Estimates on the convergence of expansions at finite baryon chemical potentials

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Based on arXiv:2403.06770

Outline

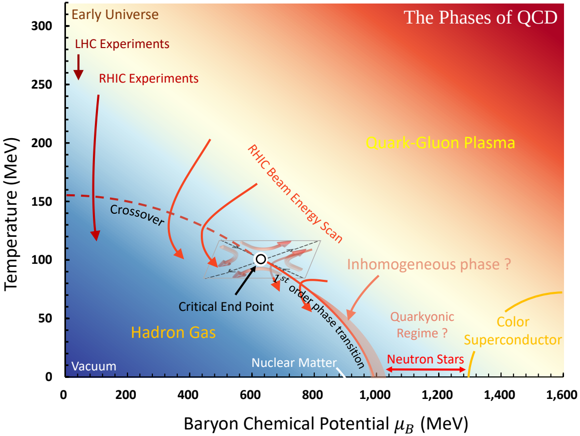
QCD phase structure

Experiments:

- LHC Experiments at low density
- RHIC Beam Energy Scan at finite density
- ...

Goals:

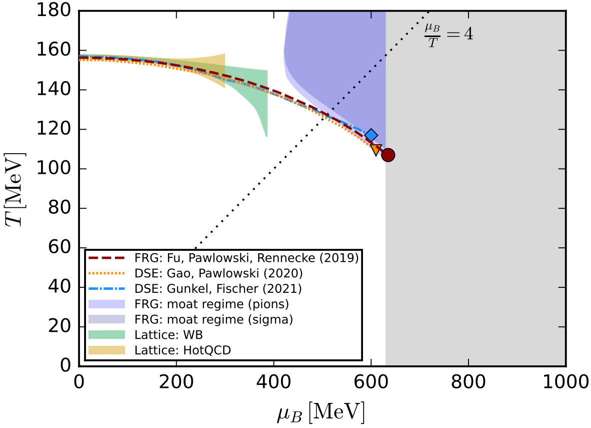
- Location of Critical End Point (CEP)
- New phases at high density region
- EoS
- ...



Nucl.Tech. 46 (2023) 04, 040002-040002

Theoretical predictions

- Lattice QCD (at vanishing chemical potential)
- Dyson-Schwinger Equations (DSE)
- Functional renormalization group (fRG)
- ...



QCD phase diagram (fQCD 2025)

Well known Advantages & Disadvantages

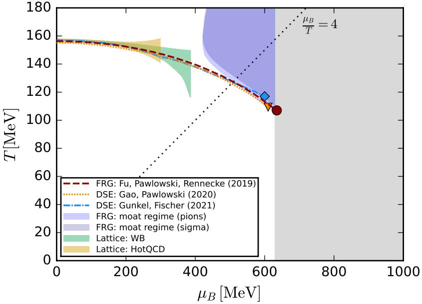
Lattice:

- Reliable at 0 chemical potential
- Expansion methods are needed to reach finite density

Functional methods:

- Can compute at finite density
- Have to use truncations to prevent an infinite number of loop diagrams

- Making reasonable use of reliable vanishing chemical potential results
- Attempt to reach higher chemical potentials

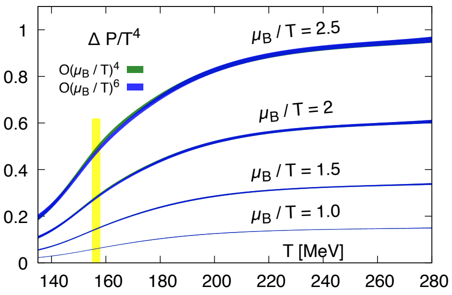


Extrapolation to finite density

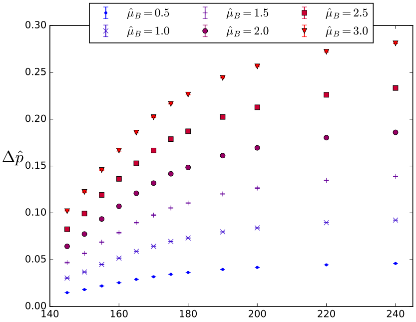
Taylor Expansion of the pressure:

$$\frac{p(T,\hat{\mu}_B)-p(T,0)}{T^4}=\sum_{n=1}^{\infty}\frac{1}{(2n)!}\chi_{2n}^{B}(T,0)\,\hat{\mu}_B^{2n} \tag{1}$$

HotQCD:



WB:



Extrapolation to finite density

Padé approximation:

$$P[m,n] = \frac{p(T,\hat{\mu}_B) - p(T,0)}{T^4} = \frac{\sum_{i=1}^{n/2} a_i\ \hat{\mu}_B^{2i}}{1 + \sum_{j=1}^{m/2} b_i\ \hat{\mu}_B^{2j}} \tag{2}$$

Here the coefficients a_i and b_i are determined by

$$\frac{\partial^i P[m,n]}{\partial \hat{\mu}_B^i} = \chi_i^B \tag{3}$$

- When $m = 0$ the Padé approximation will go back to Taylor expansion
- The poles of Padé approximation can be used to estimate the convergence radius of Taylor expansion

Extrapolation to finite density

Ratio estimator: (the pole of P[2,n])

$$r_{c,2n}^{\mathrm{ratio}} = \left| \frac{(2n+1)(2n+2)\chi_{2n}^{B}}{\chi_{2n+2}^{B}} \right|^{\frac{1}{2}} \tag{4}$$

Mercer-Roberts estimator: (the pole of P[4,n])

$$r_{c,2n}^{\mathrm{MR}} = \left| \left[\frac{\chi_{2n+2}^{B}\chi_{2n-2}^{B}}{(2n+2)!(2n-2)!} - \left(\frac{\chi_{2n}^{B}}{(2n)!} \right)^{2} \right] \right|^{\frac{1}{4}} \left| \left[\frac{\chi_{2n}^{B}\chi_{2n+4}^{B}}{(2n)!(2n+4)!} - \left(\frac{\chi_{2n+2}^{B}}{(2n+2)!} \right)^{2} \right] \right|^{-\frac{1}{4}} \tag{5}$$

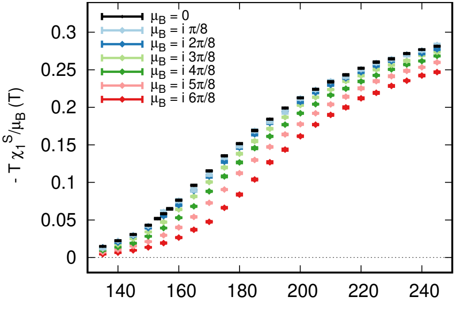
- The estimators are given by the poles of the Padé approximation
- They can give a approximation convergence radius of Taylor expansion

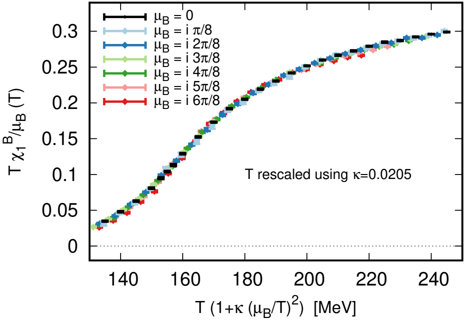
Extrapolation to finite density

T' expansion:

$$\frac{\chi_1^B(T,\hat{\mu}_B)}{\hat{\mu}_B} = \chi_2^B(T',0) \tag{6}$$

$$T'(T,\hat{\mu}_B) = T\left(1+\kappa_2(T)\,\hat{\mu}_B^2+\kappa_4(T)\,\hat{\mu}_B^4+\mathcal{O}(\hat{\mu}_B^6)\right) \tag{7}$$





Phys. Rev. Lett. 126, 232001 (2021)

Outline

Polyakov-Quark-Meson (PQM) Model

Effective action:

$$\Gamma_k = \int_x \Big\{ Z_q\, \bar{q}\big[\gamma_\mu \partial_\mu - \gamma_0(\mu + igA_0)\big] q + \frac{1}{2} Z_\phi\, (\partial_\mu \phi)^2 + h\, \bar{q}(T^0 \sigma + i\gamma_5 \vec{T}\cdot\vec{\pi}) + V_k(\rho) - c\sigma + V_{\mathrm{glue}}(L, \bar{L}) \Big\} \tag{8}$$

Here we use Local potential approximation (LPA):

$$\partial_t Z_{q/\phi} = 0 \tag{9}$$

$$\partial_t h = 0 \tag{10}$$

We only consider a simple computation of the Grand potential.

Effective potential

Flow equation of effective potential:

$$\partial_t V_k(\rho) = \frac{k^4}{4\pi^2}\left[3\, l_0^{(B)}(m_\pi^2;T) + l_0^{(B)}(m_\sigma^2;T) - 4N_c N_f l_0^{(F)}(m_f^2;\mu,T)\right] \tag{11}$$

The fermion loop for real and imaginary chemical potential

$$l_0^{(F)}(m_f^2;\mu,T) = \frac{k}{3\sqrt{k^2+m_f^2}}\left(1 - n_F(m_f^2;\mu,T;L,\bar{L}) - \bar{n}_F(m_f^2;-\mu,T;L,\bar{L})\right) \tag{12}$$

$$= \frac{k}{3\sqrt{k^2+m_f^2}}\left(1 - 2\,\mathrm{Re}\Big(n_F(m_f^2;\mu,T;L,\bar{L})\Big)\right) \tag{13}$$

Equation of State

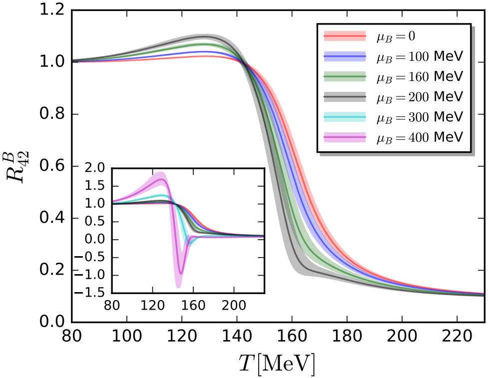
Pressure and Baryon number fluctuations:

$$p(T,\mu) = -\,\Omega(\mu, T) \tag{14}$$

$$\chi_n^B = \frac{\partial^n}{\partial \hat{\mu}_B^n}\frac{p}{T^4}, \qquad \hat{\mu}_B = \frac{\mu_B}{T} \tag{15}$$

$$R_{m,n}^B = \frac{\chi_m^B}{\chi_n^B} \tag{16}$$

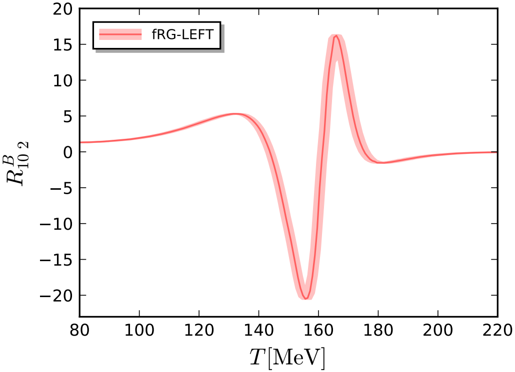
- The baryon number fluctuations at vanishing chemical potential can be used as Taylor coefficients



Phys.Rev.D 104 (2021) 9, 094047

Equation of State

- The baryon number fluctuations can be computed up to (at least) the 10th order



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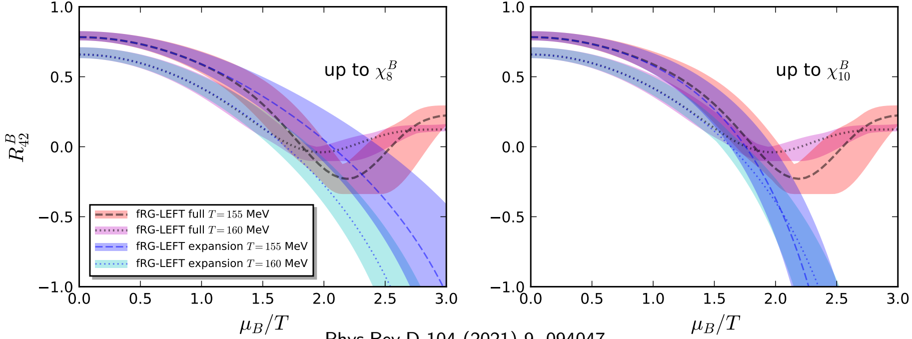
Comparison of different Expansion methods

Taylor Expansion



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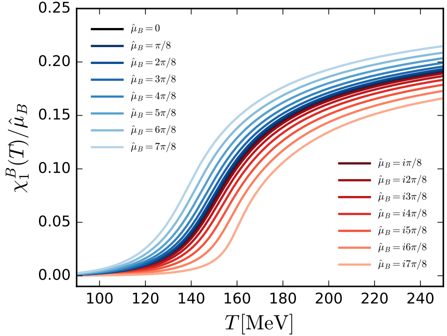
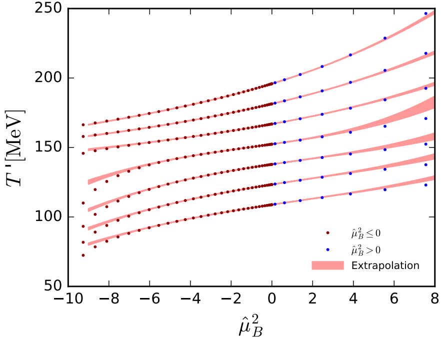
- Direct calculation vs. Taylor expansion of R_{42}^B
- The R_{42}^B around T_{pc} exhibits strong fluctuations at high chemical potential, which are difficult to capture with a finite-order Taylor expansion.

Comparison of different Expansion methods

T' Expansion





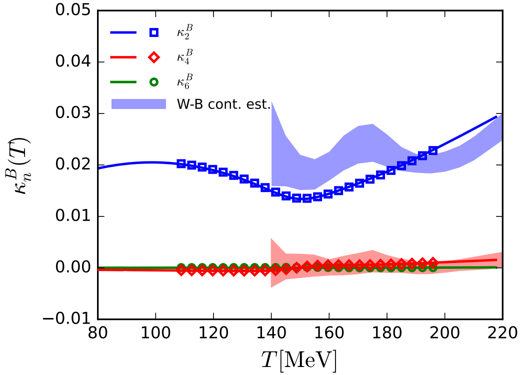
arXiv: 2403.06770

- Directly compute at imaginary chemical potential
- Apply the T' expansion to perform extrapolation

Comparison of different Expansion methods

T' Expansion

$$T' = T\Big(1 + \kappa_2^B(T)\,\hat{\mu}_B^2 + \kappa_4^B(T)\,\hat{\mu}_B^4 + \kappa_6^B(T)\,\hat{\mu}_B^6 + \cdots\Big) \tag{17}$$



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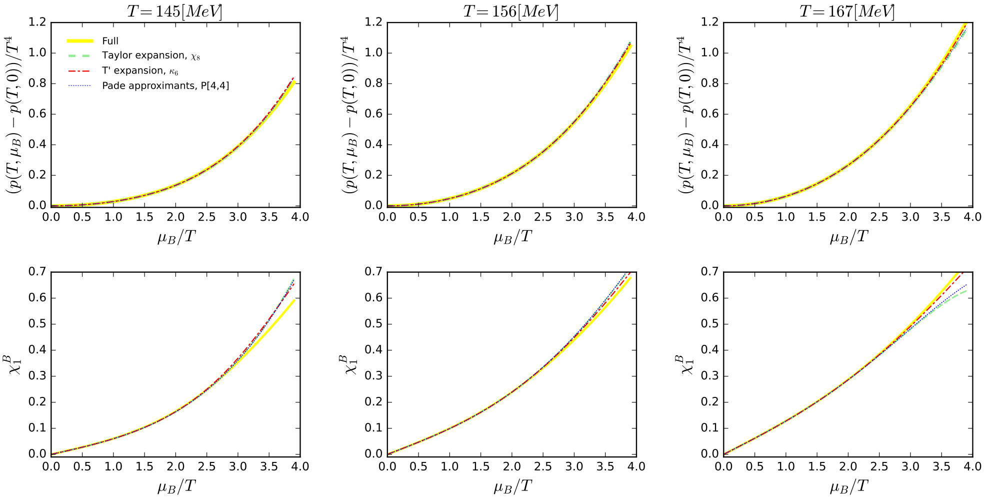
Comparison of different Expansion methods



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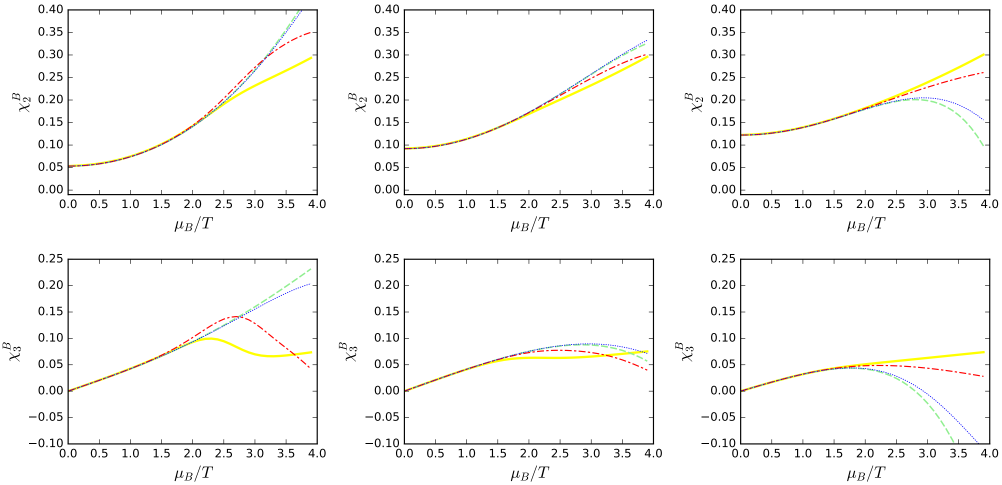

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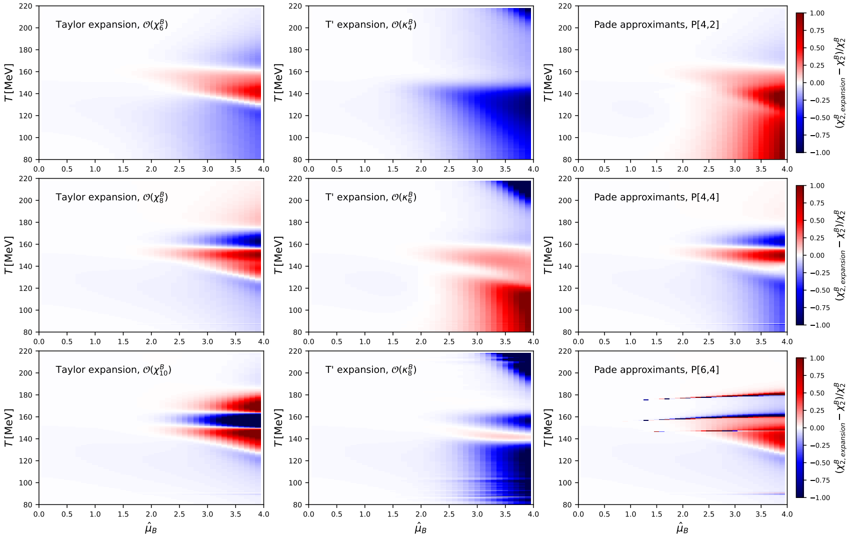
Comparison of different Expansion methods



arXiv: 2403.06770

Outline

Mention your findings here.

Happy TEXing!

Mention your findings here.

Happy $\TeX$ing!

References I

Outline

Appendix I: Title

Remark

This is what an appendix would look like. It utilises the same structure as the rest of the presentation *(section, subsection, subsubsection, etc)*.

Example

Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris.

Outline

Appendix II: Title

Relevant Title

Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi.

Example of Appendix II

Quisque ullamcorper placerat ipsum. Cras nibh. Morbi vel justo vitae lacus tincidunt ultrices. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. In hac habitasse platea dictumst. Integer tempus convallis augue. Etiam facilisis. Nunc elementum fermentum wisi. Aenean placerat. Ut imperdiet, enim sed gravida sollicitudin, felis odio placerat quam, ac pulvinar elit purus eget enim. Nunc vitae tortor. Proin tempus nibh sit amet nisl. Vivamus quis tortor vitae risus porta vehicula.