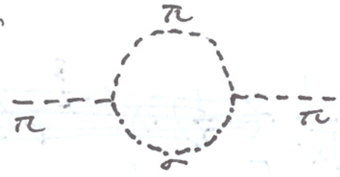
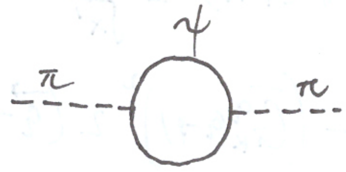
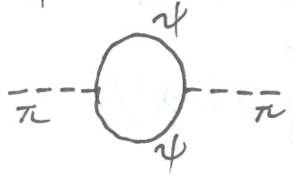
① π介子两点函数流方程.

$$\partial_t \Gamma^{(2)}_\pi = \tilde{\partial}_t \left\{ \right.$$  $$-$$  $$\left. \right\}$$

先计算夸克圈:

$$\tilde{\partial}_t$$  $$= -4N_c \delta_{ab} h_K^2 \frac{1}{Z_{q,k}^2} \tilde{\partial}_t \left\{ \bar{G}^q_k(q) \bar{G}^q_k(q-p) \left(q^F \cdot (q-p)^F + \bar{m}_f^2 \right) \right\}$$

$$\begin{aligned}
= -4N_c \delta_{ab} h^2 \frac{1}{Z_{q,k}^2} \Big\{ & \left(\tilde{\partial}_t \bar{G}^q_k(q)\right) \bar{G}^q_k(q-p) \left(q^F \cdot (q-p)^F + \bar{m}_f^2 \right) && ① \\
+ & \bar{G}^q_k(q) \left(\tilde{\partial}_t \bar{G}^q_k(q-p)\right) \left(q^F \cdot (q-p)^F + \bar{m}_f^2 \right) && ② \\
+ & \bar{G}^q_k(q) \bar{G}^q_k(q-p) \left[\left(\tilde{\partial}_t q^F\right) (q-p)^F \cancel{+\bar{m}_f^2} \right] && ③ \\
+ & \bar{G}^q_k(q) \bar{G}^q_k(q-p) \left[q^F \tilde{\partial}_t (q-p)^F \right] \Big\} && ④
\end{aligned}$$

$$\tilde{\partial}_t \bar{G}^q_k(q) = -2k^2 \left(\bar{G}^q_k(q)\right)^2 \left[(1-\eta_{q,k}) + \eta_{q,k} x^{\frac{1}{2}} \right] \theta(1-x)$$

$$\tilde{\partial}_t \bar{G}^q_k(q-p) = -\left(\bar{G}^q_k(q-p)\right)^2 \cdot 2(\vec{q}-\vec{p})^2 \left(1+r_F(x')\right)\left(\tilde{\partial}_t r_F(x')\right)$$

$$\tilde{\partial}_t r_F(x') = \frac{1}{Z_q} \partial_t \left(Z_q r_F(x') \right) = -\eta_{q,k} r_F(x') + r_F'(x')(-2)x'$$

$$\begin{aligned}
r_F'(x') &= \frac{d}{dx'}\left[\left(\frac{1}{\sqrt{x'}} - 1\right) \theta(1-x') \right] \\
&= -\frac{1}{2} x'^{-\frac{3}{2}} \theta(1-x') - \left(\frac{1}{\sqrt{x'}} - 1\right) \delta(1-x') \\
&= -\frac{1}{2} x'^{-\frac{3}{2}} \theta(1-x')
\end{aligned}$$

$$= -\eta_{q,k} \left(\frac{1}{\sqrt{x'}} - 1\right) \theta(1-x') + \frac{1}{\sqrt{x'}} \theta(1-x')$$

$$1 + r_F(x') = 1 + \left(\frac{1}{\sqrt{x'}} - 1\right) \theta(1-x')$$

② $1+\left(\frac{1}{\sqrt{x'}}-1\right)\theta(1-x') = \frac{1}{\sqrt{x'}}$

$$\partial_t \bar G^q_k(q-p) = -\left(\bar G^q_k(q-p)\right)^2 2(\vec q-\vec p)^2 \frac{1}{\sqrt{x'}} \cdot \cancel{\left(-\frac{1}{2}\right)\cdot x'^{-\frac{3}{2}}\,\theta(1-x)}$$

$$\left[-\eta_{q,k}\left(\frac{1}{\sqrt{x'}}-1\right)+\frac{1}{\sqrt{x'}}\right]\theta(1-x')$$

$$= -2\left(\bar G^q_k(q-p)\right)^2(\vec q-\vec p)^2\cdot\left[-\eta_{q,k}\left(x'^{-1}-x'^{-\frac{1}{2}}\right)+x'^{-1}\right]\theta(1-x')$$

$$= -2k^2\left(\bar G^q_k(q-p)\right)^2\left[-\eta_{q,k}\left(1-x'^{\frac{1}{2}}\right)+1\right]\theta(1-x')$$

$$= -2k^2\left(\bar G^q_k(q-p)\right)^2\left[(1-\eta_{q,k})+\eta_{q,k}x'^{\frac{1}{2}}\right]\theta(1-x')$$

$$\boxed{\begin{aligned} q_0(q_0-p_0) &= \frac{1}{2}\left[q_0^2+(q_0-p_0)^2-p_0^2\right] \\ &= \frac{1}{2}\Big[\bar G^{q\,-1}_k(q)+\bar G^{q\,-1}_k(q-p)-p_0^2 \\ &\qquad -\vec q^{\,2}(1+r_F(x))^2-(\vec q-\vec p)^2(1+r_F(x'))^2\Big]\end{aligned}}$$

第①部分：

$$\left(\tilde\partial_t \bar G^q_k(q)\right)\bar G^q_k(q-p)\left[q^F\cdot(q-p)^F+\bar m_f^2\right]$$

$$q^F\cdot(q-p)^F = q_0(q_0-p_0)+\vec q\cdot(\vec q-\vec p)(1+r_F(x))(1+r_F(x'))+\bar m_f^2$$

[1] $\left(\tilde\partial_t\bar G^q_k(q)\right)\bar G^q_k(q-p)\left[q_0(q_0-p_0)+\bar m_f^2\right]$

$$= -2k^2\left(\bar G^q_k(q)\right)^2\bar G^q_k(q-p)\left[(1-\eta_{q,k})+\eta_{q,k}x^{\frac{1}{2}}\right]\theta(1-x)$$

$$\times\frac{1}{2}\left[\bar G^{q\,-1}_k(q)+\bar G^{q\,-1}_k(q-p)-k^2-(\vec q-\vec p)^2(1+r_F(x'))^2-p_0^2\right]$$

$$= -k^2\left[(1-\eta_{q,k})+\eta_{q,k}x^{\frac{1}{2}}\right]\theta(1-x)$$

$$\times\left[\bar G^q_k(q)\bar G^q_k(q-p)+\left(\bar G^q_k(q)\right)^2-\left(k^2+(\vec q-\vec p)^2(1+r_F(x'))^2+p_0^2\right)\left(\bar G^q_k(q)\right)^2\bar G^q_k(q-p)\right]$$

$$= -k^2\left[(1-\eta_{q,k})+\eta_{q,k}x^{\frac{1}{2}}\right]\theta(1-x)$$

$$\times\left[\frac{1}{k^4}\tilde G^q_k(\tilde q)\tilde G^q_k(\tilde q-\tilde p)+\frac{1}{k^4}\left(\tilde G^q_k(\tilde q)\right)^2-\frac{1}{k^4}\left(1+x'(1+r_F(x'))^2+\tilde p_0^2\right)\left(\tilde G^q_k(\tilde q)\right)^2\tilde G^q_k(\tilde q-\tilde p)\right]$$

③ 恢复空间动量积分，求和

$$T\sum_n\int\frac{d^3q}{(2\pi)^3}\left(-\frac{1}{k^2}\right)\left[(1-\eta_{q,k})+\eta_{q,k}x^{\frac{1}{2}}\right]\theta(1-x)$$

$$\times\left[\tilde{G}^q_k(q)\,\tilde{G}^q_k(q-p)+\left(\tilde{G}^q_k(q)\right)^2-\left(1+x'(1+r_F(x'))^2+\tilde{p}_0^{\,2}\right)\left(\tilde{G}^q_k(q)\right)^2\tilde{G}^q_k(q-p)\right]$$

$$=\int\frac{d^3q}{(2\pi)^3}\left(-\frac{1}{k}\right)\left[(1-\eta_{q,k})+\eta_{q,k}x^{\frac{1}{2}}\right]\theta(1-x)$$

$$\times\left[F_1F_1^-+F_2-\left(1+x'(1+r_F(x'))^2+\tilde{p}_0^{\,2}\right)F_2F_1^-\right]$$

$$=\int_0^1dx\int_{-1}^1d\cos\theta\left(-\frac{V_3k^2}{4}\right)\left[(1-\eta_{q,k})x^{\frac{1}{2}}+\eta_{q,k}x\right]$$

$$\times\left[F_1F_1^-+F_2-\left(1+x'(1+r_F(x'))^2+\tilde{p}_0^{\,2}\right)F_2F_1^-\right]$$

---

[2] $\left(\tilde{\partial}_t\,\tilde{G}^q_k(q)\right)\tilde{G}^q_k(q-p)\quad\vec{q}\cdot(\vec{q}-\vec{p})\,(1+r_F(x'))(1+r_F(x))$

$$=-2k^2\left(\tilde{G}^q_k(q)\right)^2\tilde{G}^q_k(q-p)\left[(1-\eta_{q,k})+\eta_{q,k}x^{\frac{1}{2}}\right]\theta(1-x)$$

$$\times\,\vec{q}(\vec{q}-\vec{p})(1+r_F(x'))(1+r_F(x))$$

$$=-\frac{2}{k^2}\left(\tilde{G}^q_k(q)\right)^2\tilde{G}^q_k(q-p)\left[(1-\eta_{q,k})+\eta_{q,k}x^{\frac{1}{2}}\right]\theta(1-x)$$

$$\times\left(x^{\frac{1}{2}}-\tilde{p}\cos\theta\right)(1+r_F(x'))$$

$$=T\sum_n\int\frac{d^3q}{(2\pi)^3}\left(-\frac{2}{k^2}\right)\left(\tilde{G}^q_k(q)\right)^2\tilde{G}^q_k(q-p)\left[(1-\eta_{q,k})+\eta_{q,k}x^{\frac{1}{2}}\right]\theta(1-x)$$

$$\times\left(x^{\frac{1}{2}}-\tilde{p}\cos\theta\right)(1+r_F(x'))$$

$$=-\frac{V_3k^2}{2}\int_0^1dx\int_{-1}^1d\cos\theta\left[(1-\eta_{q,k})x^{\frac{1}{2}}+\eta_{q,k}x\right]$$

$$\times F_2F_1^-\left(x^{\frac{1}{2}}-\tilde{p}\cos\theta\right)(1+r_F(x'))$$

④ 第②部分：

$$\bar G^q_K(q)\left(\tilde\partial_t \bar G^q_K(q-p)\right)\left[q^F\cdot(q-p)^F+\bar m_f^2\right]$$

$$[1]\quad \bar G^q_K(q)\left(\tilde\partial_t \bar G^q_K(q-p)\right)\left[q_0(q_0-p_0)+\bar m_f^2\right]$$

$$=-2K^2\,\bar G^q_K(q)\left(\bar G^q_K(q-p)\right)^2\left[(1-\eta_{q,K})+\eta_{q,K}\,x'^{\frac12}\right]\theta(1-x')$$

$$\times\frac12\left[\bar G^{q\,-1}_K(q)+\bar G^{q\,-1}_K(q-p)-\left(\cancel{K^2}+(\vec q-\vec p)^2(1+r_F(x'))^2+p_0^2\right)\right]$$

$$\vec q^{\,2}(1+r_F(x))^2$$

$$=-K^2\left[(1-\eta_{q,K})+\eta_{q,K}\,x'^{\frac12}\right]\theta(1-x')$$

$$\vec q^{\,2}(1+r_F(x))^2$$

$$\times\left[\left(\bar G^q_K(q-p)\right)^2+\bar G^q_K(q)\,\bar G^q_K(q-p)-\left(\cancel{K^2}+(\vec q-\vec p)^2(1+r_F(x'))^2+p_0^2\right)\bar G^q_K(q)\left(\bar G^q_K(q-p)\right)^2\right]$$

对动量作代换：

令 $q-p=q'$ 则 $q=q'+p$.

则 $\bar G^q_K(q)\left(\tilde\partial_t \bar G^q_K(q-p)\right)\left[q^F(q-p)^F+\bar m_f^2\right]$ 可写为：

$$\bar G^q_K(q'+p)\left(\tilde\partial_t \bar G^q_K(q')\right)\left[(q'+p)^F(q'^F)+\bar m_f^2\right]$$

将 $q'=q$ 代入 则有：

$$\bar G^q_K(q+p)\left(\tilde\partial_t \bar G^q_K(q)\right)\left[q^F(q+p)^F+\bar m_f^2\right]$$

$$\cancel{[1]}\quad \bar G^q_K(q+p)\left(\tilde\partial_t \bar G^q_K(q)\right)\left[q_0(q_0+p_0)+\vec q\cdot(\vec q+\vec p)(1+r_F(x))(1+r_F(x''))+\bar m_f^2\right]$$

$$[1]=-2K^2\left(\bar G^q_K(q)\right)^2\bar G^q_K(q+p)\left[q_0(q_0+p_0)+\bar m_f^2\right]$$

$$\times\left[(1-\eta_{q,K})+\eta_{q,K}\,x^{\frac12}\right]\theta(1-x)$$

$$q_0(q_0+p_0)=q_0^2+q_0p_0$$

$$=q_0^2+\frac12\left[(q_0+p_0)^2-q_0^2-p_0^2\right]$$

$$=\frac12 q_0^2+\frac12(q_0+p_0)^2-\frac12 p_0^2$$

$$=\frac12\left[q_0^2+(q_0+p_0)^2-p_0^2\right]$$

⑤

$$= \frac{1}{2}\Big[(q_0^2+k^2+\bar{m}_f^2)-(k^2+\bar{m}_f^2)+\big((q_0+p_0)^2+(\vec{q}+\vec{p})^2(1+r_F(x''))^2+\bar{m}_f^2\big)$$

$$-\big((\vec{q}+\vec{p})^2(1+r_F(x''))^2+\bar{m}_f^2\big)-p_0^2\Big] \qquad \text{其中 } x''=\frac{(\vec{q}+\vec{p})^2}{k^2}$$

$$=\frac{1}{2}\Big[\bar{G}^{q\,-1}_{k}(q)+\bar{G}^{q\,-1}_{k}(q+p)-\big(k^2+(\vec{q}+\vec{p})^2(1+r_F(x''))^2\big)-p_0^2\Big]-\bar{m}_f^2$$

$$[1] = -2k^2\big(\bar{G}^q_k(q)\big)^2\bar{G}^q_k(q+p)\Big[(1-\eta_{q,k})+\eta_{q,k}x^{\frac{1}{2}}\Big]\theta(1-x)$$

$$\times\Big[\bar{G}^{q\,-1}_k(q)+\bar{G}^{q\,-1}_k(q+p)-\big(k^2+(\vec{q}+\vec{p})^2(1+r_F(x''))^2\big)-p_0^2\Big]$$

$$=-k^2\Big[(1-\eta_{q,k})+\eta_{q,k}x^{\frac{1}{2}}\Big]\theta(1-x)$$

$$\times\Big[\bar{G}^q_k(q)\bar{G}^q_k(q+p)+\big(\bar{G}^q_k(q)\big)^2-\big(k^2+(\vec{q}+\vec{p})^2(1+r_F(x''))^2+p_0^2\big)\big(\bar{G}^q_k(q)\big)^2\bar{G}^q_k(q+p)\Big]$$

$$=-\frac{1}{k^2}\Big[(1-\eta_{q,k})+\eta_{q,k}x^{\frac{1}{2}}\Big]\theta(1-x)$$

$$\times\Big[\tilde{G}^q_k(q)\tilde{G}^q_k(q+p)+\big(\tilde{G}^q_k(q)\big)^2-\big(1+x''(1+r_F(x''))^2+\tilde{p}_0^2\big)\big(\tilde{G}^q_k(q)\big)^2\tilde{G}^q_k(q+p)\Big]$$

$$=\frac{T}{k}\sum_n\int_0^1 dx\int_{-1}^1 d\cos\theta\left(-\frac{V_3k^2}{4}\right)\Big[(1-\eta_{q,k})x^{\frac{1}{2}}+\eta_{q,k}x\Big]\theta(1-x)$$

$$\times\Big[\tilde{G}^q_k(q)\tilde{G}^q_k(q+p)+\big(\tilde{G}^q_k(q)\big)^2-\big(1+x''(1+r_F(x''))^2+\tilde{p}_0^2\big)\big(\tilde{G}^q_k(q)\big)^2\tilde{G}^q_k(q+p)\Big]$$

$$=\int_0^1 dx\int_{-1}^1 d\cos\theta\left(-\frac{V_3k^2}{4}\right)\Big[(1-\eta_{q,k})x^{\frac{1}{2}}+\eta_{q,k}x\Big]\theta(1-x)$$

$$\times\Big[F_1F_1^{+}+F_2-\big(1+x''(1+r_F(x''))^2+\tilde{p}_0^2\big)F_2F_1^{+}\Big]$$

$$[2] = -2k^2\big(\bar{G}^q_k(q)\big)^2\bar{G}^q_k(q+p)\;\vec{q}\cdot(\vec{q}+\vec{p})(1+r_F(x))(1+r_F(x''))$$

$$\times\Big[(1-\eta_{q,k})+\eta_{q,k}x^{\frac{1}{2}}\Big]\theta(1-x)$$

⑥ $-2K^2 \cdot \left(\bar{G}^{a}_{1K}(q)\right)^2 \bar{G}^{a}_{1K}(q+p) \left[(1-\eta_{a,K}) + \eta_{a,K} x^{\frac{1}{2}}\right] \theta(1-x)$

$\times K^2 (x^{\frac{1}{2}} + \tilde{\vec{p}} \cos\theta)(1+r_F(x''))$

$= \frac{T}{K} \sum_n \int_0^1 dx \int_{-1}^1 d\cos\theta \left(-\frac{V_3 K^3}{2}\right) \frac{K^4}{K^{\cancel{6}}} \left(\tilde{G}^{a}_{1K}(q)\right)^2 \tilde{G}^{a}_{1K}(q+p) \left[(1-\eta_{a,K}) x^{\frac{1}{2}} + \eta_{a,K} x\right] \theta(1-x)$

$\times (x^{\frac{1}{2}} + \tilde{\vec{p}} \cos\theta)(1+r_F(x''))$

$= \int_0^1 dx \int_{-1}^1 d\cos\theta \left(-\frac{V_3 K^2}{2}\right) \left[(1-\eta_{a,K}) x^{\frac{1}{2}} + \eta_{a,K} x\right] \theta(1-x)$

$\times F_2 F_1^+ (x^{\frac{1}{2}} + \tilde{\vec{p}} \cos\theta)(1+r_F(x''))$

第③部分:

$\bar{G}^{a}_{1K}(q) \bar{G}^{a}_{1K}(q-p) \left[(\tilde{\partial}_t q^F) \cdot (q-p)^F\right]$

$(\tilde{\partial}_t q^F)(q-p)^F = \left[(1-\eta_{a,K}) x^{-\frac{1}{2}} + \eta_{a,K}\right] \theta(1-x)$

$\times \vec{q} \cdot (\vec{q} - \vec{p})(1+r_F(x'))$

$= K^2 \left[(1-\eta_{a,K}) + \eta_{a,K} x^{\frac{1}{2}}\right] \theta(1-x)$

$\times (x^{\frac{1}{2}} - \tilde{\vec{p}} \cos\theta)(1+r_F(x'))$

$= K^2 \bar{G}^{a}_{1K}(q) \bar{G}^{a}_{1K}(q-p) \left[(1-\eta_{a,K}) + \eta_{a,K} x^{\frac{1}{2}}\right] \theta(1-x)$

$\times (x^{\frac{1}{2}} - \tilde{\vec{p}} \cos\theta)(1+r_F(x'))$

$= \frac{1}{K^2} \left[(1-\eta_{a,K}) + \eta_{a,K} x^{\frac{1}{2}}\right] \theta(1-x)$

$\times \tilde{G}^{a}_{1K}(q) \tilde{G}^{a}_{1K}(q-p) (x^{\frac{1}{2}} - \tilde{\vec{p}} \cos\theta)(1+r_F(x'))$

$= T \sum_n \int_0^1 dx \int_{-1}^1 d\cos\theta \frac{V_3 K^3}{4} \cdot \frac{1}{K^2} \left[(1-\eta_{a,K}) x^{\frac{1}{2}} + \eta_{a,K} x\right] \theta(1-x)$

$\times \tilde{G}^{a}_{1K}(q) \tilde{G}^{a}_{1K}(q-p) (x^{\frac{1}{2}} - \tilde{\vec{p}} \cos\theta)(1+r_F(x'))$

$$= \int_0^1 dx \int_{-1}^{1} d\cos\theta \; \frac{V_3 K^2}{4} \left[(1-\eta_{q,k}) x^{\frac{1}{2}} + \eta_{q,k} x \right] \theta(1-x)$$

$$\times F_1 F_1^- \left(x^{\frac{1}{2}} - \tilde{p}\cos\theta \right)\left(1 + r_p(x')\right)$$

第④部分：

$$\bar{G}^q_{\bar{K}}(q) \; \bar{G}^q_{K}(q-p) \; q^F\left(\tilde{\partial}_t (q-p)^F\right)$$

作动量代换之后.

$$\bar{G}^q_{K}(q+p) \; \bar{G}^q_{\bar{K}}(q) \; (q+p)^F\left(\tilde{\partial}_t q^F\right)$$

$$\left(\tilde{\partial}_t q^F\right)\cdot (q+p)^F = \left[(1-\eta_{q,k}) x^{-\frac{1}{2}} + \eta_{q,k} \right] \theta(1-x) \; \vec{q}\cdot(\vec{q}+\vec{p})\left(1 + r_F(x'')\right)$$

$$= K^2 \left[(1-\eta_{q,k}) + \eta_{q,k} x^{\frac{1}{2}} \right] \theta(1-x) \left(x^{\frac{1}{2}} + \tilde{p}\cos\theta \right)\left(1 + r_F(x'')\right)$$

$$= K^2 \bar{G}^q_{\bar{K}}(q) \bar{G}^q_{\bar{K}}(q+p) \left[(1-\eta_{q,k}) + \eta_{q,k} x^{\frac{1}{2}} \right] \theta(1-x)$$

$$\times \left(x^{\frac{1}{2}} + \tilde{p}\cos\theta \right)\left(1 + r_F(x'')\right)$$

$$= T \sum_n \int \frac{d^3p}{(2\pi)^3} \frac{1}{K^2} \left[(1-\eta_{q,k}) + \eta_{q,k} x^{\frac{1}{2}} \right] \theta(1-x) \left(x^{\frac{1}{2}} + \tilde{p}\cos\theta \right)\left(1 + r_F(x'')\right) \times \bar{G}^q_{\bar{K}}(q) \bar{G}^q_{\bar{K}}(q+p)$$

$$= \frac{V_3 K^2}{4} \int_0^1 dx \int_{-1}^{1} d\cos\theta \left[(1-\eta_{q,k}) x^{\frac{1}{2}} + \eta_{q,k} x \right] \theta(1-x) \left(x^{\frac{1}{2}} + \tilde{p}\cos\theta \right)\left(1 + r_F(x'')\right)$$

$$\times F_1 F_1^+$$

4部分结合到一起：

$$
\begin{aligned}
\tilde{\partial}_t \;{-}{-}{-}\bigcirc{-}{-}{-} &= -4 N_c \delta_{ab} h_k^2 \frac{V_3 k^2}{Z_{q,k}^2} \int_0^1 dx \int_{-1}^1 d\cos\theta \left[(1-\eta_{q,k}) x^{\frac{1}{2}} + \eta_{q,k} x \right] \\
&\times \Big\{ -\frac{1}{4} \left[F_1 F_1^- + F_2 - \left(1 + x'(1+r_F(x'))^2 + \tilde{p}_0^2\right) F_2 F_1^- \right] \\
&\quad - \frac{1}{2} \left(x^{\frac{1}{2}} - \tilde{\vec{p}} \cos\theta\right)(1+r_F(x')) F_2 F_1^- \\
&\quad - \frac{1}{4} \left[F_1 F_1^+ + F_2 - \left(1 + x''(1+r_F(x''))^2 + \tilde{p}_0^2\right) F_2 F_1^+ \right] \\
&\quad - \frac{1}{2} \left(x^{\frac{1}{2}} + \tilde{\vec{p}} \cos\theta\right)(1+r_F(x'')) F_2 F_1^+ \\
&\quad + \frac{1}{4} \left(x^{\frac{1}{2}} - \tilde{\vec{p}} \cos\theta\right)(1+r_F(x')) F_1 F_1^- \\
&\quad + \frac{1}{4} \left(x^{\frac{1}{2}} + \tilde{\vec{p}} \cos\theta\right)(1+r_F(x'')) F_1 F_1^+ \Big\} \\
&= -2 N_c \delta_{ab} h_k^2 \frac{V_3 k^2}{Z_{q,k}^2} \int_0^1 dx \int_{-1}^1 d\cos\theta \left[(1-\eta_{q,k}) x^{\frac{1}{2}} + \eta_{q,k} x \right] \\
&\times \Big\{ -F_2 + \frac{1}{2} \left[-1 + \left(x^{\frac{1}{2}} - \tilde{\vec{p}} \cos\theta\right)(1+r_F(x')) \right] F_1 F_1^- \\
&\quad + \frac{1}{2} \left[\left(1 + x'(1+r_F(x'))^2 + \tilde{p}_0^2\right) - 2\left(x^{\frac{1}{2}} - \tilde{\vec{p}} \cos\theta\right)(1+r_F(x')) \right] F_2 F_1^- \\
&\quad + \frac{1}{2} \left[-1 + \left(x^{\frac{1}{2}} + \tilde{\vec{p}} \cos\theta\right)(1+r_F(x'')) \right] F_1 F_1^+ \\
&\quad + \frac{1}{2} \left[\left(1 + x''(1+r_F(x''))^2 + \tilde{p}_0^2\right) - 2\left(x^{\frac{1}{2}} + \tilde{\vec{p}} \cos\theta\right)(1+r_F(x'')) \right] F_2 F_1^+ \Big\}
\end{aligned}
$$

(9)

$$F_1 F_1^{+} = \sum \tilde{G}_{ik}^{q}(q_0, \vec{q})\ \tilde{G}_{ik}^{q}(q_0+p_0, \vec{q}+\vec{p})$$

$$= \sum \frac{1}{(q_0+i\mu)^2+E_q^2}\ \frac{1}{(q_0+p_0+i\mu)^2+E_{q+p}^2}$$

$$= \sum \frac{1}{-(iq_0-\mu)^2+E_q^2}\ \frac{1}{-(iq_0+ip_0-\mu)^2+E_{q+p}^2}$$

$$= \sum \frac{1}{(iq_0-\mu+E_q)(iq_0-\mu-E_q)}\ \frac{1}{(iq_0+ip_0-\mu+E_{q+p})(iq_0+ip_0-\mu-E_{q+p})}$$

Poles: $-E_q+\mu$, $+(E_q+\mu)$, $-ip_0-E_{q+p}+\mu$, $-ip_0+E_{q+p}+\mu$

$$= n_f(E_q+\mu)\ \frac{-1}{4E_qE_{q+p}}\left[\frac{1}{ip_0-E_q-E_{q+p}} - \frac{1}{ip_0-E_q+E_{q+p}}\right]$$

$$+ n_f(E_q+\mu)\ \frac{1}{4E_qE_{q+p}}\left[\frac{1}{ip_0+E_q-E_{q+p}} - \frac{1}{ip_0+E_q+E_{q+p}}\right]$$

$$+ n_f(-E_{q+p}+\mu)\ \frac{-1}{4E_qE_{q+p}}\left[\frac{1}{ip_0-E_q+E_{q+p}} - \frac{1}{ip_0+E_q+E_{q+p}}\right]$$

$$+ n_f(E_{q+p}+\mu)\ \frac{1}{4E_qE_{q+p}}\left[\frac{1}{ip_0-E_q-E_{q+p}} - \frac{1}{ip_0+E_q-E_{q+p}}\right]$$

$$n_f(-x) = 1 - n_f(x)$$

$$n_f(-E_q+\mu) = 1 - n_f(E_q-\mu)$$

$$n_f(-E_{q+p}+\mu) = 1 - n_f(E_{q+p}-\mu)$$

(10)

$$= \frac{K^3}{4E_q E_{q+p}} \left\{ \frac{1}{ip_0 - E_q - E_{q+p}} \left[-1 + n_f(E_q - \mu) + n_f(E_{q+p} + \mu) \right] \right.$$

无量纲化

$$+ \frac{1}{ip_0 + E_q - E_{q+p}} \left[n_f(E_q + \mu) - n_f(E_{q+p} + \mu) \right]$$

$$+ \frac{1}{ip_0 - E_q + E_{q+p}} \left[n_f(E_{q+p} - \mu) - n_f(E_q - \mu) \right]$$

$$\left. + \frac{1}{ip_0 + E_q + E_{q+p}} \left[1 - n_f(E_{q+p} - \mu) - n_f(E_q + \mu) \right] \right\}$$

$$= \frac{K^3}{4E_q E_{q+p}} \left\{ \frac{1}{ip_0 - E_q - E_{q+p}} \left[-1 + n_f(E_q, \mu) + n_f(E_{q+p}, -\mu) \right] \right.$$

$$+ \frac{1}{ip_0 + E_q - E_{q+p}} \left[n_f(E_q, -\mu) - n_f(E_{q+p}, -\mu) \right]$$

$$+ \frac{1}{ip_0 - E_q + E_{q+p}} \left[n_f(E_{q+p}, \mu) - n_f(E_q, \mu) \right]$$

$$\left. + \frac{1}{ip_0 + E_q + E_{q+p}} \left[1 - n_f(E_{q+p}, \mu) - n_f(E_q, -\mu) \right] \right\}$$

(11)

$$k\frac{\partial}{\partial k}\Gamma^{(2)} = \left(N_c \bar{h}_k^2 Z_{\phi,k} V_3 k^2\right) \int_0^1 d\frac{q^2}{k^2} \int_{-1}^1 d\cos\theta \left[(1-\eta_{q,k})\frac{q}{k} + \eta_{q,k}\frac{q^2}{k^2}\right]$$

$$\times \Big\{ -2F_2 \cdot k^3 + \left[-1 + \left(\frac{q}{k} - \frac{p}{k}\cos\theta\right)(1+r_F(x'))\right] F_1 F_1^- k^3$$

$$+ \left[\left(1 + \frac{(\vec{q}-\vec{p})^2}{k^2}(1+r_F(x'))^2 + \frac{p_0^2}{k^2}\right) - 2\left(\frac{q}{k} - \frac{p}{k}\cos\theta\right)(1+r_F(x'))\right] F_2 F_1^- k^5$$

$$+ \left[-1 + \left(\frac{q}{k} + \frac{p}{k}\cos\theta\right)(1+r_F(x''))\right] F_1 F_1^+ k^3$$

$$+ \left[\left(1 + \frac{(\vec{q}+\vec{p})^2}{k^2}(1+r_F(x''))^2 + \frac{p_0^2}{k^2}\right) - 2\left(\frac{q}{k} + \frac{p}{k}\cos\theta\right)(1+r_F(x''))\right] F_2 F_1^+ k^5 \Big\}$$

$$\frac{\partial}{\partial k}\Gamma^{(2)}_k = f(k)\, k \int_0^k \frac{2q}{k^2} dq \int_{-1}^1 d\cos\theta \left[(1-\eta_{q,k}) + \eta_{q,k}\frac{q}{k}\right]\frac{q}{k}$$

$$\times k^3 \times \Big\{ -2F_2 + \left[-1 + \frac{1}{k}(q - p\cos\theta)(1+r_F(x'))\right] F_1 F_1^-$$

$$+ \left[\left(k^2 + (\vec{q}-\vec{p})^2(1+r_F(x'))^2 + p_0^2\right) - 2k(q - p\cos\theta)(1+r_F(x'))\right] F_2 F_1^-$$

$$+ \left[-1 + \frac{1}{k}(q + p\cos\theta)(1+r_F(x''))\right] F_1 F_1^+$$

$$+ \left[\left(k^2 + (\vec{q}+\vec{p})^2(1+r_F(x''))^2 + p_0^2\right) - 2k(q + p\cos\theta)(1+r_F(x''))\right] F_2 F_1^+ \Big\}$$

$$\partial_k \Gamma^{(2)}_k = f(k) \int_0^k dq \int_{-1}^1 d\cos\theta \cdot q^2 \left[(1-\eta_{q,k})k + \eta_{q,k} q\right]$$

$$\times \{ \quad \}$$

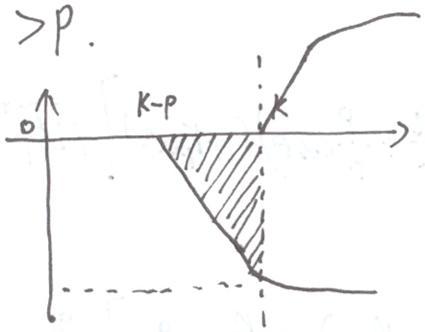
(12) $F_1F_1^-$ 和 $F_2F_1^-$ 部分与之前计算相同.

先计算 $F_1F_1^+$ 部分

$$f(k)\int_0^K dq\int_{-1}^{1} d\cos\theta\; q^2\left[(1-\eta_{q,k})k+\eta_{q,k}q\right]$$
$$\times\left[-1+\frac{1}{k}(q+p\cos\theta)(1+r_p(x''))\right]F_1F_1^+$$

$\delta(p_0-E_q+E_{q+p})$

A. $k>p$.



$1+\left(\frac{1}{\sqrt{x''}}-1\right)\theta(1-x'')$

在阴影区域 $\theta(1-x'')=0$

$\cancel{\frac{k}{\sqrt{x''}}=\sqrt{(q+p)^2}}$

[1] $E_q-\sqrt{(k+p)^2+m^2}\le p_0\le \cancel{E_q}\ 0$.

$$f(k)\int_0^K dq\int_{-1}^{1} d\cos\theta\; q^2\left[(1-\eta_{q,k})k+\eta_{q,k}q\right]$$
$$\times\left[-1+\frac{\frac{1}{k}(q+p\cos\theta)}{\cancel{\sqrt{q^2+p^2+2pq\cos\theta}}}\right]\frac{-\pi}{4E_qE_{q+p}}\left[n_f(E_{q+p},\mu)-n_f(E_q,\mu)\right]$$
$$\times\,\delta(p_0-E_q+E_{q+p})$$

$$=f(k)\int_{q_-}^K dq\int_{E_{min}}^{E_{max}} dE_{q+p}\left(+\frac{E_{q+p}}{p\cancel{q}}\right)q^{\cancel{2}}\left[(1-\eta_{q,k})k+\eta_{q,k}q\right]$$
$$\times\left[-1+\frac{1}{k}\left(q+\frac{E_{q+p}^2-q^2-p^2-m^2}{2q}\right)\right]\frac{-\pi}{4E_q\cancel{E_{q+p}}}\left[n_f(E_{q+p},\mu)-n_f(E_q,\mu)\right]\cdot\delta$$

$$=\frac{f(k)}{p}\int_{q_-}^K dq\; q\left[(1-\eta_{q,k})k+\eta_{q,k}q\right]$$
$$\times\left[-1+\frac{1}{k}\left(q+\frac{(E_q-p_0)^2-q^2-p^2-m^2}{2q}\right)\right]D_{III}(E_q,E_q-p_0)$$

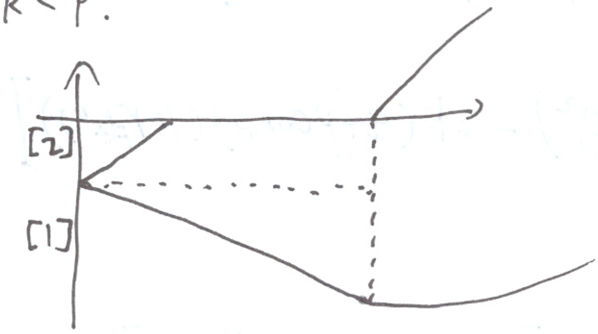
$$q_{mA1}=\sqrt{(E_q-p_0)^2-m^2}-p.$$

(13)

[2] $P_0 = 0$. 无贡献.

B、$\frac{p}{2} < K < p$.

图



[1] $-\sqrt{(K+p)^2+m^2} + E_q \leq P_0 \leq -\sqrt{p^2+m^2} + E_q$.

$$\frac{f(K)}{p}\int_{q_-}^{K} dq\, q\left[(1-\eta_{q,K})K + \eta_{q,K}\, q\right]$$

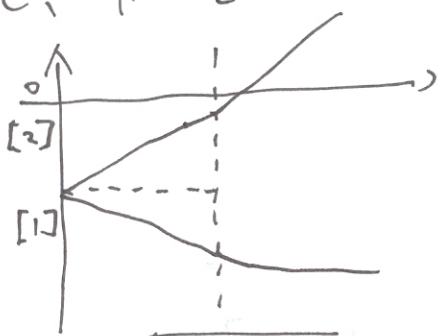
$$\times\left[-1 + \frac{1}{K}\left(q + \frac{(E_q-P_0)^2 - q^2 - p^2 - m^2}{2q}\right)\right] D_{III}\, E_q, (E_q - P_0)$$

$$q_m B1 = \sqrt{(E_q - P_0)^2 - m^2} - p$$

[2] $-\sqrt{p^2+m^2} + E_q \leq P_0 < 0$.

$$q_m B2 = p - \sqrt{(E_q - P_0)^2 - m^2}$$

C、$K < \frac{p}{2}$



[1] $-\sqrt{(p+K)^2+m^2} + E_q \leq P_0 \leq -\sqrt{p^2+m^2} + E_q$

$$q_m C1 = \sqrt{(E_q - P_0)^2 - m^2} - p$$

[2] $-\sqrt{p^2+m^2} + E_q \leq P_0 \leq -\sqrt{(p-K)^2+m^2} + E_q$

$$q_m C2 = p - \sqrt{(E_q - P_0)^2 - m^2}$$

⑭ $F_2 F_1^+$

$$f(k)\int_0^k dq\int_{-1}^1 d\cos\theta\ q^2\left[(1-\eta_{q,k})k+\eta_{q,k}q\right]$$

$$\times\left[\left(k^2+(\vec{q}+\vec{p})^2(1+r_F(x''))^2-p_0^2\right)-2k(q+p\cos\theta)(1+r_F(x''))\right]F_2F_1^+$$

$\delta(p_0-E_q+E_{q+p})$ 部分.

$$f(k)\int_{q_-}^k dq\int_{-1}^1 d\cos\theta\ q^2\left[(1-\eta_{q,k})k+\eta_{q,k}q\right]$$

$$\times\left[\left(k^2+q^2+p^2+2pq\cos\theta-p_0^2\right)-2k(q+p\cos\theta)\right]$$

$$\times\ F_2F_1^+$$

$$\lim_{\varepsilon\to 0}(\mathrm{Im}\,F_2F_1^+)=\left\{\frac{-\pi}{8E_q^2E_{q+p}}n_f'(E_q,\mu)+\frac{\pi}{8E_q^3E_{q+p}}\left[n_f(E_{q+p},\mu)-n_f(E_q,\mu)\right]\right\}$$

$$\times\,\delta(p_0-E_q+E_{q+p})$$

$$+\left\{\frac{\pi}{8E_q^2E_{q+p}}\left[n_f(E_q,\mu)-n_f(E_{q+p},\mu)\right]\right\}\delta'(p_0-E_q+E_{q+p})$$

A. $k>p$.

[1] $-\sqrt{(k+p)^2+m^2}+E_q\leq p_0<0$.

$$\frac{f(k)}{p}\int_{q_-}^k dq\ q\left[(1-\eta_{q,k})k+\eta_{q,k}q\right]$$

$$\times\left[k^2+E_{q+p}^2-m^2-p_0^2-2k\left(q+\frac{E_{q+p}^2-q^2-p^2-m^2}{2q}\right)\right]D_{\mathrm{III}}(E_q,E_{q+p})\Big|_{E_{q+p}=E_q-p_0}$$

$$=\frac{f(k)}{p}\int_{q_-}^k dq\ q\left[(1-\eta_{q,k})k+\eta_{q,k}q\right]$$

$$\times\left[k^2+(E_q-p_0)^2-m^2-p_0^2-2k\left(q+\frac{(E_q-p_0)^2-q^2-p^2-m^2}{2q}\right)\right]D_{\mathrm{III}}(E_q,E_q-p_0)$$

(15) $q_p A1 = \sqrt{(E_q - p_0)^2 - m^2} - p$.

[2] $p_0 = 0$. 边界贡献.

$$\frac{f(k)}{} \int_0^k dq \int_{-1}^{1} d\cos\theta \; q^2 \left[(1-\eta_{q,k}) k + \eta_{q,k} q \right]$$

$$\times \left[2k^2 - p_0^2 - 2k^2 \frac{q + p\cos\theta}{\sqrt{q^2+p^2+2pq\cos\theta}} \right] \frac{-\pi}{8E_q^2 E_{q+p}} n_f'(E_q, \mu) \, \delta(p_0 - E_q + E_{q+p})$$

① $0 < q < k-p$

$$\int_0^{k-p} dq \int_{-1}^{1} d\cos\theta$$

② $k-p < q < k$

$$\int_{k-p}^{k} dq \int_{\cos\theta_{min}}^{1} d\cos\theta$$

B. $\frac{p}{2} < k < k$.

[1] $-\sqrt{(k+p)^2+m^2} + E_q \le p_0 \le -\sqrt{p^2+m^2} + E_q$

$$q_p B1 = \sqrt{(E_q - p_0)^2 - m^2} - p$$

[2] $-\sqrt{p^2+m^2} + E_q \le p_0 < 0$.

$$q_p B2 = p - \sqrt{(E_q - p_0)^2 - m^2}$$

[3] $p_0 = 0$ 边界

与 A [2] ② 相同.

C. $k < \frac{p}{2}$

[1] $-\sqrt{(k+p)^2+m^2} + E_q \le p_0 \le -\sqrt{p^2+m^2} + E_q$

$$q_p C1 = \sqrt{(E_q - p_0)^2 - m^2} - p$$

[2] $-\sqrt{p^2+m^2} + E_q \le p_0 \le -\sqrt{(p-k)^2+m^2} + E_q$

$$q_p C2 = p - \sqrt{(E_q - p_0)^2 - m^2}$$

16 $\delta'(p_0 - E_q + E_{q+p})$ 部分

A、$k > p$

[1] $-\sqrt{(k+p)^2+m^2} + E_q \le p_0 < 0$.

$$\frac{f(k)}{\cancel{\phi}} \int_0^k dq \int_{-1}^{1} d\cos\theta \; f(p,q,k)^{\cos\theta} \frac{D_{p\text{III}}(E_q, E_{q+p})}{E_{q+p}} \delta'(p_0 - E_q + E_{q+p})$$

$$= \int_0^k dq \int_{E_{min}}^{E_{max}} dE_{q+p} \cdot \frac{\cancel{E_{q+p}}}{pq} f(p_0, p, q, \cos\theta, k, E_{q+p}) \frac{1}{\cancel{E_{q+p}}} \delta'(p_0 - E_q + E_{q+p})$$

$$= \int_0^k dq \int_{E_{min}}^{E_{max}} dE_{q+p} \; f(E_{q+p}) \, \delta'(p_0 - E_q + E_{q+p})$$

$$= \int_0^k dq \left[f(E_{max}) \delta(p_0 - E_q + E_{max}) - f(E_{min}) \delta(p_0 - E_q + E_{min}) - f'(E_q - p_0) \right]$$

$$= f(q_{0max}) - \int_{q_-}^{k} dq \, f(E_q) \, \delta(p_0 - 0) - \int_{q_-}^{k} dq \, f'(E_q - p_0)$$

$$q_{pA1} = \sqrt{(E_q - p_0)^2 - m^2} - p$$

+ 主值积分 + k 积分.

[2] $p_0 = 0$ 边界元贡献

B、$\frac{p}{2} < k < p$.

[1] $-\sqrt{(k+p)^2+m^2} + E_q \le p_0 \le -\sqrt{p^2+m^2} + E_q$

$$q_{pB1} = \sqrt{(E_q - p_0)^2 - m^2} - p$$

[2] $-\sqrt{p^2+m^2} + E_q \le p_0 < 0$.

$$q_{pB2} = p - \sqrt{(E_q - p_0)^2 - m^2}$$

C、$k < \frac{p}{2}$

[1] $-\sqrt{(k+p)^2+m^2} + E_q \le p_0 \le -\sqrt{p^2+m^2} + E_q$

$$q_{pC1} = \sqrt{(E_q - p_0)^2 - m^2} - p$$

[2] $-\sqrt{p^2+m^2} + E_q \le p_0 \le -\sqrt{(p-k)^2+m^2} + E_q$

$$q_{pC2} = p - \sqrt{(E_q - p_0)^2 - m^2}$$

(17)

$$
\begin{aligned}
\lim_{\varepsilon\to 0}\left(\mathrm{Im}\, F_2 F_1^{+}\right) = -\frac{K^{5}}{8E_q^2 E_{q+p}} \Bigg\{ & \frac{n_f'(E_q,\mu)}{p_0-E_q-E_{q+p}} + \frac{n_f'(E_q,-\mu)}{p_0+E_q-E_{q+p}} \\
& - \frac{n_f'(E_q,\mu)}{p_0-E_q+E_{q+p}} - \frac{n_f'(E_q,-\mu)}{p_0+E_q+E_{q+p}} \\
& + \frac{1}{(p_0-E_q-E_{q+p})^2}\left[-1+n_f(E_q,\mu)+n_f(E_{q+p},-\mu)\right] \\
& + \frac{1}{(p_0+E_q-E_{q+p})^2}\left[n_f(E_{q+p},-\mu)-n_f(E_q,-\mu)\right] \\
& + \frac{1}{(p_0-E_q+E_{q+p})^2}\left[n_f(E_{q+p},\mu)-n_f(E_q,\mu)\right] \\
& + \frac{1}{(p_0+E_q+E_{q+p})^2}\left[-1+n_f(E_q,-\mu)+n_f(E_{q+p},+\mu)\right] \Bigg\} \\
+ \frac{K^{5}}{8E_q^3 E_{q+p}} \Bigg\{ & \frac{1}{p_0-E_q-E_{q+p}}\left[-1+n_f(E_q,\mu)+n_f(E_{q+p},-\mu)\right] \\
& + \frac{1}{p_0+E_q-E_{q+p}}\left[n_f(E_q,-\mu)-n_f(E_{q+p},-\mu)\right] \\
& + \frac{1}{p_0-E_q+E_{q+p}}\left[n_f(E_{q+p},\mu)-n_f(E_q,\mu)\right] \\
& + \frac{1}{p_0+E_q+E_{q+p}}\left[1-n_f(E_q,-\mu)-n_f(E_{q+p},\mu)\right] \Bigg\}
\end{aligned}
$$

(18)

$$\lim_{\varepsilon\to 0} \mathrm{Im}\left(F_2 F_1^+\right) = \left\{ \frac{\pi}{8E_q^2 E_{q+p}} n_F'(E_q,\mu) - \frac{\pi}{8E_q^3 E_{q+p}} \left[-1+n_F(E_q,\mu)+n_F(E_{q+p},-\mu)\right] \right\}$$

$$\times \delta(p_0 - E_q - E_{q+p})$$

$$+ \left\{ \frac{\pi}{8E_q^2 E_{q+p}} n_F'(E_q,-\mu) - \frac{\pi}{8E_q^3 E_{q+p}} \left[n_F(E_q,-\mu) - n_F(E_{q+p},-\mu)\right] \right\}$$

$$\times \delta(p_0 + E_q - E_{q+p})$$

$$+ \left\{ -\frac{\pi}{8E_q^2 E_{q+p}} n_F'(E_q,\mu) - \frac{\pi}{8E_q^3 E_{q+p}} \left[n_F(E_{q+p},\mu) - n_F(E_q,\mu)\right] \right\}$$

$$\times \delta(p_0 - E_q + E_{q+p})$$

$$+ \left\{ -\frac{\pi}{8E_q^2 E_{q+p}} n_F'(E_q,-\mu) - \frac{\pi}{8E_q^3 E_{q+p}} \left[1 - n_F(E_q,-\mu) - n_F(E_{q+p},\mu)\right] \right\}$$

$$\times \delta(p_0 + E_q + E_{q+p})$$

$$+ \left\{ -\frac{\pi}{8E_q^2 E_{q+p}} \left[-1+n_F(E_q,\mu)+n_F(E_{q+p},-\mu)\right] \right\} \delta'(p_0 - E_q - E_{q+p})$$

$$+ \left\{ -\frac{\pi}{8E_q^2 E_{q+p}} \left[n_F(E_{q+p},-\mu) - n_F(E_q,-\mu)\right] \right\} \delta'(p_0 + E_q - E_{q+p})$$

$$+ \left\{ -\frac{\pi}{8E_q^2 E_{q+p}} \left[n_F(E_{q+p},\mu) - n_F(E_q,\mu)\right] \right\} \delta'(p_0 - E_q + E_{q+p})$$

$$+ \left\{ -\frac{\pi}{8E_q^2 E_{q+p}} \left[-1+n_F(E_q,-\mu)+n_F(E_{q+p},\mu)\right] \right\} \delta'(p_0 + E_q + E_{q+p})$$

① 区分 $Z_0(\vec{p})$ 和 $Z_s(\vec{p})$ 谱函数介子圈计算

$$\tilde{\partial}_t$$ 

$$= \left(V_{2\pi\sigma}(\rho)\right)^2 \left[\left(\tilde{\partial}_t G^{\pi}_{k}(q)\right) G^{\sigma}_{k}(q-p) + G^{\pi}_{k}(q)\left(\tilde{\partial}_t G^{\sigma}_{k}(q-p)\right)\right]$$

$$= \left(V_{2\pi\sigma}(\rho)\right)^2 \left[\left(\tilde{\partial}_t G^{\pi}_{k}(q)\right) G^{\sigma}_{k}(q-p) + G^{\pi}_{k}(q+p)\left(\tilde{\partial}_t G^{\sigma}_{k}(q)\right)\right]$$

$$G^{\pi/\sigma}_{k}(q) = \frac{1}{\left(Z_0(\vec{q})\, q_0^2 + Z_s(\vec{q})\, \vec{q}^{\,2}\right) + Z_s(0)\, \vec{q}^{\,2}\, r_B(x) + m_\pi^2}$$

$$= \frac{1}{Z_0(\vec{q})} \frac{1}{q_0^2 + \frac{Z_s(\vec{q})}{Z_0(\vec{q})}\vec{q}^{\,2} + \frac{Z_s(0)}{Z_0(\vec{q})}\vec{q}^{\,2} r_B(x) + \frac{m_\pi^2}{Z_0(\vec{q})}}$$

$$= \frac{1}{Z_0(\vec{q})} \cdot \frac{1}{q_0^2 + \vec{q}^{\,2}\left(\frac{Z_s(\vec{q})}{Z_0(\vec{q})} + \frac{Z_s(0)}{Z_0(\vec{q})} r_B(x)\right) + \frac{m_\pi^2}{Z_0(\vec{q})}}$$

$$\tilde{\partial}_t \bar{G}^{\pi/\sigma}_{k}(\vec{q}) = \tilde{\partial}_t \frac{1}{q_0^2 + \vec{q}^{\,2}\left(\frac{Z_s(\vec{q})}{Z_0(\vec{q})} + \frac{Z_s(0)}{Z_0(\vec{q})} r_B(x)\right) + \frac{m_\pi^2}{Z_0(\vec{q})}}$$

$$= -\left(\bar{G}^{\pi/\sigma}_{k}(q)\right)^2 \cdot \partial_t\left[\vec{q}^{\,2} \frac{Z_s(0)}{Z_0(\vec{q})} r_B(x)\right]$$

$$= -\left(\bar{G}^{\pi/\sigma}_{k}(q)\right)^2 \cdot \frac{\vec{q}^{\,2}}{Z_0(\vec{q})}\, \partial_t\left[Z_s(0) r_B(x)\right]$$

$$= -\left(\bar{G}^{\pi/\sigma}_{k}(q)\right)^2 \frac{\vec{q}^{\,2}}{Z_0(\vec{q})} \cdot \left[-Z_s(0)\, \eta_\phi\, r_B(x) + Z_s(0)\left(\partial_t r_B(x)\right)\right]$$

$$= -\left(\bar{G}^{\pi/\sigma}_{k}(q)\right)^2 \frac{Z_s(0)}{Z_0(\vec{q})} \vec{q}^{\,2} \left[-\eta_\phi\left(\frac{1}{x} - 1\right) + 2\frac{1}{x}\right] \theta(1-x)$$

$$= -k^2 \left(\bar{G}^{\pi/\sigma}_{k}(q)\right)^2 \frac{Z_s(0)}{Z_0(\vec{q})} \left[(x-1)\eta_\phi + 2\right] \theta(1-x)$$

$$= -k^2 \left(\bar{G}^{\pi/\sigma}_{k}(q)\right)^2 \frac{Z_s(0)}{Z_0(\vec{q})} \left[(2-\eta_\phi) + x\eta_\phi\right] \theta(1-x)$$

2

$$G_{IK}^{\sigma}(q-p) = \frac{1}{Z_0(\vec{q}-\vec{p})\,(q_0-p_0)^2 + Z_s(\vec{q}-\vec{p})\,(\vec{q}-\vec{p})^2 + Z_s(0)\,(\vec{q}-\vec{p})^2\, r_B(x') + m_\sigma^2}$$

$$= \frac{1}{Z_0(\vec{q}-\vec{p})}\;\frac{1}{(q_0-p_0)^2 + \frac{Z_s(\vec{q}-\vec{p})}{Z_0(\vec{q}-\vec{p})}(\vec{q}-\vec{p})^2 + \frac{Z_s(0)}{Z_0(\vec{q}-\vec{p})}(\vec{q}-\vec{p})^2\, r_B(x') + \frac{m_\sigma^2}{Z_0(\vec{q}-\vec{p})}}$$

$$= \frac{1}{Z_0(\vec{q}-\vec{p})}\;\frac{1}{(q_0-p_0)^2 + (\vec{q}-\vec{p})^2\left[\frac{Z_s(\vec{q}-\vec{p})}{Z_0(\vec{q}-\vec{p})} + \frac{Z_s(0)}{Z_0(\vec{q}-\vec{p})} r_B(x')\right] + \frac{m_\sigma^2}{Z_0(\vec{q}-\vec{p})}}$$

$$G_{IK}^{\pi}(q+p) = \frac{1}{Z_0(\vec{q}+\vec{p})\,(q_0+p_0)^2 + Z_s(\vec{q}+\vec{p})\,(\vec{q}+\vec{p})^2 + Z_s(0)\,(\vec{q}+\vec{p})^2\, r_B(x'') + m_\pi}$$

$$= \frac{1}{Z_0(\vec{q}+\vec{p})}\;\frac{1}{(q_0+p_0)^2 + \frac{Z_s(\vec{q}+\vec{p})}{Z_0(\vec{q}+\vec{p})}(\vec{q}+\vec{p})^2 + \frac{Z_s(0)}{Z_0(\vec{q}+\vec{p})}(\vec{q}+\vec{p})^2\, r_B(x'') + \frac{m_\pi^2}{Z_0(\vec{q}+\vec{p})}}$$

$\tilde{\partial}_t$  $= \left(V_{2\pi\sigma}(p)\right)^2\left[\left(\tilde{\partial}_t G_{IK}^{\pi}(q)\right) G_{IK}^{\sigma}(q-p) + G_{IK}^{\pi}(q+p)\left(\tilde{\partial}_t G_{IK}^{\sigma}(q)\right)\right]$

$$= \left(V_{2\pi\sigma}(p)\right)^2\left[\frac{1}{Z_0(\vec{q})\,Z_0(\vec{q}-\vec{p})}\left(\tilde{\partial}_t \bar{G}_{K}^{\pi}(q)\right)\bar{G}_{IK}^{\sigma}(q-p) + \frac{1}{Z_0(\vec{q})\,Z_0(\vec{q}+\vec{p})}\bar{G}_{K}^{\pi}(q+p)\left(\tilde{\partial}_t \bar{G}_{IK}^{\sigma}(q)\right)\right]$$

$$= \left(V_{2\pi\sigma}(p)\right)^2\left\{\frac{1}{Z_0(\vec{q})\,Z_0(\vec{q}-\vec{p})}\cdot\frac{Z_s(0)}{Z_0(\vec{q})\,\cancel{Z_0(\vec{q}-\vec{p})}}(-k^2)\left(\bar{G}_{IK}^{\pi}(q)\right)^2\bar{G}_{IK}^{\sigma}(q-p)\left[(2-\eta_\phi)+\eta_\phi x\right]\right.$$

$$\left. + \frac{1}{Z_0(\vec{q})\,Z_0(\vec{q}+\vec{p})}\,\frac{Z_s(0)}{Z_0(\vec{q})}(-k^2)\left(\bar{G}_{IK}^{\sigma}(q)\right)^2\bar{G}_{K}^{\pi}(q+p)\left[(2-\eta_\phi)+\eta_\phi x\right]\right\}$$

$$\times\,\theta(1-x)$$

③ $$= -K^2 \frac{\left(V_{2\pi\sigma}(p)\right)^2 Z_s(0)}{Z_0(\vec{q})^2} \left[(2-\eta_\phi)+\eta_\phi x\right]\theta(1-x)$$

$$\times \left\{ \frac{1}{Z_0(\vec{q}-\vec{p})} \left(\tilde{G}^{\pi}_{K}(q)\right)^2 \tilde{G}^{\sigma}_{K}(q-p) + \frac{1}{Z_0(\vec{q}+\vec{p})} \tilde{G}^{\pi}_{K}(q+p)\left(\tilde{G}^{\sigma}_{K}(q)\right)^2 \right\}$$

$$= -\frac{\left(V_{2\pi\sigma}(p)\right)^2 Z_s(0)}{Z_0(\vec{q})^2} \cdot \frac{1}{K^4} \left[(2-\eta_\phi)+\eta_\phi x\right]\theta(1-x)$$

$$\times \left\{ \frac{1}{Z_0(\vec{q}-\vec{p})} \left(\tilde{G}^{\pi}_{K}(q)\right)^2 \tilde{G}^{\sigma}_{K}(q-p) + \frac{1}{Z_0(\vec{q}+\vec{p})} \tilde{G}^{\pi}_{K}(q+p)\left(\tilde{G}^{\sigma}_{K}(q)\right)^2 \right\}$$

恢复积分

$$= -\left(V_{2\pi\sigma}(p)\right)^2 Z_s(0) \cdot \frac{V_3}{4} K^3 \int_0^1 dx \int_{-1}^{1} d\cos\theta \cdot \frac{1}{K^4} \left[(2-\eta_\phi)x^{\frac{1}{2}}+\eta_\phi x^{\frac{3}{2}}\right] \frac{1}{Z_0(\vec{q})^2}$$

$$\times K\, T_K \sum \left\{ \frac{1}{Z_0(\vec{q}-\vec{p})} \left(\tilde{G}^{\pi}_{K}(q)\right)^2 \tilde{G}^{\sigma}_{K}(q-p) + \frac{1}{Z_0(\vec{q}+\vec{p})} \tilde{G}^{\pi}_{K}(q+p)\left(\tilde{G}^{\sigma}_{K}(q)\right)^2 \right\}$$

$$= -\frac{V_3}{4}\left(V_{2\pi\sigma}(p)\right)^2 Z_s(0) \int_0^1 dx \int_{-1}^{1} d\cos\theta \left[(2-\eta_\phi)x^{\frac{1}{2}}+\eta_\phi x^{\frac{3}{2}}\right]$$

$$\times \left\{ \frac{1}{Z_0(\vec{q})^2 Z_0(\vec{q}-\vec{p})} B_2B_1(\tilde{m}_\pi^2, \tilde{m}_\sigma^2; \vec{q}-\vec{p}) + \frac{1}{Z_0(\vec{q})^2 Z_0(\vec{q}+\vec{p})} B_1B_2(\tilde{m}_\pi^2, \tilde{m}_\sigma^2; \vec{q}+\vec{p}) \right\}$$

④ $B_1 B_1 (m_1^2, m_2^2) = \sum \tilde{G}^1_K(q_0, \vec{q})\, \tilde{G}^2_K(q_0+p_0, \vec{q}+\vec{p})$

$$= \sum \frac{1}{q_0^2 + E_q^2} \frac{1}{(q_0+p_0)^2 + E_{q+p}^2}$$

$$= \sum \frac{1}{-(iq_0)^2 + E_q^2} \frac{1}{-(iq_0+ip_0)^2 + E_{q+p}^2}$$

$$= \sum \frac{1}{(iq_0+E_q)(iq_0-E_q)} \frac{1}{(iq_0+ip_0+E_{q+p})(iq_0+ip_0-E_{q+p})}$$

Poles: $-E_q$, E_q, $-ip_0 - E_{q+p}$, $-ip_0 + E_{q+p}$.

$$= -\Big\{ n_b(-E_q) \frac{-1}{2E_q} \frac{1}{(ip_0 - E_q + E_{q+p})(ip_0 - E_q - E_{q+p})}$$

$$+ n_b(E_q) \frac{1}{2E_q} \frac{1}{(ip_0 + E_q + E_{q+p})(ip_0 + E_q - E_{q+p})}$$

$$+ n_b(-E_{q+p}) \frac{-1}{2E_{q+p}} \frac{1}{(ip_0 - E_q + E_{q+p})(ip_0 + E_q + E_{q+p})}$$

$$+ n_b(E_{q+p}) \frac{1}{2E_{q+p}} \frac{1}{(ip_0 - E_q - E_{q+p})(ip_0 + E_q - E_{q+p})} \Big\}$$

$$\boxed{\begin{aligned} n_b(-x) &= \frac{1}{e^{-x}-1} = \frac{e^x}{1-e^x} = \frac{e^x - 1 + 1}{1-e^x} \\ &= -\frac{e^x - 1 + 1}{e^x - 1} = -(1 + n_b(x)) \end{aligned}}$$

5

$$= -\Bigg\{ n_b(-E_q) \frac{-1}{4E_q E_{q+p}} \left[\frac{1}{ip_0 - E_q - E_{q+p}} - \frac{1}{ip_0 - E_q + E_{q+p}} \right]$$

$$+ n_b(E_q) \frac{1}{4E_q E_{q+p}} \left[\frac{1}{ip_0 + E_q - E_{q+p}} - \frac{1}{ip_0 + E_q + E_{q+p}} \right]$$

$$+ n_b(-E_{q+p}) \frac{-1}{4E_q E_{q+p}} \left[\frac{1}{ip_0 - E_q + E_{q+p}} - \frac{1}{ip_0 + E_q + E_{q+p}} \right]$$

$$+ n_b(E_{q+p}) \frac{1}{4E_q E_{q+p}} \left[\frac{1}{ip_0 - E_q - E_{q+p}} - \frac{1}{ip_0 + E_q - E_{q+p}} \right] \Bigg\}$$

$$= \frac{-K^3}{4E_q E_{q+p}} \Bigg\{ \frac{1}{ip_0 - E_q - E_{q+p}} \left[1 + n_b(E_q) + n_b(E_{q+p}) \right]$$

$$+ \frac{1}{ip_0 + E_q - E_{q+p}} \left[n_b(E_q) - n_b(E_{q+p}) \right]$$

$$+ \frac{1}{ip_0 - E_q + E_{q+p}} \left[n_b(E_{q+p}) - n_b(E_q) \right]$$

$$+ \frac{1}{ip_0 + E_q + E_{q+p}} \left[-1 - n_b(E_q) - n_b(E_{q+p}) \right] \Bigg\}$$

(b)

$$G_K^B(q) = \frac{1}{q_0^2 + \frac{Z_s(\vec{q})}{Z_0(\vec{q})}\vec{q}^2 + \frac{Z_s(0)}{Z_0(\vec{q})}\vec{q}^2 r_B(x) + \frac{m^2}{Z_0(\vec{q})}}$$

$$= \frac{1}{\tilde{q}_0^2 + E_B^2(\vec{q})}\frac{1}{K^2}$$

$$E_B(\vec{q}) = K\sqrt{\frac{Z_s(\vec{q})}{Z_0(\vec{q})}\frac{\vec{q}^2}{K^2} + \frac{Z_s(0)}{Z_0(\vec{q})}\frac{\vec{q}^2}{K^2}\cdot\left(\frac{1}{x}-1\right)\theta(1-x) + \frac{m^2}{Z_0(\vec{q})K^2}}$$

$$= K\sqrt{\frac{1}{Z_0(\vec{q})}\left[Z_s(\vec{q})x + Z_s(0)\,x\left(\frac{1}{x}-1\right)\right] + \frac{m^2}{Z_0(\vec{q})K^2}}$$

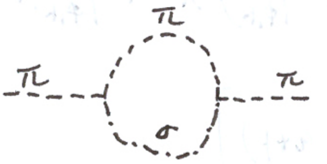
$$= K\sqrt{\frac{1}{Z_0(\vec{q})}\left[Z_s(0) + x\left(Z_s(\vec{q}) - Z_s(0)\right)\right] + \frac{m^2}{Z_0(\vec{q})K^2}}$$

$$= \sqrt{\frac{Z_s(0)}{Z_0(\vec{q})}K^2 + \frac{Z_s(\vec{q}) - Z_s(0)}{Z_0(\vec{q})}\vec{q}^2 + \frac{m^2}{Z_0(\vec{q})}}$$

$$G_K^B(q\pm p) = \frac{1}{(q_0 \pm p_0)^2 + \frac{Z_s(\vec{q}\pm\vec{p})}{Z_0(\vec{q}\pm\vec{p})}(\vec{q}\pm\vec{p})^2 + \frac{Z_s(0)}{Z_0(\vec{q}\pm\vec{p})}(\vec{q}\pm\vec{p})^2 r_B(x') + \frac{m^2}{Z_0(\vec{q}\pm\vec{p})}}$$

$$E_B(\vec{q}\pm\vec{p}) = K\sqrt{\frac{Z_s(\vec{q}\pm\vec{p})}{Z_0(\vec{q}\pm\vec{p})}x + \frac{Z_s(0)}{Z_0(\vec{q}\pm\vec{p})}x\left(\frac{1}{x}-1\right)\theta(1-x) + \frac{m^2}{Z_0(\vec{q}\pm\vec{p})K^2}}$$

① 谱函数介子圈

$\tilde{\partial}_t$  $= \dfrac{\left(V_{2\pi\sigma}(\rho)\right)^2}{Z_{\phi,k}^2}\left[\left(\tilde{\partial}_t \bar{G}^{\pi}_{k}(q)\right)\bar{G}^{\sigma}_{k}(q-p)+\bar{G}^{\pi}_{k}(q)\left(\tilde{\partial}_t \bar{G}^{\sigma}_{k}(q-p)\right)\right]$

$$\tilde{\partial}_t \bar{G}^{\pi}_{k}(q) = -\vec{q}^{\,2}\left(\bar{G}^{\pi}_{k}(q)\right)^2\left[\frac{1}{x}(2-\eta_{\phi,k})+\eta_{\phi,k}\right]\theta(1-x)$$

$$= -k^2\left(\bar{G}^{\pi}_{k}(q)\right)^2\left[(2-\eta_{\phi,k})+\eta_{\phi,k}x\right]\theta(1-x)$$

$$\left(\tilde{\partial}_t \bar{G}^{\pi}_{k}(q)\right)\bar{G}^{\sigma}_{k}(q-p)$$

$$= -k^2\left(\bar{G}^{\pi}_{k}(q)\right)^2\bar{G}^{\sigma}_{k}(q-p)\left[(2-\eta_{\phi,k})+\eta_{\phi,k}x\right]\theta(1-x)$$

$\bar{G}^{\pi}_{k}(q)\left(\tilde{\partial}_t \bar{G}^{\sigma}_{k}(q-p)\right)$ 对动量作代换：$q-p=q'$ 则 $q=q'+p$.

则上式可写为：$\bar{G}^{\pi}_{k}(q'+p)\left(\tilde{\partial}_t \bar{G}^{\sigma}_{k}(q')\right)$ 并令 $q'=q$.

则 $\bar{G}^{\pi}_{k}(q)\left(\tilde{\partial}_t \bar{G}^{\sigma}_{k}(q-p)\right) = \bar{G}^{\pi}_{k}(q+p)\left(\tilde{\partial}_t \bar{G}^{\sigma}_{k}(q)\right)$

$$= -k^2\,\bar{G}^{\pi}_{k}(q+p)\left(\bar{G}^{\sigma}_{k}(q)\right)^2\left[(2-\eta_{\phi,k})+\eta_{\phi,k}x\right]\theta(1-x)$$

则

$\tilde{\partial}_t$  $= -\dfrac{\left(V_{2\pi\sigma}(\rho)\right)^2}{Z_{\phi,k}^2}k^2\left[(2-\eta_{\phi,k})+\eta_{\phi,k}x\right]\theta(1-x)$

$$\times\left\{\left(\bar{G}^{\pi}_{k}(q)\right)^2\bar{G}^{\sigma}_{k}(q-p)+\bar{G}^{\pi}_{k}(q+p)\left(\bar{G}^{\sigma}_{k}(q)\right)^2\right\}$$

求和：$k\frac{T}{k}\sum_n$

$$= -\frac{\left(V_{2\pi\sigma}(\rho)\right)^2}{Z_{\phi,k}^2\,k^3}\left[(2-\eta_{\phi,k})+\eta_{\phi,k}x\right]\theta(1-x)$$

$$\times\left[B_2B_1(\tilde{m}_\pi^2,\tilde{m}_\sigma^2,q-p)+B_2B_1(\tilde{m}_\sigma^2,\tilde{m}_\pi^2,q+p)\right]$$

② 积分：

$$= -\frac{V_3}{4}\cancel{k^3}\int_0^1 dx\int_{-1}^1 d\cos\theta\,\frac{(V_{2\pi\sigma}(\bar\rho))^2}{Z_{\phi,k}^2\cancel{k^3}}\left[(2-\eta_{\phi,k})x^{\frac{1}{2}}+\eta_{\phi,k}x^{\frac{3}{2}}\right]$$

$$\times\left[B_2B_1(\tilde m_\pi^2,\tilde m_\sigma^2,q-p)+B_2B_1(\tilde m_\sigma^2,\tilde m_\pi^2,q+p)\right]$$

$$= -\frac{V_3}{2}\,\bar\rho\,\bar\lambda_{2,k}^2\cdot Z_{\phi,k}\int_0^1 dx\int_{-1}^1 d\cos\theta\left[(2-\eta_{\phi,k})x^{\frac{1}{2}}+\eta_{\phi,k}x^{\frac{3}{2}}\right]$$

$$\times\left[B_2B_1(\tilde m_\pi^2,\tilde m_\sigma^2,q-p)+B_2B_1(\tilde m_\sigma^2,\tilde m_\pi^2,q+p)\right]$$

当 $|\vec p|=0$ 时.

$$= -\frac{V_3}{\cancel{2}}\,\bar\rho\,\bar\lambda_{2,k}^2 Z_{\phi,k}\cdot\cancel{\frac{4}{15}(\eta_{\phi,k}-4)}\,\frac{4}{15}(5-\eta_{\phi,k})\cdot\cancel{2}.$$

$$\times\left[B_2B_1(\tilde m_\pi^2,\tilde m_\sigma^2,q_0-p_0)+B_2B_1(\tilde m_\sigma^2,\tilde m_\pi^2,q_0+p_0)\right]$$

$$= -\frac{4}{15}V_3\,\bar\rho\,\bar\lambda_{2,k}^2 Z_{\phi,k}(5-\eta_{\phi,k})\left[B_2B_1(\pi,\sigma,q_0-p_0)+B_2B_1(\sigma,\pi,q_0+p_0)\right]$$

$$B_1B_1(q_0-p_0) = \sum G_k(q_0)G_k(q_0-p_0)$$

$$= \sum\frac{1}{q_0^2+E_q^2}\,\frac{1}{(q_0-p_0)^2+E_{qmp}^2}$$

$$= \sum\frac{1}{-(iq_0)^2+E_q^2}\,\frac{1}{-(iq_0-ip_0)^2+E_{qmp}^2}$$

$$= \sum\frac{1}{(iq_0+E_q)(iq_0-E_q)}\,\frac{1}{(iq_0-ip_0+E_{qmp})(iq_0-ip_0-E_{qmp})}$$

Poles: $-E_q$, E_q, ip_0-E_{qmp}, ip_0+E_{qmp}

③

$$= -\Big\{ n_b(-E_q)\,\frac{-1}{2E_q}\,\frac{(-1)(-1)}{(iP_0+E_q-E_{qmp})(iP_0+E_q+E_{qmp})}$$

$$+ n_b(E_q)\,\frac{1}{2E_q}\,\frac{(-1)(-1)}{(iP_0-E_q-E_{qmp})(iP_0-E_q+E_{qmp})}$$

$$+ n_b(-E_{qmp})\,\frac{-1}{2E_{qmp}}\,\frac{1}{(iP_0+E_q-E_{qmp})(iP_0-E_q-E_{qmp})}$$

$$+ n_b(E_{qmp})\,\frac{1}{2E_{qmp}}\,\frac{1}{(iP_0+E_q+E_{qmp})(iP_0-E_q+E_{qmp})}\Big\}$$

$$= -\Big\{ n_b(-E_q)\cdot\frac{-1}{4E_qE_{q-p}}\left[\frac{1}{iP_0+E_q-E_{q-p}}-\frac{1}{iP_0+E_q+E_{q-p}}\right]$$

$$+ n_b(E_q)\,\frac{1}{4E_qE_{q-p}}\left[\frac{1}{iP_0-E_q-E_{q-p}}-\frac{1}{iP_0-E_q+E_{q-p}}\right]$$

$$+ n_b(-E_{q-p})\,\frac{-1}{4E_qE_{q-p}}\left[\frac{1}{iP_0-E_q-E_{q-p}}-\frac{1}{iP_0+E_q-E_{q-p}}\right]$$

$$+ n_b(E_{q-p})\,\frac{1}{4E_qE_{q-p}}\left[\frac{1}{iP_0-E_q+E_{q-p}}-\frac{1}{iP_0+E_q+E_{q-p}}\right]\Big\}$$

④

$$= -\frac{K^3}{4E_qE_{q-p}}\Bigg\{\frac{1}{ip_0-E_q-E_{q-p}}\left[1+n_b(E_q)+n_b(E_{q-p})\right]$$

$$+\frac{1}{ip_0+E_q-E_{q-p}}\left[n_b(E_q)-n_b(E_{q-p})\right]$$

$$+\frac{1}{ip_0-E_q+E_{q-p}}\left[n_b(E_{q-p})-n_b(E_q)\right]$$

$$+\frac{1}{ip_0+E_q+E_{q-p}}\left[-1-n_b(E_q)-n_b(E_{q-p})\right]\Bigg\}$$

$$B_1B_1(q_0+p_0)=\sum G_K(q_0)\,G_K(q_0+p_0)$$

$$=\sum\frac{1}{q_0^2+E_q^2}\,\frac{1}{(q_0+p_0)^2+E_{q+p}^2}$$

$$=\sum\frac{1}{(iq_0)^2-E_q^2}\,\frac{1}{(iq_0+ip_0)^2-E_{q+p}^2}$$

$$=\sum\frac{1}{(iq_0+E_q)(iq_0-E_q)}\,\frac{1}{(iq_0+ip_0+E_{q+p})(iq_0+ip_0-E_{q+p})}$$

Poles : $-E_q$, E_q , $-ip_0-E_{q+p}$, $-ip_0+E_{q+p}$.

$$
\begin{aligned}
= -\Big\{ & n_b(-E_q) \frac{-1}{2E_q} \frac{1}{(ip_0 - E_q + E_{q+p})} \frac{1}{(ip_0 - E_q - E_{q+p})} \\
& + n_b(E_q) \frac{1}{2E_q} \frac{1}{(ip_0 + E_q + E_{q+p})} \frac{1}{(ip_0 + E_q - E_{q+p})} \\
& + n_b(-E_{q+p}) \frac{-1}{2E_{q+p}} \frac{-1}{(ip_0 - E_q + E_{q+p})} \frac{-1}{(ip_0 + E_q + E_{q+p})} \\
& + n_b(E_{q+p}) \frac{1}{2E_{q+p}} \frac{-1}{(ip_0 - E_q - E_{q+p})} \frac{-1}{(ip_0 + E_q - E_{q+p})} \Big\}
\end{aligned}
$$

$$
\begin{aligned}
= -\Big\{ & (-1 - n_b(E_q)) \frac{-1}{4E_q E_{q+p}} \left[\frac{1}{ip_0 - E_q - E_{q+p}} - \frac{1}{ip_0 - E_q + E_{q+p}} \right] \\
& + n_b(E_q) \frac{1}{4E_q E_{q+p}} \left[\frac{1}{ip_0 + E_q - E_{q+p}} - \frac{1}{ip_0 + E_q + E_{q+p}} \right] \\
& + (-1 - n_b(E_{q+p})) \frac{-1}{4E_q E_{q+p}} \left[\frac{1}{ip_0 - E_q + E_{q+p}} - \frac{1}{ip_0 + E_q + E_{q+p}} \right] \\
& + n_b(E_{q+p}) \frac{1}{4E_q E_{q+p}} \left[\frac{1}{ip_0 - E_q - E_{q+p}} - \frac{1}{ip_0 + E_q - E_{q+p}} \right] \Big\}
\end{aligned}
$$

⑥

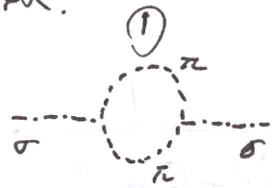
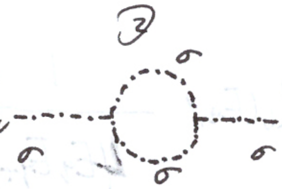
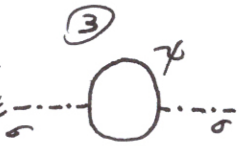
$$= -\frac{K^3}{4E_q E_{q+p}} \left\{ \frac{1}{ip_0 - E_q - E_{q+p}} \left[1 + n_b(E_q) + n_b(E_{q+p}) \right] \right.$$

$$+ \frac{1}{ip_0 + E_q - E_{q+p}} \left[n_b(E_q) - n_b(E_{q+p}) \right]$$

$$+ \frac{1}{ip_0 - E_q + E_{q+p}} \left[n_b(E_{q+p}) - n_b(E_q) \right]$$

$$\left. + \frac{1}{ip_0 + E_q + E_{q+p}} \left[-1 - n_b(E_q) - n_b(E_{q+p}) \right] \right\}$$

Sigma 介子谱函数.

$\partial_t \Gamma^{(2)}_{\sigma\sigma} = -\frac{1}{4}\tilde{\partial}_t$  $-\frac{1}{4}\tilde{\partial}_t$  $+\frac{1}{2}\tilde{\partial}_t$ 

① : 

$$= (V_{2\pi\sigma}(\rho))^2 \, G^\pi_k(q) \, G^\pi_k(q-p)$$

$$= \frac{2\rho \lambda_{2,k}^2}{Z_{\phi,k}^2} \, \bar{G}^\pi_k(q) \, \bar{G}^\pi_k(q-p)$$

$$= 2\bar{\rho}\,\bar{\lambda}_{2,k}^2 Z_{\phi,k} \, \bar{G}^\pi_k(q) \, \bar{G}^\pi_k(q-p)$$

$\tilde{\partial}_t$  $= \underset{(N_f^2-1)}{\wedge} 2\bar{\rho}\,\bar{\lambda}_{2,k}^2 Z_{\phi,k} \left[\left(\tilde{\partial}_t \bar{G}^\pi_k(q)\right) \bar{G}^\pi_k(q-p) + \bar{G}^\pi_k(q) \left(\tilde{\partial}_t \bar{G}^\pi_k(q-p)\right) \right]$

⑦

$$= 2(N_f^2-1)\,\bar\rho\,\bar\lambda_{2,k}^2 Z_{\phi,k}\left[\left(\tilde\partial_t \bar G^\pi_k(q)\right)\bar G^\pi_k(q-p) + \bar G^\pi_k(q+p)\left(\tilde\partial_t \bar G^\pi_k(q)\right)\right]$$

$$= -2(N_f^2-1)\,\bar\rho\,\bar\lambda_{2,k}^2 Z_{\phi,k}\, k^2\left[(2-\eta_{\phi,k}) + \eta_{\phi,k}\,x\right]\theta(1-x)$$

$$\times\left[\left(\bar G^\pi_k(q)\right)^2 \bar G^\pi_k(q-p) + \bar G^\pi_k(q+p)\left(\bar G^\pi_k(q)\right)^2\right]$$

求和

$$= -2(N_f^2-1)\,\bar\rho\,\bar\lambda_{2,k}^2 Z_{\phi,k}\,\frac{1}{k^3}\left[(2-\eta_{\phi,k}) + \eta_{\phi,k}\,x\right]\theta(1-x)$$

$$\times\left[B_2B_1(\tilde m_\pi^2, q-p) + B_2B_1(\tilde m_\pi^2, q+p)\right]$$

积分:

$$= -\frac{v_3}{2}(N_f^2-1)\,\bar\rho\,\bar\lambda_{2,k}^2 Z_{\phi,k}\int_0^1 dx\int_{-1}^1 d\cos\theta\left[(2-\eta_{\phi,k})x^{\frac{1}{2}} + \eta_{\phi,k}\,x^{\frac{3}{2}}\right]$$

$$\times\left[B_2B_1(\tilde m_\pi^2, q-p) + B_2B_1(\tilde m_\pi^2, q+p)\right]$$

当 $|\vec p|=0$ 时.

$$= -\frac{v_3}{\cancel{2}}(N_f^2-1)\,\bar\rho\,\bar\lambda_{2,k}^2 Z_{\phi,k}\cdot\cancel{2}\cdot\frac{4}{15}(5-\eta_{\phi,k})\left[B_2B_1(\tilde m_\pi^2, \tilde q_0-\tilde p_0) + B_2B_1(\tilde m_\pi^2, \tilde q_0+\tilde p_0)\right]$$

$$= -\frac{4}{15}v_3(N_f^2-1)\,\bar\rho\,\bar\lambda_{2,k}^2 Z_{\phi,k}(5-\eta_{\phi,k})\left[B_2B_1(\tilde m_\pi^2, \tilde q_0-\tilde p_0) + B_2B_1(\tilde m_\pi^2, \tilde q_0+\tilde p_0)\right]$$

---

②:

$$= \left(V_{3\sigma}(\rho)\right)^2 \bar G^\sigma_k(q)\,\bar G^\sigma_k(q-p)$$

$$= \frac{\left(3(2\rho)^{\frac{1}{2}}\lambda_2 + (2\rho)^{\frac{3}{2}}\lambda_3\right)^2}{Z_{\phi,k}^2}\,\bar G^\sigma_k(q)\,\bar G^\sigma_k(q-p)$$

$$= \left[18\bar\rho\,\bar\lambda_{2,k}^2 + 24\bar\lambda_{2,k}\bar\lambda_{3,k}\bar\rho^2 + 8\bar\lambda_{3,k}^2\bar\rho^3\right] Z_{\phi,k}\,\bar G^\sigma_k(q)\,\bar G^\sigma_k(q-p)$$

$\tilde{\partial}_t$  $= \left[18\bar{\rho}\bar{\lambda}_{2,k}^2 + 24\bar{\lambda}_{2,k}\bar{\lambda}_{3,k}\bar{\rho}^2 + 8\bar{\lambda}_{3,k}^2\bar{\rho}^3\right] Z_{\phi,k}$

$$\times\left[\left(\tilde{\partial}_t \bar{G}_k^\sigma(q)\right)\bar{G}_k^\sigma(q-p) + \bar{G}_k^\sigma(q+p)\left(\tilde{\partial}_t \bar{G}_k^\sigma(q)\right)\right]$$

$$= \left[18\bar{\rho}\bar{\lambda}_{2,k}^2 + 24\bar{\lambda}_{2,k}\bar{\lambda}_{3,k}\bar{\rho}^2 + 8\bar{\lambda}_{3,k}^2\bar{\rho}^3\right] Z_{\phi,k} \cdot$$

$$\times(-k^2)\left[(2-\eta_{\phi,k}) + \eta_{\phi,k}x\right]\theta(1-x)$$

$$\times\left\{\left(\bar{G}_k^\sigma(q)\right)^2\bar{G}_k^\sigma(q-p) + \left(\bar{G}_k^\sigma(q)\right)^2\bar{G}_k^\sigma(q+p)\right\}$$

求和：

$$= -\frac{1}{k^3}\left[18\bar{\rho}\bar{\lambda}_{2,k}^2 + 24\bar{\lambda}_{2,k}\bar{\lambda}_{3,k}\bar{\rho}^2 + 8\bar{\lambda}_{3,k}^2\bar{\rho}^3\right] Z_{\phi,k} \cdot$$

$$\times\left[(2-\eta_{\phi,k}) + \eta_{\phi,k}x\right]\theta(1-x)\left[B_2B_1(\tilde{m}_\sigma^2, q-p) + B_2B_1(\tilde{m}_\sigma^2, q+p)\right]$$

积分：

$$= -\frac{V_3}{4}\left[18\bar{\rho}\bar{\lambda}_{2,k}^2 + 24\bar{\lambda}_{2,k}\bar{\lambda}_{3,k}\bar{\rho}^2 + 8\bar{\lambda}_{3,k}^2\bar{\rho}^3\right] Z_{\phi,k}\int_0^1 dx\int_{-1}^1 d\cos\theta$$

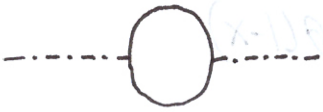
$$\times\left[(2-\eta_{\phi,k})x^{\frac{1}{2}} + \eta_{\phi,k}x^{\frac{3}{2}}\right]\left(B_2B_1(\tilde{m}_\sigma^2, q-p) + B_2B_1(\tilde{m}_\sigma^2, q+p)\right)$$

$\downarrow$ 当 $|\vec{p}|=0$ 时

$$= -\frac{2}{15}V_3 Z_{\phi,k}\left[18\bar{\rho}\bar{\lambda}_{2,k}^2 + 24\bar{\lambda}_{2,k}\bar{\lambda}_{3,k}\bar{\rho}^2 + 8\bar{\lambda}_{3,k}^2\bar{\rho}^3\right](5-\eta_{\phi,k})$$

$$\times\left(B_2B_1(\tilde{m}_\sigma^2, \tilde{q}_0-\tilde{p}_0) + B_2B_1(\tilde{m}_\sigma^2, \tilde{q}_0+\tilde{p}_0)\right)$$

(9)


$$= \mathrm{Tr}\left\{ h_K T^0 G^q_{iK}(q)\, h_K T^0 G^q_{iK}(q-p) \right\}$$

$$= \frac{N_c}{N_f} h_K^2 \cdot \mathrm{Tr}\left\{ G^q_{iK}(q)\, G^q_{iK}(q-p) \right\}$$

$$= \frac{N_c}{N_f} h_K^2 \frac{1}{Z_q^2} \mathrm{Tr}\left[\left(-i q_F^{\mu} \gamma^{\mu} + \bar{m}_f\right) \bar{G}^q_{iK}(q) \left(-i (q-p)_F^{\mu} \gamma^{\mu} + \bar{m}_f\right) \bar{G}^q_{iK}(q-p) \right]$$

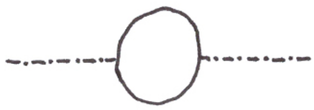
$$= \frac{N_c}{N_f} \frac{h_K^2}{Z_q^2} \bar{G}^q_{iK}(q)\, \bar{G}^q_{iK}(q-p)\, \mathrm{Tr}\left[\left(-i q^F_{\mu} \gamma_{\mu} + \bar{m}_f^2\right) \left(-i (q-p)^F_{\mu} \gamma_{\mu} + \bar{m}_f^2\right) \right]$$

$$\mathrm{Tr}\left[\left(-i q^F_{\mu} \gamma_{\mu} + \bar{m}_f\right) \left(-i (q-p)^F_{\mu} \gamma_{\mu} + \bar{m}_f\right) \right]$$

$$= \mathrm{Tr}\left[- q^F_{\mu} \gamma_{\mu} (q-p)^F_{\mu} \gamma_{\mu} + \bar{m}_f^2 \right]$$

$$= 4 \left(-q_F \cdot (q-p)_F + \bar{m}_f^2 \right)$$

$$= \frac{4 N_c}{N_f} \bar{h}_K^2 Z_{\phi,K}\, \bar{G}^q_{iK}(q)\, \bar{G}^q_{iK}(q-p) \left[-q_F \cdot (q-p)_F + \bar{m}_f^2 \right]$$

$\tilde{\partial}_t$ 
$$\begin{aligned} = -\frac{4 N_c}{N_f} \bar{h}_K^2 Z_{\phi,K} \Big\{ & \left(\tilde{\partial}_t \bar{G}^q_{iK}(q)\right) \bar{G}^q_{iK}(q-p) \left[q_F \cdot (q-p)_F - \bar{m}_f^2 \right] && (1) \\ & + \bar{G}^q_{iK}(q) \left(\tilde{\partial}_t \bar{G}^q_{iK}(q-p)\right) \left[q_F \cdot (q-p)_F - \bar{m}_f^2 \right] && (2) \\ & + \bar{G}^q_{iK}(q)\, \bar{G}^q_{iK}(q-p) \left(\tilde{\partial}_t q_F\right) (q-p)_F && (3) \\ & + \bar{G}^q_{iK}(q)\, \bar{G}^q_{iK}(q-p)\, q_F\, \tilde{\partial}_t (q-p)_F \Big\} && (4) \end{aligned}$$

$$⑩\quad \tilde{\partial}_t \bar{G}^q_k(q) = -2k^2\left(\bar{G}^q_k(q)\right)^2\left[(1-\eta_{q,k})+\eta_{q,k}x^{\frac{1}{2}}\right]\theta(1-x)$$

$$①:\quad \left(\tilde{\partial}_t \bar{G}^q_k(q)\right)\bar{G}^q_k(q-p)\left[q_F(q-p)_F-\bar{m}_f^2\right]$$

$$[1] = \left(\tilde{\partial}_t \bar{G}^q_k(q)\right)\bar{G}^q_k(q-p)\left[q_0(q_0-p_0)-\bar{m}_f^2\right]$$

$$= -2k^2\left(\bar{G}^q_k(q)\right)^2\bar{G}^q_k(q-p)\left[(1-\eta_{q,k})+\eta_{q,k}x^{\frac{1}{2}}\right]\theta(1-x)$$

$$\times\left\{\frac{1}{2}\left[\bar{G}^{q\,-1}_k(q)+\bar{G}^{q\,-1}_k(q-p)-k^2-(\vec{q}-\vec{p})^2(1+r_F(x'))^2-p_0^2\right]-2\bar{m}_f^2\right\}$$

$$= -k^2\left(\bar{G}^q_k(q)\right)^2\bar{G}^q_k(q-p)\left[(1-\eta_{q,k})+\eta_{q,k}x^{\frac{1}{2}}\right]\theta(1-x)$$

$$\times\left[\bar{G}^{q\,-1}_k(q)+\bar{G}^{q\,-1}_k(q-p)-\left(k^2+(\vec{q}-\vec{p})^2(1+r_F(x'))^2+p_0^2+4\bar{m}_f^2\right)\right]$$

$$= -k^2\left[(1-\eta_{q,k})+\eta_{q,k}x^{\frac{1}{2}}\right]\theta(1-x)$$

$$\times\left[\bar{G}^q_k(q)\bar{G}^q_k(q-p)+\left(\bar{G}^q_k(q)\right)^2-\left(k^2+(\vec{q}-\vec{p})^2(1+r_F(x'))^2+p_0^2+4\bar{m}_f^2\right)\left(\bar{G}^q_k(q)\right)^2\bar{G}^q_k(q-p)\right]$$

求和:

$$= -\frac{1}{k}\left[(1-\eta_{q,k})+\eta_{q,k}x^{\frac{1}{2}}\right]\theta(1-x)$$

$$\times\left[F_1F_1^-+F_2-\left(1+x'(1+r_F(x'))^2+\tilde{p}_0^2+4\bar{m}_f^2\right)F_2F_1^-\right]$$

積分:

$$= -\frac{V_3}{4}k^2\int_0^1 dx\int_{-1}^1 d\cos\theta\left[(1-\eta_{q,k})x^{\frac{1}{2}}+\eta_{q,k}x\right]$$

$$\times\left[F_1F_1^-+F_2-\left(1+x'(1+r_F(x'))^2+\tilde{p}_0^2+4\bar{m}_f^2\right)F_2F_1^-\right]$$

⑪ $[2] = \left(\tilde{\partial}_t \bar{G}^q_{\bar{\imath}k}(q)\right) \bar{G}^q_{\bar{\imath}k}(q-p)\ \vec{q}\cdot(\vec{q}-\vec{p})(1+r_F(x))(1+r_F(x'))$

$$= -2K^2\left(\bar{G}^q_{\bar{\imath}k}(q)\right)^2 \bar{G}^q_{\bar{\imath}k}(q-p)\left[(1-\eta_{q,k})+\eta_{q,k}x^{\frac{1}{2}}\right]\theta(1-x)\times(\quad)$$

$$= -\frac{2}{K^2}\left(\tilde{G}^q_{\bar{\imath}k}(q)\right)^2 \tilde{G}^q_{\bar{\imath}k}(q-p)\left[(1-\eta_{q,k})+\eta_{q,k}x^{\frac{1}{2}}\right]\theta(1-x)$$

$$\times\left(x^{\frac{1}{2}}-\tilde{p}'\cos\theta\right)(1+r_F(x'))$$

求和：

$$= -\frac{2}{K}\left[(1-\eta_{q,k})+\eta_{q,k}x^{\frac{1}{2}}\right]\theta(1-x)\left(x^{\frac{1}{2}}-\tilde{p}'\cos\theta\right)(1+r_F(x'))F_2F_1^-$$

积分：

$$= -\frac{v_3}{2}K^2\int_0^1 dx\int_{-1}^1 d\cos\theta\left[(1-\eta_{q,k})x^{\frac{1}{2}}+\eta_{q,k}x\right]$$

$$\times F_2F_1^-\left(x^{\frac{1}{2}}-\tilde{p}'\cos\theta\right)(1+r_F(x'))$$

---

②： $\bar{G}^q_{\bar{\imath}k}(q)\left(\tilde{\partial}_t\bar{G}^q_{\bar{\imath}k}(q-p)\right)\left[q_F\cdot(q-p)_F-\bar{m}_f^2\right]$

$\big\downarrow$ 动量代换： $q-p\to q'$ 则 $q=q'+p$.

$$\bar{G}^q_{\bar{\imath}k}(q+p)\left(\tilde{\partial}_t\bar{G}^q_{\bar{\imath}k}(q)\right)\left[q_F\cdot(q+p)_F-\bar{m}_f^2\right]$$

$[1]$： $\left(\tilde{\partial}_t\bar{G}^q_{\bar{\imath}k}(q)\right)\bar{G}^q_{\bar{\imath}k}(q+p)\left[q_0(q_0+p_0)-\bar{m}_f^2\right]$

$$= -2K^2\left(\bar{G}^q_{\bar{\imath}k}(q)\right)^2\bar{G}^q_{\bar{\imath}k}(q+p)\left[(1-\eta_{q,k})+\eta_{q,k}x^{\frac{1}{2}}\right]\theta(1-x)$$

$$\times\frac{1}{2}\left[\bar{G}^{q\,-1}_{\bar{\imath}k}(q)+\bar{G}^{q\,-1}_{\bar{\imath}k}(q+p)-\left(K^2+(\vec{q}+\vec{p})^2(1+r_F(x''))^2+p_0^2+4\bar{m}_f^2\right)\right]$$

$$= -K^2\left[(1-\eta_{q,k})+\eta_{q,k}x^{\frac{1}{2}}\right]\theta(1-x)$$

$$\times\left[\bar{G}^q_{\bar{\imath}k}(q)\bar{G}^q_{\bar{\imath}k}(q+p)+\left(\bar{G}^q_{\bar{\imath}k}(q)\right)^2-\left(K^2+(\vec{q}+\vec{p})^2(1+r_F(x''))^2+p_0^2+4\bar{m}_f^2\right)\left(\bar{G}^q_{\bar{\imath}k}(q)\right)^2\bar{G}^q_{\bar{\imath}k}(q+p)\right]$$

⑫ 求和

$$= -\frac{1}{K}\left[(1-\eta_{q,k})+\eta_{q,k}x^{\frac{1}{2}}\right]\theta(1-x)$$

$$\times\left[F_1F_1^+ + F_2 - (1+x''(1+r_F(x''))^2+\tilde{p}_0^2+4\tilde{m}_f^2)F_2F_1^+\right]$$

积分：

$$= -\frac{V_3}{4}K^2\int_0^1 dx\int_{-1}^1 d\cos\theta\left[(1-\eta_{q,k})x^{\frac{1}{2}}+\eta_{q,k}x\right]$$

$$\times\left[F_1F_1^+ + F_2 - (1+x''(1+r_F(x''))^2+\tilde{p}_0^2+4\tilde{m}_f^2)F_2F_1^+\right]$$

$$[2] = -2K^2(\bar{G}^q_{k}(q))^2\bar{G}^q_{k}(q+p)\left[(1-\eta_{q,k})+\eta_{q,k}x^{\frac{1}{2}}\right]\theta(1-x)$$

$$\times\vec{q}\cdot(\vec{q}+\vec{p})(1+r_F(x))(1+r_F(x''))$$

$$= -\frac{V_3}{2}K^2\int_0^1 dx\int_{-1}^1 d\cos\theta\left[(1-\eta_{q,k})x^{\frac{1}{2}}+\eta_{q,k}x\right]$$

$$\times F_2F_1^+(x^{\frac{1}{2}}+\tilde{p}\cos\theta)(1+r_F(x''))$$

---

③ : $\bar{G}^q_{k}(q)\,\bar{G}^q_{k}(q-p)\left[(\tilde{\partial}_t q_F)(q-p)_F\right]$

$$(\tilde{\partial}_t q_F)(q-p)_F = K^2\left[(1-\eta_{q,k})+\eta_{q,k}x^{\frac{1}{2}}\right]\theta(1-x)$$

$$\times(x^{\frac{1}{2}}-\tilde{p}\cos\theta)(1+r_F(x'))$$

$$= K^2\bar{G}^q_{k}(q)\bar{G}^q_{k}(q-p)\left[(1-\eta_{q,k})+\eta_{q,k}x^{\frac{1}{2}}\right]\theta(1-x)$$

$$\times(x^{\frac{1}{2}}-\tilde{p}\cos\theta)(1+r_F(x'))$$

积分求和：

$$= \frac{V_3}{4}K^2\int_0^1 dx\int_{-1}^1 d\cos\theta\left[(1-\eta_{q,k})x^{\frac{1}{2}}+\eta_{q,k}x\right]\theta(1-x)$$

$$\times F_1F_1^-(x^{\frac{1}{2}}-\tilde{p}\cos\theta)(1+r_F(x'))$$

⑬

④: $\bar{G}^q_k(q)\,\bar{G}^q_k(q-p)\; q_F \cdot \tilde{\partial}_t (q-p)_F$

$\downarrow$ 动量代换

$\bar{G}^q_k(q+p)\,\bar{G}^q_k(q)\;(q+p)_F \cdot (\tilde{\partial}_t q_F)$

$$(\tilde{\partial}_t q_F)(q+p)_F = k^2\left[(1-\eta_{q,k}) + \eta_{q,k} x^{\frac{1}{2}}\right]\theta(1-x) \times \left(x^{\frac{1}{2}} + \tilde{p}\cos\theta\right)(1+r_F(x''))$$

$\downarrow$

求和积分

$$= \frac{V_3}{4} k^2 \int_0^1 dx \int_{-1}^1 d\cos\theta \left[(1-\eta_{q,k})x^{\frac{1}{2}} + \eta_{q,k} x\right]\theta(1-x) \times F_1 F_1^+ \left(x^{\frac{1}{2}} + \tilde{p}\cos\theta\right)(1+r_F(x''))$$

4部分结合到一起:

$$\tilde{\partial}_t \cdots\bigcirc\cdots = -\frac{4N_c}{N_f}\bar{h}_k^2 Z_{\phi,k} V_3 k^2 \int_0^1 dx \int_{-1}^1 d\cos\theta \left[(1-\eta_{q,k})x^{\frac{1}{2}} + \eta_{q,k} x\right]$$

$$\times\Big\{ -\frac{1}{4}\left[F_1F_1^- + F_2 - \left(1 + x'(1+r_F(x'))^2 + \tilde{p}_0^2 + 4\tilde{m}_f^2\right)F_2F_1^-\right]$$

$$-\frac{1}{2} F_2F_1^-\left(x^{\frac{1}{2}} - \tilde{p}\cos\theta\right)(1+r_F(x'))$$

$$-\frac{1}{4}\left[F_1F_1^+ + F_2 - \left(1 + x''(1+r_F(x''))^2 + \tilde{p}_0^2 + 4\tilde{m}_f^2\right)F_2F_1^+\right]$$

$$-\frac{1}{2} F_2F_1^+\left(x^{\frac{1}{2}} + \tilde{p}\cos\theta\right)(1+r_F(x''))$$

$$+\frac{1}{4} F_1F_1^-\left(x^{\frac{1}{2}} - \tilde{p}\cos\theta\right)(1+r_F(x'))$$

$$+\frac{1}{4} F_1F_1^+\left(x^{\frac{1}{2}} + \tilde{p}\cos\theta\right)(1+r_F(x''))\Big\}$$

$$
(14) \quad = -\frac{2N_c}{N_f}\bar{h}_k^2 Z_{\phi,k} V_3 k^2 \int_0^1 dx \int_{-1}^{1} d\cos\theta \left[(1-\eta_{q,k})x^{\frac{1}{2}} + \eta_{q,k}x\right]
$$

$$
\times \Bigg\{ -F_2 + \frac{1}{2}\left[-1 + \left(x^{\frac{1}{2}} - \tilde{\bar{p}}\cos\theta\right)\left(1+r_F(x')\right)\right] F_1 F_1^-
$$

$$
+ \frac{1}{2}\left[\left(1 + x'(1+r_F(x'))^2 + \tilde{p}_0^2 + 4\tilde{m}_f^2\right) - 2\left(x^{\frac{1}{2}} - \tilde{\bar{p}}\cos\theta\right)\left(1+r_F(x')\right)\right] F_2 F_1^-
$$

$$
+ \frac{1}{2}\left[-1 + \left(x^{\frac{1}{2}} + \tilde{\bar{p}}\cos\theta\right)\left(1+r_F(x'')\right)\right] F_1 F_1^+
$$

$$
+ \frac{1}{2}\left[\left(1 + x''(1+r_F(x''))^2 + \tilde{p}_0^2 + 4\tilde{m}_f^2\right) - 2\left(x^{\frac{1}{2}} + \tilde{\bar{p}}\cos\theta\right)\left(1+r_F(x'')\right)\right] F_2 F_1^+ \Bigg\}
$$