# 1 Flow equation

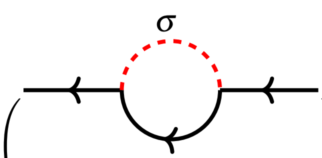
$$\bar{m}(p) = \frac{1}{2}\bar{h}_l \bar{\sigma}_l \tag{1}$$

$$\partial_t h_l(p) = (\eta_q + \frac{1}{2}\eta_\phi)\bar{h}_l(p) - \bar{m}_\pi^2 \dot{\bar{A}}(p) + \frac{1}{\sigma_l}\mathrm{Re}[\overline{\mathrm{Flow}^{(2)}_{(\bar{q}T^L q)}}](p) \tag{2}$$

$$\dot{\bar{A}}(p) = -\frac{1}{h_l(p)}\overline{\mathrm{Flow}^{(4)}_{(\bar{q}T^L q)(\bar{q}T^L q)}}(s) \tag{3}$$

## 1.1 A

$$\frac{1}{\sigma_l}\mathrm{Re}[\overline{\mathrm{Flow}^{(2)}_{(\bar{q}T^L q)}}](p)^\sigma = -\frac{1}{\sigma_l}\tilde{\partial}_t\left(\frac{1}{4}tr\left(\right.\right.$$




$$\left.\left.\right)\Big|_p\right)$$

$$\begin{aligned} &= -\tilde{\partial}_t \sum\!\!\!\!\!\!\int \left(\frac{1}{Z_{\phi,k}Z_{q,k}}\frac{h_k^3}{4}\bar{G}_k^q(q)\bar{G}_k^\sigma(q-p)\right) \\ &= -\frac{1}{Z_{\phi,k}Z_{q,k}}\frac{h_k^3}{4}\sum\!\!\!\!\!\!\int\left(\tilde{\partial}_t\bar{G}_k^q(q)\bar{G}_k^\sigma(q-p) + \bar{G}_k^q(q-p)\tilde{\partial}_t\bar{G}_k^\sigma(q)\right)\end{aligned} \tag{4}$$

$$\tilde{\partial}_t\bar{G}_k^q(q) = -2k^2(\bar{G}_k^q(q))^2[(1-\eta_q)+\eta_q x^{\frac{1}{2}}]\theta(1-x) \tag{5}$$

$$\tilde{\partial}_t\bar{G}_k^\sigma(q) = -k^2(\bar{G}_k^\sigma(q))^2[(2-\eta_\phi)+\eta_\phi x]\theta(1-x) \tag{6}$$

$$\begin{aligned} &T\sum_n\int\frac{d^3q}{(2\pi)^3}(\tilde{\partial}_t\bar{G}_k^q(q))\bar{G}_k^\sigma(q-p) \\ =&T\sum_n\int\frac{d^3q}{(2\pi)^3}(-2)\frac{1}{k^2}(\tilde{G}_k^q(q))^2[(1-\eta_q)+\eta_q x^{\frac{1}{2}}]\theta(1-x)\frac{1}{k^2}\tilde{G}_k^\sigma(q-p) \\ =&-\frac{2}{k^4}\frac{1}{(2\pi)^3}\int_0^\infty q^2dq\int_{-1}^1 d\cos\theta(2\pi)[(1-\eta_q)+\eta_q x^{\frac{1}{2}}]\theta(1-x)T\sum_n\tilde{G}_k^q(q))^2\tilde{G}_k^\sigma(q-p) \\ =&-\frac{1}{(2\pi)^2}\int_0^1 x^{\frac{1}{2}}[(1-\eta_q)+\eta_q x^{\frac{1}{2}}]dx\int_{-1}^1 d\cos\theta\frac{T}{k}\sum_n\tilde{G}_k^q(q))^2\tilde{G}_k^\sigma(q-p)\end{aligned} \tag{7}$$

here we note that

$$\frac{T}{k}\sum_n \tilde{G}_k^q(q))^2\tilde{G}_k^\phi(q-p)=\mathscr{F}2\mathscr{B}1(m_q;m_{\phi,q-p}) \tag{8}$$

Then

$$above=-\frac{1}{(2\pi)^2}\int_0^1 x^{\frac{1}{2}}[(1-\eta_q)+\eta_q x^{\frac{1}{2}}]dx\int_{-1}^1 d\cos\theta\mathscr{F}2\mathscr{B}1(m_q;m_{\sigma,q-p}) \tag{9}$$

$$\begin{aligned}&T\sum_n\int\frac{d^3q}{(2\pi)^3}\bar{G}_k^q(q-p)\tilde{\partial}_t\bar{G}_k^\sigma(q)\\&=-\frac{1}{2(2\pi)^2}\int_0^1 x^{\frac{1}{2}}[(2-\eta_\phi)+\eta_\phi x]dx\int_{-1}^1 d\cos\theta\mathscr{F}1\mathscr{B}2(m_{q,q-p};m_\sigma)\end{aligned} \tag{10}$$

Then

$$\begin{aligned}\frac{1}{\sigma_l}\mathrm{Re}[\overline{\mathrm{Flow}^{(2)}_{(\bar{q}T^Lq)}}](p)^\sigma=&\frac{\bar{h}_k^3}{8(2\pi)^2}\Bigg(2\int_0^1 x^{\frac{1}{2}}[(1-\eta_q)+\eta_q x^{\frac{1}{2}}]dx\int_{-1}^1 d\cos\theta\mathscr{F}2\mathscr{B}1(m_q;m_{\sigma,q-p})\\&+\int_0^1 x^{\frac{1}{2}}[(2-\eta_\phi)+\eta_\phi x]dx\int_{-1}^1 d\cos\theta\mathscr{F}1\mathscr{B}2(m_{q,q-p};m_\sigma)\Bigg)\end{aligned} \tag{11}$$

similar

$$\begin{aligned}\frac{1}{\sigma_l}\mathrm{Re}[\overline{\mathrm{Flow}^{(2)}_{(\bar{q}T^Lq)}}](p)^\pi=&\frac{3\bar{h}_k^3}{8(2\pi)^2}\Bigg(2\int_0^1 x^{\frac{1}{2}}[(1-\eta_q)+\eta_q x^{\frac{1}{2}}]dx\int_{-1}^1 d\cos\theta\mathscr{F}2\mathscr{B}1(m_q;m_{\pi,q-p})\\&+\int_0^1 x^{\frac{1}{2}}[(2-\eta_\phi)+\eta_\phi x]dx\int_{-1}^1 d\cos\theta\mathscr{F}1\mathscr{B}2(m_{q,q-p};m_\pi)\Bigg)\end{aligned} \tag{12}$$

$$\begin{aligned}\frac{1}{\sigma_l}\mathrm{Re}[\overline{\mathrm{Flow}^{(2)}_{(\bar{q}T^Lq)}}](p)^A=&-\frac{3\bar{h}_k\bar{g}_k^2C_2(N_c)}{2(2\pi)^2}\Bigg(2\int_0^1 x^{\frac{1}{2}}[(1-\eta_q)+\eta_q x^{\frac{1}{2}}]dx\int_{-1}^1 d\cos\theta\mathscr{F}2\mathscr{B}1(m_q;m_{A,q-p})\\&+\int_0^1 x^{\frac{1}{2}}[(2-\eta_A)+\eta_A x]dx\int_{-1}^1 d\cos\theta\mathscr{F}1\mathscr{B}2(m_{q,q-p};m_A)\Bigg)\end{aligned} \tag{13}$$

## 1.2 B

$$
\begin{aligned}
&\overline{\mathrm{Flow}^{(4)}_{(\bar{q}T^Lq)(\bar{q}T^Lq)}}(s)^{(A)} \\
=&-\frac{3}{2}C_c(N_c)\big(\frac{3}{4}-\frac{1}{N_c^2}\big)g_k^4\tilde{\partial}_t\big\{(\bar{G}_k^A(q))^2\overrightarrow{(q+p)}_F\cdot\overrightarrow{(q-p)}_F\bar{G}_k^q(q+p)\bar{G}_k^q(q-p)\big\} \\
=&-\frac{3}{2}C_c(N_c)\big(\frac{3}{4}-\frac{1}{N_c^2}\big)g_k^4\big\{2\bar{G}_k^A(q)(\tilde{\partial}_t\bar{G}_k^A(q))\overrightarrow{(q+p)}_F\cdot\overrightarrow{(q-p)}_F\bar{G}_k^q(q+p)\bar{G}_k^q(q-p) \\
&+(\bar{G}_k^A(q))^2(\tilde{\partial}_t\overrightarrow{(q+p)}_F)\cdot\overrightarrow{(q-p)}_F\bar{G}_k^q(q+p)\bar{G}_k^q(q-p) \\
&+(\bar{G}_k^A(q))^2\overrightarrow{(q+p)}_F\cdot(\tilde{\partial}_t\overrightarrow{(q-p)}_F)\bar{G}_k^q(q+p)\bar{G}_k^q(q-p) \\
&+(\bar{G}_k^A(q))^2\overrightarrow{(q+p)}_F\cdot\overrightarrow{(q-p)}_F(\tilde{\partial}_t\bar{G}_k^q(q+p))\bar{G}_k^q(q-p) \\
&+(\bar{G}_k^A(q))^2\overrightarrow{(q+p)}_F\cdot\overrightarrow{(q-p)}_F\bar{G}_k^q(q+p)(\tilde{\partial}_t\bar{G}_k^q(q-p))\big\} \\
=&-\frac{3}{2}C_c(N_c)\big(\frac{3}{4}-\frac{1}{N_c^2}\big)g_k^4\big\{2\bar{G}_k^A(q)(\tilde{\partial}_t\bar{G}_k^A(q))\overrightarrow{(q+p)}_F\cdot\overrightarrow{(q-p)}_F\bar{G}_k^q(q+p)\bar{G}_k^q(q-p) \\
&+(\bar{G}_k^A(q-p))^2(\tilde{\partial}_t\vec{q}_F)\cdot\overrightarrow{(q-2p)}_F\bar{G}_k^q(q)\bar{G}_k^q(q-2p) \\
&+(\bar{G}_k^A(q+p))^2\overrightarrow{(q+2p)}_F\cdot(\tilde{\partial}_t\vec{q}_F)\bar{G}_k^q(q+2p)\bar{G}_k^q(q) \\
&+(\bar{G}_k^A(q-q))^2\vec{q}_F\cdot\overrightarrow{(q-2p)}_F(\tilde{\partial}_t\bar{G}_k^q(q))\bar{G}_k^q(q-2p) \\
&+(\bar{G}_k^A(q+p))^2\overrightarrow{(q+2p)}_F\cdot\vec{q}_F\bar{G}_k^q(q+2p)(\tilde{\partial}_t\bar{G}_k^q(q))\big\}
\end{aligned}
\tag{14}
$$

$$(\tilde{\partial}_t\vec{q}_F)=\vec{q}(\tilde{\partial}_t r_F)=\vec{q}\frac{1}{Z_{q,k}}\partial_t(Z_{q,k}r_F)=[(1-\eta_q)x^{-\frac{1}{2}}+\eta_q]\theta(1-x)\vec{q} \tag{15}$$

$$\tilde{\partial}_t\bar{G}_k^q(q)=-2k^2(\bar{G}_k^q(q))^2[(1-\eta_q)+\eta_q x^{\frac{1}{2}}]\theta(1-x) \tag{16}$$

$$\tilde{\partial}_t\bar{G}_k^A(q)=-k^2(\bar{G}_k^A(q))^2[(2-\eta_A)+\eta_A x]\theta(1-x) \tag{17}$$

So, we need

$$\frac{T}{k}\sum_n(\tilde{G}_k^A(q))^3\tilde{G}_k^q(q+p)\tilde{G}_k^q(q-p)=\mathscr{B}3\mathscr{F}1\mathscr{F}1(m_{A,q};m_{q,q+p};m_{q,q-p}) \tag{18}$$

$$\frac{T}{k}\sum_n(\tilde{G}_k^A(q-p))^2\tilde{G}_k^q(q)\tilde{G}_k^q(q-2p)=\mathscr{B}2\mathscr{F}1\mathscr{F}1(m_{A,q-p};m_{q,q};m_{q,q-2p}) \tag{19}$$

$$\frac{T}{k}\sum_n (\tilde{G}^A_k(q+p))^2 \tilde{G}^q_k(q+2p)\tilde{G}^q_k(q) = \mathscr{B}2\mathscr{F}1\mathscr{F}1(m_{A,q+p};m_{q,q+2p};m_{q,q}) \tag{20}$$

$$\frac{T}{k}\sum_n (\tilde{G}^A_k(q-p))^2 (\tilde{G}^q_k(q))^2\tilde{G}^q_k(q-2p) = \mathscr{B}2\mathscr{F}2\mathscr{F}1(m_{A,q-p};m_{q,q};m_{q,q-2p}) \tag{21}$$

$$\frac{T}{k}\sum_n (\tilde{G}^A_k(q+p))^2 (\tilde{G}^q_k(q))^2\tilde{G}^q_k(q+2p) = \mathscr{B}2\mathscr{F}2\mathscr{F}1(m_{A,q+p};m_{q,q};m_{q,q+2p}) \tag{22}$$

We need

$$\mathscr{B}1\mathscr{F}1\mathscr{F}1(m_a;m_b;m_c) \tag{23}$$

$$\begin{aligned}
&\overline{\mathrm{Flow}^{(4)}_{(\bar{q}T^Lq)(\bar{q}T^Lq)}}(s)^{(meson)}\\
=&-\frac{2}{32N_c}h_k^4\big\{\tilde{\partial}_t[\overrightarrow{(q+p)}_F\cdot\overrightarrow{(q-p)}_F G^\pi_k(q)G^\sigma_k(q)G^q_k(q+p)\bar{G}^q_k(q-p)]\\
&-\tilde{\partial}_t[\overrightarrow{(q+p)}_F\cdot\overrightarrow{(q-p)}_F(G^\pi_k(q))^2\bar{G}^q_k(q+p)\bar{G}^q_k(q-p)]\big\}
\end{aligned} \tag{24}$$

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