0.1 定义

现将介子波函数重整化系数$Z_{\Sigma,k}$分开,我们定义的介子波函数重整化

$$Z_{\Sigma,k} \equiv \begin{pmatrix} Z_{\phi_L,k} & Z_{\phi_L,k} & Z_{\phi_S,k} \\ Z_{\phi_L,k} & Z_{\phi_L,k} & Z_{\phi_S,k} \\ Z_{\phi_S,k} & Z_{\phi_S,k} & Z_{\phi_S,k} \end{pmatrix} \tag{1}$$

所以介子场

$$\bar{\phi}_i = Z^{\frac{1}{2}}_{\phi_L,k}\phi_i, \quad \text{for} \quad i = 1,2,3 \tag{2}$$

$$\bar{\phi}_i = Z^{\frac{1}{2}}_{\phi_S,k}\phi_i, \quad \text{for} \quad i = 4,5,6,7 \tag{3}$$

$$\bar{\phi}_L = Z^{\frac{1}{2}}_{\phi_L,k}\phi_L, \tag{4}$$

$$\bar{\phi}_S = Z^{\frac{1}{2}}_{\phi_S,k}\phi_S, \tag{5}$$

其中$\phi = \pi/\sigma$表示赝标/标量场，这里L,S与08基之间存在转动关系：

$$\begin{pmatrix} \phi_L \\ \phi_S \end{pmatrix} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & \sqrt{2} \\ -\sqrt{2} & 1 \end{pmatrix} \begin{pmatrix} \phi_8 \\ \phi_0 \end{pmatrix} \tag{6}$$

介子的质量由Hessian矩阵来计算：

$$H_{ij} = \frac{\partial^2 \tilde{U}_k}{\partial\phi_i \partial\phi_j} \tag{7}$$

1 方案1

对Hessian矩阵进行重整化：

$$\bar{H}_{ij} = \frac{\partial^2 \bar{\tilde{U}}_k}{\partial\bar{\phi}_i \partial\bar{\phi}_j} \tag{8}$$

即

$$\bar{H}_{ii} = \frac{H_{ii}}{Z_{\phi_L}}, \quad \text{for} \quad i = 1,2,3,L \tag{9}$$

$$\bar{H}_{ii} = \frac{H_{ii}}{Z_{\phi_S}}, \quad \text{for} \quad i = 4,5,6,7,S \tag{10}$$

$$\bar{H}_{LS} = \frac{H_{LS}}{Z^{1/2}_{\phi_L} Z^{1/2}_{\phi_S}} \tag{11}$$

Hessian矩阵在LS基下存在非零的非对角元$\bar{H}_{LS}$和$\bar{H}_{SL}$，所以介子质量

$$\bar{m}_{f0}^2 = \cos^2\varphi_s \bar{H}_{s,SS} + \sin^2\varphi_s \bar{H}_{s,LL} - 2\sin\varphi_s\cos\varphi_s \bar{H}_{s,LS} \tag{12}$$

$$\bar{m}_{\sigma}^2 = \sin^2\varphi_s \bar{H}_{s,SS} + \cos^2\varphi_s \bar{H}_{s,LL} + 2\sin\varphi_s\cos\varphi_s \bar{H}_{s,LS} \tag{13}$$

$$\bar{m}_{a0}^2 = \bar{H}_{s,11} \tag{14}$$

$$\bar{m}_{\kappa}^2 = \bar{H}_{s,55} \tag{15}$$

$$\bar{m}_{\eta}^2 = \cos^2\varphi_p \bar{H}_{p,SS} + \sin^2\varphi_p \bar{H}_{p,LL} - 2\sin\varphi_p\cos\varphi_p \bar{H}_{p,LS} \tag{16}$$

$$\bar{m}_{\eta'}^2 = \sin^2\varphi_p \bar{H}_{p,SS} + \cos^2\varphi_p \bar{H}_{p,LL} + 2\sin\varphi_p\cos\varphi_p \bar{H}_{s,LS} \tag{17}$$

$$\bar{m}_{\pi}^2 = \bar{H}_{p,11} \tag{18}$$

$$\bar{m}_{K}^2 = \bar{H}_{p,55} \tag{19}$$

其中混合角

$$\varphi_{s/p} = \frac{1}{2}\arctan\left(\frac{2\bar{H}_{s/p,LS}}{\bar{H}_{s/p,SS} - \bar{H}_{s/p,LL}}\right) \tag{20}$$

对混合化简一下

$$\bar{m}_{f0/\eta}^2 = \frac{\bar{H}_{s/p,LL} + \bar{H}_{s/p,SS}}{2} + \sqrt{(\bar{H}_{s/p,LL} - \bar{H}_{s/p,SS})^2 + 4\bar{H}_{s/p,LS}^2} \tag{21}$$

$$\bar{m}_{\sigma/\eta'}^2 = \frac{\bar{H}_{s/p,LL} + \bar{H}_{s/p,SS}}{2} - \sqrt{(\bar{H}_{s/p,LL} - \bar{H}_{s/p,SS})^2 + 4\bar{H}_{s/p,LS}^2} \tag{22}$$

这种方案下$\bar{m}_{f0/\eta}^2$和$\bar{m}_{\sigma/\eta'}^2$无法写成简单的$\bar{m}^2 = \frac{m^2}{Z_\phi}$,或者说混合的$Z_\phi$比较复杂，不能由$Z_{\phi_L}, Z_{\phi_S}$简单表示。这使得规制函数前面的波函数重整化系数难以写出，需要近似。

2 方案2

我们对Hessian矩阵统一用Z_{ϕ_L}重整化：

$$\bar{H}_{ij} = \frac{H_{ij}}{Z_{\phi_L}} \tag{23}$$

此时

$$\bar{H}_{ii} = \frac{\partial^2 \bar{\tilde{U}}_k}{\partial \bar{\phi}_i \partial \bar{\phi}_i}, \quad \text{for} \quad i = 1,2,3,L \tag{24}$$

$$\bar{H}_{ii} = \frac{1}{z_{LS}} \frac{\partial^2 \bar{\tilde{U}}_k}{\partial \bar{\phi}_i \partial \bar{\phi}_i}, \quad \text{for} \quad i = 4,5,6,7,S \tag{25}$$

$$\bar{H}_{LS} = \frac{1}{z_{LS}^{1/2}} \frac{\partial^2 \bar{\tilde{U}}_k}{\partial \bar{\phi}_L \partial \bar{\phi}_S} \tag{26}$$

这里$z_{LS} \equiv Z_L/Z_S$,这样，式(12-19)与裸的情况下完全一致。方案1的困难自然没有，但是以K介子为例：

$$\bar{m}_K^2 = \bar{H}_{p,55} = \frac{1}{z_{LS}} \frac{\partial^2 \bar{\tilde{U}}_k}{\partial \bar{\pi}_5 \partial \bar{\pi}_5} \tag{27}$$

在引入

$$\phi_S^* = \sqrt{\frac{Z_{\phi_L,k}}{Z_{\phi_S,k}}} \bar{\phi}_S \tag{28}$$

会发现回到不区分的情况，也就是白忙活。。。