c} + \lambda_{2\sigma 2\pi}(\sigma_c\pi_{i,c} + \sigma_2\pi_{i,2})$$

$$(\Gamma_k^{(2)\pi\sigma})^{2c}_{i} = (F^{\pi\sigma})^{2c}_{i}$$

$$= \lambda_{1\sigma 2\pi}\,\pi_{i,c} + \lambda_{2\sigma 2\pi}(\sigma_c\pi_{i,c} + \sigma_2\pi_{i,2})$$

$$(\Gamma_k^{(2)\sigma\pi})^{c2}_{j} = (F^{\sigma\pi})^{c2}_{j}$$

$$= (\Gamma_k^{(2)\pi\sigma})^{2c}_{j}$$

$$= \lambda_{1\sigma 2\pi}\,\pi_{j,c} + \lambda_{2\sigma 2\pi}(\sigma_c\pi_{j,c} + \sigma_2\pi_{j,2})$$

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$$\tilde{P} = \begin{pmatrix} 0 & 0 & Z_{\phi,k}(q^2)q^2 - m_\sigma^2 & 0 \\ 0 & 0 & 0 & (Z_{\phi,k}(q^2)q^2 - m_\pi^2 \\ Z_{\phi,k}(q^2)q^2 - m_\sigma^2 & 0 & 0 & 0 \\ 0 & (Z_{\phi,k}(q^2)q^2 - m_\pi^2)\delta_{ij} & 0 & 0 \end{pmatrix}$$

暂时引入 3d regulators

$$\Delta S_k = \int \frac{d^4q}{(2\pi)^4}\Big[Z_{\phi,k}(q^2)\,\sigma_q(-q)\Big(-\vec{q}^{\,2}\, r_B\Big(\frac{\vec{q}^{\,2}}{k^2}\Big)\Big)\sigma_c(q)$$

$$+ Z_{\phi,k}(q^2)\,\pi_{i,q}(-q)\Big(-\vec{q}^{\,2}\, r_B\Big(\frac{\vec{q}^{\,2}}{k^2}\Big)\Big)\pi_{i,c}(q)\Big]$$

$$(\Delta S_k^{(2)\sigma\sigma})^{qc} = \frac{\delta^2 \Delta S_k}{\delta\sigma_q(q')\,\delta\sigma_c(q)}$$

$$= Z_{\phi,k}(q^2)\Big(-\vec{q}^{\,2}\, r_B\Big(\frac{\vec{q}^{\,2}}{k^2}\Big)\Big)(2\pi)^4\delta^4(q'+q)$$

$$= R_k^{\sigma\sigma}(q',q)$$

$$(\Delta S_k^{(2)\pi\pi})^{qc}_{ij} = \frac{\delta^2 \Delta S_k}{\delta\pi_{i,q}(q')\,\delta\pi_{j,c}(q)}$$

$$= Z_{\phi,k}(q^2)\Big(-\vec{q}^{\,2}\, r_B\Big(\frac{\vec{q}^{\,2}}{k^2}\Big)\Big)\delta_{ij}(2\pi)^4\delta^4(q+q')$$

$$= (R_k^{\pi\pi})_{ij}(q',q)$$

$$P = \tilde{P} + R_k$$

$$
P = \begin{pmatrix}
0 & 0 & Z_{\phi,h}(z^2)(z_0^2-\vec{z}^2(1+r_B))-m_\pi^2 & 0 \\
0 & 0 & 0 & (Z_{\phi,h}(z^2)(z_0^2-\vec{z}^2(1+r_B))-m_\pi^2)\delta_{ij} \\
Z_{\phi,h}(z^2)(z_0^2-\vec{z}^2(1+r_B))-m_\pi^2 & 0 & 0 & 0 \\
0 & (Z_{\phi,h}(z^2)(z_0^2-\vec{z}^2(1+r_B))-m_\pi^2)\delta_{ij} & 0 & 0
\end{pmatrix}
$$

此外在最低阶近似的情况下，还需要添上一个无穷小虚部

即

$$
P = \begin{pmatrix}
0 & 0 & Z_{\phi,h}(z^2)(z_0^2-\vec{z}^2(1+r_B))-m_\pi^2 - \mathrm{sgn}(z_0)i\varepsilon & 0 \\
0 & 0 & 0 & (Z_{\phi,h}(z^2)(z_0^2-\vec{z}^2(1+r_B))-m_\pi^2-\mathrm{sgn}(z_0)i\varepsilon)\delta_{ij} \\
Z_{\phi,h}(z^2)(z_0^2-\vec{z}^2(1+r_B))-m_\pi^2 + \mathrm{sgn}(z_0)i\varepsilon & 0 & 0 & 0 \\
0 & (Z_{\phi,h}(z^2)(z_0^2-\vec{z}^2(1+r_B))-m_\pi^2+\mathrm{sgn}(z_0)i\varepsilon)\delta_{ij} & 0 & 0
\end{pmatrix}
$$

在热平衡的情况下，由于涨落—耗散关系(FDR)的限制，上面传播子的还还需要添加涨落的相应贡献

即 $$P = \begin{pmatrix} 0 & P^A \\ P^R & P^K \end{pmatrix}$$

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其中

$$P^R = \begin{pmatrix} Z_{\phi,k}(\vec{q}^2)(q_0^2 - \vec{q}^2(1+r_B)) - m_\sigma^2 + \mathrm{sgn}(q_0)\, i\varepsilon & 0 \\ 0 & \delta_{ij}\left(Z_{\phi,k}(\vec{q}^2)(q_0^2 - \vec{q}^2(1+r_B)) - m_\pi^2 + \mathrm{sgn}(q_0)\right. \end{pmatrix}$$

$$P^A = (P^R)^*$$

$$P^K = \begin{pmatrix} 2i\overset{\mathrm{sgn}(q_0)}{\varepsilon}\coth\left(\frac{\beta q^0}{2}\right) & 0 \\ 0 & 2i\overset{\mathrm{sgn}(q_0)}{\varepsilon}\coth\left(\frac{\beta q^0}{2}\right)\delta_{ij} \end{pmatrix}$$

$$G = P^{-1} = \begin{pmatrix} -(P^R)^{-1}P^K(P^A)^{-1} & (P^R)^{-1} \\ (P^A)^{-1} & 0 \end{pmatrix}$$

$$= \begin{pmatrix} G^K & G^R \\ G^A & 0 \end{pmatrix}$$

$$G^R = (P^R)^{-1}$$

$$= \begin{pmatrix} \dfrac{1}{Z_{\phi,k}(\vec{q}^2)(q_0^2 - \vec{q}^2(1+r_B)) - m_\sigma^2 + \mathrm{sgn}(q_0)\, i\varepsilon} & 0 \\ 0 & \dfrac{\delta_{ij}}{Z_{\phi,k}(\vec{q}^2)(q_0^2 - \vec{q}^2(1+r_B)) - m_\pi^2 + \mathrm{sgn}(q_0)\, i\varepsilon} \end{pmatrix}$$

$$G^A = (P^A)^{-1} = G^{R*}$$

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$$G^K=\begin{pmatrix}\dfrac{-2i\varepsilon\,\mathrm{sgn}(\varepsilon_0)}{[Z_{t,h}(\varepsilon^2)(\varepsilon_0^2-\vec{\varepsilon}^2(1+r_B))-m_\pi^2]^2+\varepsilon^2} & 0\\ 0 & \dfrac{-2i\varepsilon\,\mathrm{sgn}(\varepsilon_0)\,\delta_{ij}}{[Z_{t,h}(\varepsilon^2)(\varepsilon_0^2-\vec{\varepsilon}^2(1+r_B))-m_\pi^2]^2+\varepsilon^2}\end{pmatrix}$$

$$\times\coth\left(\frac{\beta\varepsilon^0}{2}\right)$$

$$G^R-G^A$$

$$=\begin{pmatrix}\dfrac{-2i\varepsilon\,\mathrm{sgn}(\varepsilon_0)}{[Z_{t,h}(\varepsilon^2)(\varepsilon_0^2-\vec{\varepsilon}^2(1+r_B))-m_\pi^2]^2+\varepsilon^2} & 0\\ 0 & \dfrac{-2i\varepsilon\,\mathrm{sgn}(\varepsilon_0)\,\delta_{ij}}{[Z_{t,h}(\varepsilon^2)(\varepsilon_0^2-\vec{\varepsilon}^2(1+r_B))-m_\pi^2]^2+\varepsilon^2}\end{pmatrix}$$

所以有

$$G^K=(G^R-G^A)\coth\left(\frac{\beta\varepsilon^0}{2}\right)$$

由前面的讨论我们知道

$$T_k^{(2)}+R_k=\tilde{P}+F+R_k$$

$$=(\tilde{P}+R_k)+F=P+F.$$

其中 $P=\tilde{P}+R_k$

$$=\begin{pmatrix}0 & \tilde{P}^A\\ \tilde{P}^R & \tilde{P}^K\end{pmatrix}+\begin{pmatrix}0 & R_k^A\\ R_k^R & 0\end{pmatrix}$$

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$$= \begin{pmatrix} 0 & \tilde{P}^A + R_k^A \\ \tilde{P}^R + R_k^R & \tilde{P}^K \end{pmatrix} = \begin{pmatrix} 0 & P^A \\ P^R & P^K \end{pmatrix}$$

下面计算 F 矩阵 cc 和 22 矩阵元

$$\frac{\delta^2 \Gamma_{k,U}}{\delta\phi_{i,c}\,\delta\phi_{j,c}}$$

$$= -\int \frac{\partial^3 U_k}{\partial\phi_{k,2}\,\partial\phi_{i,c}\,\partial\phi_{j,c}}\Big|_{\bar\phi} \phi_{k,2} + \frac{\partial^4 U_k}{\partial\phi_{l,2}\,\partial\phi_{i,c}\,\partial\phi_{j,c}\,\partial\phi_{k,c}}\Big|_{\bar\phi} \phi_{l,2}(\phi_{k,c} - \bar\phi_{k,c})$$

$$+ \cdots \Big]$$

$$\frac{\partial^3 U_k}{\partial\phi_{0,c}\,\partial\phi_{0,c}\,\partial\phi_{k,2}}\Big|_{\bar\phi} \phi_{k,2}$$

$$= \Big[\frac{1}{2} V_k^{(2)}(\rho_{0,c})(\bar\phi_{k,c} + 2\delta_{0k}\bar\phi_{0,c}) + \frac{1}{4} V_k^{(3)}(\rho_{0,c})\bar\phi_{0,c}^2\bar\phi_{k,c}\Big]\phi_{k,2}$$

$$= \Big[3(\rho_{0,c})^{\frac{1}{2}} V_k^{(2)}(\rho_{0,c}) + 2(\rho_{0,c})^{\frac{3}{2}} V_k^{(3)}(\rho_{0,c})\Big]\sigma_2$$

$$= -\lambda_{3a}\sigma_2$$

$$\frac{\partial^4 U_k}{\partial\phi_{l,2}\,\partial\phi_{0,c}\,\partial\phi_{0,c}\,\partial\phi_{k,c}}\Big|_{\bar\phi} \phi_{l,2}(\phi_{k,c} - \bar\phi_{k,c})$$

$$= \Big[\frac{1}{2} V_k^{(2)}(\rho_{0,c})(\delta_{kl} + 2\delta_{0k}\delta_{0l})$$

$$+ \frac{1}{4} V_k^{(3)}(\rho_{0,c})(\bar\phi_{k,c}\bar\phi_{l,c} + \delta_{kl}\bar\phi_{0,c}\bar\phi_{0,c} + 2\delta_{0k}\bar\phi_{0,c}\bar\phi_{l,c} + 2\delta_{0l}\bar\phi_{0c}\bar\phi_{k,c}$$

$$+ \frac{1}{8} V_k^{(4)}(\rho_{0,c})\bar\phi_{0,c}\bar\phi_{0,c}\bar\phi_{k,c}\bar\phi_{l,c}\Big]\phi_{l,2}(\phi_{k,c} - \bar\phi_{k,c})$$

$$= \frac{1}{2} V_k^{(2)}(\rho_{0,c})\left(\sigma_a \sigma_c + \pi_a \cdot \pi_c + 2 \sigma_a \sigma_c \right)$$

$$+ \frac{1}{4} V_k^{(3)}(\rho_{0,c})\left(\bar{\phi}_{0,c}^2 \sigma_a \sigma_c + \bar{\phi}_{0,c}^2 (\sigma_a \sigma_c + \pi_a \cdot \pi_c) + 4 \bar{\phi}_{0,c}^2 \sigma_a \sigma_c \right)$$

$$+ \frac{1}{8} V_k^{(4)}(\rho_{0,c}) \bar{\phi}_{0,c}^4 \sigma_a \sigma_c$$

$$= \frac{1}{2} V_k^{(2)} (3\sigma_a \sigma_c + \pi_a \cdot \pi_c)$$

$$+ \rho_{0,c} V_k^{(3)} (6 \sigma_a \sigma_c + \pi_a \pi_c)$$

$$+ 2 \rho_{0,c}^2 V_k^{(4)} \sigma_a \sigma_c$$

$$= \left[\frac{3}{2} V_k^{(2)} + 6 \rho_{0,c} V_k^{(3)} + 2 \rho_{0,c}^2 V_k^{(4)} \right] \sigma_a \sigma_c$$

$$+ \left[\frac{1}{2} V_k^{(2)} + \rho_{0,c} V_k^{(3)} \right] \pi_a \pi_c$$

$$= -\lambda_{4\sigma} \sigma_a \sigma_c - \lambda_{2\sigma 2\pi} \pi_a \cdot \pi_c$$

所以有

$$(\Gamma_{k,U}^{\sigma\sigma})^{cc} = \frac{\delta^2 \Gamma_{k,U}}{\delta \sigma_c \delta \sigma_c} = \frac{\delta^2 \Gamma_{k,U}}{\delta \phi_{0,c} \delta \phi_{0,c}}$$

$$= \lambda_{3\sigma} \sigma_a + \lambda_{4\sigma} \sigma_a \sigma_c + \lambda_{2\sigma 2\pi} \pi_a \cdot \pi_c$$

$$\frac{\partial^3 U_k}{\partial \pi_{i,c} \partial \pi_{j,c} \partial \phi_{k,a}} \bigg|_{\bar{\phi}} \phi_{k,a}$$

$$= \left[\frac{1}{2} V_k^{(2)} (\delta_{ij} \bar{\phi}_{k,c} + \delta_{ik} \bar{\phi}_{j,c} + \delta_{jk} \bar{\phi}_{i,c}) \right.$$

$$\left. + \frac{1}{4} V_k^{(3)} \bar{\phi}_{i,c} \bar{\phi}_{j,c} \bar{\phi}_{k,c} \right] \phi_{k,a}$$

$$= \frac{1}{2} V_k^{(2)} \delta_{ij} \bar{\phi}_{0,c} \sigma_a = \rho_{0,c}^{\frac{1}{2}} V_k^{(2)} \delta_{ij} \sigma_a = -\lambda_{1\sigma 2\pi} \sigma_a \delta_{ij}$$

$$\frac{\partial^4 U_k}{\partial\phi_{l,2}\,\partial\pi_{i,c}\,\partial\pi_{j,c}\,\partial\phi_{k,c}}\bigg|_{\bar\phi}\phi_{l,2}(\phi_{k,c}-\bar\phi_{k,c})$$

$$=\Big[\frac{1}{2}V_k^{(2)}(\delta_{ij}\delta_{kl}+\delta_{ik}\delta_{jl}+\delta_{jk}\delta_{il})$$

$$+\frac{1}{4}V_k^{(3)}(\delta_{ij}\bar\phi_{k,c}\bar\phi_{l,c}+\delta_{kl}\cancel{\bar\phi_{i,c}}\bar\phi_{j,c}+\delta_{ik}\cancel{\bar\phi_{j,c}}\bar\phi_{l,c}+\delta_{jl}\cancel{\bar\phi_{i,c}}\bar\phi_{k,c}$$

$$+\delta_{jk}\cancel{\bar\phi_{i,c}}\bar\phi_{l,c}+\delta_{il}\cancel{\bar\phi_{j,c}}\bar\phi_{k,c})$$

$$+\frac{1}{8}V_k^{(4)}\bar\phi_{i,c}\bar\phi_{j,c}\bar\phi_{k,c}\bar\phi_{l,c}\Big]\phi_{l,2}(\phi_{k,c}-\bar\phi_{k,c})$$

$$=\frac{1}{2}V_k^{(2)}(\delta_{ij}(\sigma_2\sigma_c+\pi_2\cdot\pi_c)+\pi_{j,2}\pi_{i,c}+\pi_{i,2}\pi_{j,c})$$

$$+\frac{1}{4}V_k^{(3)}(\delta_{ij}\bar\phi_{0,c}^2\sigma_2\sigma_c)$$

$$=\left(\frac{1}{2}V_k^{(2)}+\rho_{0,c}V_k^{(3)}\right)\delta_{ij}\sigma_2\sigma_c+\frac{1}{2}V_k^{(2)}(\delta_{ij}\pi_2\cdot\pi_c+\pi_{i,2}\pi_{j,c}+\pi_{i,c}\pi_{j,2})$$

$$=-\lambda_{2\sigma2\pi}\sigma_2\sigma_c\delta_{ij}-\lambda_{4\pi}(\delta_{ij}\pi_2\cdot\pi_c+\pi_{i,2}\pi_{j,c}+\pi_{i,c}\pi_{j,2})$$

所

$$(\Gamma_{k,0}^{\pi\pi})_{ij}^{cc}=\frac{\delta^2\Gamma_{k,0}}{\delta\pi_{i,c}\,\delta\pi_{j,c}}$$

$$=\lambda_{1\sigma2\pi}\sigma_2\delta_{ij}+\lambda_{2\sigma2\pi}\sigma_2\sigma_c\delta_{ij}$$

$$+\lambda_{4\pi}(\delta_{ij}\pi_2\cdot\pi_c+\pi_{i,2}\pi_{j,c}+\pi_{i,c}\pi_{j,2})$$

$$\frac{\partial^3 U}{\partial\phi_{0,c}\,\partial\pi_{j,c}\,\partial\phi_{k,2}}\bigg|_{\bar\phi}\phi_{k,2}$$

$$=\Big[\frac{1}{2}V_k^{(2)}(\cancel{\delta_{0j}}\bar\phi_{k,c}+\delta_{0k}\bar\phi_{j,c}+\delta_{jk}\bar\phi_{0,c})$$

$$+\frac{1}{4}V_k^{(3)}\bar\phi_{0,c}\cancel{\bar\phi_{j,c}}\bar\phi_{k,c}\Big]\phi_{k,2}=\rho_{0,c}^{\frac{1}{2}}V_k^{(2)}\pi_{j,2}=-\lambda_{1\sigma2\pi}\pi_{j,2}$$

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$$\frac{\partial^4 U}{\partial\phi_{l,2}\,\partial\phi_{0,c}\,\partial\pi_{j,c}\,\partial\phi_{k,c}}\bigg|_{\bar\phi}\phi_{l,2}(\phi_{k,c}-\bar\phi_{k,c})$$

$$=\Big[\frac{1}{2}V_k^{(2)}\big(\cancel{\delta_{0j}}\delta_{kl}+\delta_{0k}\delta_{jl}+\delta_{0l}\delta_{jk}\big)$$

$$+\frac{1}{4}V_k^{(3)}\big(\cancel{\delta_{0j}}\bar\phi_{k,c}\bar\phi_{l,c}+\delta_{kl}\bar\phi_{0,c}\cancel{\bar\phi_{j,c}}+\delta_{0k}\cancel{\bar\phi_{j,c}}\bar\phi_{l,c}$$

$$+\delta_{jl}\bar\phi_{0,c}\bar\phi_{k,c}+\delta_{0l}\cancel{\bar\phi_{j,c}}\bar\phi_{k,c}+\delta_{jk}\bar\phi_{0,c}\bar\phi_{l,c}\big)$$

$$+\frac{1}{8}V_k^{(4)}\bar\phi_{0,c}\cancel{\bar\phi_{j,c}}\bar\phi_{k,c}\bar\phi_{l,c}\Big]\phi_{l,2}(\phi_{k,c}-\bar\phi_{k,c})$$

$$=\frac{1}{2}V_k^{(2)}\big(\pi_{j,2}\sigma_c+\pi_{j,c}\sigma_2\big)$$

$$+\frac{1}{4}V_k^{(3)}\big(\bar\phi_{0,c}^2\pi_{j,2}\sigma_c+\bar\phi_{0,c}^2\pi_{j,c}\sigma_2\big)$$

$$=\Big(\frac{1}{2}V_k^{(2)}+\rho_{0,c}V_k^{(3)}\Big)\big(\pi_{j,2}\sigma_c+\pi_{j,c}\sigma_2\big)$$

$$=-\lambda_{2a2a}\big(\pi_{j,2}\sigma_c+\pi_{j,c}\sigma_2\big)$$

$$(\Gamma_{k,U}^{(2a)})_j^{cc}=\frac{\delta^2\Gamma_{k,U}}{\delta\phi_{0,c}\,\delta\pi_{j,c}}$$

$$=\lambda_{1a2a}\pi_{j,2}+\lambda_{2a2a}\big(\pi_{j,2}\sigma_c+\pi_{j,c}\sigma_2\big)$$

$$\frac{\delta^2 \Gamma_{k,U}}{\delta\phi_{i,2}\,\delta\phi_{j,2}}$$

$$= -\left\{ \frac{\partial^3 U_k}{\partial\phi_{i,2}\,\partial\phi_{j,2}\,\partial\phi_{k,2}}\bigg|_{\bar\phi} \phi_{k,2} \right.$$

$$\left. + \frac{\partial^4 U_k}{\partial\phi_{i,2}\,\partial\phi_{l,c}\,\partial\phi_{j,2}\,\partial\phi_{k,2}}\bigg|_{\bar\phi} \phi_{k,2}\left(\phi_{l,c}-\bar\phi_{l,c}\right) + \cdots \right\}$$

$$\frac{\partial^3 U}{\partial\phi_{0,2}\,\partial\phi_{0,2}\,\partial\phi_{k,2}}\bigg|_{\bar\phi} \phi_{k,2}$$

$$= \Big[\frac{1}{2}V_k^{(2)}\left(\delta_{00}\bar\phi_{k,c} + \delta_{0k}\bar\phi_{0,c} + \delta_{0k}\bar\phi_{0,c}\right)$$

$$+ \frac{1}{4}V_k^{(3)}\bar\phi_{0,c}\bar\phi_{0,c}\bar\phi_{k,c}\Big]\phi_{k,2}$$

$$= \frac{3}{2}\bar\phi_{0,c}V_k^{(2)}\sigma_2 + \frac{1}{4}\bar\phi_{0,c}^{3}V_k^{(3)}\sigma_2$$

$$= \left(3\rho_{0,c}^{\frac{1}{2}}V_k^{(2)} + 2\rho_{0,c}^{\frac{3}{2}}V_k^{(3)}\right)\sigma_2 = -\lambda_{3\sigma}\sigma_2$$

$$\frac{\partial^4 U_k}{\partial\phi_{0,2}\,\partial\phi_{l,c}\,\partial\phi_{0,2}\,\partial\phi_{k,2}}\bigg|_{\bar\phi} \phi_{k,2}\left(\phi_{l,c}-\bar\phi_{l,c}\right)$$

$$= -\lambda_{4\sigma}\sigma_2\sigma_c - \lambda_{2\sigma2\pi}\pi_2\cdot\pi_c$$

所以有

$$\left(\Gamma_{k,U}^{\sigma\sigma}\right)^{22} = \frac{\delta^2\Gamma_{k,U}}{\delta\sigma_2\,\delta\sigma_2} = \frac{\delta^2\Gamma_{k,U}}{\delta\phi_{0,2}\,\delta\phi_{0,2}}$$

$$= \lambda_{3\sigma}\sigma_2 + \lambda_{4\sigma}\sigma_2\sigma_c + \lambda_{2\sigma2\pi}\pi_2\cdot\pi_c$$

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$$\frac{\partial^3 U}{\partial \pi_{i,2}\,\partial \pi_{j,2}\,\partial \phi_{k,2}}\Big|_{\bar\phi}\ \phi_{k,2}$$

$$= \Big[\frac{1}{2}V_k^{(2)}\big(\delta_{ij}\bar\phi_{k,c} + \delta_{ik}\bar\phi_{j,2} + \delta_{jk}\bar\phi_{i,c}\big)$$

$$+\frac{1}{4}V_k^{(3)}\cancel{\bar\phi_{i,c}}\,\bar\phi_{j,c}\,\bar\phi_{k,c}\Big]\phi_{k,2}$$

$$= \frac{1}{2}V_k^{(2)}\bar\phi_{0,c}\,\delta_{ij}\,\sigma_2$$

$$= \rho_{0,c}^{\frac{1}{2}}V_k^{(2)}\delta_{ij}\sigma_2 = -\lambda_{1\sigma 2\pi}\sigma_2\delta_{ij}$$

$$\frac{\partial^4 U}{\partial \pi_{i,2}\,\partial \phi_{l,c}\,\partial \pi_{j,2}\,\partial \phi_{k,2}}\Big|_{\bar\phi}\ \phi_{k,2}(\phi_{l,c}-\bar\phi_{l,c})$$

$$= -\lambda_{2\sigma 2\pi}\sigma_2\sigma_c\delta_{ij} - \lambda_{4\pi}\big(\delta_{ij}\pi_2\cdot\pi_c + \pi_{i,2}\pi_{j,c} + \pi_{i,c}\pi_{j,2}\big)$$

故に

$$(\Gamma_{k,U}^{\pi\pi})_{ij}^{22} = \frac{\delta^2\Gamma_{k,U}}{\delta\pi_{i,2}\,\delta\pi_{j,2}}$$

$$= \lambda_{1\sigma 2\pi}\sigma_2\delta_{ij} + \lambda_{2\sigma 2\pi}\sigma_2\sigma_c\delta_{ij}$$

$$+\lambda_{4\pi}\big(\delta_{ij}\pi_2\cdot\pi_c + \pi_{i,2}\pi_{j,c} + \pi_{i,c}\pi_{j,2}\big)$$

$$\frac{\partial^3 U}{\partial \phi_{0,2}\,\partial \pi_{j,2}\,\partial \phi_{k,2}}\Big|_{\bar\phi}\ \phi_{k,2}$$

$$= \Big[\frac{1}{2}V_k^{(2)}\big(\cancel{\delta_{0j}}\bar\phi_{k,c} + \delta_{0k}\cancel{\bar\phi_{j,c}} + \delta_{jk}\bar\phi_{0,c}\big)$$

$$+\frac{1}{4}V_k^{(3)}\bar\phi_{0,c}\,\cancel{\bar\phi_{j,c}}\,\bar\phi_{k,c}\Big]\phi_{k,2} = \rho_{0,c}^{\frac{1}{2}}V_k^{(2)}\pi_{j,2} = -\lambda_{1\sigma 2\pi}\pi_{j,2}$$

$$\frac{\partial^4 U_m}{\partial\phi_{a,2}\,\partial\phi_{b,c}\,\partial\pi_{j,2}\,\partial\phi_{m,2}}\Big|_{\bar\phi}\ \phi_{m,2}\left(\phi_{b,c}-\bar\phi_{b,c}\right)$$

$$= -\lambda_{2a2\pi}\left(\pi_{j,2}\,\sigma_c + \pi_{j,c}\,\sigma_2\right)$$

$$\left(\Gamma^{\alpha\pi}_{m,n}\right)^{22}_{j} = \frac{\delta^2\Gamma_{m,n}}{\delta\phi_{a,2}\,\delta\pi_{j,2}}$$

$$= \lambda_{1a2\pi}\,\pi_{j,2} + \lambda_{2a2\pi}\left(\pi_{j,2}\,\sigma_c + \pi_{j,c}\,\sigma_2\right)$$

最终得到F矩阵：

$$F = \begin{pmatrix} F^C & F^A \\ F^R & F^K \end{pmatrix}$$

$$F^R = \begin{pmatrix} (F^{\alpha\alpha})^{2c} & (F^{\alpha\pi})^{2c}_j \\ (F^{\pi\alpha})^{2c}_i & (F^{\pi\pi})^{2c}_{ij} \end{pmatrix}$$

$$(F^{\alpha\alpha})^{2c} = \lambda_{3a}\,\sigma_c + \frac{1}{2}\lambda_{4a}\left(\sigma_c^2+\sigma_2^2\right) + \frac{1}{2}\lambda_{2a2\pi}\left(\pi_c^2+\pi_2^2\right)$$

$$(F^{\pi\pi})^{2c}_{ij} = \lambda_{1a2\pi}\,\sigma_c\,\delta_{ij} + \frac{1}{2}\lambda_{2a2\pi}\,\delta_{ij}\left(\sigma_c^2+\sigma_2^2\right) + \frac{1}{2}\lambda_{4\pi}\left(\pi_c^2+\pi_2^2\right)\delta_{ij} + \lambda_{4\pi}\left(\pi_{i,c}\pi_{j,c} + \pi_{i,2}\pi_{j,2}\right)$$

$$(F^{\alpha\pi})^{2c}_{j} = \lambda_{1a2\pi}\,\pi_{j,c} + \lambda_{2a2\pi}\left(\sigma_c\,\pi_{j,c} + \sigma_2\,\pi_{j,2}\right)$$

$$(F^{\pi\alpha})^{2c}_{i} = \lambda_{1a2\pi}\,\pi_{i,c} + \lambda_{2a2\pi}\left(\sigma_c\,\pi_{i,c} + \sigma_2\,\pi_{i,2}\right)$$

$$F^A = \begin{pmatrix} (F^{\alpha\alpha})^{c2} & (F^{\alpha\pi})^{c2}_j \\ (F^{\pi\alpha})^{c2}_i & (F^{\pi\pi})^{c2}_{ij} \end{pmatrix}$$

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$$(F^{\sigma\sigma})^{c2} = (F^{\sigma\sigma})^{2c}$$
$$(F^{\pi\pi})^{c2}_{ij} = (F^{\pi\pi})^{2c}_{ij}$$

$$(F^{\sigma\pi})^{c2}_{j} = (F^{\sigma\pi})^{2c}_{j}$$
$$(F^{\pi\sigma})^{c2}_{i} = (F^{\pi\sigma})^{2c}_{i}$$

$$F^{c} = \begin{pmatrix} (F^{\sigma\sigma})^{cc} & (F^{\sigma\pi})^{cc}_{j} \\ (F^{\pi\sigma})^{cc}_{i} & (F^{\pi\pi})^{cc}_{ij} \end{pmatrix}$$

$$(F^{\sigma\sigma})^{cc} = \lambda_{3\sigma}\sigma_2 + \lambda_{4\sigma}\sigma_2\sigma_c + \lambda_{2\sigma2\pi}\pi_2\cdot\pi_c$$
$$(F^{\pi\pi})^{cc}_{ij} = \lambda_{1\sigma2\pi}\sigma_2\delta_{ij} + \lambda_{2\sigma2\pi}\sigma_2\sigma_c\delta_{ij}$$
$$+ \lambda_{4\pi}(\delta_{ij}\pi_2\cdot\pi_c + \pi_{i,2}\pi_{j,c} + \pi_{i,c}\pi_{j,2})$$

$$(F^{\sigma\pi})^{cc}_{j} = \lambda_{1\sigma2\pi}\pi_{j,2} + \lambda_{2\sigma2\pi}(\pi_{j,2}\sigma_c + \pi_{j,c}\sigma_2)$$
$$(F^{\pi\sigma})^{cc}_{i} = (F^{\sigma\pi})^{cc}_{i}$$
$$= \lambda_{1\sigma2\pi}\pi_{i,2} + \lambda_{2\sigma2\pi}(\pi_{i,2}\sigma_c + \pi_{i,c}\sigma_2)$$

$$F^{K} = \begin{pmatrix} (F^{\sigma\sigma})^{22} & (F^{\sigma\pi})^{22}_{j} \\ (F^{\pi\sigma})^{22}_{i} & (F^{\pi\pi})^{22}_{ij} \end{pmatrix}$$

$$(F^{\sigma\sigma})^{22} = (F^{\sigma\sigma})^{cc}$$

$$(F^{\pi\pi})^{22}_{ij} = (F^{\pi\pi})^{cc}_{ij}$$
$$(F^{\sigma\pi})^{22}_{j} = (F^{\sigma\pi})^{cc}_{j}$$

$$(F^{\pi\sigma})^{22}_{i} = (F^{\pi\sigma})^{cc}_{i}$$

$$G_k^{-1} = T_k^{(2)} + R_k$$
$$= P + F$$
$$= \tilde{P} + R_k + F$$
$$= \begin{pmatrix} 0 & \tilde{P}^A + R_k^A \\ \tilde{P}^R + R_k^R & P^K \end{pmatrix} + \begin{pmatrix} F^c & F^A \\ F^R & F^K \end{pmatrix}$$

$$\tilde{\partial}_t \ln G_k^{-1}$$

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$$= G_{k,ab}\, \tilde{\partial}_t (G_k^{-1})_{bc}$$
$$= \begin{pmatrix} G_k^{cc} & G_k^{cq} \\ G_k^{qc} & G_k^{qq} \end{pmatrix} \begin{pmatrix} 0 & \tilde{\partial}_t R_k^A \\ \tilde{\partial}_t R_k^R & 0 \end{pmatrix}^T$$
$$= \begin{pmatrix} G_k^{cq} \tilde{\partial}_t R_k^A & G_k^{cc} \tilde{\partial}_t R_k^R \\ G_k^{qq} \tilde{\partial}_t R_k^A & G_k^{qc} \tilde{\partial}_t R_k^R \end{pmatrix}$$

所以

$$\mathrm{STr}\left\{ \tilde{\partial}_t \ln G_k^{-1} \right\}$$
$$= \mathrm{STr} \begin{pmatrix} G_k^K & G_k^R \\ G_k^A & G_k^Q \end{pmatrix} \begin{pmatrix} 0 & \tilde{\partial}_t R_k^A \\ \tilde{\partial}_t R_k^R & 0 \end{pmatrix}^T = \mathrm{STr} \begin{pmatrix} G_k^R \tilde{\partial}_t R_k^A & G_k^K \tilde{\partial}_t R_k \\ G_k^Q \tilde{\partial}_t R_k^A & G_k^A \tilde{\partial}_t R_k \end{pmatrix}$$

$$= \mathrm{STr}\left(G_k^R \tilde{\partial}_t R_k^A \right) + \mathrm{STr}\left(G_k^A \tilde{\partial}_t R_k^R \right)$$

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有效作用量的流方程

$$\partial_\tau \Gamma_k[\Phi]$$

$$= \frac{i}{2} \mathrm{STr}\left\{ \widetilde{\partial_\tau} \ln G_K^{-1} \right\}$$

$$= \frac{i}{2} \mathrm{STr}\, \widetilde{\partial_\tau} \ln(P+F)$$

$$= \frac{i}{2} \mathrm{STr}\, \widetilde{\partial_\tau} \ln P\left(1+\frac{1}{P}F\right)$$

$$= \frac{i}{2} \mathrm{STr}\, \widetilde{\partial_\tau} \ln P + \frac{i}{2} \mathrm{STr}\, \widetilde{\partial_\tau} \ln\left(1+\frac{1}{P}F\right)$$

$$\ln\left(1+\frac{1}{P}F\right)$$

$$= \frac{1}{P}F - \frac{1}{2}\left(\frac{1}{P}F\right)^2 + \frac{1}{3}\left(\frac{1}{P}F\right)^3 - \frac{1}{4}\left(\frac{1}{P}F\right)^4 + \cdots$$

所以有

$$\partial_\tau \Gamma_k[\Phi]$$

$$= \frac{i}{2} \mathrm{STr}\, \widehat{\partial_\tau} \ln P + \frac{i}{2} \mathrm{STr}\, \widehat{\partial_\tau}\left(\frac{1}{P}F\right)$$

$$- \frac{i}{4} \mathrm{STr}\, \widehat{\partial_\tau}\left(\frac{1}{P}F\right)^2 + \frac{i}{6} \mathrm{STr}\, \widehat{\partial_\tau}\left(\frac{1}{P}F\right)^3$$

$$- \frac{i}{8} \mathrm{STr}\, \widehat{\partial_\tau}\left(\frac{1}{P}F\right)^4 + \cdots$$

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显然

$$\frac{1}{P} = G_k[\bar{\Phi}=0] = \begin{pmatrix} G_k^K[\bar{\Phi}=0] & G_k^R[\bar{\Phi}=0] \\ G_k^A[\bar{\Phi}=0] & 0 \end{pmatrix}$$

$$= \begin{pmatrix} G_k^K & G_k^R \\ G_k^A & 0 \end{pmatrix}$$ 在不引起误会的情况下仍用这个记号。

$$= G$$

$$GF = \begin{pmatrix} G^K & G^R \\ G^A & 0 \end{pmatrix} \begin{pmatrix} F^c & F^A \\ F^R & F^K \end{pmatrix}$$

$$= \begin{pmatrix} G^K F^c + G^R F^R & G^K F^A + G^R F^K \\ G^A F^c & G^A F^A \end{pmatrix}$$

其中 $$G^R = \begin{pmatrix} (G^a)^{c2} & 0 \\ 0 & (G^\pi)^{c2}_{ij} \end{pmatrix}$$

$$G^A = \begin{pmatrix} (G^a)^{2c} & \\ & (G^\pi)^{2c}_{ij} \end{pmatrix}$$

$$G^K = \begin{pmatrix} (G^a)^{cc} & 0 \\ 0 & (G^\pi)^{cc}_{ij} \end{pmatrix}$$

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$$G^kF^c = \begin{pmatrix} (G^\alpha)^{cc} & 0 \\ 0 & (G^\pi)^{cc}_{ik} \end{pmatrix} \begin{pmatrix} (F^{\alpha\alpha})^{cc} & (F^{\alpha\pi})^{cc}_{j} \\ (F^{\pi\alpha})^{cc}_{k} & (F^{\pi\pi})^{cc}_{kj} \end{pmatrix}$$

$$= \begin{pmatrix} (G^\alpha)^{cc}(F^{\alpha\alpha})^{cc} & (G^\alpha)^{cc}(F^{\alpha\pi})^{cc}_{j} \\ [(G^\pi)^{cc}(F^{\pi\alpha})^{cc}]_i & [(G^\pi)^{cc}(F^{\pi\pi})^{cc}]_{ij} \end{pmatrix}$$

$$\cancel{= (G^\alpha_{cc} F}$$

$$= \begin{pmatrix} G^{cc}_\alpha F^{cc}_{\alpha\alpha} & (G^{cc}_\alpha F^{cc}_{\alpha\pi})_j \\ (G^{cc}_\pi F^{cc}_{\pi\alpha})_i & (G^{cc}_\pi F^{cc}_{\pi\pi})_{ij} \end{pmatrix}$$

$$G^RF^R = \begin{pmatrix} G^{c2}_\alpha & 0 \\ 0 & (G^{c2}_\pi)_{ik} \end{pmatrix} \begin{pmatrix} F^{2c}_{\alpha\alpha} & (F^{2c}_{\alpha\pi})_j \\ (F^{2c}_{\pi\alpha})_k & (F^{2c}_{\pi\pi})_{kj} \end{pmatrix}$$

$$= \begin{pmatrix} G^{c2}_\alpha F^{2c}_{\alpha\alpha} & (G^{c2}_\alpha F^{2c}_{\alpha\pi})_j \\ (G^{c2}_\pi F^{2c}_{\pi\alpha})_i & (G^{c2}_\pi F^{2c}_{\pi\pi})_{ij} \end{pmatrix}$$

故

$$GF = \begin{pmatrix} (GF)^{cc} & (GF)^{c2} \\ (GF)^{2c} & (GF)^{22} \end{pmatrix}$$

$$(GF)^{cc} = G^K F^c + G^R F^R$$

$$= \begin{pmatrix} G_\sigma^{cc} F_{\sigma\sigma}^{cc} + G_\sigma^{cq} F_{\sigma\sigma}^{qc} & (G_\sigma^{cc} F_{\sigma\pi}^{cc})_j + (G_\sigma^{cq} F_{\sigma\pi}^{qc})_j \\ (G_\pi^{cc} F_{\pi\sigma}^{cc})_i + (G_\pi^{cq} F_{\pi\sigma}^{qc})_i & (G_\pi^{cc} F_{\pi\pi}^{qc})_{ij} + (G_\pi^{cq} F_{\pi\pi}^{qc})_{ij} \end{pmatrix}$$

$$= \begin{pmatrix} (GF)_\sigma^{cc} & [(GF)_{\sigma\pi}^{cc}]_j \\ ((GF)_{\pi\sigma}^{cc})_i & ((GF)_{\pi\pi}^{cc})_{ij} \end{pmatrix}$$

$$(GF)_\sigma^{cc} = G_\sigma^{cc} F_{\sigma\sigma}^{cc} + G_\sigma^{cq} F_{\sigma\sigma}^{qc}$$
$$= \lambda_{3\sigma} G_\sigma^{cc} \sigma_q + \lambda_{4\sigma} G_\sigma^{cc} \sigma_q \sigma_c + \lambda_{2\sigma 2\pi} G_\sigma^{cc} (\pi_q \cdot \pi_c)$$
$$+ \lambda_{3\sigma} G_\sigma^{cq} \sigma_c + \frac{1}{2} \lambda_{4\sigma} G_\sigma^{cq} (\sigma_c^2 + \sigma_q^2) + \frac{1}{2} \lambda_{2\sigma 2\pi} G_\sigma^{cq} (\pi_c^2 + \pi_q^2$$

$$((GF)_{\pi\pi}^{cc})_{ij} = (G_\pi^{cc} F_{\pi\pi}^{cc})_{ij} + (G_\pi^{cq} F_{\pi\pi}^{qc})_{ij}$$
$$= (G_\pi^{cc})_{ik} (F_{\pi\pi}^{cc})_{kj} + (G_\pi^{cq})_{ik} (F_{\pi\pi}^{qc})_{kj}$$
$$= \lambda_{1\sigma 2\pi} (G_\pi^{cc})_{ij} \sigma_q + \lambda_{2\sigma 2\pi} (G_\pi^{cc})_{ij} \sigma_q \sigma_c$$
$$+ \lambda_{4\pi} \left[(G_\pi^{cc})_{ij} \pi_q \cdot \pi_c + (G_\pi^{cc})_{ik} \pi_{k,q} \pi_{j,c} + (G_\pi^{cc})_{ik} \pi_{k,c} \pi_{j,q} \right]$$
$$+ \lambda_{1\sigma 2\pi} (G_\pi^{cq})_{ij} \sigma_c + \frac{1}{2} \lambda_{2\sigma 2\pi} (G_\pi^{cq})_{ij} (\sigma_c^2 + \sigma_q^2)$$
$$+ \frac{1}{2} \lambda_{4\pi} (G_\pi^{cq})_{ij} (\pi_c^2 + \pi_q^2) + \lambda_{4\pi} \left[(G_\pi^{cq})_{ik} \pi_{k,c} \pi_{j,c} + (G_\pi^{cq})_{ik} \pi_{k,q} \pi_{j,q} \right]$$

$$= \lambda_{1\sigma 2\pi} \left((G_\pi^{cc})_{ij} \sigma_q + (G_\pi^{cq})_{ij} \sigma_c \right)$$
$$+ \lambda_{2\sigma 2\pi} \left((G_\pi^{cc})_{ij} \sigma_q \sigma_c + \frac{1}{2} (G_\pi^{cq})_{ij} (\sigma_c^2 + \sigma_q^2) \right)$$
$$+ \lambda_{4\pi} \left[(G_\pi^{cc})_{ij} \pi_q \cdot \pi_c + \frac{1}{2} (G_\pi^{cq})_{ij} (\pi_c^2 + \pi_q^2) + (G_\pi^{cc})_{ik} (\pi_{k,q} \pi_{j,c} + \pi_{k,c} \pi_{j,q}) + (G_\pi^{cq})_{ik} (\pi_{k,c} \pi_{j,c} + \pi_{k,q} \pi_{j,q}) \right]$$

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$$((GF)^{cc}_{\sigma\pi})_j$$

$$= (G^{cc}_{\sigma} F^{cc}_{\sigma\pi})_j + (G^{c\ell}_{\sigma} F^{\ell c}_{\sigma\pi})_j$$

$$= \lambda_{1\alpha2\pi} G^{cc}_{\sigma} \pi_{j,\ell} + \lambda_{2\alpha2\pi} G^{cc}_{\sigma} (\pi_{j,\ell} \sigma_c + \pi_{j,c} \sigma_\ell)$$
$$+ \lambda_{1\alpha2\pi} G^{c\ell}_{\sigma} \pi_{j,c} + \lambda_{2\alpha2\pi} G^{c\ell}_{\sigma} (\sigma_c \pi_{j,c} + \sigma_\ell \pi_{j,\ell})$$
$$= \lambda_{1\alpha2\pi} (G^{cc}_{\sigma} \pi_{j,\ell} + G^{c\ell}_{\sigma} \pi_{j,c})$$
$$+ \lambda_{2\alpha2\pi} [G^{cc}_{\sigma} (\sigma_c \pi_{j,\ell} + \sigma_\ell \pi_{j,c}) + G^{c\ell}_{\sigma} (\sigma_c \pi_{j,c} + \sigma_\ell \pi_{j,\ell})]$$

$$((GF)^{cc}_{\pi\alpha})_i = (G^{cc}_{\pi} F^{cc}_{\pi\sigma})_i + (G^{c\ell}_{\pi} F^{\ell c}_{\pi\sigma})_i$$
$$(G^{cc}_{\pi})_{ij} (\lambda_{1\alpha2\pi} \pi_{j,\ell} + \lambda_{2\alpha2\pi} (\pi_{j,\ell} \sigma_c + \pi_{j,c} \sigma_\ell))$$
$$(G^{c\ell}_{\pi})_{ij} (\lambda_{1\alpha2\pi} \pi_{j,c} + \lambda_{2\alpha2\pi} (\sigma_c \pi_{j,c} + \sigma_\ell \pi_{j,\ell}))$$

$$= \lambda_{1\alpha2\pi} ((G^{cc}_{\pi})_{ij} \pi_{j,\ell} + (G^{c\ell}_{\pi})_{ij} \pi_{j,c})$$
$$+ \lambda_{2\alpha2\pi} [(G^{cc}_{\pi})_{ij} (\pi_{j,\ell} \sigma_c + \pi_{j,c} \sigma_\ell)$$
$$+ (G^{c\ell}_{\pi})_{ij} (\pi_{j,c} \sigma_c + \pi_{j,\ell} \sigma_\ell)]$$

$$(GF)^{c\ell} = G^K F^A + G^R F^K$$

$$G^K F^A = \begin{pmatrix} G^{cc}_{\sigma} & 0 \\ 0 & (G^{cc}_{\pi})_{ik} \end{pmatrix} \begin{pmatrix} F^{c\ell}_{\sigma\sigma} & (F^{c\ell}_{\sigma\pi})_j \\ (F^{c\ell}_{\pi\sigma})_k & (F^{c\ell}_{\pi\pi})_{kj} \end{pmatrix}$$

$$= \begin{pmatrix} G^{cc}_{\sigma} F^{c\ell}_{\sigma\sigma} & G^{cc}_{\sigma} (F^{c\ell}_{\sigma\pi})_j \\ (G^{cc}_{\pi})_{ik} (F^{c\ell}_{\pi\sigma})_k & (G^{cc}_{\pi})_{ik} (F^{c\ell}_{\pi\pi})_{kj} \end{pmatrix}$$

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$G^k F^k$

$$= \begin{pmatrix} G_\alpha^{c\varepsilon} & 0 \\ 0 & (G_\pi^{c\varepsilon})_{ik} \end{pmatrix} \begin{pmatrix} F^{\varepsilon\varepsilon}_{\alpha\alpha} & (F^{\varepsilon\varepsilon}_{\alpha\pi})_j \\ (F^{\varepsilon\varepsilon}_{\pi\alpha})_k & (F^{\varepsilon\varepsilon}_{\pi\pi})_{kj} \end{pmatrix}$$

$$= \begin{pmatrix} G_\alpha^{c\varepsilon} F^{\varepsilon\varepsilon}_{\alpha\alpha} & G_\alpha^{c\varepsilon}(F^{\varepsilon\varepsilon}_{\alpha\pi})_j \\ (G_\pi^{c\varepsilon})_{ik}(F^{\varepsilon\varepsilon}_{\pi\alpha})_k & (G_\pi^{c\varepsilon})_{ik}(F^{\varepsilon\varepsilon}_{\pi\pi})_{kj} \end{pmatrix}$$

$$(GF)^{c\varepsilon} = \begin{pmatrix} G^{cc}_\alpha F^{c\varepsilon}_{\alpha\alpha} + G^{c\varepsilon}_\alpha F^{\varepsilon\varepsilon}_{\alpha\alpha} & G^{cc}_\alpha (F^{c\varepsilon}_{\alpha\pi})_j + G^{c\varepsilon}_\alpha (F^{\varepsilon\varepsilon}_{\alpha\pi})_j \\ (G^{cc}_\pi)_{ik}(F^{c\varepsilon}_{\pi\alpha})_k + (G^{c\varepsilon}_\pi)_{ik}(F^{\varepsilon\varepsilon}_{\pi\alpha})_k , & (G^{cc}_\pi)_{ik}(F^{c\varepsilon}_{\pi\pi})_{kj} + (G^{c\varepsilon}_\pi)_{ik}(F^{\varepsilon\varepsilon}_{\pi\pi} \end{pmatrix}$$

$$= \begin{pmatrix} (GF)^{c\varepsilon}_\alpha & ((GF)^{c\varepsilon}_{\alpha\pi})_j \\ ((GF)^{c\varepsilon}_{\pi\alpha})_i & ((GF)^{c\varepsilon}_{\pi\pi})_{ij} \end{pmatrix}$$

$$\begin{aligned}
(GF)^{c\varepsilon}_\alpha &= G^{cc}_\alpha F^{c\varepsilon}_{\alpha\alpha} + G^{c\varepsilon}_\alpha F^{\varepsilon\varepsilon}_{\alpha\alpha} \\
&= G^{cc}_\alpha \left(\lambda_{3\alpha}\sigma_c + \tfrac{1}{2}\lambda_{4\alpha}(\sigma_c^2 + \sigma_\varepsilon^2) + \tfrac{1}{2}\lambda_{2\alpha2\pi}(\pi_c^2 + \pi_\varepsilon^2) \right) \\
&\quad + G^{c\varepsilon}_\alpha \left(\lambda_{3\alpha}\sigma_\varepsilon + \lambda_{4\alpha}\sigma_\varepsilon\sigma_c + \lambda_{2\alpha2\pi}\pi_\varepsilon\cdot\pi_c \right) \\
&= \lambda_{3\alpha}\left(G^{cc}_\alpha\sigma_c + G^{c\varepsilon}_\alpha\sigma_\varepsilon \right) \\
&\quad + \lambda_{4\alpha}\left(\tfrac{1}{2}G^{cc}_\alpha(\sigma_c^2+\sigma_\varepsilon^2) + G^{c\varepsilon}_\alpha\sigma_\varepsilon\sigma_c \right) \\
&\quad + \lambda_{2\alpha2\pi}\left(\tfrac{1}{2}G^{cc}_\alpha(\pi_c^2+\pi_\varepsilon^2) + G^{c\varepsilon}_\alpha\pi_\varepsilon\cdot\pi_c \right)
\end{aligned}$$

$$\begin{aligned}
((GF)^{c\varepsilon}_{\alpha\pi})_j &= G^{cc}_\alpha (F^{c\varepsilon}_{\alpha\pi})_j + G^{c\varepsilon}_\alpha (F^{\varepsilon\varepsilon}_{\alpha\pi})_j \\
&= G^{cc}_\alpha \left(\lambda_{1\alpha2\pi}\pi_{j,c} + \lambda_{2\alpha2\pi}(\sigma_c\pi_{j,c} + \sigma_\varepsilon\pi_{j,\varepsilon}) \right) \\
&\quad + G^{c\varepsilon}_\alpha \left(\lambda_{1\alpha2\pi}\pi_{j,\varepsilon} + \lambda_{2\alpha2\pi}(\pi_{j,\varepsilon}\sigma_c + \pi_{j,c}\sigma_\varepsilon) \right) \\
&= \lambda_{1\alpha2\pi}\left(G^{cc}_\alpha\pi_{j,c} + G^{c\varepsilon}_\alpha\pi_{j,\varepsilon} \right) \\
&\quad + \lambda_{2\alpha2\pi}\left[G^{cc}_\alpha(\sigma_c\pi_{j,c} + \sigma_\varepsilon\pi_{j,\varepsilon}) + G^{c\varepsilon}_\alpha(\pi_{j,\varepsilon}\sigma_c + \pi_{j,c}\sigma_\varepsilon) \right]
\end{aligned}$$

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$$\left((GF)^{c2}_{\pi\alpha}\right)_i = (G^{cc}_\pi)_{ik}(F^{c2}_{\pi\alpha})_k + (G^{c2}_\pi)_{ik}(F^{22}_{\pi\alpha})_k$$
$$= (G^{cc}_\pi)_{ik}\left(\lambda_{1\alpha2\pi}\pi_{k,c} + \lambda_{2\alpha2\pi}(\sigma_c\pi_{k,c} + \sigma_2\pi_{k,2})\right)$$
$$+ (G^{c2}_\pi)_{ik}\left(\lambda_{1\alpha2\pi}\pi_{k,2} + \lambda_{2\alpha2\pi}(\pi_{k,2}\sigma_c + \pi_{k,c}\sigma_2)\right)$$
$$= \lambda_{1\alpha2\pi}\left((G^{cc}_\pi)_{ik}\pi_{k,c} + (G^{c2}_\pi)_{ik}\pi_{k,2}\right)$$
$$+ \lambda_{2\alpha2\pi}\left[(G^{cc}_\pi)_{ik}(\sigma_c\pi_{k,c} + \sigma_2\pi_{k,2})\right.$$
$$\left. + (G^{c2}_\pi)_{ik}(\pi_{k,2}\sigma_c + \pi_{k,c}\sigma_2)\right]$$

$$\left((GF)^{c2}_{\pi\pi}\right)_{ij} = \lambda_{1\alpha2\pi}\left((G^{cc}_\pi)_{ij}\sigma_c + (G^{c2}_\pi)_{ij}\sigma_2\right)$$
$$+ \lambda_{2\alpha2\pi}\left((G^{c2}_\pi)_{ij}\sigma_c\sigma_2 + \frac{1}{2}(G^{cc}_\pi)_{ij}(\sigma_c^2 + \sigma_2^2)\right)$$
$$+ \lambda_{4\pi}\left[(G^{c2}_\pi)_{ij}\pi_2\cdot\pi_c + \frac{1}{2}(G^{cc}_\pi)_{ij}(\pi_c^2 + \pi_2^2)\right.$$
$$+ (G^{c2}_\pi)_{ik}(\pi_{k,2}\pi_{j,c} + \pi_{k,c}\pi_{j,2})$$
$$\left. + (G^{cc}_\pi)_{ik}(\pi_{k,c}\pi_{j,c} + \pi_{k,2}\pi_{j,2})\right]$$

$$(GF)^{2c}_\alpha = \lambda_{3\alpha}G^{2c}_\alpha\sigma_2 + \lambda_{4\alpha}G^{2c}_\alpha\sigma_c\sigma_2 + \lambda_{2\alpha2\pi}G^{2c}_\alpha(\pi_2\cdot\pi_c)$$
$$\left((GF)^{2c}_{\alpha\pi}\right)_j = \lambda_{1\alpha2\pi}G^{2c}_\alpha\pi_{j,2} + \lambda_{2\alpha2\pi}G^{2c}_\alpha(\sigma_2\pi_{j,c} + \sigma_c\pi_{j,2})$$
$$\left((GF)^{2c}_{\pi\alpha}\right)_i = \lambda_{1\alpha2\pi}(G^{2c}_\pi)_{ik}\pi_{k,2} + \lambda_{2\alpha2\pi}(G^{2c}_\pi)_{ik}(\sigma_2\pi_{k,c} + \sigma_c\pi_{k,2})$$
$$\left((GF)^{2c}_{\pi\pi}\right)_{ij} = \lambda_{1\alpha2\pi}(G^{2c}_\pi)_{ij}\sigma_2 + \lambda_{2\alpha2\pi}(G^{2c}_\pi)_{ij}\sigma_c\sigma_2$$
$$+ \lambda_{4\pi}\left[(G^{2c}_\pi)_{ik}(\pi_{k,2}\pi_{j,c} + \pi_{k,c}\pi_{j,2})\right.$$
$$\left. + (G^{2c}_\pi)_{ij}\pi_2\cdot\pi_c\right]$$

$$(GF)^{22}_\alpha = \lambda_{3\alpha}G^{2c}_\alpha\sigma_c + \frac{1}{2}\lambda_{4\alpha}G^{2c}_\alpha(\sigma_c^2 + \sigma_2^2)$$
$$+ \frac{1}{2}\lambda_{2\alpha2\pi}G^{2c}_\alpha(\pi_c^2 + \pi_2^2)$$

$$\left((GF)^{22}_{\alpha\pi}\right)_j = \lambda_{1\alpha2\pi}G^{2c}_\alpha\pi_{j,c} + \lambda_{2\alpha2\pi}G^{2c}_\alpha(\sigma_c\pi_{j,c} + \sigma_2\pi_{j,2})$$
$$\left((GF)^{22}_{\pi\alpha}\right)_i = \lambda_{1\alpha2\pi}(G^{2c}_\pi)_{ik}\pi_{k,c} + \lambda_{2\alpha2\pi}(G^{2c}_\pi)_{ik}$$
$$\times(\sigma_c\pi_{k,c} + \sigma_2\pi_{k,2})$$

$$((GF)^{22}_{\pi\pi})_{ij} = \lambda_{1\alpha2\pi} (G^{2c}_{\pi})_{ij}\, \sigma_c + \frac{1}{2} \lambda_{2\alpha2\pi} (G^{2c}_{\pi})_{ij} (\sigma_c^2 + \sigma_2^2)$$
$$+ \lambda_{4\pi} \Big[(G^{2c}_{\pi})_{ik} (\pi_{k,c}\, \pi_{j,c} + \pi_{k,2}\, \pi_{j,2})$$
$$+ \frac{1}{2} (G^{2c}_{\pi})_{ij} (\pi_c^2 + \pi_2^2) \Big]$$

$\mathrm{Str}(GF)$

$$= (GF)^{cc}_{\sigma} + ((GF)^{cc}_{\pi\pi})_{ii} + (GF)^{22}_{\sigma} + ((GF)^{22}_{\pi\pi})_{ii}$$

$$= \lambda_{3\sigma} (G^{c2}_{\sigma} \sigma_c + G^{2c}_{\sigma} \sigma_c + G^{cc}_{\sigma} \sigma_2)$$
$$+ \lambda_{4\sigma} \Big[\big(G^{c2}_{\sigma} (\sigma_c^2 + \sigma_2^2) + G^{2c}_{\sigma} (\sigma_c^2 + \sigma_2^2) \big) \frac{1}{2} + G^{cc}_{\sigma} \sigma_2 \sigma_c$$
$$+ \lambda_{2\alpha2\pi} \Big[\big(G^{c2}_{\sigma} (\pi_c^2 + \pi_2^2) + G^{2c}_{\sigma} (\pi_c^2 + \pi_2^2) \big) \frac{1}{2} + G^{cc}_{\sigma}\, \pi_2 \cdot \pi_c$$
$$+ \lambda_{1\alpha2\pi} \Big[(G^{cc}_{\pi})_{ii} \sigma_2 + \big((G^{c2}_{\pi})_{ii} + (G^{2c}_{\pi})_{ii} \big) \sigma_c \Big]$$
$$+ \lambda_{2\alpha2\pi} \Big[(G^{cc}_{\pi})_{ii} \sigma_2 \sigma_c + \frac{1}{2} \big((G^{c2}_{\pi})_{ii} + (G^{2c}_{\pi})_{ii} \big) (\sigma_c^2 + \sigma_2^2) \Big]$$
$$+ \lambda_{4\pi} \Big[(G^{cc}_{\pi})_{ii}\, \pi_2 \cdot \pi_c + \frac{1}{2} \big((G^{c2}_{\pi})_{ii} + (G^{2c}_{\pi})_{ii} \big) (\pi_c^2 + \pi_2^2)$$
$$+ (G^{cc}_{\pi})_{ik} (\pi_{k,2}\, \pi_{i,c} + \pi_{k,c}\, \pi_{i,2})$$
$$+ \big((G^{c2}_{\pi})_{ik} + (G^{2c}_{\pi})_{ik} \big) (\pi_{k,c}\, \pi_{i,c} + \pi_{k,2}\, \pi_{i,2}) \Big]$$

$\mathrm{STr}\, \tilde{\partial}_t \ln P$

$$= \mathrm{STr} (\partial_t P) G = \mathrm{STr} \left[\begin{pmatrix} 0 & \tilde{\partial}_t R^A_k \\ \tilde{\partial}_t R^R_k & 0 \end{pmatrix} \begin{pmatrix} G^K_k & G^R_k \\ G^A_k & 0 \end{pmatrix} \right]$$

$$= \mathrm{STr} \begin{pmatrix} \tilde{\partial}_t R^R_k G^A_k & 0 \\ \tilde{\partial}_t R^A_k G^K_k & \tilde{\partial}_t R^R_k G^R_k \end{pmatrix} = \mathrm{STr} (\tilde{\partial}_t R^R_k G^A_k) + \mathrm{STr} (\tilde{\partial}_t R^A_k G^R_k)$$

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$$\mathrm{Str}\left((\tilde{\partial}_t R_k^{A}) G_k^R \right)$$

$$= \mathrm{Str} \begin{pmatrix} -\vec{q}^{\,2}\,\partial_t \left(Z_{\phi,k}\, r_B\left(\frac{\vec{q}^{\,2}}{k^2}\right)\right) & 0 \\ 0 & \delta_{ik}(-\vec{q}^{\,2})\,\partial_t\left(Z_{\phi,k}\, r_B\left(\frac{\vec{q}^{\,2}}{k^2}\right)\right) \end{pmatrix} \begin{pmatrix} G_\sigma^R(q) & 0 \\ 0 & \delta_{kj}\, G_\pi^R(q) \end{pmatrix}$$

其中

$$G_\sigma^R(q) = \frac{1}{Z_{\phi,k}\left(q_0^2 - \vec{q}^{\,2}(1+r_B)\right) - m_\sigma^2 + \mathrm{sgn}(q_0)\, i\varepsilon}$$

$$G_\pi^R(q) = \frac{1}{Z_{\phi,k}\left(q_0^2 - \vec{q}^{\,2}(1+r_B)\right) - m_\pi^2 + \mathrm{sgn}(q_0)\, i\varepsilon}$$

$$\mathrm{Str}\left((\tilde{\partial}_t R_k^{A}) G_k^R \right)$$

$$= \int \frac{d^4q}{(2\pi)^4} (-\vec{q}^{\,2})\, \partial_t \left(Z_{\phi,k}\, r_B\left(\frac{\vec{q}^{\,2}}{k^2}\right)\right) G_\sigma^R(q)$$

$$+ (N-1) \int \frac{d^4q}{(2\pi)^4} (-\vec{q}^{\,2})\, \partial_t \left(Z_{\phi,k}\, r_B\left(\frac{\vec{q}^{\,2}}{k^2}\right)\right) G_\pi^R(q)$$

同理

$$\mathrm{Str}\left((\tilde{\partial}_t R_k^{R}) G_k^A \right)$$

$$= \int \frac{d^4q}{(2\pi)^4} (-\vec{q}^{\,2})\, \partial_t \left(Z_{\phi,k}\, r_B\left(\frac{\vec{q}^{\,2}}{k^2}\right)\right) G_\sigma^A(q)$$

$$+ (N-1) \int \frac{d^4q}{(2\pi)^4} (-\vec{q}^{\,2})\, \partial_t \left(Z_{\phi,k}\, r_B\left(\frac{\vec{q}^{\,2}}{k^2}\right)\right) G_\pi^A(q)$$

$$\int \frac{d^4q}{(2\pi)^4} (-\vec{q}^{\,2})\, \partial_t \left(Z_{\phi,k}\, r_B\left(\frac{\vec{q}^{\,2}}{k^2}\right)\right) G_\sigma^R(q)$$

$$= \int \frac{d^3q}{(2\pi)^3} \frac{dq_0}{2\pi} (-\vec{q}^{\,2})\, \partial_t \left(Z_{\phi,k}\, r_B\left(\frac{\vec{q}^{\,2}}{k^2}\right)\right) \frac{1}{Z_{\phi,k}\left(q_0^2 - \vec{q}^{\,2}(1+r_B)\right) - m_\sigma^2 + \mathrm{sgn}(q_0)\, i\varepsilon}$$

$$= 0$$

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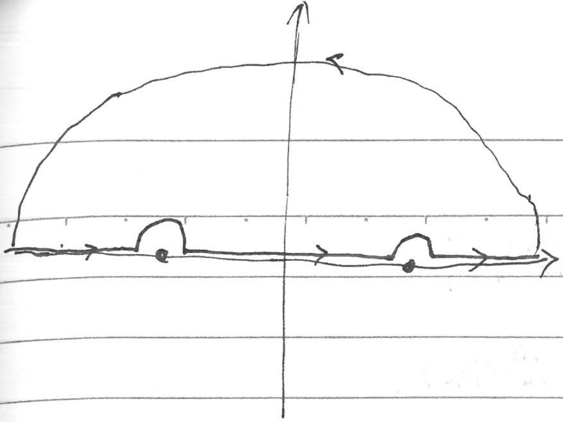

因此, $\mathrm{Str}\left((\hat{\partial}_t R_k^{A})\, G_k^R\right) = 0$

$\mathrm{Str}\left((\hat{\partial}_t R_k^{R})\, G_k^A\right) = 0$

$\mathrm{STr}\, \hat{\partial}_t \ln P = 0$

同样道理，我们得到

$\mathrm{STr}(GF)$

$= \lambda_{3\alpha} G_\alpha^{cc} \sigma_2 + \lambda_{4\alpha} G_\alpha^{cc} \sigma_2 \sigma_c + \lambda_{2\alpha 2\pi} G_\alpha^{cc} \pi_2 \cdot \pi_c$

$+ \lambda_{1\alpha 2\pi} (G_\pi^{cc})_{ii} \sigma_2 + \lambda_{2\alpha 2\pi} (G_\pi^{cc})_{ii} \sigma_2 \sigma_c$

$+ \lambda_{4\pi} \left[(G_\pi^{cc})_{ii} \pi_2 \cdot \pi_c + (G_\pi^{cc})_{ik} (\pi_{k,2} \pi_{i,c} + \pi_{k,c} \pi_{i,2}) \right]$

$$U_k(\phi_c, \phi_2) = \left.\frac{\partial U_k}{\partial \phi_{i,2}}\right|_{\bar{\phi}} \phi_{i,2} + \cdots$$

$$= V_k'(\rho_{0,c}) \bar{\phi}_{i,c} \phi_{i,2} + \cdots$$

$$= V_k'(\rho_{0,c}) \bar{\phi}_{0,c} \sigma_2 + \cdots$$

$$= 2 \rho_{0,c}^{\frac{1}{2}} V_k'(\rho_{0,c}) \sigma_2 + \cdots$$

由 $\partial_z \Gamma_k = \frac{1}{2} \mathrm{STr}\, \hat{\partial}_z (GF) + \cdots$

方程两边 $\frac{\delta}{\delta \sigma_2}$，并且 $\Phi \to 0$，式等价于

$\Rightarrow$

$$-\partial_z \left(2 \rho_{0,c}^{\frac{1}{2}} V_k'(\rho_{0,c}) \right) \sigma_2 + \cdots$$

$$= \frac{1}{2} \hat{\partial}_z \left[\left(\lambda_{3\alpha} G_\alpha^{cc} + \lambda_{1\alpha 2\pi} (G_\pi^{cc})_{ii} \right) \sigma_2 \right] + \cdots$$

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$$\Rightarrow \partial_t \left(2\rho_{0,c}^{\frac{1}{2}} V_k'(\rho_{0,c}) \right)$$

$$= -\frac{1}{2} \hat{\partial}_t \left[\lambda_{3\sigma} G_\sigma^{cc} + \lambda_{1\sigma 2\pi} (G_\pi^{cc})_{ii} \right]$$

$$\lambda_{3\sigma} = -\left(3\rho_{0,c}^{\frac{1}{2}} V_k^{(2)}(\rho_{0,c}) + 2\rho_{0,c}^{\frac{3}{2}} V_k^{(3)}(\rho_{0,c}) \right)$$

$$\lambda_{1\sigma 2\pi} = -\rho_{0,c}^{\frac{1}{2}} V_k^{(2)}(\rho_{0,c})$$

$$m_\sigma^2 = V_k'(\rho_{0,c}) + 2\rho_{0,c} V_k^{(2)}(\rho_{0,c})$$

$$m_\pi^2 = V_k'(\rho_{0,c})$$

$$\frac{dm_\sigma^2}{d\rho_{0,c}} = V_k^{(2)}(\rho_{0,c}) + 2V_k^{(2)}(\rho_{0,c}) + 2\rho_{0,c} V_k^{(3)}(\rho_{0,c})$$

$$= 3V_k^{(2)}(\rho_{0,c}) + 2\rho_{0,c} V_k^{(3)}(\rho_{0,c})$$

$$\frac{dm_\pi^2}{d\rho_{0,c}} = V_k^{(2)}(\rho_{0,c})$$

所以

$$\lambda_{3\sigma} = -\rho_{0,c}^{\frac{1}{2}} \frac{\partial m_\sigma^2}{\partial \rho_{0,c}} \qquad \lambda_{1\sigma 2\pi} = -\rho_{0,c}^{\frac{1}{2}} \frac{\partial m_\pi^2}{\partial \rho_{0,c}}$$

我们有

$$\partial_t V_k'(\rho_{0,c}) = \frac{1}{4} \tilde{\partial}_t \left[\frac{\partial m_\sigma^2}{\partial \rho_{0,c}} G_\sigma^{cc} + \frac{\partial m_\pi^2}{\partial \rho_{0,c}} (G_\pi^{cc})_{ii} \right]$$

$$\tilde{\partial}_t \left(\frac{\partial m_\sigma^2}{\partial \rho_{0,c}} G_\sigma^{cc} \right)$$

$$\frac{\partial m_\sigma^2}{\partial \rho_{0,c}} \tilde{\partial}_t \int \frac{d^4 q}{(2\pi)^4} \frac{-2i\varepsilon\, \mathrm{sgn}(q_0)}{[Z_{\phi,k}(q_0^2 - \vec{q}^2(1+r_B)) - m_\sigma^2]^2 + \varepsilon^2} \coth\left(\frac{\beta q^0}{2}\right)$$

$$= \frac{\partial m_\sigma^2}{\partial \rho_{0c}} \int \frac{d^4 q}{(2\pi)^4} \partial_t(-\vec{q}^2 Z_{\phi,k} r_B)(-1) \frac{2[Z_{\phi,k}(q_0^2 - \vec{q}^2(1+r_B)) - m_\sigma^2]}{([Z_{\phi,k}(q_0^2 - \vec{q}^2(1+r_B)) - m_\sigma^2]^2 + \varepsilon^2)^2}$$

$$\times (-2i\varepsilon\, \mathrm{sgn}(q_0)) \coth\left(\frac{\beta q^0}{2}\right)$$

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$$= \int \frac{d^4 q}{(2\pi)^4} \partial_t \left(-\vec{q}^2 Z_{\phi,k} r_B\right) \left(-\frac{\partial}{\partial \rho_{0,c}}\right) \frac{1}{[Z_{\phi,k}(q_0^2 - \vec{q}^2(1+r_B)) - m_\alpha^2]^2 + \varepsilon^2}$$

$$\times (-2i\varepsilon \, \mathrm{sgn}(q_0)) \coth\left(\frac{\beta q^0}{2}\right)$$

$$= \left(-\frac{\partial}{\partial \rho_{0,c}}\right) \int \frac{d^4 q}{(2\pi)^4} \partial_t \left(-\vec{q}^2 Z_{\phi,k} r_B\right) G_\alpha^{cc}$$

$$\boxed{\lim_{\varepsilon \to 0} \frac{1}{\pi} \frac{\varepsilon}{x^2+\varepsilon^2} = \delta}$$

同样

$$\tilde{\partial}_t \left(\frac{\partial m_\pi^2}{\partial \rho_{0,c}} (G_\pi^{cc})_{ii}\right)$$

$$= \left(-\frac{\partial}{\partial \rho_{0,c}}\right) \int \frac{d^4 q}{(2\pi)^4} \partial_t \left(-\vec{q}^2 Z_{\phi,k} r_B\right) (G_\pi^{cc})_{ii}$$

$\Rightarrow$

$$\frac{\partial}{\partial \rho_{0,c}} \left(\partial_t V_k(\rho_{0,c})\right) = -\frac{\partial}{\partial \rho_{0,c}} \left\{ \frac{i}{4} \int \frac{d^4 q}{(2\pi)^4} \left[\partial_t \left(-\vec{q}^2 Z_{\phi,k} r_B\right) G_\alpha^{cc} + \partial_t \left(-\vec{q}^2 Z_{\phi,k} r_B\right) (G_\pi^{cc})_{ii} \right] \right\}$$

$$\Rightarrow \partial_t V_k(\rho_{0,c}) = -\frac{i}{4} \int \frac{d^4 q}{(2\pi)^4} \left[\partial_t \left(-\vec{q}^2 Z_{\phi,k} r_B\right) G_\alpha^{cc} + \partial_t \left(-\vec{q}^2 Z_{\phi,k} r_B\right) (G_\pi^{cc})_{ii} \right]$$

下面考虑最简单的情况 $Z_{\phi,k}$ 不依赖外动量

$$-\frac{i}{4} \int \frac{d^4 q}{(2\pi)^4} \partial_t \left(-\vec{q}^2 Z_{\phi,k} r_B\right) G_\alpha^{cc}$$

$$= -\frac{i}{4} \int \frac{d^4 q}{(2\pi)^4} \partial_t \left(-\vec{q}^2 Z_{\phi,k} r_B\right) \frac{-2i\varepsilon \, \mathrm{sgn}(q_0)}{[Z_{\phi,k}(q_0^2 - \vec{q}^2(1+r_B)) - m_\alpha^2]^2 + \varepsilon^2} \coth\left(\frac{\beta q^0}{2}\right)$$

$$= \frac{i}{4} \int \frac{d^4 q}{(2\pi)^4} \vec{q}^2 Z_{\phi,k} \left[\frac{k^2}{\vec{q}^2}(2-\eta_{\phi,k}) + \eta_{\phi,k}\right] \theta\left(1 - \frac{\vec{q}^2}{k^2}\right)$$

$$\times \frac{1}{Z_{\phi,k}} \frac{-2i\varepsilon \, \mathrm{sgn}(q_0)}{[q_0^2 - k^2 - \bar{m}_\alpha^2]^2 + \varepsilon^2} \coth\left(\frac{\beta q^0}{2}\right)$$

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$$E_{\sigma,k}^2 = k^2 + \bar{m}_\sigma^2$$

$$= \frac{i}{4}\int \frac{d^4q}{(2\pi)^4} Z_{\phi,k}\left[k^2(2-\eta_{\phi,k}) + \eta_{\phi,k}\vec{q}^{\,2}\right]\theta\left(1-\frac{\vec{q}^{\,2}}{k^2}\right)$$
$$\frac{1}{Z_{\phi,k}}(-i)\,2\pi\,\delta(q_0^2 - \cancel{} E_{\sigma,k}^2)\,\mathrm{sgn}(q_0)\coth\left(\frac{\beta q^0}{2}\right)$$

$$= \frac{i}{4}\int \frac{d^3q}{(2\pi)^3} Z_{\phi,k}\left[k^2(2-\eta_{\phi,k}) + \eta_{\phi,k}\vec{q}^{\,2}\right]\theta\left(1-\frac{\vec{q}^{\,2}}{k^2}\right)$$
$$\int \frac{dq_0}{(2\pi)}\frac{1}{Z_{\phi,k}}(-i)\,2\pi\,\frac{1}{2E_{\sigma,k}}\left(\delta(q_0 - E_{\sigma,k}) + \delta(q_0 + E_{\sigma,k})\right)\mathrm{sgn}(q_0)\coth\left(\frac{\beta q^0}{2}\right)$$

$$= \frac{i}{4}\int \frac{d^3q}{(2\pi)^3} Z_{\phi,k}\left[k^2(2-\eta_{\phi,k}) + \eta_{\phi,k}\vec{q}^{\,2}\right]\theta\left(1-\frac{\vec{q}^{\,2}}{k^2}\right)$$
$$\frac{1}{Z_{\phi,k}}(-i)\frac{1}{2E_{\sigma,k}}\,2\coth\left(\frac{\beta E_{\sigma,k}}{2}\right)$$

$$= \frac{1}{4E_{\sigma,k}}\coth\left(\frac{\beta E_{\sigma,k}}{2}\right)\int\frac{d^3q}{(2\pi)^3}\left[k^2(2-\eta_{\phi,k}) + \eta_{\phi,k}\vec{q}^{\,2}\right]\theta\left(1-\frac{\vec{q}^{\,2}}{k^2}\right)$$

$$= \frac{1}{4E_{\sigma,k}}\coth\left(\frac{\beta E_{\sigma,k}}{2}\right)\frac{k^5}{2\pi^2}\left(\frac{1}{3}(2-\eta_{\phi,k}) + \frac{1}{5}\eta_{\phi,k}\right)$$

$$= \frac{1}{4\pi^2}\frac{k^5}{E_{\sigma,k}}\frac{\coth\left(\frac{\beta E_{\sigma,k}}{2}\right)}{2}\,\frac{2}{3}\left(1-\frac{1}{5}\eta_{\phi,k}\right)$$

$$= \frac{1}{4\pi^2}\frac{k^5}{E_{\sigma,k}}\left(\frac{1}{2} + n_b(E_{\sigma,k})\right)\frac{2}{3}\left(1-\frac{\eta_{\phi,k}}{5}\right)$$

同样

$$-\frac{i}{4}\int\frac{d^4q}{(2\pi)^4}\partial_t\left(-\vec{q}^{\,2} Z_{\phi,k} r_B\right)\left(G_\pi^{cc}\right)_{ii}$$

$$= (N-1)\frac{1}{4\pi^2}\frac{k^5}{E_{\pi,k}}\frac{\coth\left(\frac{\beta E_{\pi,k}}{2}\right)}{2}\,\frac{2}{3}\left(1-\frac{\eta_{\phi,k}}{5}\right)$$

其中 $E_{\pi,k} = \sqrt{k^2 + \bar{m}_\pi^2}$

所以有

$$\partial_c V_k = \frac{1}{4\pi^2}\frac{k^5}{E_{\sigma,k}}\frac{\coth\left(\frac{\beta E_{\sigma,k}}{2}\right)}{2}\frac{2}{3}\left(1-\frac{\eta_{\sigma,k}}{5}\right)$$

$$+(N-1)\frac{1}{4\pi^2}\frac{k^5}{E_{\pi,k}}\frac{\coth\left(\frac{\beta E_{\pi,k}}{2}\right)}{2}\frac{2}{3}\left(1-\frac{\eta_{\pi,k}}{5}\right)$$

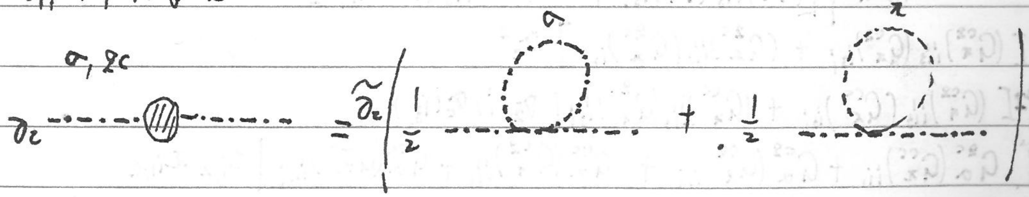
对两点关联函数作如下投影

$$\partial_c\left(\frac{\delta^2\Gamma_k}{\delta\sigma_a\delta\sigma_c}\right) = \frac{1}{2}\tilde{\partial}_c\,\mathrm{STr}\,\frac{\delta^2(GF)}{\delta\sigma_a\delta\sigma_c} + \cdots$$

$$\frac{\delta^2\Gamma_k}{\delta\sigma_a\delta\sigma_c} = \left(\Gamma_k^{(2)\sigma\sigma}\right)^{ac}$$

$$\frac{\delta^2\,\mathrm{STr}(GF)}{\delta\sigma_a\delta\sigma_c} = \lambda_{4\sigma}G_\sigma^{cc} + \lambda_{2\sigma2\pi}\left(G_\pi^{cc}\right)_{ii}$$

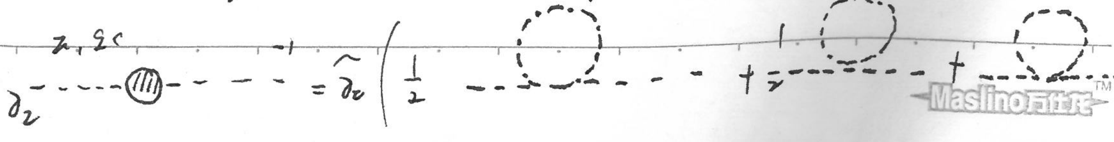
用图表示为



$$\frac{\delta^2\,\mathrm{STr}(GF)}{\delta\pi_{i,a}\delta\pi_{j,c}} = \lambda_{2\sigma2\pi}G_\sigma^{cc}\delta_{ij} + \lambda_{4\pi}\left(G_\pi^{cc}\right)_{i'i'}\delta_{ij}$$

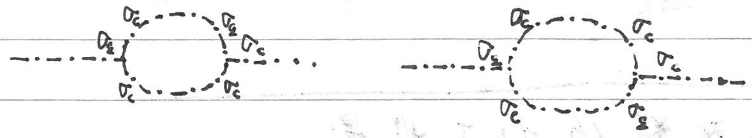
$$+\,2\lambda_{4\pi}\left(G_\pi^{cc}\right)_{ij}$$

$$\partial_c\left(\left(\Gamma_k^{(2)\pi\pi}\right)^{ac}_{ij}\right) = \frac{1}{2}\tilde{\partial}_c\frac{\delta^2\,\mathrm{STr}(GF)}{\delta\pi_{i,a}\delta\pi_{j,c}} + \cdots$$

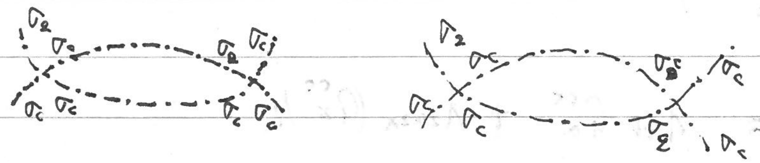


$$
\mathrm{Str}(GF)^2 \sim \lambda_{3\alpha}^2 \Big[(G_\alpha^{c2})^2 \sigma_c^2 + (G_\alpha^{2c})^2 \sigma_c^2 + 2\, G_\alpha^{cc} G_\alpha^{c2} \sigma_c \sigma_2 + 2\, G_\alpha^{cc} G_\alpha^{2c} \sigma_c \sigma_2 + (G_\alpha^{cc})^2 \sigma_2^2 + 2\, G_\alpha^{c2} G_\alpha^{2c} \sigma_2^2 \Big]
$$

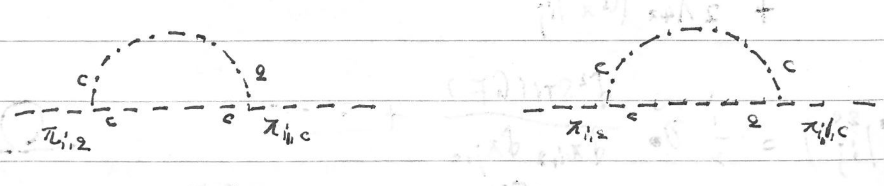
$$
= \lambda_{3\alpha}^2 \Big[\big((G_\alpha^{c2})^2 + (G_\alpha^{2c})^2 \big) \sigma_c^2 + 2 \big(G_\alpha^{cc} G_\alpha^{c2} + G_\alpha^{cc} G_\alpha^{2c} \big) \sigma_c \sigma_2 + \big((G_\alpha^{cc})^2 + 2\, G_\alpha^{c2} G_\alpha^{2c} \big) \sigma_2^2 \Big]
$$



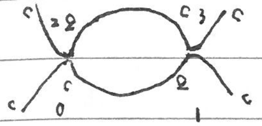
$$
\mathrm{Str}(GF)^2 \sim \lambda_{4\alpha}^2 \Big\{ \frac{1}{4} \big((G_\alpha^{c2})^2 + (G_\alpha^{2c})^2 \big) \sigma_c^4 + \big(G_\alpha^{cc} G_\alpha^{c2} + G_\alpha^{cc} G_\alpha^{2c} \big) \sigma_c^3 \sigma_2
+ \frac{1}{2} \big[2 (G_\alpha^{cc})^2 + (G_\alpha^{c2})^2 + (G_\alpha^{2c})^2 + 4\, G_\alpha^{c2} G_\alpha^{2c} \big] \sigma_c^2 \sigma_2^2
+ \big(G_\alpha^{cc} G_\alpha^{c2} + G_\alpha^{cc} G_\alpha^{2c} \big) \sigma_c \sigma_2^3 + \frac{1}{4} \big((G_\alpha^{c2})^2 + (G_\alpha^{2c})^2 \big) \sigma_2^4 \Big\}
$$

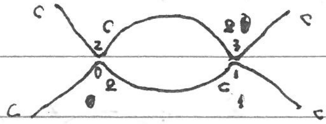


$$
\mathrm{Str}(GF)^2 \sim \lambda_{1\sigma 2\pi}^2 \Big\{ \big[(G_\pi^{cc})_{ii_3} (G_\pi^{cc})_{i_3 i} + 2 (G_\pi^{c2})_{ii_3} (G_\pi^{2c})_{i_3 i} \big] \sigma_2^2
+ \big[(G_\pi^{c2})_{ii_3} (G_\pi^{c2})_{i_3 i} + (G_\pi^{2c})_{ii_3} (G_\pi^{2c})_{i_3 i} \big] \sigma_c^2
+ 2 \big[(G_\pi^{c2})_{ii_3} (G_\pi^{cc})_{i_3 i} + (G_\pi^{cc})_{ii_3} (G_\pi^{2c})_{i_3 i} \big] \sigma_2(i) \sigma_c(i_3)
+ 2 \big[G_\alpha^{2c} (G_\pi^{cc})_{ii_1} + G_\alpha^{c2} (G_\pi^{cc})_{ii_1} + G_\alpha^{cc} (G_\pi^{c2})_{ii_1} + G_\alpha^{cc} (G_\pi^{2c})_{i_1 i} \big] \pi_{i,2} \pi_{i_1,c}
+ 2 \big[G_\alpha^{cc} (G_\pi^{cc})_{ii_1} + G_\alpha^{2c} (G_\pi^{c2})_{ii_1} + G_\alpha^{c2} (G_\pi^{2c})_{ii_1} \big] \pi_{i,2} \pi_{i_1,2}
+ 2 \big[G_\alpha^{c2} (G_\pi^{c2})_{ii_1} + G_\alpha^{2c} (G_\pi^{2c})_{ii_1} \big] \pi_{i,c} \pi_{i_1,c} \Big\}
$$

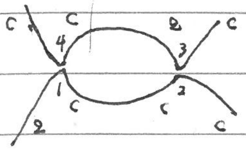


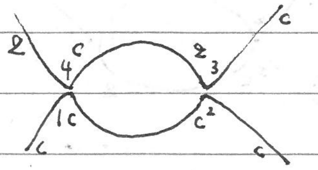
$$\mathrm{Str}(GF)^2 \sim \lambda_{4\pi}^2 \Big\{ \Big[(G_\pi^{c2})_{i_1 i_1} (G_\pi^{c2})_{i_3 i_2} + (G_\pi^{2c})_{i_1 i_1} (G_\pi^{2c})_{i_3 i_2} \Big] \pi_{i_1,c}\, \pi_{i_1,c}\, \pi_{i_2,c}\, \pi_{i_3,c}$$

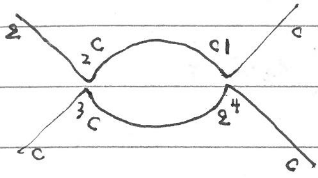


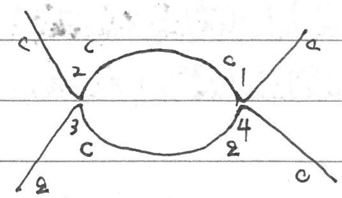


$$\mathrm{Str}(GF)^2 \sim \lambda_{4\pi}^2 \Big\{ 2 (G_\pi^{cc})_{i_1 i_2} (G_\pi^{2c})_{i_3 i_4} \big(\pi_{i_1,c}\, \pi_{i_2,c}\, \pi_{i_3,c}\, \pi_{i_4,2} + \pi_{i_2,c}\, \pi_{i_3,c}\, \pi_{i_4,c}\, \pi_{i_1,2} \big)$$
$$+ 2 (G_\pi^{cc})_{i_1 i_2} (G_\pi^{c2})_{i_3 i_4} \big(\pi_{i_1,c}\, \pi_{i_3,c}\, \pi_{i_4,c}\, \pi_{i_2,2} + \pi_{i_1,c}\, \pi_{i_2,c}\, \pi_{i_4,c}\, \pi_{i_3,2} \big) \Big\}$$

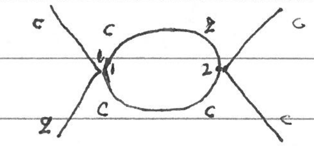


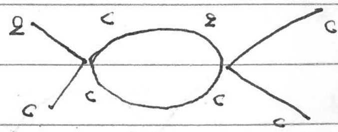


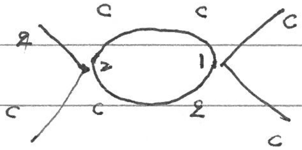


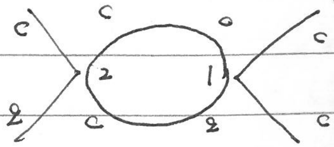


$$\mathrm{Str}(GF)^2 \sim \lambda_{4\pi}^2 \Big\{ \Big[(G_\pi^{cc})_{i_1 i_2} (G_\pi^{c2})_{i_2 i_1} + (G_\pi^{cc})_{i_1 i_2} (G_\pi^{2c})_{i_2 i_1} \Big] \pi_c^2\, \pi_2 \cdot \pi_c \Big\}$$

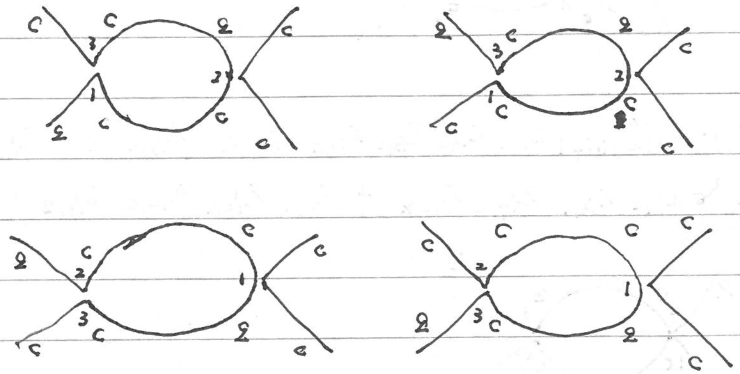




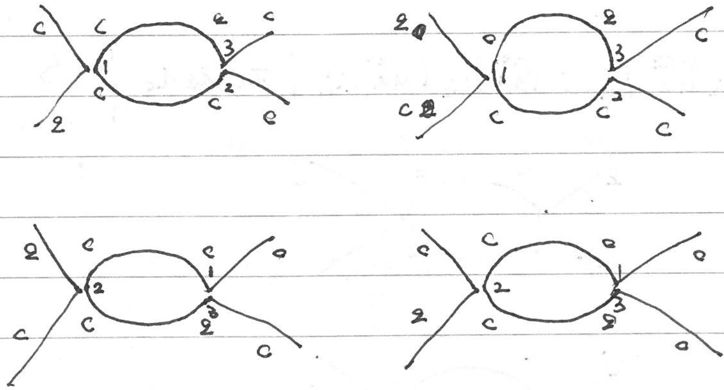




$$\mathrm{STr}(GF)^2 \simeq \frac{\lambda^2}{4\pi}\Big\{ \Big[(G_\pi^{cc})_{i_1 i_2} (G_\pi^{2c})_{i_2 i_3} \left(\pi_{i_3,c}\, \pi_{i_1,2} + \pi_{i_1,c}\, \pi_{i_3,2} \right) + (G_\pi^{cc})_{i_1 i_2} (G_\pi^{c2})_{i_3 i_1} \left(\pi_{i_3,c}\, \pi_{i_2,2} + \pi_{i_2,c}\, \pi_{i_3,2} \right) \Big] \pi_c^2 \Big\}$$



$$\mathrm{STr}(GF)^2 \simeq \frac{\lambda^2}{4\pi}\Big\{ 2\Big[(G_\pi^{cc})_{i_1 i_2} (G_\pi^{c2})_{i_2 i_3}\, \pi_{i_1,c}\, \pi_{i_3,c} + (G_\pi^{cc})_{i_1 i_2} (G_\pi^{2c})_{i_3 i_1}\, \pi_{i_2,c}\, \pi_{i_3,c} \Big] \pi_2 \cdot \pi_c \Big\}$$



由 $\partial_t \Gamma_k[\phi] \sim -\frac{i}{4}\, \mathrm{STr}\, \tilde{\partial}_t (GF)^2$

$$\frac{\delta^4 \Gamma_k}{\delta\pi_{i,a}\,\delta\pi_{j,c}\,\delta\pi_{k,c}\,\delta\pi_{l,c}} = \lambda_{4\pi}\left(\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{jk}\delta_{il}\right)$$

令 $f_1 = (G_\pi^{cc})_{i_1 i_2} (G_\pi^{ac})_{i_3 i_4}\, \pi_{i_1,a}\, \pi_{i_2,c}\, \pi_{i_3,c}\, \pi_{i_4,c}$

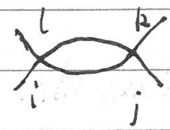
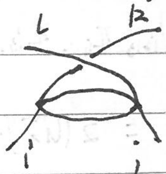
$$\frac{\delta^4 f_1}{\delta\pi_{i,a}\,\delta\pi_{j,c}\,\delta\pi_{k,c}\,\delta\pi_{l,c}} = (G_\pi^{cc})_{i_1 i_2} (G_\pi^{ac})_{i_3 i_4}\, \delta_{i_1 i}\big(\delta_{i_2 j}\delta_{i_3 k}\delta_{i_4 l} + \delta_{i_2 j}\delta_{i_3 l}\delta_{i_4 k} + \delta_{i_2 k}\delta_{i_3 j}\delta_{i_4 l} + \delta_{i_2 k}\delta_{i_3 l}\delta_{i_4 j} + \delta_{i_2 l}\delta_{i_3 k}\delta_{i_4 j} + \delta_{i_2 l}\delta_{i_3 j}\delta_{i_4 k}\big)$$

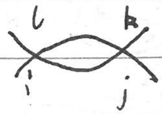
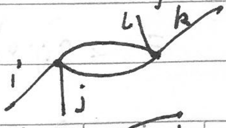
$$= \delta_{ij}\delta_{kl}\left(G_\pi^{cc}(i,j)\, G_\pi^{ac}(k,l) + G_\pi^{cc}(i,j)\, G_\pi^{ac}(l,k)\right)$$
$$+ \delta_{ik}\delta_{jl}\left(G_\pi^{cc}(i,k)\, G_\pi^{ac}(j,l) + G_\pi^{cc}(i,k)\, G_\pi^{ac}(l,j)\right)$$
$$+ \delta_{il}\delta_{kj}\left(G_\pi^{cc}(i,l)\, G_\pi^{ac}(k,j) + G_\pi^{cc}(i,l)\, G_\pi^{ac}(j,k)\right)$$

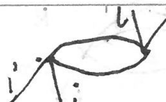
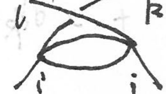
$f_2 = (G_\pi^{cc})_{i_1 i_2} (G_\pi^{ac})_{i_3 i_4}\, \pi_{i_1,c}\, \pi_{i_2,c}\, \pi_{i_3,c}\, \pi_{i_4,a}$

$$\frac{\delta^4 f_2}{\delta\pi_{i,a}\,\delta\pi_{j,c}\,\delta\pi_{k,c}\,\delta\pi_{l,c}} = (G_\pi^{cc})_{i_1 i_2} (G_\pi^{ac})_{i_3 i_4}\, \delta_{i_4 i}\big(\delta_{i_3 j}\delta_{i_1 k}\delta_{i_2 l} + \delta_{i_3 j}\delta_{i_1 l}\delta_{i_2 k} + \delta_{i_3 k}\delta_{i_1 j}\delta_{i_2 l} + \delta_{i_3 k}\delta_{i_1 l}\delta_{i_2 j} + \delta_{i_3 l}\delta_{i_1 k}\delta_{i_2 j} + \delta_{i_3 l}\delta_{i_1 j}\delta_{i_2 k}\big)$$

$$= \delta_{ij}\delta_{kl}\left(G_\pi^{cc}(k,l)\, G_\pi^{ac}(j,i) + G_\pi^{cc}(l,k)\, G_\pi^{ac}(j,i)\right)$$
$$+ \delta_{ik}\delta_{jl}\left(G_\pi^{cc}(j,l)\, G_\pi^{ac}(k,i) + G_\pi^{cc}(l,j)\, G_\pi^{ac}(k,i)\right)$$
$$+ \delta_{il}\delta_{kj}\left(G_\pi^{cc}(k,j)\, G_\pi^{ac}(l,i) + G_\pi^{cc}(j,k)\, G_\pi^{ac}(l,i)\right)$$

即 $\frac{\delta^4 f_1}{\delta\pi_{i,a}\,\delta\pi_{j,c}\,\delta\pi_{k,c}\,\delta\pi_{l,c}} = \delta_{ij}\delta_{kl}\Big($  $+$  $\Big)$

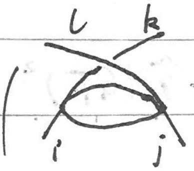
$+\, \delta_{ik}\delta_{jl}\Big($  $+$  $\Big)$

$+\, \delta_{il}\delta_{kj}\Big($  $+$  $\Big)$

NO.

Date

$$\frac{\partial^4 f_2}{\partial\pi_{i,2}\,\partial\pi_{j,c}\,\partial\pi_{k,c}\,\partial\pi_{l,c}} = \delta_{ij}\delta_{kl}\Big($$



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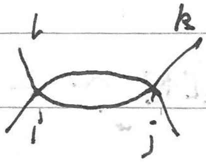
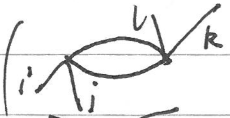


$\Big)$

$+\ \delta_{ik}\delta_{jl}\Big($



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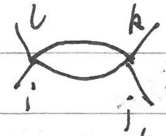

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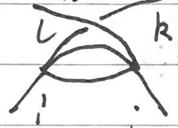
$+\ \delta_{il}\delta_{kj}\Big($



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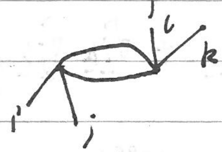


$\Big)$

$$f_3 = (G_\pi^{cc})_{i_1 i_2}(G_\pi^{2c})_{i_2 i_3}\,\pi_{i_3,c}\,\pi_{i_1,2}\,\pi_c^2$$

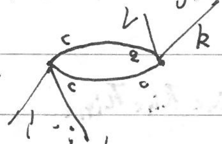
$$\frac{\partial^4 f_3}{\partial\pi_{i,2}\,\partial\pi_{j,c}\,\partial\pi_{k,c}\,\partial\pi_{l,c}} = 2(G_\pi^{cc})_{i_1 i_2}(G_\pi^{2c})_{i_2 i_3}\,\delta_{i_1 i}\left(\delta_{i_3 j}\delta_{kl} + \delta_{i_3 k}\delta_{jl} + \delta_{i_3 l}\delta_{jk}\right)$$

$$= 2\delta_{ij}\delta_{kl}\, G_\pi^{cc}(i,k)\, G_\pi^{2c}(l,j)$$

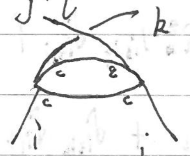
$$+\, 2\delta_{ik}\delta_{jl}\, G_\pi^{cc}(i,j)\, G_\pi^{2c}(l,k)$$

$$+\, 2\delta_{il}\delta_{jk}\, G_\pi^{cc}(i,j)\, G_\pi^{2c}(k,l)$$

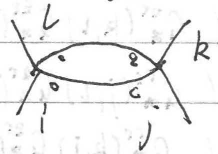
$= 2\delta_{ij}\delta_{kl}$



$+\, 2\delta_{ik}\delta_{jl}$



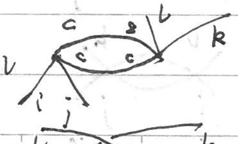
$+\, 2\delta_{il}\delta_{jk}$



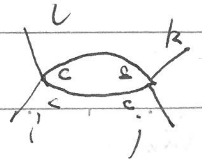
$$f_4 = (G_\pi^{cc})_{i_1 i_2}(G_\pi^{2c})_{i_2 i_3}\,\pi_{i_1,c}\,\pi_{i_3,2}\,\pi_c^2$$

$$\frac{\partial^4 f_4}{\partial\pi_{i,2}\,\partial\pi_{j,c}\,\partial\pi_{k,c}\,\partial\pi_{l,c}} = 2(G_\pi^{cc})_{i_1 i_2}(G_\pi^{2c})_{i_2 i_3}\,\delta_{i_3 i}\left(\delta_{i_1 j}\delta_{kl} + \delta_{i_1 k}\delta_{jl} + \delta_{i_1 l}\delta_{jk}\right)$$

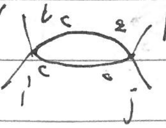
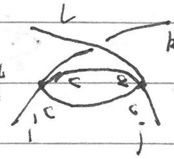
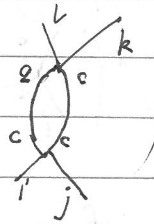
$= 2\delta_{ij}\delta_{kl}$



$+\, 2\delta_{ik}\delta_{jl}$

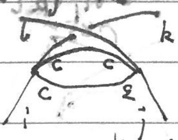
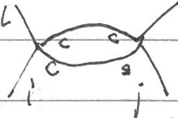
$+\,2\,\delta_{il}\delta_{jk}$ 

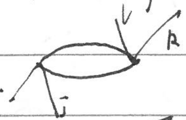
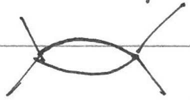
$$\frac{\delta^4(2f_1+2f_2+f_3+f_4)}{\delta\pi_{i,s}\,\delta\pi_{j,c}\,\delta\pi_{k,c}\,\delta\pi_{l,c}}$$

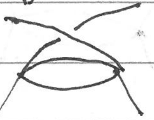
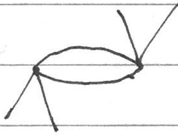
$= 4(\delta_{ij}\delta_{kl}+\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk})\Big($  $+$  $+$ 

$$f_1' = (G_\pi^{cc})_{i_1 i_2}(G_\pi^{cs})_{i_3 i_4}\,\pi_{i_1,c}\,\pi_{i_3,c}\,\pi_{i_4,c}\,\pi_{i_2,s}$$

$$\frac{\delta^4 f_1'}{\delta\pi_{i,s}\,\delta\pi_{j,c}\,\delta\pi_{k,c}\,\delta\pi_{l,c}} = (G_\pi^{cc})_{i_1 i_2}(G_\pi^{cs})_{i_3 i_4}\,\delta_{i_2 i}\big(\delta_{i_1 j}\delta_{i_3 k}\delta_{i_4 l}+\delta_{i_1 j}\delta_{i_3 l}\delta_{i_4 k}$$
$$+\,\delta_{i_1 k}\delta_{i_3 j}\delta_{i_4 l}+\delta_{i_1 k}\delta_{i_3 l}\delta_{i_4 j}+\delta_{i_1 l}\delta_{i_3 k}\delta_{i_4 j}+\delta_{i_1 l}\delta_{i_3 j}\delta_{i_4 k}\big)$$

$= \delta_{ij}\delta_{kl}\Big($  $+$  $\Big)$

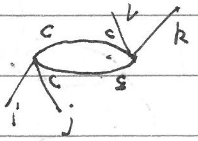
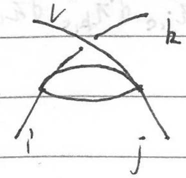
$+\,\delta_{ik}\delta_{jl}\Big($  $+$  $\Big)$

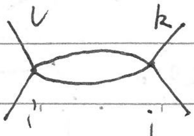
$+\,\delta_{il}\delta_{kj}\Big($  $+$  $\Big)$

$$f_2' = (G_\pi^{cc})_{i_1 i_2}(G_\pi^{cs})_{i_3 i_4}\,\pi_{i_1,c}\,\pi_{i_2,c}\,\pi_{i_4,c}\,\pi_{i_3,s}$$

$$f_3' = (G_\pi^{cc})_{i_1 i_2}(G_\pi^{cs})_{i_3 i_1}\,\pi_{i_3,c}\,\pi_{i_2,s}\,\pi_c^2$$

$$\frac{\delta^4 f_3'}{\delta\pi_{i,s}\,\delta\pi_{j,c}\,\delta\pi_{k,c}\,\delta\pi_{l,c}} = 2(G_\pi^{cc})_{i_1 i_2}(G_\pi^{cs})_{i_3 i_1}\,\delta_{i_2 i}(\delta_{i_3 j}\delta_{kl}+\delta_{i_3 k}\delta_{jl}+\delta_{i_3 l}\delta_{jk})$$

$= 2\,\delta_{ij}\delta_{kl}$  $+\,2\,\delta_{ik}\delta_{jl}$ 

$+\,2\,\delta_{il}\delta_{jk}$ 

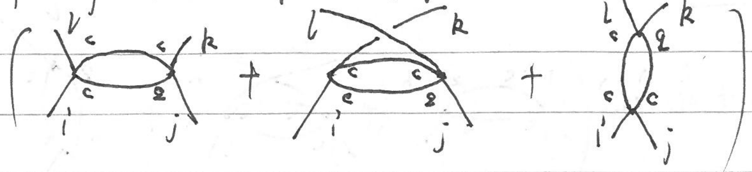
$$f_4' = (G_\pi^{cc})_{i_1 i_2}(G_\pi^{cs})_{i_3 i_1}\,\pi_{i_2,c}\,\pi_{i_3,s}\,\pi_c^2$$

所以有

$$\delta^4(2f_1'+2f_2'+f_3'+f_4')/(\delta\pi_{i,2}\,\delta\pi_{j,c}\,\delta\pi_{k,c}\,\delta\pi_{l,c})$$

$$=4(\delta_{ij}\delta_{kl}+\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk})$$

$\times$

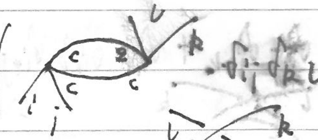


$$f_5=(G_\pi^{cc})_{i_1i_2}(G_\pi^{2c})_{i_2i_1}\,\pi_c^2\,\pi_2\cdot\pi_c$$

$$=(N-1)\,G_\pi^{cc}G_\pi^{2c}\,\pi_c^2\,\pi_2\cdot\pi_c$$

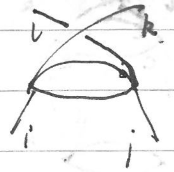
$$\frac{\delta^4 f_5}{\delta\pi_{i,2}\,\delta\pi_{j,c}\,\delta\pi_{k,c}\,\delta\pi_{l,c}}=(N-1)\,G_\pi^{cc}G_\pi^{2c}\,2(\delta_{ij}\delta_{kl}+\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk})$$

$=(N-1)2\Big($

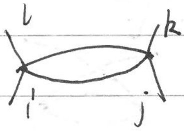


$\delta_{ij}\delta_{kl}$

$+\ \delta_{ik}\delta_{jl}$



$+\ \delta_{il}\delta_{jk}$



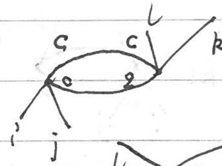
$\Big)$

$$f_5'=(G_\pi^{cc})_{i_1i_2}(G_\pi^{c2})_{i_2i_1}\,\pi_c^2\,\pi_2\cdot\pi_c$$

$$=(N-1)\,G_\pi^{c2}G_\pi^{cc}\,\pi_c^2\,\pi_2\cdot\pi_c$$

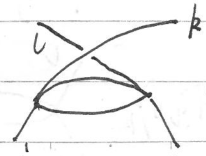
$$\frac{\delta^4 f_5'}{\delta\pi_{i,2}\,\delta\pi_{j,c}\,\delta\pi_{k,c}\,\delta\pi_{l,c}}=(N-1)\,G_\pi^{c2}G_\pi^{cc}\,2(\delta_{ij}\delta_{kl}+\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk})$$

$=(N-1)2\Big($

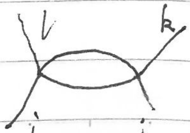


$\delta_{ij}\delta_{kl}$

$+\ \delta_{ik}\delta_{jl}$



$+\ \delta_{il}\delta_{jk}$

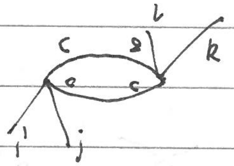


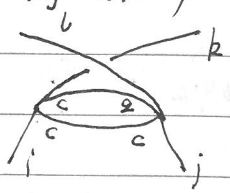
$\Big)$

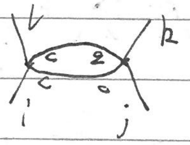
$$f_6 = (G_\pi^{cc})_{i_1 i_2} (G_\pi^{2c})_{i_3 i_1} \pi_{i_2,c} \pi_{i_3,c} \pi_2 \cdot \pi_c$$

$$\frac{\delta^4 f_6}{\delta\pi_{i,2}\, \delta\pi_{j,c}\, \delta\pi_{k,c}\, \delta\pi_{l,c}}$$

$$= (G_\pi^{cc})_{i_1 i_2} (G_\pi^{2c})_{i_3 i_1} \big(\delta_{ij}\delta_{i_2 k}\delta_{i_3 l} + \delta_{ij}\delta_{i_2 l}\delta_{i_3 k}$$
$$+ \delta_{ik}\delta_{i_2 j}\delta_{i_3 l} + \delta_{ik}\delta_{i_2 l}\delta_{i_3 j}$$
$$+ \delta_{il}\delta_{i_2 k}\delta_{i_3 j} + \delta_{il}\delta_{i_2 j}\delta_{i_3 k} \big)$$

$= 2\delta_{ij}\delta_{kl}$ 

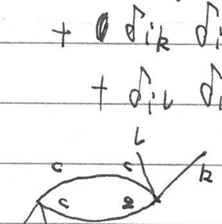
$+ 2\delta_{ik}\delta_{jl}$ 

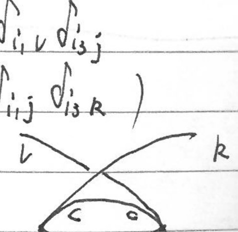
$+ 2\delta_{il}\delta_{kj}$ 

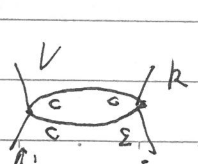
$$f_6' = (G_\pi^{cc})_{i_1 i_2} (G_\pi^{c2})_{i_2 i_3} \pi_{i_1,c} \pi_{i_3,c} \pi_2 \cdot \pi_c$$

$$\frac{\delta^4 f_6'}{\delta\pi_{i,2}\, \delta\pi_{j,c}\, \delta\pi_{k,c}\, \delta\pi_{l,c}}$$

$$= (G_\pi^{c2})_{i_2 i_3} (G_\pi^{cc})_{i_1 i_2} \big(\delta_{ij}\delta_{i_1 k}\delta_{i_3 l} + \delta_{ij}\delta_{i_1 l}\delta_{i_3 k}$$
$$+ \delta_{ik}\delta_{i_1 j}\delta_{i_3 l} + \delta_{ik}\delta_{i_1 l}\delta_{i_3 j}$$
$$+ \delta_{il}\delta_{i_1 k}\delta_{i_3 j} + \delta_{il}\delta_{i_1 j}\delta_{i_3 k} \big)$$

$= 2\delta_{ij}\delta_{kl}$ 

$+ 2\delta_{ik}\delta_{jl}$ 

$+ 2\delta_{il}\delta_{kj}$ 

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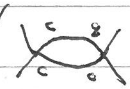
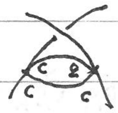
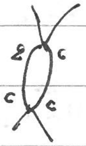
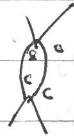
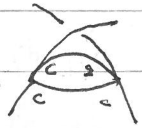
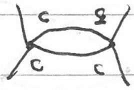
$$\frac{\delta^4 S_{T1}(GF)^2}{\delta\pi_{i,2}\,\delta\pi_{j,c}\,\delta\pi_{k,c}\,\delta\pi_{l,c}} \sim \frac{\delta^4}{\delta\pi_{i,2}\,\delta\pi_{j,c}\,\delta\pi_{k,c}\,\delta\pi_{l,c}}\big(2f_1+2f_2+f_3+f_4+f_5+2f_6$$

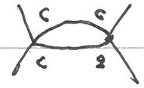
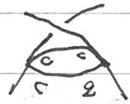
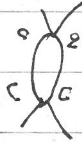
$$+2f_1'+2f_2'+f_3'+f_4'+f_5'+2f_6'\big)$$

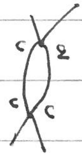
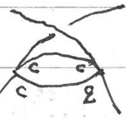
$\lambda_{4\pi}^2$

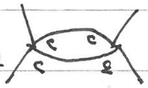
$= \{ 4(\delta_{ij}\delta_{kl}+\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk})\Big($  $+$  $+$  $\Big)$

$[(N-1)+2]\,2\,\delta_{ij}\delta_{kl}$  $+[(N-1)+2]\,2\,\delta_{ik}\delta_{jl}$ 

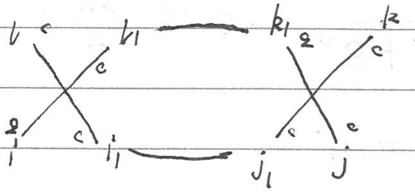
$+[(N-1)+2]\,2\,\delta_{il}\delta_{jk}$ 

$+4(\delta_{ij}\delta_{kl}+\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk})\Big($  $+$  $+$  $\Big)$

$+[(N-1)+2]\,2\,\delta_{ij}\delta_{kl}$  $+[(N-1)+2]\,2\,\delta_{ik}\delta_{jl}$ 

$+[(N-1)+2]\,2\,\delta_{il}\delta_{jk}$  $\}$

利用费曼规则作一个演示计算:



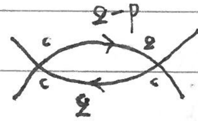
$$i\lambda_{4\pi}(\delta_{ii_1}\delta_{ll_1}+\delta_{il}\delta_{i_1l_1}+\delta_{il_1}\delta_{li_1})\,i(G_\pi^{cc})_{i_1j_1}\,i(G_\pi^{qc})_{k_1l_1}$$
$$\times i\lambda_{4\pi}(\delta_{jj_1}\delta_{kk_1}+\delta_{jk_1}\delta_{kj_1}+\delta_{jk}\delta_{j_1k_1})$$

$$=\lambda_{4\pi}^2(\delta_{ii_1}\delta_{ll_1}+\delta_{il}\delta_{i_1l_1}+\delta_{il_1}\delta_{li_1})\delta_{i_1j_1}\delta_{k_1l_1}(\delta_{jj_1}\delta_{kk_1}+\delta_{jk_1}\delta_{kj_1}+\delta_{jk}\delta_{j_1k_1})$$
$$\times G_\pi^{cc}G_\pi^{qc}$$

$$=\lambda_{4\pi}^2\{2(\delta_{ij}\delta_{kl}+\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk})+\delta_{il}\delta_{jk}[(N-1)+2]\}G_\pi^{cc}G_\pi^{qc}$$

$$\partial_t\left(\frac{\delta^4\Gamma_k}{\delta\pi_{i,q}\,\delta\pi_{j,c}\,\delta\pi_{k,c}\,\delta\pi_{l,c}}\right)$$

$$=-\frac{i}{4}\tilde{\partial}_t\frac{\delta^4\,\mathrm{STr}(GF)^2}{\delta\pi_{i,q}\,\delta\pi_{j,c}\,\delta\pi_{k,c}\,\delta\pi_{l,c}}$$



$$=\int\frac{d^4q}{(2\pi)^4}G_\pi^{cc}(q)\,G_\pi^{qc}(q-p)$$

$$=\int\frac{d^4q}{(2\pi)^4}\frac{-2i\varepsilon\,\mathrm{sgn}(q_0)}{[Z_{\phi,k}(q_0^2-\vec{q}^{\,2}(1+r_B))-m_\pi^2]^2+\varepsilon^2}\coth\left(\frac{\beta q^0}{2}\right)$$

$$\times\frac{1}{Z_{\phi,k}((q_0-p_0)^2-(\vec{q}-\vec{p})^2(1+r_B))-m_\pi^2-\mathrm{sgn}(\underbrace{q}_{q_0-p_0})i\varepsilon}$$

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$$G_\pi^{cc}(q)$$

$$= \frac{-2i\varepsilon\,\mathrm{sgn}(q_0)}{\left[Z_{\phi,k}\left(q_0^2-\vec{q}^{\,2}\left(1+r_B\left(\frac{\vec{q}^{\,2}}{k^2}\right)\right)\right)-m_\pi^2\right]^2+\varepsilon^2}\coth\left(\frac{\beta q^0}{2}\right)$$

令 $E_{\pi,k}^2(\vec{q}^{\,2}) = \vec{q}^{\,2}\left(1+r_B\left(\frac{\vec{q}^{\,2}}{k^2}\right)\right)+\bar{m}_\pi^2$

$$= \vec{q}^{\,2}\left[1+\left(\frac{k^2}{\vec{q}^{\,2}}-1\right)\theta\left(1-\frac{\vec{q}^{\,2}}{k^2}\right)\right]+\bar{m}_\pi^2$$

$$= \begin{cases} \vec{q}^{\,2}+\bar{m}_\pi^2 & \vec{q}^{\,2}>k^2 \\ k^2+\bar{m}_\pi^2 & \vec{q}^{\,2}\leq k^2 \end{cases}$$

其中 $\bar{m}_\pi^2 = \frac{m_\pi^2}{Z_{\phi,k}}$

$$G_\pi^{cc}(q)$$

$$= \frac{1}{Z_{\phi,k}}\frac{-2i\varepsilon\,\mathrm{sgn}(q_0)}{\left(q_0^2-E_{\pi,k}^2(\vec{q}^{\,2})\right)^2+\varepsilon^2}\coth\left(\frac{\beta q^0}{2}\right)$$

$$= \frac{1}{Z_{\phi,k}}(-i)\,2\pi\,\delta\left(q_0^2-E_{\pi,k}^2(\vec{q}^{\,2})\right)\mathrm{sgn}(q_0)\coth\left(\frac{\beta q^0}{2}\right)$$

$$= \frac{1}{Z_{\phi,k}}(-i)\,2\pi\frac{1}{2E_{\pi,k}(\vec{q}^{\,2})}\left(\delta(q_0-E_\pi)+\delta(q_0+E_\pi)\right)\mathrm{sgn}(q_0)\coth\left(\frac{\beta q^0}{2}\right)$$

$$G_\pi^{qc}(q-p)$$

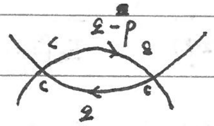
$$= \frac{1}{Z_{\phi,k}\left((q_0-p_0)^2-(\vec{q}-\vec{p})^2\left(1+r_B\left(\frac{(\vec{q}-\vec{p})^2}{k^2}\right)\right)\right)-m_\pi^2-\mathrm{sgn}(q_0-p_0)\,i\varepsilon}$$

$$= \frac{1}{Z_{\phi,k}}\frac{1}{(q_0-p_0)^2-E_{\pi,k}^2((\vec{q}-\vec{p})^2)-\mathrm{sgn}(q_0-p_0)\,i\varepsilon}$$

其中 $E_{\pi,k}^2((\vec{q}-\vec{p})^2) = (\vec{q}-\vec{p})^2\left(1+r_B\left(\frac{(\vec{q}-\vec{p})^2}{k^2}\right)\right)+\bar{m}_\pi^2$

$$= \begin{cases} (\vec{q}-\vec{p})^2+\bar{m}_\pi^2 & (\vec{q}-\vec{p})^2>k^2 \\ k^2+\bar{m}_\pi^2 & (\vec{q}-\vec{p})^2\leq k^2 \end{cases}$$

下面我们将 $E_{\pi,k}(\vec{q}^2)$，$E_{\pi,k}((\vec{q}-\vec{p})^2)$ 简记为 $E_{\pi,k}(q)$，$E_{\pi,k}(q-$



$$= \int \frac{d^4q}{(2\pi)^4} G_\pi^{cc}(q)\, G_\pi^{cc}(q-p)$$

$$= \int \frac{d^4q}{(2\pi)^4} \frac{1}{Z_{\phi,k}} (-i)\, 2\pi \frac{1}{2E_{\pi,k}(q)} \Big(\delta(q_0 - E_{\pi,k}(q)) + \delta(q_0 + E_{\pi,k}(q)) \Big)\, \mathrm{sgn}(q_0) \coth\Big(\frac{\beta q_0}{2}$$

$$\times \frac{1}{Z_{\phi,k}} \frac{1}{(q_0-p_0)^2 - E_{\pi,k}^2(q-p) - \mathrm{sgn}(q_0-p_0)\, i\varepsilon}$$

$$= (-i)\frac{1}{Z_{\phi,k}^2} \int \frac{d^3q}{(2\pi)^3} \frac{1}{2E_{\pi,k}(q)} \coth\Big(\frac{\beta E_{\pi,k}(q)}{2}\Big) \Big\{ \frac{1}{(E_{\pi,k}(q)-p_0)^2 - E_{\pi,k}^2(q-p) - \mathrm{sgn}(E_{\pi,k}(q)-}$$

$$+ \frac{1}{(-E_{\pi,k}(q)-p_0)^2 - E_{\pi,k}^2(q-p) - \mathrm{sgn}(-E_{\pi,k}(q)-p_0)\, i\varepsilon} \Big\}$$

$$G_\pi^{cc}(q) = \frac{1}{Z_{\phi,k}} \frac{1}{q_0^2 - E_{\pi,k}^2(q) - \mathrm{sgn}(q_0)\, i\varepsilon}$$

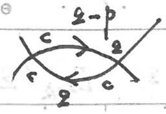
$$= \frac{1}{Z_{\phi,k}} \frac{1}{(q_0 - E_{\pi,k}(q) - i\varepsilon)(q_0 + E_{\pi,k}(q) - i\varepsilon)}$$

$$= \frac{1}{Z_{\phi,k}} \frac{1}{2E_{\pi,k}(q)} \Big(\frac{1}{q_0 - E_{\pi,k}(q) - i\varepsilon} - \frac{1}{q_0 + E_{\pi,k}(q) - i\varepsilon} \Big)$$

$$= \frac{1}{Z_{\phi,k}} \frac{1}{2E_{\pi,k}(q)} \Big(P\frac{1}{q_0 - E_{\pi,k}(q)} - P\frac{1}{q_0 + E_{\pi,k}(q)} \Big)$$

$$+ \frac{1}{Z_{\phi,k}} \frac{1}{2E_{\pi,k}(q)}\, i\pi \Big(\delta(q_0 - E_{\pi,k}(q)) - \delta(q_0 + E_{\pi,k}(q)) \Big)$$

其中上面 $P\frac{1}{q_0 - E_{\pi,k}(q)}$ 和 $P\frac{1}{q_0 + E_{\pi,k}(q)}$ 是其主值



$$= (-i)\frac{1}{Z_{\phi,k}^2}\int\frac{d^3q}{(2\pi)^3}\frac{1}{2E_{\pi,k}(q)}\coth\left(\frac{\beta E_{\pi,k}(q)}{2}\right)\Big\{\frac{1}{2E_{\pi,k}(q-p)}\left(\frac{1}{E_{\pi,k}(q)-p_0-E_{\pi,k}(q-p)}-\frac{1}{E_{\pi,k}(q)-p_0+E_{\pi,k}(q-p)}\right)$$

$$+\frac{1}{2E_{\pi,k}(q-p)}\,i\pi\Big(\delta(E_{\pi,k}(q)-p_0-E_{\pi,k}(q-p))-\delta(E_{\pi,k}(q)-p_0+E_{\pi,k}(q-p))\Big)$$

$$+\frac{1}{2E_{\pi,k}(q-p)}\left(\frac{1}{-E_{\pi,k}(q)-p_0-E_{\pi,k}(q-p)}-\frac{1}{-E_{\pi,k}(q)-p_0+E_{\pi,k}(q-p)}\right)$$

$$+\frac{1}{2E_{\pi,k}(q-p)}\,i\pi\Big(\delta(-E_{\pi,k}(q)-p_0-E_{\pi,k}(q-p))-\delta(-E_{\pi,k}(q)-p_0+E_{\pi,k}(q-p))\Big)\Big\}$$

$$= (-i)\frac{1}{Z_{\phi,k}^2}\int\frac{d^3q}{(2\pi)^3}\frac{1}{2E_{\pi,k}(q)}\frac{1}{2E_{\pi,k}(q-p)}\coth\left(\frac{\beta E_{\pi,k}(q)}{2}\right)\Big\{\frac{1}{E_{\pi,k}(q)-p_0-E_{\pi,k}(q-p)}-\frac{1}{E_{\pi,k}(q)-p_0+E_{\pi,k}(q-p)}$$

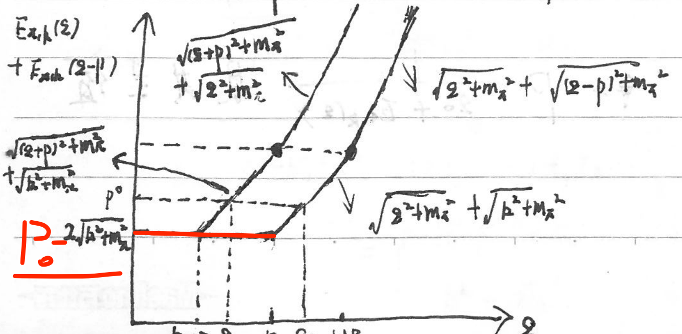
$$+\frac{1}{-E_{\pi,k}(q)-p_0-E_{\pi,k}(q-p)}-\frac{1}{-E_{\pi,k}(q)-p_0+E_{\pi,k}(q-p)}$$

$$+i\pi\Big[\delta(E_{\pi,k}(q)-p_0-E_{\pi,k}(q-p))-\delta(E_{\pi,k}(q)-p_0+E_{\pi,k}(q-p))$$

$$+\delta(-E_{\pi,k}(q)-p_0-E_{\pi,k}(q-p))-\delta(-E_{\pi,k}(q)-p_0+E_{\pi,k}(q-p))\Big]\Big\}$$

$$① \equiv \int\frac{d^3q}{(2\pi)^3}\frac{1}{2E_{\pi,k}(q)}\frac{1}{2E_{\pi,k}(q-p)}\coth\left(\frac{\beta E_{\pi,k}(q)}{2}\right)\delta(E_{\pi,k}(q)-p_0+E_{\pi,k}(q-p))$$

I. 当 $k > p$，存在里 $P \equiv |\vec{p}|$



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(1). 当 $p^0 < 2\sqrt{k^2+m_\pi^2}$ ① $= 0$

(2) 当 $2\sqrt{k^2+m_\pi^2} \le p^0 < \sqrt{(k+p)^2+m_\pi^2} + \sqrt{k^2+m_\pi^2}$

$$p^0 = \sqrt{(q_-+p)^2+m_\pi^2} + \sqrt{k^2+m_\pi^2}$$

$\Rightarrow$

$$q_- = \left[(p^0-\sqrt{k^2+m_\pi^2})^2 - m_\pi^2\right]^{\frac{1}{2}} - p$$

$$q_+ = q_- + p$$

$$= \left[(p_0-\sqrt{k^2+m_\pi^2})^2 - m_\pi^2\right]^{\frac{1}{2}}$$

(3) 当 $p^0 \ge \sqrt{(k+p)^2+m_\pi^2} + \sqrt{k^2+m_\pi^2}$

$$p^0 = \sqrt{(q_-+p)^2+m_\pi^2} + \sqrt{q_-^2+m_\pi^2}$$

$$\Rightarrow \quad q_- = \frac{-p(p_0^2-p^2) + \sqrt{p_0^2(p_0^2-p^2)(p_0^2-p^2-4m_\pi^2)}}{2(p_0^2-p^2)}$$

$$= -\frac{p}{2} + \frac{\sqrt{p_0^2(p_0^2-p^2)(p_0^2-p^2-4m_\pi^2)}}{2(p_0^2-p^2)}$$

$$q_+ = q_- + p$$

$$E_{\pi,k}(q-p) = \sqrt{q^2+p^2-2qp\cos\theta+m_\pi^2}$$

$$dE_{\pi,k}(q-p) = \frac{1}{2}\frac{-2qp\,d\cos\theta}{\sqrt{q^2+p^2-2qp\cos\theta+m_\pi^2}} = \frac{-qp\,d\cos\theta}{E_{\pi,k}(q-p)}$$

$$① = \frac{1}{(2\pi)^2}\int dq\, q^2\int_{-1}^{1} d\cos\theta\, \frac{1}{2E_{\pi,k}(q)}\frac{1}{2E_{\pi,k}(q-p)}\coth\left(\frac{\beta E_{\pi,k}(q)}{2}\right)\delta(E_{\pi,k}(q)-p_0$$

$$+E_{\pi,k}(q-p)).$$

$$= \frac{1}{(2\pi)^2}\frac{1}{q}\int dq\, q^2\,\frac{dE_{\pi,k}(q-p)}{qp}\,E_{\pi,k}(q-p)\frac{1}{2E_{\pi,k}(q)}\frac{1}{2E_{\pi,k}(q-p)}\coth\left(\frac{\beta E_{\pi,k}(q)}{2}\right)$$

$$\delta(E_{\pi,k}(q)-p_0+E_{\pi,k}(q-p))$$

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$E_{\pi,k}(q)=$

$$=\frac{1}{(2\pi)^2}\left(\frac{1}{4p}\right)\int_{q_-}^{q_+}dq\,\frac{q}{E_{\pi,k}(q)}\coth\left(\frac{\beta E_{\pi,k}(q)}{2}\right)$$

当 $q_- > k$

$$E_{\pi,k}(q)=(q^2+m_\pi^2)^{\frac{1}{2}}$$

$$dE_{\pi,k}(q)=\frac{q\,dq}{E_{\pi,k}(q)}$$

$$① = \frac{1}{(2\pi)^2}\frac{1}{4p}\int_{E_{\pi,k}(q_-)}^{E_{\pi,k}(q_+)}dE_{\pi,k}(q)\coth\left(\frac{\beta E_{\pi,k}(q)}{2}\right)$$

$$=\frac{1}{(2\pi)^2}\frac{1}{4p}\frac{2}{\beta}\ln\frac{\sinh\frac{\beta E_{\pi,k}(q_+)}{2}}{\sinh\frac{\beta E_{\pi,k}(q_-)}{2}}$$

$$=\frac{1}{(2\pi)^2}\frac{1}{4p}\frac{2}{\beta}\ln\frac{e^{\frac{\beta}{2}E_{\pi,k}(q_+)}\left(1-e^{-\beta E_{\pi,k}(q_+)}\right)}{e^{\frac{\beta}{2}E_{\pi,k}(q_-)}\left(1-e^{-\beta E_{\pi,k}(q_-)}\right)}$$

$$=\frac{1}{(2\pi)^2}\frac{1}{4p}\left(\left(E_{\pi,k}(q_+)-E_{\pi,k}(q_-)\right)+\frac{2}{\beta}\ln\frac{1-e^{-\beta E_{\pi,k}(q_+)}}{1-e^{-\beta E_{\pi,k}(q_-)}}\right)$$

$$=\frac{1}{(2\pi)^2}\frac{1}{4p}\left(\left((q_+^2+m_\pi^2)^{\frac{1}{2}}-(q_-^2+m_\pi^2)^{\frac{1}{2}}\right)+\frac{2}{\beta}\ln\frac{1-e^{-\beta\sqrt{q_+^2+m_\pi^2}}}{1-e^{-\beta\sqrt{q_-^2+m_\pi^2}}}\right) \quad *1$$

$$\equiv A(q_+,q_-)$$

当 $q_- < k \qquad q_+ > k$

$$\int_{q_-}^{q_+}dq\,\frac{q}{E_{\pi,k}(q)}\coth\left(\frac{\beta E_{\pi,k}(q)}{2}\right)$$

$$=\int_{q_-}^{k}dq\,\frac{q}{E_{\pi,k}(q)}\coth\left(\frac{\beta E_{\pi,k}(q)}{2}\right)+\int_{k}^{q_+}dq\,\frac{q}{E_{\pi,k}(q)}\coth\left(\frac{\beta E_{\pi,k}(q)}{2}\right)$$

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$$= \frac{1}{\sqrt{k^2+m_\pi^2}}\coth\left(\frac{\beta\sqrt{k^2+m_\pi^2}}{2}\right)\int_{q_-}^{k} dq\, q$$

$$+\int_{k}^{q_+} dq\, \frac{q}{E_{\pi,k}(q)}\coth\left(\frac{\beta E_{\pi,k}(q)}{2}\right)$$

$$= \frac{1}{\sqrt{k^2+m_\pi^2}}\coth\left(\frac{\beta\sqrt{k^2+m_\pi^2}}{2}\right)\frac{1}{2}(k^2-q_-^2)$$

$$+(q_+^2+m_\pi^2)^{\frac{1}{2}}-(k^2+m_\pi^2)^{\frac{1}{2}}+\frac{2}{\beta}\ln\frac{1-e^{-\beta\sqrt{q_+^2+m_\pi^2}}}{1-e^{-\beta\sqrt{k^2+m_\pi^2}}}$$

$$① = \frac{1}{(2\pi)^2}\frac{1}{4p}\left\{\frac{1}{\sqrt{k^2+m_\pi^2}}\coth\left(\frac{\beta\sqrt{k^2+m_\pi^2}}{2}\right)\frac{1}{2}(k^2-q_-^2)\right.$$

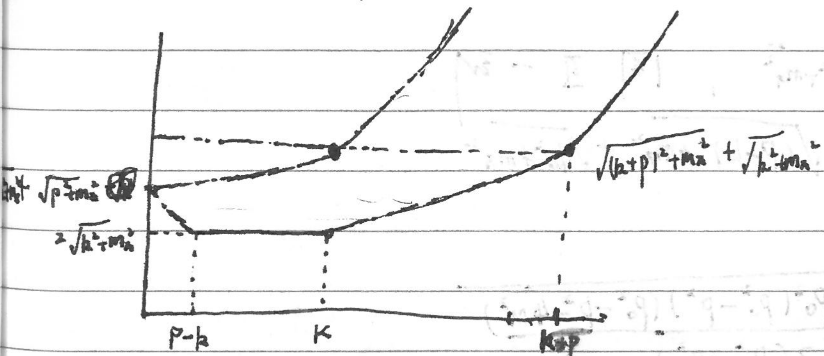
$$\left.+(q_+^2+m_\pi^2)^{\frac{1}{2}}-(k^2+m_\pi^2)^{\frac{1}{2}}+\frac{2}{\beta}\ln\frac{1-e^{-\beta\sqrt{q_+^2+m_\pi^2}}}{1-e^{-\beta\sqrt{k^2+m_\pi^2}}}\right\}\times 2$$

$$\equiv B(q_+, q_-, k)$$

当 $q_+ < k$

$① = 0$

Ⅱ 当 $\frac{p}{2} < k \leq p$



(1) 当 $p^0 \geq \sqrt{(k+p)^2+m_\pi^2}+\sqrt{k^2+m_\pi^2}$，同前面计算相同，

(2) 当 $\sqrt{p^2+m_\pi^2}+\sqrt{k^2+m_\pi^2} \leq p_0 < \sqrt{(k+p)^2+m_\pi^2}+\sqrt{k^2+m_\pi^2}$

$$p_0 = \sqrt{(q_-+p)^2+m_\pi^2}+\sqrt{k^2+m_\pi^2}$$

$$q_- = \left[(p_0-\sqrt{k^2+m_\pi^2})^2-m_\pi^2\right]^{\frac{1}{2}}-p \qquad q_+ = q_-+p$$

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同前面计算一致。

(3) 当 $2\sqrt{k^2+m^2} \leqslant p_0 < \sqrt{p^2+m_\pi^2} + \sqrt{k^2+m_\pi^2}$

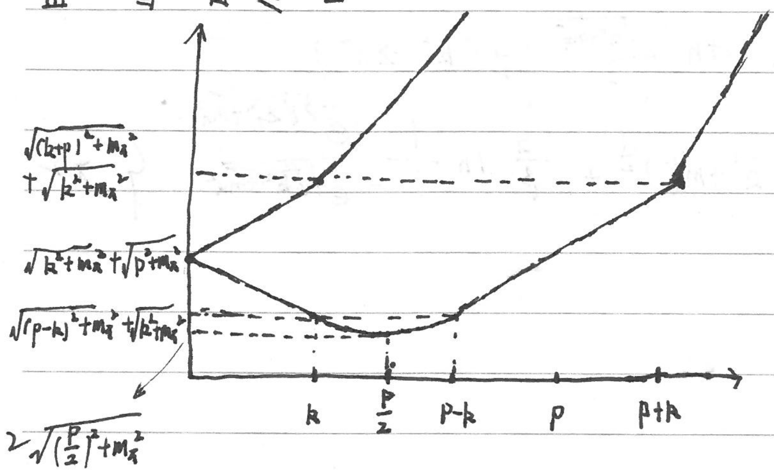
$p_0 = \sqrt{(p-q_-)^2+m_\pi^2} + \sqrt{k^2+m_\pi^2}$

$q_- = p - [(p_0-\sqrt{k^2+m_\pi^2})^2 - m_\pi^2]^{\frac{1}{2}}$

$q_+ = [(p_0-\sqrt{k^2+m_\pi^2})^2 - m_\pi^2]^{\frac{1}{2}}$

① = $\overset{0}{\cancel{\times}}$ 2

III 当 $k \leqslant \frac{p}{2}$



当 $p_0 > \sqrt{(p-k)^2+m_\pi^2} + \sqrt{k^2+m_\pi^2}$，同 II 一到

当 $2\sqrt{(\frac{p}{2})^2+m_\pi^2} \leqslant p_0 < \sqrt{(p-k)^2+m_\pi^2} + \sqrt{k^2+m_\pi^2}$

① = $\times 1$

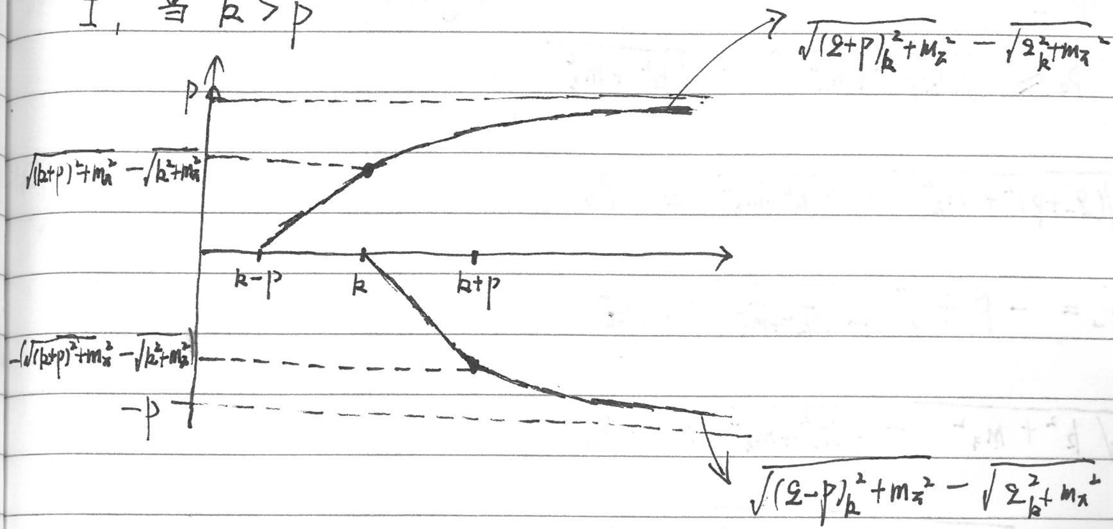
其中 $q_- = \frac{p}{2} - \frac{\sqrt{p_0^2(p_0^2-p^2)(p_0^2-p^2-4m^2)}}{2(p_0^2-p^2)}$

$q_+ = \frac{p}{2} + \frac{\sqrt{p_0^2(p_0^2-p^2)(p_0^2-p^2-4m^2)}}{2(p_0^2-p^2)}$

$$② = \int \frac{d^3q}{(2\pi)^3} \frac{1}{2E_{\pi,k}(q)} \frac{1}{2E_{\pi,k}(q-p)} \coth\left(\frac{\beta E_{\pi,k}(q)}{2}\right) \delta(-E_{\pi,k}(q) - p_0 + E_{\pi,k}(q-p))$$

$$③ = \int \frac{d^3q}{(2\pi)^3} \frac{1}{2E_{\pi,k}(q)} \frac{1}{2E_{\pi,k}(q-p)} \coth\left(\frac{\beta E_{\pi,k}(q)}{2}\right) \delta(E_{\pi,k}(q) - p_0 - E_{\pi,k}(q-p))$$

I, 当 $k > p$



(1) 当 $\sqrt{(k+p)^2+m_\pi^2} - \sqrt{k^2+m_\pi^2} < p_0 \le p$

求解 $\sqrt{(q+p)^2+m_\pi^2} - \sqrt{q^2+m_\pi^2} = p_0$

$$q_- = -\frac{p}{2} - \frac{\sqrt{p_0^2(p_0^2-p^2)(p_0^2-p^2-4m^2)}}{2(p_0^2-p^2)}$$

$$\sqrt{(q_+-p)^2+m_\pi^2} - \sqrt{q_+^2+m_\pi^2} = -p_0$$

$$q_+ = \frac{p}{2} - \frac{\sqrt{p_0^2(p_0^2-p^2)(p_0^2-p^2-4m^2)}}{2(p_0^2-p^2)} = q_- + p$$

$$② = \frac{1}{(2\pi)^2} \int_{q_-}^{\infty} dq\, q^2 \int_{-1}^{1} d\cos\theta \frac{1}{2E_{\pi,k}(q)} \frac{1}{2E_{\pi,k}(q-p)} \coth\left(\frac{\beta E_{\pi,k}(q)}{2}\right) \delta(-E_{\pi,k}(q) - p_0 + E_{\pi,k}(q-p))$$

$$= \frac{1}{(2\pi)^2} \frac{1}{4p} \int_{q_-}^{p} dq \frac{q}{E_{\pi,k}(q)} \coth\left(\frac{\beta E_{\pi,k}(q)}{2}\right)$$

$$③ = \frac{1}{(2\pi)^2} \frac{1}{4p} \int_{q_+}^{\infty} dq \frac{q}{E_{\pi,k}(q)} \coth\left(\frac{\beta E_{\pi,k}(q)}{2}\right)$$

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②-③

$$= \frac{1}{(2\pi)^2}\frac{1}{4p}\int_{q_-}^{q_+} dq\, \frac{q}{E_{\pi,k}(q)}\coth\left(\frac{\beta E_{\pi,k}(q)}{2}\right)$$

$$= \frac{1}{(2\pi)^2}\frac{1}{4p}\left[\left((q_+^2+m_\pi^2)^{\frac{1}{2}}-(q_-^2+m_\pi^2)^{\frac{1}{2}}\right)+\frac{2}{\beta}\ln\frac{1-e^{-\beta\sqrt{q_+^2+m_\pi^2}}}{1-e^{-\beta\sqrt{q_-^2+m_\pi^2}}}\right] \quad *1$$

(2) 当 $p_0 \leqslant \sqrt{(k+p)^2+m_\pi^2} - \sqrt{k^2+m_\pi^2}$

$$\sqrt{(q_-+p)^2+m_\pi^2} - \sqrt{k^2+m_\pi^2} = p_0$$

$$q_- = -p + \sqrt{(p_0+\sqrt{k^2+m_\pi^2})^2 - m_\pi^2}$$

$$\sqrt{k^2+m_\pi^2} - \sqrt{q_+^2+m_\pi^2} = -p_0$$

$$q_+ = \sqrt{(p_0+\sqrt{k^2+m_\pi^2})^2 - m_\pi^2} = q_- + p$$

②-③

$$= \frac{1}{(2\pi)^2}\frac{1}{4p}\int_{q_-}^{q_+} dq\, \frac{q}{E_{\pi,k}(q)}\coth\left(\frac{\beta E_{\pi,k}(q)}{2}\right)$$

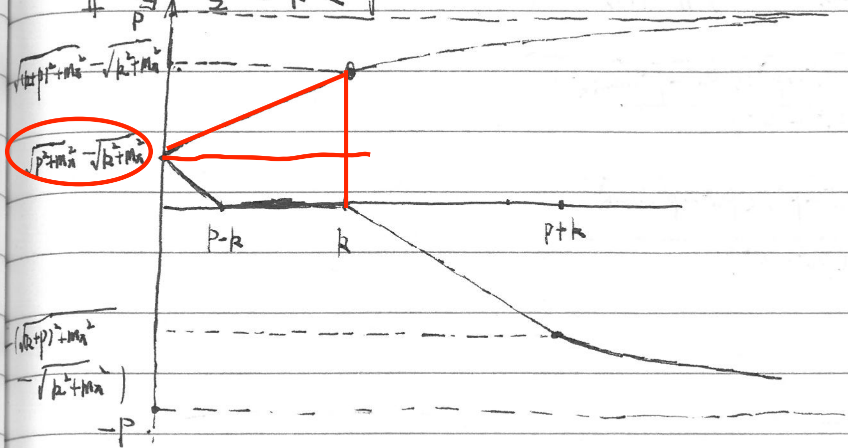
$$= \frac{1}{(2\pi)^2}\frac{1}{4p}\left(\int_{q_-}^{k} dq\, \frac{q}{E_{\pi,k}(q)}\coth\left(\frac{\beta E_{\pi,k}(q)}{2}\right)\right.$$

$$\left.+\int_{k}^{q_+} dq\, \frac{q}{E_{\pi,k}(q)}\coth\left(\frac{\beta E_{\pi,k}(q)}{2}\right)\right)$$

$= *2$

(1) 当 $p_0 > p$ ② = 0 ③ = 0

II 当 $\frac{p}{2} < k \le p$



当 $p_0 > p$ ② - ③ = 0

当 $\sqrt{(k+p)^2+m_\pi^2}-\sqrt{k^2+m_\pi^2} < p_0 \le p$

② - ③ = *1，同前面一致

当 $\sqrt{p^2+m_\pi^2}-\sqrt{k^2+m_\pi^2} < p_0 \le \sqrt{(k+p)^2+m_\pi^2}-\sqrt{k^2+m_\pi^2}$

② - ③ = *2，同前面一致

当 $p_0 \le \sqrt{p^2+m_\pi^2}-\sqrt{k^2+m_\pi^2}$

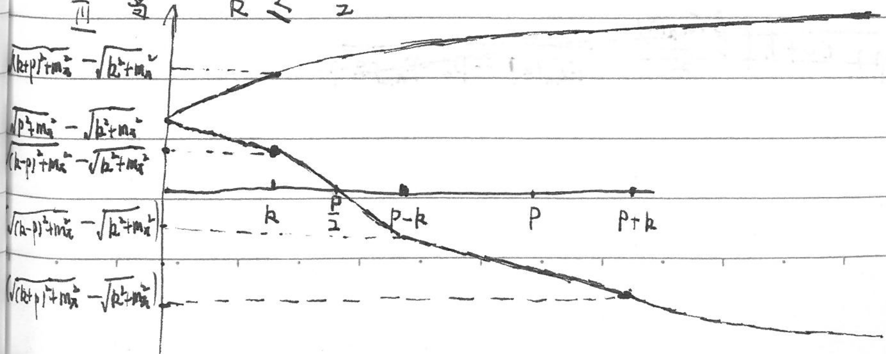
$$\sqrt{(q_- - p)^2+m_\pi^2}-\sqrt{k^2+m_\pi^2} = p_0$$

$$q_- = p - \left[(p_0+\sqrt{k^2+m_\pi^2})^2 - m_\pi^2\right]^{\frac{1}{2}}$$

同理 $q_+ = \left[(p_0+\sqrt{k^2+m_\pi^2})^2 - m_\pi^2\right]^{\frac{1}{2}}$

② - ③ = *2

III 当 $k \le \frac{p}{2}$



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当 $p_0 > \sqrt{(k-p)^2+m_\pi^2} - \sqrt{k^2+m_\pi^2}$，同Ⅱ一致

当 $p_0 \leq \sqrt{(k-p)^2+m_\pi^2} - \sqrt{k^2+m_\pi^2}$

$$q_- = \frac{p}{2} + \frac{\sqrt{p_0^2(p_0^2-p^2)(p_0^2-p^2-4m_\pi^2)}}{2(p_0^2-p^2)}$$

$$q_+ = \frac{p}{2} - \frac{\sqrt{p_0^2(p_0^2-p^2)(p_0^2-p^2-4m_\pi^2)}}{2(p_0^2-p^2)}$$

$② - ③ = \times 1$

令 $F_1(p_0,p,k)$

$$= \int\frac{d^3q}{(2\pi)^3}\frac{1}{2E_{\pi,k}(q)}\frac{1}{2E_{\pi,k}(q-p)}\coth\left(\frac{\beta E_{\pi,k}(q)}{2}\right)\Big[\delta(E_{\pi,k}(q)-p_0+E_{\pi,k}(q-p))$$

$$-\delta(-E_{\pi,k}(q)-p_0-E_{\pi,k}(q-p))\Big]$$

$F_2(p_0,p,k)$

$$=\int\frac{d^3q}{(2\pi)^3}\frac{1}{2E_{\pi,k}(q)}\frac{1}{2E_{\pi,k}(q-p)}\coth\left(\frac{\beta E_{\pi,k}(q)}{2}\right)\Big[\delta(-E_{\pi,k}(q)-p_0+E_{\pi,k}(q-p))$$

$$-\delta(E_{\pi,k}(q)-p_0-E_{\pi,k}(q-p))\Big]$$

显然有 $F_1(-p_0,p,k) = -F_1(p_0,p,k)$

$F_2(-p_0,p,k) = -F_2(p_0,p,k)$

$$\cancel{\int\frac{d^3q}{(2\pi)^3}\frac{1}{2E_{\pi,k}(q)}\frac{1}{2E_{\pi,k}(q-p)}\coth\left(\frac{\beta E_{\pi,k}(q)}{2}\right)\frac{1}{E_{\pi,k}(q)-p_0-E_{\pi,k}(q-p)}}$$

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$$\int dp_0' \frac{1}{p_0'-p_0} F_1(p_0', p, k)$$

$$= \int \frac{d^3 q}{(2\pi)^3} \frac{1}{2E_{\pi,k}(q)} \frac{1}{2E_{\pi,k}(q-p)} \coth\left(\frac{\beta E_{\pi,k}(q)}{2}\right) \int dp_0' \Big[\frac{1}{p_0'-p_0} \delta(E_{\pi,k}(q) - p_0' + E_{\pi,k}(q-p))$$

$$- \frac{1}{p_0'-p_0} \delta(-E_{\pi,k}(q) - p_0' - E_{\pi,k}(q-p)) \Big]$$

$$= \int \frac{d^3 q}{(2\pi)^3} \frac{1}{2E_{\pi,k}(q)} \frac{1}{2E_{\pi,k}(q-p)} \coth\left(\frac{\beta E_{\pi,k}(q)}{2}\right) \Big[\frac{1}{E_{\pi,k}(q) - p_0 + E_{\pi,k}(q-p)} - \frac{1}{-E_{\pi,k}(q) - p_0 - E_{\pi,k}(q-p)}$$

$$\int dp_0' \frac{1}{p_0'-p_0} F_2(p_0', p, k)$$

$$= \int \frac{d^3 q}{(2\pi)^3} \frac{1}{2E_{\pi,k}(q)} \frac{1}{2E_{\pi,k}(q-p)} \coth\left(\frac{\beta E_{\pi,k}(q)}{2}\right) \int dp_0' \Big[\frac{1}{p_0'-p_0} \delta(-E_{\pi,k}(q) - p_0' + E_{\pi,k}(q-p))$$

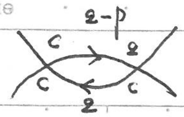
$$- \frac{1}{p_0'-p_0} \delta(E_{\pi,k}(q) - p_0' - E_{\pi,k}(q-p)) \Big]$$

$$= \int \frac{d^3 q}{(2\pi)^3} \frac{1}{2E_{\pi,k}(q)} \frac{1}{2E_{\pi,k}(q-p)} \coth\left(\frac{\beta E_{\pi,k}(q)}{2}\right) \Big[\frac{1}{-E_{\pi,k}(q) - p_0 + E_{\pi,k}(q-p)} - \frac{1}{E_{\pi,k}(q) - p_0 - E_{\pi,k}(q-p)}$$

令 $\tilde{F}_1(p_0, p, k) = \int dp_0' \frac{1}{p_0'-p_0} F_1(p_0', p, k)$

$\tilde{F}_2(p_0, p, k) = \int dp_0' \frac{1}{p_0'-p_0} F_2(p_0', p, k)$

显然 $\tilde{F}_1(-p_0, p, k) = \tilde{F}_1(p_0, p, k) \qquad \tilde{F}_2(-p_0, p, k) = \tilde{F}_2(p_0, p, k)$



$$= (-i)\frac{1}{Z_{\pi k}^2}\Big\{ -\tilde{F}_1(p_0,p,k) - \tilde{F}_2(p_0,p,k)$$
$$+ i\pi\left[-F_1(p_0,p,k) - F_2(p_0,p,k)\right]\Big\}$$

$$= \frac{i}{Z_{\pi k}^2}\Big\{ \tilde{F}_1(p_0,p,k) + \tilde{F}_2(p_0,p,k)$$
$$+ i\pi\left[F_1(p_0,p,k) + F_2(p_0,p,k)\right]\Big\}$$

前面①，②－③的计算不完整，首先来看①

①：

I. 当 $k > p$

(2) 当 $2\sqrt{k^2+m_\pi^2} \le p^0 < \sqrt{(k+p)^2+m_\pi^2} + \sqrt{k^2+m_\pi^2}$

右边界上还需要积分，在右边界上 $E_{\pi,k}(q-p) = \sqrt{k^2+m_\pi^2}$

所以 $|\vec{q}-\vec{p}|^2 \le k^2$

$$\Rightarrow \quad q^2+p^2-2qp\cos\theta \le k^2$$

$$\Rightarrow \quad \cos\theta \ge \frac{q^2+p^2-k^2}{2qp} = \cos\theta_{min}$$

$$① \sim \frac{1}{(2\pi)^2}\int dq\, q^2 \int_{\cos\theta_{min}}^{1} d\cos\theta\, \frac{1}{2E_{\pi,k}(q)}\frac{1}{2E_{\pi,k}(k)} \coth\left(\frac{\beta E_{\pi,k}(q)}{2}\right)\delta\big(E_{\pi,k}(q) - p_0$$
$$+ E_{\pi,k}(k)\big)$$

令 $\bar{E}_{\pi,k}(q) = p_0 - E_{\pi,k}(k) = p_0 - \sqrt{k^2+m_\pi^2}$

$$\bar{q} = (\bar{E}_\pi^2 - m_\pi^2)^{\frac{1}{2}}$$
$$= \left[\left(p_0 - \sqrt{k^2+m_\pi^2}\right)^2 - m_\pi^2\right]^{\frac{1}{2}}$$
$$= q_+$$

令 $E_+ = \bar{E}_{\pi,k}(q)$

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$$①\sim \frac{1}{(2\pi)^2}\frac{1}{4}\int_{\cos\theta_{min}}^{1} d\cos\theta \frac{1}{E_\pi(k)}\int dq\, q^2 \frac{1}{E_\pi(q)} \coth\left(\frac{\beta E_\pi(q)}{2}\right)\delta(E_\pi(q)-p_0+E_\pi(k)$$

$$=\frac{1}{(2\pi)^2}\frac{1}{4}\int_{\cos\theta_{min}}^{1} d\cos\theta \frac{1}{E_\pi(k)}\int dE_\pi(q)\, q \coth\left(\frac{\beta E_\pi(q)}{2}\right)\delta(E_\pi(q)-p_0+E_\pi(k)$$

$$=\frac{1}{(2\pi)^2}\frac{1}{4E_\pi(k)}\int_{\cos\theta_{min}}^{1} d\cos\theta\; q_+ \coth\left(\frac{\beta E_+}{2}\right)$$

$$=\frac{1}{(2\pi)^2}\frac{1}{4E_\pi(k)}\; q_+ \coth\left(\frac{\beta E_+}{2}\right)(1-\cos\theta_{min})$$

$$=\frac{1}{(2\pi)^2}\frac{1}{4E_\pi(k)}\; q_+ \coth\left(\frac{\beta E_+}{2}\right)\left(1-\frac{q_+^2+p^2-k^2}{2q_+p}\right) \qquad (*3)$$

II、当 $\frac{p}{2}<k\leq p$

当 $2\sqrt{k^2+m_\pi^2}\leq p_0<\sqrt{(k+p)^2+m_\pi^2}+\sqrt{k^2+m_\pi^2}$，在右边界上的积分贡献为

$① \sim (*3)$

当 $k\leq\frac{p}{2}$

当 $\sqrt{(p-k)^2+m_\pi^2}+\sqrt{k^2+m_\pi^2}\leq p_0<\sqrt{(k+p)^2+m_\pi^2}+\sqrt{k^2+m_\pi^2}$，在右边界上的积分贡献为

$① \sim (*3)$

下面讨论 ②－③ 的计算

I、当 $k>p$，

当 $p_0\leq\sqrt{(k+p)^2+m_\pi^2}-\sqrt{k^2+m_\pi^2}$，下边界上的积分贡献，所

$② - ③ \sim -(*3)$

II 当 $\frac{p}{2}<k\leq p$

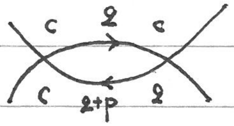
当 $p_0\leq\sqrt{(k+p)^2+m_\pi^2}-\sqrt{k^2+m_\pi^2}$，下边界上的积分贡献为

$② - ③ \sim -(*3)$

当Ⅲ: $k \le \frac{p}{2}$

当 $\sqrt{(k-p)^2+m_\pi^2}-\sqrt{k^2+m_\pi^2}<p_0\le\sqrt{(k+p)^2+m_\pi^2}-\sqrt{k^2+m_\pi^2}$，下边界上的积分贡献为

②−③ ~ −(×3)



$$= \int \frac{d^4q}{(2\pi)^4}\, G_\pi^{cc}(q)\, G_\pi^{cq}(q+p)$$

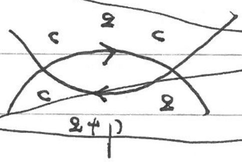
$$G_\pi^{cq}(q+p)$$

$$= \frac{1}{Z_{\pi,k}\left((q_0+p_0)^2-(\vec{q}+\vec{p})^2\left(1+r_B\left(\frac{(\vec{q}+\vec{p})^2}{k^2}\right)\right)\right)-m_\pi^2+\mathrm{sgn}(q_0+p_0)i\varepsilon}$$

$$= \frac{1}{Z_{\pi,k}}\,\frac{1}{(q_0+p_0)^2-E_{\pi,k}^2((\vec{q}+\vec{p})^2)+\mathrm{sgn}(q_0+p_0)i\varepsilon}$$

其中

$$E_{\pi,k}^2((\vec{q}+\vec{p})^2) = (\vec{q}+\vec{p})^2\left(1+r_B\left(\frac{(\vec{q}+\vec{p})^2}{k^2}\right)\right)+\bar{m}_\pi^2$$



$$\cancel{= \int \frac{d^4q}{(2\pi)^4}\, G_\pi^{cc}(q)\, G_\pi^{cq}(q+p)}$$

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的移

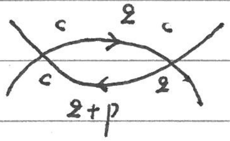
$$G_\pi^{c2}(q) = \frac{1}{Z_{\phi,k}} \frac{1}{q_0^2 - E_{\pi,k}^2(q) + \mathrm{sgn}(q_0)\, i\varepsilon}$$

$$= \frac{1}{Z_{\phi,k}} \frac{1}{(q_0 - E_{\pi,k}(q) + i\varepsilon)(q_0 + E_{\pi,k}(q) + i\varepsilon)}$$

$$= \frac{1}{Z_{\phi,k}} \frac{1}{2E_{\pi,k}(q)} \left(\frac{1}{q_0 - E_{\pi,k}(q) + i\varepsilon} - \frac{1}{q_0 + E_{\pi,k}(q) + i\varepsilon} \right)$$

$$= \frac{1}{Z_{\phi,k}} \frac{1}{2E_{\pi,k}(q)} \left(P\frac{1}{q_0 - E_{\pi,k}(q)} - P\frac{1}{q_0 + E_{\pi,k}(q)} \right)$$

$$+ \frac{1}{Z_{\phi,k}} \frac{1}{2E_{\pi,k}(q)} (-i\pi) \left(\delta(q_0 - E_{\pi,k}(q)) - \delta(q_0 + E_{\pi,k}(q)) \right)$$



$$= \int \frac{d^4q}{(2\pi)^4} G_\pi^{cc}(q)\, G_\pi^{c2}(q+p)$$

$$= \int \frac{d^4q}{(2\pi)^4} \frac{1}{Z_{\phi,k}} (-i)\, 2\pi \frac{1}{2E_{\pi,k}(q)} \coth\left(\frac{\beta E_{\pi,k}(q)}{2}\right) \left(\delta(q_0 - E_{\pi,k}(q)) + \delta(q_0 + E_{\pi,k}(q)) \right)$$

$$\times \frac{1}{Z_{\phi,k}} \frac{1}{2E_{\pi,k}(q+p)} \left\{ \frac{1}{q_0+p_0 - E_{\pi,k}(q+p)} - \frac{1}{q_0+p_0+E_{\pi,k}(q+p)} \right.$$

$$\left. + (-i\pi)\left(\delta(q_0+p_0 - E_{\pi,k}(q+p)) - \delta(q_0+p_0+E_{\pi,k}(q+p)) \right) \right\}$$

$$= (-i) \frac{1}{Z_{\phi,k}^2} \int \frac{d^3q}{(2\pi)^3} \frac{1}{2E_{\pi,k}(q)} \frac{1}{2E_{\pi,k}(q+p)} \coth\left(\frac{\beta E_{\pi,k}(q)}{2}\right) \left\{ \frac{1}{E_{\pi,k}(q) + p_0 - E_{\pi,k}(q+p)} \right.$$

$$- \frac{1}{E_{\pi,k}(q) + p_0 + E_{\pi,k}(q+p)} + \frac{1}{-E_{\pi,k}(q) + p_0 - E_{\pi,k}(q+p)} - \frac{1}{-E_{\pi,k}(q) + p_0 + E_{\pi,k}(q+p)}$$

$$+ (-i\pi)\Big[\delta(E_{\pi,k}(q) + p_0 - E_{\pi,k}(q+p)) - \delta(E_{\pi,k}(q) + p_0 + E_{\pi,k}(q+p))$$

$$\left. + \delta(-E_{\pi,k}(q) + p_0 - E_{\pi,k}(q+p)) - \delta(-E_{\pi,k}(q) + p_0 + E_{\pi,k}(q+p)) \Big] \right\}$$

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$\vec{q} \to -\vec{q}$

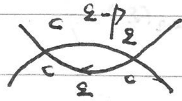
$$= (-i)\frac{1}{Z_{\phi,k}^2}\int\frac{d^3q}{(2\pi)^3}\frac{1}{2E_{\pi,k}(q)}\frac{1}{2E_{\pi,k}(q-p)}\coth\left(\frac{\beta E_{\pi,k}(q)}{2}\right)\Big\{\frac{1}{E_{\pi,k}(q)-p_0-E_{\pi,k}(q-p)}$$

$$-\frac{1}{E_{\pi,k}(q)-p_0+E_{\pi,k}(q-p)}+\frac{1}{-E_{\pi,k}(q)-p_0-E_{\pi,k}(q-p)}-\frac{1}{-E_{\pi,k}(q)-p_0+E_{\pi,k}(q-p)}$$

$$+(i\pi)\Big[\delta(E_{\pi,k}(q)-p_0-E_{\pi,k}(q-p))-\delta(E_{\pi,k}(q)-p_0+E_{\pi,k}(q-p))$$

$$+\delta(-E_{\pi,k}(q)-p_0-E_{\pi,k}(q-p))-\delta(-E_{\pi,k}(q)-p_0+E_{\pi,k}(q-p))\Big]$$

=



$$= \frac{i}{Z_{\phi,k}^2}\Big\{\tilde{F}_1(p_0,p,k)+\tilde{F}_2(p_0,p,k)$$

$$+i\pi\left[F_1(p_0,p,k)+F_2(p_0,p,k)\right]\Big\}$$

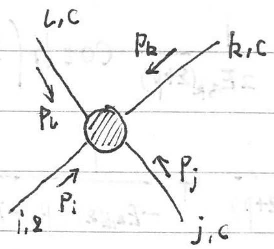
由前面的讨论

$$\partial_t\left(\frac{\delta^4\Gamma_k}{\delta\pi_{i,2}\,\delta\pi_{j,c}\,\delta\pi_{k,c}\,\delta\pi_{l,c}}\right)$$

$$= -\frac{i}{4}\tilde{\partial}_t\frac{\delta^4\,\mathrm{STr}(GF)^2}{\delta\pi_{i,2}\,\delta\pi_{j,c}\,\delta\pi_{k,c}\,\delta\pi_{l,c}}$$

令

$$i\frac{\delta^4\Gamma_k}{\delta\pi_{i,2}\,\delta\pi_{j,c}\,\delta\pi_{k,c}\,\delta\pi_{l,c}} =$$

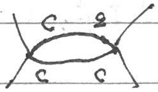
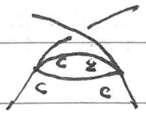
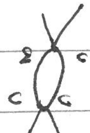


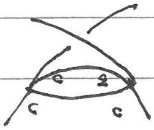
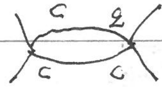
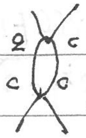
$$= i\left[\lambda_{4\pi,\mathrm{eff}}(p_i,p_j,p_k,p_l)\,\delta_{il}\,\delta_{jk}\right.$$

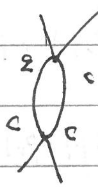
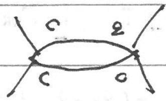
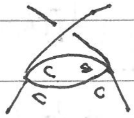
$$+\lambda_{4\pi,\mathrm{eff}}(p_i,p_k,p_l,p_j)\,\delta_{ij}\,\delta_{kl}$$

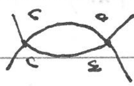
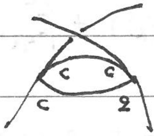
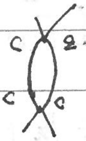
$$\left.+\lambda_{4\pi,\mathrm{eff}}(p_i,p_l,p_j,p_k)\,\delta_{ik}\,\delta_{jl}\right]$$

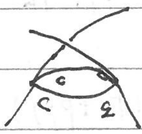
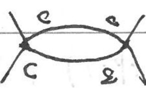
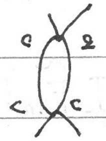
$$-\frac{i}{4}\frac{\delta^4 \, \mathrm{STr}(GF)^2}{\delta\pi_{i,\ell}\,\delta\pi_{j,c}\,\delta\pi_{k,c}\,\delta\pi_{l,c}}$$

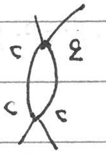
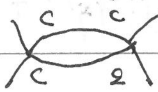
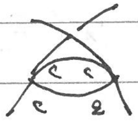
$= -i\lambda_{4\pi}^2 \Big\{ \Big[\left(\frac{1}{2}[(N-1)+2]+1\right)$  $+$  $+$  $\Big]\delta_{il}\delta_{jk}$

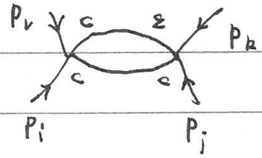
$+ \Big[\left(\frac{1}{2}[(N-1)+2]+1\right)$  $+$  $+$  $\Big]\delta_{ik}\delta_{jl}$

$+ \Big[\left(\frac{1}{2}[(N-1)+2]+1\right)$  $+$  $+$  $\Big]\delta_{ij}\delta_{kl}$

$+ \Big[\left(\frac{1}{2}[(N-1)+2]+1\right)$  $+$  $+$  $\Big]\delta_{il}\delta_{jk}$

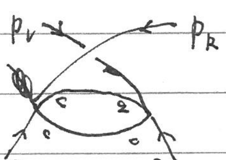
$+ \Big[\left(\frac{1}{2}[(N-1)+2]+1\right)$  $+$  $+$  $\Big]\delta_{ik}\delta_{jl}$

$+ \Big[\left(\frac{1}{2}[(N-1)+2]+1\right)$  $+$  $+$  $\Big]\delta_{ij}\delta_{kl}$



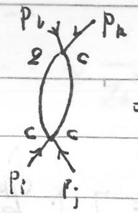
$= \frac{i}{Z_{\phi,k}^2}\Big\{ \tilde{F}_{1,k}(-(p_i+p_l)) + \tilde{F}_{2,k}(-(p_i+p_l))$

$+ i\pi\left[F_{1,k}(-(p_i+p_l)) + F_{2,k}(-(p_i+p_l)) \right] \Big\}$



$= \frac{i}{Z_{\phi,k}^2}\Big\{ \tilde{F}_{1,k}(-(p_i+p_k)) + \tilde{F}_{2,k}(-(p_i+p_k))$

$+ i\pi\left[F_{1,k}(-(p_i+p_k)) + F_{2,k}(-(p_i+p_k)) \right] \Big\}$



$$= \frac{i}{z_{4,k}^2}\Big\{ \tilde{F}_{1,k}(-(p_i+p_j)) + \tilde{F}_{2,k}(-(p_i+p_j)) + i\pi\big[F_{1,k}(-(p_i+p_j)) + F_{2,k}(-(p_i+p_j))\big]\Big\}$$

得到

$$-\frac{i}{4}\frac{\delta^4 STr(GF)^2}{\delta\pi_{i,2}\,\delta\pi_{j,c}\,\delta\pi_{k,c}\,\delta\pi_{l,c}}$$

$$\begin{aligned}
=2\frac{\lambda_{4,k}^2}{z_{4,k}^2}\Big\{ \Big[&\Big(\tfrac{1}{2}[(N-1)+2]+1\Big)\Big[\tilde{F}_{1,k}(-(p_i+p_l)) + \tilde{F}_{2,k}(-(p_i+p_l)) \\
&\quad + i\pi\big(F_{1,k}(-(p_i+p_l)) + F_{2,k}(-(p_i+p_l))\big)\Big] \\
&+ \Big[\tilde{F}_{1,k}(-(p_i+p_k)) + \tilde{F}_{2,k}(-(p_i+p_k)) \\
&\quad + i\pi\big(F_{1,k}(-(p_i+p_k)) + F_{2,k}(-(p_i+p_k))\big)\Big] \\
&+ \Big[\tilde{F}_{1,k}(-(p_i+p_j)) + \tilde{F}_{2,k}(-(p_i+p_j)) \\
&\quad + i\pi\big(F_{1,k}(-(p_i+p_j)) + F_{2,k}(-(p_i+p_j))\big)\Big]\Big]\delta_{il}\delta_{jk} \\
+ \Big[&\Big(\tfrac{1}{2}[(N-1)+2]+1\Big)\Big[\tilde{F}_{1,k}(-(p_i+p_k)) + \tilde{F}_{2,k}(-(p_i+p_k)) \\
&\quad + i\pi\big(F_{1,k}(-(p_i+p_k)) + F_{2,k}(-(p_i+p_k))\big)\Big] \\
&+ \Big[\tilde{F}_{1,k}(-(p_i+p_l)) + \tilde{F}_{2,k}(-(p_i+p_l)) \\
&\quad + i\pi\big(F_{1,k}(-(p_i+p_l)) + F_{2,k}(-(p_i+p_l))\big)\Big] \\
&+ \Big[\tilde{F}_{1,k}(-(p_i+p_j)) + \tilde{F}_{2,k}(-(p_i+p_j)) \\
&\quad + i\pi\big(F_{1,k}(-(p_i+p_j)) + F_{2,k}(-(p_i+p_j))\big)\Big]\Big]\delta_{ik}\delta_{jl}
\end{aligned}$$

NO.

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$$+\Big[\left(\tfrac{1}{2}[(N-1)+2]+1\right)\Big[\tilde{\hat{F}}_{1,k}(-(p_i+p_j))+\hat{F}_{2,k}(-(p_i+p_j))$$

$$+i\pi\left(F_{1,k}(-(p_i+p_j))+F_{2,k}(-(p_i+p_j))\right)\Big]$$

$$+\Big[\tilde{F}_{1,k}(-(p_i+p_v))+\tilde{F}_{2,k}(-(p_i+p_v))$$

$$+i\pi\left(F_{1,k}(-(p_i+p_v))+F_{2,k}(-(p_i+p_v))\right)\Big]$$

$$+\Big[\tilde{\hat{F}}_{1,k}(-(p_i+p_k))+\tilde{\hat{F}}_{2,k}(-(p_i+p_k))$$

$$+i\pi\left(F_{1,k}(-(p_i+p_k))+F_{2,k}(-(p_i+p_k))\right)\Big]\Big]\delta_{ij}\delta_{kv}\Big\}$$

得到

$$\partial_t \lambda_{4\pi,\mathrm{eff}}(p_i,p_j,p_k,p_v)$$

$$=\frac{2\lambda_{4\pi}^2}{Z_{\phi,k}^2}\tilde{\partial}_t\Big\{\left(\tfrac{1}{2}[(N-1)+2]+1\right)\Big[\tilde{\hat{F}}_{1,k}(-(p_i+p_v))+\tilde{\hat{F}}_{2,k}(-(p_i+p_v))$$

$$+i\pi\left(F_{1,k}(-(p_i+p_v))+F_{2,k}(-(p_i+p_v))\right)\Big]$$

$$+\Big[\hat{F}_{1,k}(-(p_i+p_k))+\tilde{F}_{2,k}(-(p_i+p_k))$$

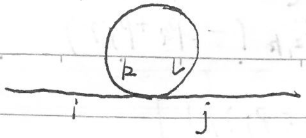
$$+i\pi\left(F_{1,k}(-(p_i+p_k))+F_{2,k}(-(p_i+p_k))\right)\Big]$$

$$+\Big[\hat{F}_{1,k}(-(p_i+p_j))+\hat{F}_{2,k}(-(p_i+p_j))$$

$$+i\pi\left(F_{1,k}(-(p_i+p_j))+F_{2,k}(-(p_i+p_j))\right)\Big]\Big\}$$

NO.

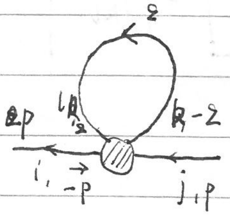
Date 演示计算



$$= i\lambda_{4\pi}(\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})\, i(G^{cc}_{\pi})_{kl}$$

$$= i\lambda_{4\pi}(\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})(iG^{cc}_{\pi})\delta_{kl}$$

$$= (i\lambda_{4\pi})(iG^{cc}_{\pi})(\delta_{ij}(N-1) + 2\delta_{ij})$$

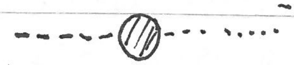


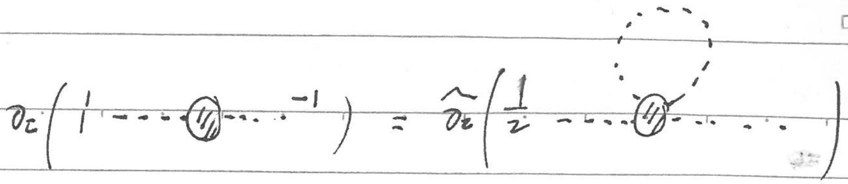
$$= i\Big[\lambda_{4\pi,\mathrm{eff}}(-p, p, -q, q)\,\delta_{il}\delta_{jk}$$
$$+ \lambda_{4\pi,\mathrm{eff}}(-p, -q, q, p)\,\delta_{ij}\delta_{kl}$$
$$+ \lambda_{4\pi,\mathrm{eff}}(-p, q, p, -q)\,\delta_{ik}\delta_{jl}\Big]\, iG^{cc}_{\pi}(q)\,\delta_{kl}$$

$$= i^2 G^{cc}_{\pi}(q)\Big[\lambda_{4\pi,\mathrm{eff}}(-p, p, -q, q)\,\delta_{ij}$$
$$+ \lambda_{4\pi,\mathrm{eff}}(-p, -q, q, p)\,\delta_{ij}(N-1)$$
$$+ \lambda_{4\pi,\mathrm{eff}}(-p, q, p, -q)\,\delta_{ij}\Big]$$

$$= i^2 G^{cc}_{\pi}(q)\Big[(N-1)\lambda_{4\pi,\mathrm{eff}}(-p, -q, q, p)$$
$$+ \lambda_{4\pi,\mathrm{eff}}(-p, p, -q, q) + \lambda_{4\pi,\mathrm{eff}}(-p, q, p, -q)\Big]\delta_{ij}$$

$$i\frac{\delta^2\Gamma_k}{\delta\pi_{i,q}(p')\,\delta\pi_{j,c}(p)} = i(Z_{\phi,k}(p^2)p^2 - m^2_{\pi,k})\,\delta_{ij}$$

$= i$  $^{-1}$



$$\Rightarrow$$

$$\partial_\tau\left(Z_{\phi,k}(p^2)\,p^2 - m^2_{\pi,k}\right)$$

$$= \frac{1}{2}\left(\tilde{\partial}_\tau G^{cc}_\pi(q)\right)\Big[(N-1)\,\lambda_{4\pi,\mathrm{eff}}(-p,-q,q,p) + \lambda_{4\pi,\mathrm{eff}}(-p,p,-q,q) + \lambda_{4\pi,\mathrm{eff}}(-p,q,p,-q)\Big]$$

$$\left(\tilde{\partial}_\tau G^{cc}_\pi(q)\right)$$

$$= \tilde{\partial}_\tau\left(\frac{-2i\varepsilon}{\left[Z_{\phi,k}(q_0^2 - \vec{q}^{\,2}(1+r_B)) - m^2_{\pi,k}\right]^2 + \varepsilon^2}\,\mathrm{sgn}(q_0)\coth\left(\frac{\beta q^0}{2}\right)\right)$$

$$= \partial_\tau\left(-\vec{q}^{\,2} Z_{\phi,k} r_B\right)(-1)\frac{2\left[Z_{\phi,k}(q_0^2 - \vec{q}^{\,2}(1+r_B)) - m^2_{\pi,k}\right]}{\left(\left[Z_{\phi,k}(q_0^2 - \vec{q}^{\,2}(1+r_B)) - m^2_{\pi,k}\right]^2 + \varepsilon^2\right)^2}\left(-2i\varepsilon\,\mathrm{sgn}(q_0)\right)$$

$$\times \coth\left(\frac{\beta q^0}{2}\right)$$

$$= \partial_\tau\left(-\vec{q}^{\,2} Z_{\phi,k} r_B\right)\left(-\frac{\partial}{\partial m^2_{\pi,k}}\frac{1}{\left[Z_{\phi,k}(q_0^2 - \vec{q}^{\,2}(1+r_B)) - m^2_{\pi,k}\right]^2 + \varepsilon^2}\right)\left(-2i\varepsilon\,\mathrm{sgn}(q_0)\right)$$

$$\times \coth\left(\frac{\beta q^0}{2}\right)$$

$$= \left(-\frac{\partial}{\partial m^2_{\pi,k}}\right)\partial_\tau\left(-\vec{q}^{\,2} Z_{\phi,k} r_B\right) G^{cc}_\pi(q)$$

$$\partial_\tau\left(Z_{\phi,k} r_B\right) = Z_{\phi,k}\left[\frac{k^2}{\vec{q}^{\,2}}(2-\eta_{\phi,k}) + \eta_{\phi,k}\right]\theta\left(1 - \frac{\vec{q}^{\,2}}{k^2}\right)$$

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$\hat{\frac{}{}}$

$$V(p^0,\vec{p},q^0,\vec{q})$$
$$= V(p^0,|\vec{p}|,q^0,|\vec{q}|,\cos\theta)$$
$$= (N-1)\lambda_{4\pi,\mathrm{eff}}(-p,-q,q,p) + \lambda_{4\pi,\mathrm{eff}}(-p,p,-q,q)$$
$$+ \lambda_{4\pi,\mathrm{eff}}(-p,q,p,-q)$$

$$(\Gamma_k^{(2)\pi\pi})^{sc}(p) \equiv Z_{\phi,k}(p^2)p^2 - m_{\pi,k}^2$$

$$\partial_\tau(\Gamma_k^{(2)\pi\pi})^{sc}(p)$$
$$= \frac{1}{2}\int\frac{d^4q}{(2\pi)^4}(\hat{\partial}_\tau G_\pi^{cc}(q))\, V(p^0,|\vec{p}|,q^0,|\vec{q}|,\cos\theta)$$
$$= \frac{1}{2}\int\frac{d^4q}{(2\pi)^4}\Big(\partial_\tau(-\vec{q}^{\,2}Z_{\phi,k}r_B)\Big)\Big[\Big(-\frac{\partial}{\partial m_{\pi,k}^2}\Big)G_\pi^{cc}(q)\Big]\, V(p^0,|\vec{p}|,q^0,|\vec{q}|,\cos\theta)$$
$$= \frac{1}{2}\frac{\partial}{\partial m_{\pi,k}^2}\int\frac{d^4q}{(2\pi)^4}\vec{q}^{\,2}\partial_\tau(Z_{\phi,k}r_B)\, G_\pi^{cc}(q)\, V(p^0,|\vec{p}|,q^0,|\vec{q}|,\cos\theta)$$
$$= \frac{1}{2}\frac{\partial}{\partial m_{\pi,k}^2}\int\frac{d^4q}{(2\pi)^4}Z_{\phi,k}\Big[k^2(2-\eta_{\phi,k}) + \vec{q}^{\,2}\eta_{\phi,k}\Big]\theta\Big(1-\frac{\vec{q}^{\,2}}{k^2}\Big)$$
$$\times\frac{1}{Z_{\phi,k}}(-i)2\pi\frac{1}{2E_{\pi,k}}\big(\delta(q_0-E_\pi)+\delta(q_0+E_\pi)\big)\coth\Big(\frac{\beta E_{\pi,k}}{2}\Big)$$
$$\times V(p^0,|\vec{p}|,q^0,|\vec{q}|,\cos\theta)$$

$$= \frac{1}{2}\frac{\partial}{\partial m_{\pi,k}^2}\int\frac{d^3q}{(2\pi)^3}\Big[k^2(2-\eta_{\phi,k}) + \vec{q}^{\,2}\eta_{\phi,k}\Big]\theta\Big(1-\frac{\vec{q}^{\,2}}{k^2}\Big)$$
$$\times\frac{1}{2E_{\pi,k}}\coth\Big(\frac{\beta E_{\pi,k}}{2}\Big)\Big(V\big|_{q^0=E_\pi} + V\big|_{q^0=-E_\pi}\Big)$$

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$$= \frac{1}{4}\frac{\partial}{\partial m_{\pi,k}^2}\left\{\frac{1}{E_{\pi,k}}\coth\left(\frac{\beta E_{\pi,k}}{2}\right)\int\frac{d^3q}{(2\pi)^3}\left[k^2(2-\eta_{\phi,k})+\vec{q}^{\,2}\eta_{\phi,k}\right]\theta\left(1-\frac{q^2}{k^2}\right)\right.$$
$$\left.\times\left(V\big|_{q^0=E_\pi}+V\big|_{q^0=-E_\pi}\right)\right\}$$

因为 $E_{\pi,k}=(k^2+\bar{m}_\pi^2)^{\frac{1}{2}}$ $\quad \bar{m}_{\pi,k}^2=\frac{m_{\pi,k}^2}{Z_{\phi,k}}$

$$\frac{\partial}{\partial m_{\pi,k}^2}=\frac{\partial E_{\pi,k}}{\partial m_{\pi,k}^2}\frac{\partial}{\partial E_{\pi,k}}=\frac{1}{2Z_{\phi,k}E_{\pi,k}}\frac{\partial}{\partial E_{\pi,k}}$$

所以

$$\partial_t\left(\Gamma_k^{(2)\pi\pi}\right)^{ge}(p)$$
$$=\frac{1}{8}\frac{1}{Z_{\phi,k}}\frac{1}{E_{\pi,k}}\frac{\partial}{\partial E_{\pi,k}}\left\{\frac{1}{E_{\pi,k}}\coth\left(\frac{\beta E_{\pi,k}}{2}\right)\int\frac{d^3q}{(2\pi)^3}\left[k^2(2-\eta_{\phi,k})+\vec{q}^{\,2}\eta_{\phi,k}\right]\theta\left(1-\frac{q^2}{k^2}\right)\right.$$
$$\left.\times\left(V\big|_{q^0=E_\pi}+V\big|_{q^0=-E_\pi}\right)\right\}$$

$$=\frac{1}{8}\frac{1}{(2\pi)^2}\frac{1}{Z_{\phi,k}}\frac{1}{E_{\pi,k}}\frac{\partial}{\partial E_{\pi,k}}\left\{\frac{1}{E_{\pi,k}}\coth\left(\frac{\beta E_{\pi,k}}{2}\right)\int_0^k dq\,q^2\int_{-1}^1 d\cos\theta\left[k^2(2-\eta_{\phi,k})+q^2\eta_{\phi,k}\right]\right.$$
$$\left.\times\left(V\big|_{q^0=E_\pi}+V\big|_{q^0=-E_\pi}\right)\right\}$$

$$=\frac{1}{8}\frac{1}{(2\pi)^2}\frac{1}{Z_{\phi,k}}\left[-\frac{\coth\left(\frac{\beta E_{\pi,k}}{2}\right)}{E_{\pi,k}^3}-\frac{\beta\left(\mathrm{csch}\left(\frac{\beta E_{\pi,k}}{2}\right)\right)^2}{2E_{\pi,k}^2}\right]\int_0^k dq\,q^2\int_{-1}^1 d\cos\theta$$
$$\times\left[k^2(2-\eta_{\phi,k})+q^2\eta_{\phi,k}\right]\left(V\big|_{q^0=E_\pi}+V\big|_{q^0=-E_\pi}\right)$$

$$+\frac{1}{8}\frac{1}{(2\pi)^2}\frac{1}{Z_{\phi,k}}\frac{1}{E_{\pi,k}^2}\coth\left(\frac{\beta E_{\pi,k}}{2}\right)\int_0^k dq\,q^2\int_{-1}^1 d\cos\theta\left[k^2(2-\eta_{\phi,k})+q^2\eta_{\phi,k}\right]$$
$$\times\left(\frac{\partial V}{\partial q^0}\bigg|_{q^0=E_\pi}-\frac{\partial V}{\partial q^0}\bigg|_{q^0=-E_\pi}\right)$$

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$$\tilde{F}_1(p_0, p, k) = \int_{-p}^{p} dp_0' \frac{1}{p_0' - p_0} F_1(p_0', p, k)$$

$$= \int_{-p}^{p} dp_0' \frac{F_1(p_0', p, k) - F_1(p_0, p, k) + F_1(p_0, p, k)}{p_0' - p_0}$$

$$= \int_{-p}^{p} dp_0' \frac{F_1(p_0', p, k) - F_1(p_0, p, k)}{p_0' - p_0}$$

$$= \int_{0}^{p} dp_0' \frac{F_1(p_0', p, k) - F_1(p_0, p, k)}{p_0' - p_0}$$

$$+ \int_{-p}^{0} dp_0' \frac{F_1(p_0', p, k) - F_1(p_0, p, k)}{p_0' - p_0}$$

$$= \int_{0}^{p} dp_0' \frac{F_1(p_0', p, k) - F_1(p_0, p, k)}{p_0' - p_0}$$

$$+ \int_{0}^{p} dp_0' \frac{-F_1(p_0', p, k) - F_1(p_0, p, k)}{-p_0' - p_0}$$

$$= \int_{0}^{p} dp_0' \left\{ \frac{F_1(p_0', p, k) - F_1(p_0, p, k)}{p_0' - p_0} + \frac{F_1(p_0', p, k) + F_1(p_0, p, k)}{p_0' + p_0} \right\}$$

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$$\left(\Gamma^{(2)\pi\pi}\right)^{RC}(p) \equiv \left(\Gamma^{(2)\pi\pi}_{k=0}\right)^{RC}(p)$$

$$\left(\Gamma^{(2)\pi\pi}\right)^{RC}(p) = \Gamma^{(2)\pi\pi}_{R}(p_0,\vec{p}) + i\,\Gamma^{(2)\pi\pi}_{I}(p_0,\vec{p})$$

在自由的情况下

$$\Gamma^{(2)\pi\pi}_{0}(p_0,\vec{p}) = p_0^2-\vec{p}^{\,2}-m_\pi^2 + \mathrm{sgn}(p_0)\, i\varepsilon$$

计算

$$\frac{1}{\Gamma(\vec{p})} = -i\left(\frac{\partial\Gamma^{(2)\pi\pi}(p_0,\vec{p})}{\partial p_0}\right)\Bigg|_{p_0=0} = -i\,\frac{\partial\Gamma^{(2)\pi\pi}_{R}(p_0,\vec{p})}{\partial p_0}\Bigg|_{p_0=0} + \frac{\partial\Gamma^{(2)\pi\pi}_{I}(p_0,\vec{p})}{\partial p_0}\Bigg|_{p_0=0}$$

$$-\Gamma^{(2)\pi\pi}_{0}(p_0,\vec{p}) = -p_0^2+\vec{p}^{\,2}+m_\pi^2 - i\left(\mathrm{sgn}(p_0)\varepsilon\right)$$

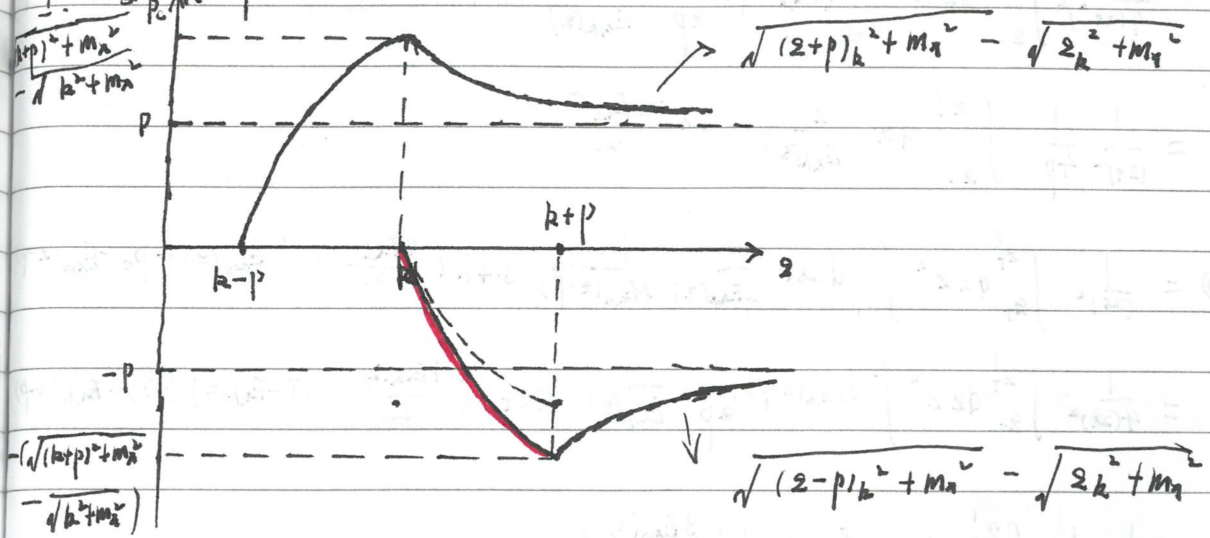
$$\frac{1}{-\Gamma^{(2)\pi\pi}_{0}(p_0,\vec{p})} = \frac{1}{-p_0^2+\vec{p}^{\,2}+m_\pi^2-i\left(\mathrm{sgn}(p_0)\varepsilon\right)}$$

$$= \frac{1}{-p_0^2+\vec{p}^{\,2}+m_\pi^2}\;\frac{1}{1-i\,\mathrm{sgn}(p_0)\dfrac{\varepsilon}{-p_0^2+\vec{p}^{\,2}+m_\pi^2}}$$

$$\omega(|\vec{p}|) = {}^{D}\Gamma(\vec{p})\left(-\Gamma^{(2)\pi\pi}(p_0=0,\vec{p})\right)$$

前面对②和③的计算没有考虑 $m_\pi^2<0$ 的情况，下面我们考虑该情况

I. 当 $k>p$



(1) 当 $p < p_0 \le \sqrt{(k+p)^2+m_\pi^2}-\sqrt{k^2+m_\pi^2}$

求解

$$\sqrt{(q_-+p)^2+m_\pi^2}-\sqrt{k^2+m_\pi^2}=p_0$$

$$\Rightarrow q_- = -p+\sqrt{(p_0+\sqrt{k^2+m_\pi^2})^2-m_\pi^2}$$

$$\sqrt{(q_-'+p)^2+m_\pi^2}-\sqrt{q_-'^2+m_\pi^2}=p_0$$

$$\Rightarrow q_-' = -\frac{p}{2}+\frac{\sqrt{p_0^2(p_0^2-p^2)(p_0^2-p^2-4m^2)}}{2(p_0^2-p^2)}$$

$$\sqrt{k^2+m_\pi^2}-\sqrt{q_+^2+m_\pi^2}=-p_0$$

$$\Rightarrow q_+ = q_-+p = \sqrt{(p_0+\sqrt{k^2+m_\pi^2})^2-m_\pi^2}$$

$$\sqrt{(q_+'-p)^2+m_\pi^2}-\sqrt{q_+'^2+m_\pi^2}=-p_0$$

$$\Rightarrow q_+' = q_-'+p = \frac{p}{2}+\frac{\sqrt{p_0^2(p_0^2-p^2)(p_0^2-p^2-4m^2)}}{2(p_0^2-p^2)}$$

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$$② = \frac{1}{(2\pi)^2}\int_{q_-}^{q_-'} dq\, q^2 \int_{-1}^{1} d\cos\theta \frac{1}{2E_{\pi,k}(q)}\frac{1}{2E_{\pi,k}(q-p)}\coth\left(\frac{\beta E_{\pi,k}(q)}{2}\right)\delta(-E_{\pi,k}(q)-p_0+E_{\pi,k}(q-p))$$

$$= \frac{1}{4(2\pi)^2}\int_{q_-}^{q_-'} dq\, q^2 \int dE_{\pi,k}(q-p)\frac{1}{qp}\frac{1}{E_{\pi,k}(q)}\coth\left(\frac{\beta E_{\pi,k}(q)}{2}\right)\delta(-E_{\pi,k}(q)-p_0+E_{\pi,k}(q-p))$$

$$= \frac{1}{(2\pi)^2}\frac{1}{4p}\int_{q_-}^{q_-'} dq\,\frac{q}{E_{\pi,k}(q)}\coth\left(\frac{\beta E_{\pi,k}(q)}{2}\right)$$

$$③ = \frac{1}{(2\pi)^2}\int_{q_+}^{q_+'} dq\, q^2 \int_{-1}^{1} d\cos\theta \frac{1}{2E_{\pi,k}(q)}\frac{1}{2E_{\pi,k}(q-p)}\coth\left(\frac{\beta E_{\pi,k}(q)}{2}\right)\delta(E_{\pi,k}(q)-p_0-E_{\pi,k}(q-p))$$

$$= \frac{1}{4(2\pi)^2}\int_{q_+}^{q_+'} dq\, q^2 \int dE_{\pi,k}(q-p)\frac{1}{qp}\frac{1}{E_{\pi,k}(q)}\coth\left(\frac{\beta E_{\pi,k}(q)}{2}\right)\delta(-E_{\pi,k}(q)+p_0+E_{\pi,k}(q-p))$$

$$= \frac{1}{(2\pi)^2}\frac{1}{4p}\int_{q_+}^{q_+'} dq\,\frac{q}{E_{\pi,k}(q)}\coth\left(\frac{\beta E_{\pi,k}(q)}{2}\right)$$

② − ③

$$= \frac{1}{(2\pi)^2}\frac{1}{4p}\int_{q_-}^{p} dq\,\frac{q}{E_{\pi,k}(q)}\coth\left(\frac{\beta E_{\pi,k}(q)}{2}\right)$$

$$- \frac{1}{(2\pi)^2}\frac{1}{4p}\int_{q_-'}^{p} dq\,\frac{q}{E_{\pi,k}(q)}\coth\left(\frac{\beta E_{\pi,k}(q)}{2}\right)$$

$$- \frac{1}{(2\pi)^2}\frac{1}{4p}\int_{q_+}^{p} dq\,\frac{q}{E_{\pi,k}(q)}\coth\left(\frac{\beta E_{\pi,k}(q)}{2}\right)$$

$$+ \frac{1}{(2\pi)^2}\frac{1}{4p}\int_{q_+'}^{p} dq\,\frac{q}{E_{\pi,k}(q)}\coth\left(\frac{\beta E_{\pi,k}(q)}{2}\right)$$

$$= \frac{1}{(2\pi)^2}\frac{1}{4p}\int_{q_-}^{q_+} dq\,\frac{q}{E_{\pi,k}(q)}\coth\left(\frac{\beta E_{\pi,k}(q)}{2}\right)$$

$$- \frac{1}{(2\pi)^2}\frac{1}{4p}\int_{q_-'}^{q_+'} dq\,\frac{q}{E_{\pi,k}(q)}\coth\left(\frac{\beta E_{\pi,k}(q)}{2}\right)$$

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$p_0+E_{\pi,k}(q-p)$

$= B(q_+, q_-, k) - A(q_+', q_-')$

(2) 当 $p_0 \le p$

② − ③ $= B(q_+, q_-, k)$

$p_0+E_{\pi,k}(q-p)$

此外 在圆中红线边界上对③的积分有贡献，在红线上

$E_{\pi,k}(q-p) = \sqrt{k^2+m_\pi^2}$ ，即 $|\vec{q}-\vec{p}|^2 \le k^2$

$\Rightarrow q^2+p^2-2qp\cos\theta \le k^2$

$\Rightarrow \cos\theta \ge \dfrac{q^2+p^2-k^2}{2qp} = \cos\theta_{min}$

$-E_{\pi,k}(q-p)$

$+E_{\pi,k}(q-p)$

$$③ = \int \frac{d^3q}{(2\pi)^3} \frac{1}{2E_{\pi,k}(q)} \frac{1}{2E_{\pi,k}(q-p)} \coth\left(\frac{\beta E_{\pi,k}(q)}{2}\right) \delta(E_{\pi,k}(q) - p_0 - E_{\pi,k}(q-p))$$

$$= \frac{1}{(2\pi)^2} \int_k^\infty dq\, q^2 \int_{\cos\theta_{min}}^1 d\cos\theta \frac{1}{2E_{\pi,k}(q)} \frac{1}{2E_{\pi,k}(k)} \coth\left(\frac{\beta E_{\pi,k}(q)}{2}\right)$$
$$\delta(E_{\pi,k}(q) - p_0 - E_{\pi,k}(k))$$

$E_{\pi,k}(q) = (q^2+m_\pi^2)^{\frac{1}{2}}$ $\quad dE_{\pi,k} = \dfrac{dq\, q}{E_{\pi,k}}$

$$③ = \frac{1}{(2\pi)^2} \int_{E_{\pi,k}(k)}^\infty dE_{\pi,k}\, E_{\pi,k}\, q \int_{\cos\theta_{min}}^1 d\cos\theta \frac{1}{2E_{\pi,k}(q)} \frac{1}{2E_{\pi,k}(k)} \coth\left(\frac{\beta E_{\pi,k}(q)}{2}\right)$$
$$\delta(E_{\pi,k}(q) - p_0 - E_{\pi,k}(k))$$

$$= \frac{1}{4}\frac{1}{(2\pi)^2} \frac{q_+}{E_{\pi,k}(k)} \coth\left(\frac{\beta E_{\pi,k}(q_+)}{2}\right) \int_{\cos\theta_{min}}^1 d\cos\theta$$

$$= \frac{1}{(2\pi)^2} \frac{1}{4E_{\pi,k}(k)} q_+ \coth\left(\frac{\beta E_{\pi,k}(q_+)}{2}\right) (1-\cos\theta_{min})$$

$$= \frac{1}{(2\pi)^2} \frac{1}{4E_{\pi,k}(k)} q_+ \coth\left(\frac{\beta E_{\pi,k}(q_+)}{2}\right) \left(1 - \frac{q_+^2+p^2-k^2}{2q_+p}\right)$$

$$= C(q_+, k)$$

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修改 (0) 当 $p_0 > \sqrt{(k+p)^2+m_\pi^2} - \sqrt{k^2+m_\pi^2}$

② − ③ = 0

(1) $p < p_0 \leq \sqrt{(k+p)^2+m_\pi^2} - \sqrt{k^2+m_\pi^2}$

② − ③ = $B(q_+, q_-, k) - A(q_+', q_-') - C(q_+, k)$

(2) $p_0 \leq p$

② − ③ = $B(q_+, q_-, k) - C(q_+, k)$