

1 Effective Action

$$\begin{aligned}
\Gamma_k = \int_x \Bigg\{ & \frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + Z_c (\partial_\mu \bar{c}^a) D_\mu^{ab} c^b + \frac{1}{2\xi} (\partial_\mu A_\mu^a)^2 \\
& + \frac{1}{2} \int_p A_\mu^a(-p) (\Gamma_{AA\mu\nu}^{(2)ab} - Z_A \Pi_{\mu\nu}^\perp \delta^{ab} p^2) A_\nu^b(p) \\
& + \bar{q} [Z_q (\gamma_\mu D_\mu - \gamma_0 (\hat{\mu} + igA_0))] q - \lambda_q \sum_{a=0}^8 [(\bar{q} T_a q)^2 + (\bar{q} i\gamma_5 T_a q)^2] \\
& + \bar{q} h_k^{1/2} \cdot \Sigma_5 \cdot h_k^{1/2} q + tr(Z_{\Sigma,k}^{1/2} \cdot \partial_\mu \Sigma \cdot Z_{\Sigma,k}^{1/2} \cdot \partial_\mu \Sigma^\dagger) + \tilde{U}_k(\Sigma, \Sigma^\dagger) + V_{glue}(L, \bar{L}) \Bigg\}
\end{aligned} \tag{1}$$

here, the meson field :

$$\Sigma = T^a (\sigma^a + i\pi^a). \quad (a = 0, 1, \dots, 8) \tag{2}$$

and

$$\Sigma_5 = T^a (\sigma^a + i\gamma_5 \pi^a). \quad (a = 0, 1, \dots, 8) \tag{3}$$

with $T^a = \lambda^a/2 (a = 1, \dots, 8)$ and $T^0 = \frac{1}{\sqrt{2N_f}} \mathbb{I}_{N_f \times N_f}$ are generators of $SU(N_f = 3)$. σ^a and π^a mean the scalar and pseudoscalar fields, respectively. The physical meson can be written obviously:

$$\Sigma = \frac{1}{2} \begin{pmatrix} a_0^0 + \sigma_L + i\pi^0 + i\eta_L & \sqrt{2}(a_0^+ + i\pi^+) & \sqrt{2}(\kappa^+ + iK^+) \\ \sqrt{2}(a_0^- + i\pi^-) & -a_0^0 + \sigma_L - i\pi^0 + i\eta_L & \sqrt{2}(\kappa^0 + iK^0) \\ \sqrt{2}(\kappa^- + iK^-) & \sqrt{2}(\bar{\kappa}^0 + i\bar{K}^0) & \sqrt{2}(\sigma_S + i\eta_S) \end{pmatrix} \tag{4}$$

the meson effective potential can be divided into three parts

$$\tilde{U}_k(\Sigma) = U_k(\rho_1, \rho_2) - c_A \xi - c_L \sigma_L - c_S \sigma_S, \tag{5}$$

here $U_k(\rho_1, \rho_2)$ is an arbitrary function of chiral symmetry invariant variables ρ_1, ρ_2 . $c_A \xi$ is Kobayashi-Maskawa-'t Hooft term which breaks $U_A(1)$ symmetry. The last two terms of Eq.(5) are linear sigma terms, which break the chiral symmetry. The ρ_1, ρ_2 are defined as:

$$\rho_1 = \text{tr}(\Sigma \cdot \Sigma^\dagger) \tag{6}$$

$$\rho_2 = \sqrt{6 \cdot \text{tr} \left(\Sigma \cdot \Sigma^\dagger - \frac{1}{3} \rho_1 \mathbb{I}_{3 \times 3} \right)^2} \tag{7}$$

And the effective potential is Taylor expanded as

$$U_k(\rho_1, \rho_2) = \sum_{i,j=0}^N \frac{\lambda_{ij,k}}{i!j!} (\rho_1 - \kappa_1)^i (\rho_2 - \kappa_2)^j \quad (8)$$

Here, we choose the expansion order $N = 5$.

On the vacuum expectation value, ρ_1, ρ_2 are given as:

$$\rho_1 = \frac{1}{2}(\sigma_l^2 + \sigma_s^2) \quad (9)$$

$$\rho_2 = \frac{1}{2}(2\sigma_s^2 - \sigma_l^2) \quad (10)$$

The Yukawa coupling

$$h_k = \begin{pmatrix} h_{l,k} & 0 & 0 \\ 0 & h_{l,k} & 0 \\ 0 & 0 & h_{s,k} \end{pmatrix} \quad (11)$$

and meson and quark wave function renormalization

$$Z_{\sigma,k} = \begin{pmatrix} Z_{\phi_l,k} & 0 & 0 \\ 0 & Z_{\phi_l,k} & 0 \\ 0 & 0 & Z_{\phi_s,k} \end{pmatrix} \quad Z_{q,k} = \begin{pmatrix} Z_{l,k} & 0 & 0 \\ 0 & Z_{l,k} & 0 \\ 0 & 0 & Z_{s,k} \end{pmatrix} \quad (12)$$

At present, we assume $Z_{\sigma,k} = Z_{\pi,k}$.

The quark masses are given as (Appx(5)):

$$m_l = \frac{h_{l,k}}{2} \sigma_l \quad (13)$$

$$m_s = \frac{h_{s,k}}{\sqrt{2}} \sigma_s \quad (14)$$

2 Flow Equations

The the Wetterich equation with dynamical hadronisation reads

$$\partial_t \Gamma_k[\Phi] + \int \langle \partial_t \hat{\phi}_{k,i} \rangle \left(\frac{\delta \Gamma_k[\Phi]}{\delta \phi_i} + c_\sigma \delta_{i\sigma} \right) = \frac{1}{2} \text{Tr}(G_k[\Phi] \partial_t R_k) + \text{Tr} \left(G_{\phi\Phi_j}[\Phi] \frac{\delta \langle \partial_t \hat{\phi}_{k,i} \rangle}{\delta \Phi_j} R_\phi \right) \quad (15)$$

we assume

$$\begin{aligned} \langle \partial_t \hat{\phi}_k \rangle &= \dot{A}_{l,k}[(\bar{q}T_a q) + (\bar{q}i\gamma_5 T_a q)] + \dot{A}_{s,k}[(\bar{q}T_b q) + (\bar{q}i\gamma_5 T_b q)] + \dot{B}_k \Sigma, \\ \text{for } a &= L, 1, \dots, 3, b = 4, \dots, 7, S \end{aligned} \quad (16)$$

here T^L, T^S are given in Appendix 7 As pointed out in ref [] , we choose $\dot{B}_k = 0$. By taking the derivative of of each side of Eq. (15)

$$\frac{\overrightarrow{\delta}}{\delta(\bar{q}T^a q)}(Eq.(15)) \frac{\overleftarrow{\delta}}{\delta(\bar{q}T^a q)}, \quad (17)$$

we get

$$-\partial_t \lambda_q + \dot{A}h_k = -\text{Flow}_{(\bar{q}T^a q)(\bar{q}T^a q)}^{(4)} \quad (18)$$

with the condication

$$\lambda_q \equiv 0, \quad \forall k \quad (19)$$

we get the renormalised hadronisation function

$$\dot{A} = -\frac{1}{\bar{h}_k} \overline{\text{Flow}}_{(\bar{q}T^a q)(\bar{q}T^a q)}^{(4)} \quad (20)$$

we split the expression

$$\dot{A}_{l,k} = -\frac{1}{\bar{h}_{l,k}} \overline{\text{Flow}}_{(\bar{q}T^L q)(\bar{q}T^L q)}^{(4)} \quad (21)$$

$$\dot{A}_{s,k} = -\frac{1}{\bar{h}_{s,k}} \overline{\text{Flow}}_{(\bar{q}T^S q)(\bar{q}T^S q)}^{(4)} \quad (22)$$

And to calculate the yukawa flow equation:

$$\frac{\delta}{\delta \sigma^a} \frac{\delta}{\delta(\bar{q}T^a q)}(Eq.(15)) \quad a = L/S \quad (23)$$

we get

$$\partial \bar{h}_{l,k} = \left(\eta_{l,k} + \frac{1}{2} \eta_{\phi,k} \right) - \frac{\delta^2 \bar{U}(\Sigma)}{(\delta \bar{\sigma}_L)^2} \dot{A}_{l,k} + \overline{\text{Flow}}_{(\bar{q}T^L q)\sigma_L}^{(3)} \quad (24)$$

$$\partial \bar{h}_{s,k} = \left(\eta_{s,k} + \frac{1}{2} \eta_{\phi,k} \right) - \frac{\delta^2 \bar{U}(\Sigma)}{(\delta \bar{\sigma}_S)^2} \dot{A}_{s,k} + \overline{\text{Flow}}_{(\bar{q}T^S q)\sigma_S}^{(3)} \quad (25)$$

A simpler way given in []

$$\frac{1}{\sigma^a} \frac{\delta}{\delta(\bar{q} T^a q)} (Eq.(15)) \quad a = L/S \quad (26)$$

and we get

$$\partial \bar{h}_{l,k} = \left(\eta_{l,k} + \frac{1}{2} \eta_{\phi,k} \right) - \frac{1}{\bar{\sigma}_L} \frac{\delta \bar{U}(\Sigma)}{\delta \bar{\sigma}_L} \dot{A}_{l,k} + \frac{1}{\bar{\sigma}_L} \text{Re} \overline{\text{Flow}}_{(\bar{q} T^L q)}^{(2)} \quad (27)$$

$$\partial \bar{h}_{s,k} = \left(\eta_{s,k} + \frac{1}{2} \eta_{\phi,k} \right) - \frac{1}{\bar{\sigma}_S} \frac{\delta \bar{U}(\Sigma)}{\delta \bar{\sigma}_S} \dot{A}_{s,k} + \frac{1}{\bar{\sigma}_S} \text{Re} \overline{\text{Flow}}_{(\bar{q} T^S q)}^{(2)} \quad (28)$$

the next step is to calculate the $\overline{\text{Flow}}$ terms.

3 Result

Pressure:

$$\frac{p}{T^4} = \frac{U(0,0) - U((T,\mu))}{T^4} \quad (29)$$

and n-th order cumulats

$$\chi_n^B = \frac{\partial^n}{\partial (\mu_B/T)^n} \frac{p}{T^4} \quad (30)$$

4 Appendix.A

The meson masses can be obtained by Hessian matrix:

$$H_{p,LL} = \frac{c_A \sigma_S}{\sqrt{2}} + U^{(1,0)} - U^{(0,1)} \quad (31)$$

$$H_{p,LS} = \frac{c_A \sigma_L}{\sqrt{2}} \quad (32)$$

$$H_{p,SS} = U^{(1,0)} + 2U^{(0,1)} \quad (33)$$

$$H_{p,11} = -\frac{c_A \sigma_S}{\sqrt{2}} + U^{(1,0)} - U^{(0,1)} \quad (34)$$

$$H_{p,44} = -\frac{c_A \sigma_L}{2} + U^{(1,0)} + \frac{\sigma_L^2 - 3\sqrt{2}\sigma_L\sigma_S + 4\sigma_S^2}{2\sigma_S^2 - \sigma_L^2} U^{(0,1)} \quad (35)$$

$$H_{s,LL} = -\frac{c_A \sigma_S}{\sqrt{2}} + U^{(1,0)} - U^{(0,1)} + (U^{(2,0)} - 2U^{(1,1)} + U^{(0,2)}) \sigma_L^2 \quad (36)$$

$$H_{s,LS} = -\frac{c_A \sigma_L}{\sqrt{2}} + (U^{(2,0)} + U^{(1,1)} - 2U^{(0,2)}) \sigma_L \sigma_S \quad (37)$$

$$H_{s,SS} = U^{(1,0)} + 2U^{(0,1)} + (4U^{(2,0)} + 4U^{(1,1)} + U^{(0,2)}) \sigma_S^2 \quad (38)$$

$$H_{s,11} = \frac{c_A \sigma_S}{\sqrt{2}} + U^{(1,0)} + \frac{7\sigma_L^2 - 2\sigma_S^2}{2\sigma_S^2 - \sigma_L^2} U^{(0,1)} \quad (39)$$

$$H_{s,44} = \frac{c_A \sigma_L}{2} + U^{(1,0)} + \frac{\sigma_L^2 + 3\sqrt{2}\sigma_L \sigma_S + 4\sigma_S^2}{2\sigma_S^2 - \sigma_L^2} U^{(0,1)} \quad (40)$$

Because the nonvanishing nondiagonal element $H_{s/p,LS}$ we introduce the mixing angles between LS and physical basis:

$$\begin{pmatrix} f_0 \\ \sigma \end{pmatrix} = \begin{pmatrix} \cos \varphi_s & -\sin \varphi_s \\ \sin \varphi_s & \cos \varphi_s \end{pmatrix} \begin{pmatrix} \sigma_L \\ \sigma_S \end{pmatrix}, \quad (41)$$

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos \varphi_p & -\sin \varphi_p \\ \sin \varphi_p & \cos \varphi_p \end{pmatrix} \begin{pmatrix} \eta_L \\ \eta_S \end{pmatrix}. \quad (42)$$

here

$$\varphi_{s/p} = \frac{1}{2} \arctan \left(\frac{2H_{s/p,LS}}{H_{s/p,SS} - H_{s/p,LL}} \right) \quad (43)$$

so the square of meson mass are given as

$$m_{f_0}^2 = \cos^2 \varphi_s H_{s,SS} + \sin^2 \varphi_s H_{s,LL} - 2 \sin \varphi_s \cos \varphi_s H_{s,LS} \quad (44)$$

$$m_{\sigma}^2 = \sin^2 \varphi_s H_{s,SS} + \cos^2 \varphi_s H_{s,LL} + 2 \sin \varphi_s \cos \varphi_s H_{s,LS} \quad (45)$$

$$m_{a_0}^2 = H_{s,11} \quad (46)$$

$$m_K^2 = H_{s,44} \quad (47)$$

$$m_{\eta}^2 = \cos^2 \varphi_p H_{p,SS} + \sin^2 \varphi_p H_{p,LL} - 2 \sin \varphi_p \cos \varphi_p H_{p,LS} \quad (48)$$

$$m_{\eta'}^2 = \sin^2 \varphi_p H_{p,SS} + \cos^2 \varphi_p H_{p,LL} + 2 \sin \varphi_p \cos \varphi_p H_{p,LS} \quad (49)$$

$$m_{\pi}^2 = H_{p,11} \quad (50)$$

$$m_K^2 = H_{p,44} \quad (51)$$

We can simplify them as

$$m_{f_0/\eta}^2 = \frac{H_{s/p,LL} + H_{s/p,SS}}{2} + \sqrt{(H_{s/p,LL} - H_{s/p,SS})^2 + 4H_{s/p,LS}^2} \quad (52)$$

$$m_{\sigma/\eta'}^2 = \frac{H_{s/p,LL} + H_{s/p,SS}}{2} - \sqrt{(H_{s/p,LL} - H_{s/p,SS})^2 + 4H_{s/p,LS}^2} \quad (53)$$

And the diagonal element of meson field become:

$$\Sigma_{(1,1)} = \frac{1}{2}(a_0^0 + \cos \varphi_s f_0 + \sin \varphi_s \sigma + i\pi^0 + i \cos \varphi_p \eta + i \sin \varphi_p \eta') \quad (54)$$

$$\Sigma_{(2,2)} = \frac{1}{2}(-a_0^0 + \cos \varphi_s f_0 + \sin \varphi_s \sigma - i\pi^0 + i \cos \varphi_p \eta + i \sin \varphi_p \eta') \quad (55)$$

$$\Sigma_{(3,3)} = \frac{1}{\sqrt{2}}(-\sin \varphi_s f_0 + \cos \varphi_s - i \sin \varphi_p \eta + i \cos \varphi_p \eta') \quad (56)$$

The coefficients in Eq.(28) are given as

$$\frac{1}{\sigma_L} \frac{\delta \tilde{U}(\Sigma)}{\delta \sigma_L} = -\frac{c_A \sigma_S}{\sqrt{2}} + U^{(1,0)} - U^{(0,1)} \quad (57)$$

$$\frac{1}{\sigma_S} \frac{\delta \tilde{U}(\Sigma)}{\delta \sigma_L} = -\frac{c_A \sigma_L^2}{2\sqrt{2}\sigma_S} + U^{(1,0)} + 2U^{(0,1)} \quad (58)$$

One interesting thing is that $\frac{1}{\sigma_L} \frac{\delta \tilde{U}(\Sigma)}{\delta \sigma_L} = m_\pi^2$.

the three-point meson vertex are defined as

$$\lambda_{\phi_i, \phi_j, \phi_l, k} = \left. \frac{\partial^3 U_k(\Sigma)}{\partial \phi_i, \partial \phi_j, \partial \phi_l} \right|_{\phi_0} \quad (59)$$

because we assume

$$Z_\phi = Z_{\pi^+} \quad (60)$$

we choose the three-point meson vertex involve one π^+ , which are given as

$$\lambda_{\pi^+ \pi^- f_0, k} = \frac{c_A}{\sqrt{2}} \sin \phi_S + \cos \phi_S (U^{(0,2)} - 2U^{(1,1)} + U^{(2,0)}) \sigma_L + \sin \phi_S (2U^{(0,2)} - U^{(1,1)} - U^{(2,0)}) \sigma_S \quad (61)$$

$$\lambda_{\pi^+ \pi^- \sigma, k} = -\frac{c_A}{\sqrt{2}} \cos \phi_S + \sin \phi_S (U^{(0,2)} - 2U^{(1,1)} + U^{(2,0)}) \sigma_L - \cos \phi_S (2U^{(0,2)} - U^{(1,1)} - U^{(2,0)}) \sigma_S \quad (62)$$

$$\lambda_{\pi^+ a_0^- \eta, k} = \sin \phi_P \frac{c_A}{\sqrt{2}} + 6 \cos \phi_P U^{(0,1)} \frac{\sigma_L}{2\sigma_S - \sigma_L} \quad (63)$$

$$\lambda_{\pi^+ a_0^- \eta', k} = -\cos \phi_P \frac{c_A}{\sqrt{2}} + 6 \sin \phi_P U^{(0,1)} \frac{\sigma_L}{2\sigma_S - \sigma_L} \quad (64)$$

$$\lambda_{\pi^+ \kappa^- K^0, k} = \lambda_{\pi^+ K^- \kappa^0, k} = \frac{c_A}{\sqrt{2}} + 6U^{(0,1)} \frac{\sigma_S}{2\sigma_S - \sigma_L} \quad (65)$$

5 Appendix.B

The quark masses are given as

$$M_q = \begin{pmatrix} m_{l,k} & 0 & 0 \\ 0 & m_{l,k} & 0 \\ 0 & 0 & m_{s,k} \end{pmatrix} = \frac{\vec{\delta}}{\delta \bar{q}} \Gamma_k \frac{\overleftarrow{\delta}}{\delta q} \Big|_{\Sigma=\Sigma_0} \quad (66)$$

with the condication Eq.(19),

$$M_q = \frac{\vec{\delta}}{\delta \bar{q}} \bar{q} h^{\frac{1}{2}} \Sigma_0 h^{\frac{1}{2}} q \frac{\overleftarrow{\delta}}{\delta q} = \begin{pmatrix} \frac{h_{l,k}}{2} \sigma_L & 0 & 0 \\ 0 & \frac{h_{l,k}}{2} \sigma_l & 0 \\ 0 & 0 & \frac{h_{s,k}}{\sqrt{2}} \sigma_s \end{pmatrix} \quad (67)$$

which are shown in Eq.(13).

The meson quark vertex are given by

$$V_{\bar{q}q\phi_i} = \frac{\delta}{\delta \phi_i} \frac{\vec{\delta}}{\delta \bar{q}} \Gamma_k \frac{\overleftarrow{\delta}}{\delta q} \quad (68)$$

here, we force on u and s quark anomalous dimension and Yukawa coupling, so meson quark vertex which are used:

$$V_{\bar{u}u\phi_i} = \frac{\delta}{\delta \phi_i} h_l \Sigma_{5(1,1)} \quad V_{\bar{u}d\phi_i} = \frac{\delta}{\delta \phi_i} h_l \Sigma_{5(1,2)} \quad V_{\bar{u}s\phi_i} = \frac{\delta}{\delta \phi_i} h_l^{1/2} h_s^{1/2} \Sigma_{5(1,3)} \quad (69)$$

$$V_{\bar{s}u\phi_i} = \frac{\delta}{\delta \phi_i} h_l^{1/2} h_s^{1/2} \Sigma_{5(3,1)} \quad V_{\bar{s}d\phi_i} = \frac{\delta}{\delta \phi_i} h_l^{1/2} h_s^{1/2} \Sigma_{5(3,2)} \quad V_{\bar{s}s\phi_i} = \frac{\delta}{\delta \phi_i} h_s \Sigma_{5(3,3)} \quad (70)$$

then we get

$$V_{\bar{u}uf_0} = h_l \cos \varphi_s / 2 \quad V_{\bar{u}u\sigma} = h_l \sin \varphi_s / 2 \quad V_{\bar{u}ua_0} = h_l / 2 \quad (71)$$

6 Appendix.C

The Kobayashi-Maskawa-'t Hooft coupling \bar{c}_A , should decrease at high scalar and high temperatures. However, if we keep c_A a constant, and

$$\bar{c}_A = \frac{c_A}{Z_\phi^{3/2}} \quad (72)$$

\bar{c}_A will increase with scalar and temperature. One scheme is given in ref [?], which assume that \bar{c}_A is a constant. However, this scheme cause a very sharp phase transitions. We assume c_A is a infrared enhancement function

$$c_A = c_{A,IR} \frac{1}{e^{\frac{k-k_{cut}}{\Delta_k}} + 1} \quad (73)$$

here $k_{cut} = 1GeV$ and $\Delta_k = 20MeV$. And \bar{c}_A still dressed as $\bar{c}_A = c_A/Z_\phi^{3/2}$.

7 Appendix.D

As we known that

$$T_0 = \frac{1}{\sqrt{6}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad T_8 = \frac{1}{2\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \quad (74)$$

We have

$$\Sigma_0 = T_0 \sigma_0 + T_8 \sigma_8 = \begin{pmatrix} \frac{1}{\sqrt{6}} \sigma_0 + \frac{1}{2\sqrt{3}} \sigma_8 & 0 & 0 \\ 0 & \frac{1}{\sqrt{6}} \sigma_0 + \frac{1}{2\sqrt{3}} \sigma_8 & 0 \\ 0 & 0 & \frac{1}{\sqrt{6}} \sigma_0 - \frac{1}{\sqrt{3}} \sigma_8 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} \sigma_L & 0 & 0 \\ 0 & \sigma_L & 0 \\ 0 & 0 & \sqrt{2} \sigma_S \end{pmatrix} = T_L \sigma_L + T_S \sigma_S \quad (75)$$

so

$$T_L = \frac{1}{2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad T_S = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (76)$$

8 Appendix.E:mesons anomalous dimension

As assumed above,

$$\eta_\phi = \eta_{\pi^+} \quad (77)$$

and we get

$$\begin{aligned}
\eta_\phi(0) = & \frac{\bar{Z}_\phi}{Z_\phi(0)} \left\{ \frac{1}{3\pi^2 k^2} [\bar{\lambda}_{\pi^+\pi^-f_0}^2 \mathcal{B}\mathcal{B}_{(2,2)}^{(\pi,f_0)} + \lambda_{\pi^+\pi^-\sigma}^2 \mathcal{B}\mathcal{B}_{(2,2)}^{(\pi,\sigma)} + \lambda_{\pi^+a_0^-}^2 \mathcal{B}\mathcal{B}_{(2,2)}^{(a_0,\eta)} \right. \\
& + \lambda_{\pi^+a_0^-\eta'}^2 \mathcal{B}\mathcal{B}_{(2,2)}^{(a_0,\eta')} + (\lambda_{\pi^+\kappa^-K^0}^2 + \lambda_{\pi^+K^-\kappa^0}^2) \mathcal{B}\mathcal{B}_{(2,2)}^{(K,\kappa)}] \\
& \left. + \frac{N_c h_{l,k}^2}{6\pi^2} [(2\eta_{l,k} - 3) \mathcal{F}_{(2)}(\tilde{m}_{l,k}^2; T, \mu_q) - 4(\eta_{l,k} - 2) \mathcal{F}_{(3)}(\tilde{m}_{l,k}^2; T, \mu_q)] \right\} \quad (78)
\end{aligned}$$

$$\begin{aligned}
\eta_\phi(0, k) = & \frac{1}{3\pi^2 k^2} [\bar{\lambda}_{\pi^+\pi^-f_0}^2 \mathcal{B}\mathcal{B}_{(2,2)}^{(\pi,f_0)} + \lambda_{\pi^+\pi^-\sigma}^2 \mathcal{B}\mathcal{B}_{(2,2)}^{(\pi,\sigma)} + \lambda_{\pi^+a_0^-}^2 \mathcal{B}\mathcal{B}_{(2,2)}^{(a_0,\eta)} \\
& + \lambda_{\pi^+a_0^-\eta'}^2 \mathcal{B}\mathcal{B}_{(2,2)}^{(a_0,\eta')} + (\lambda_{\pi^+\kappa^-K^0}^2 + \lambda_{\pi^+K^-\kappa^0}^2) \mathcal{B}\mathcal{B}_{(2,2)}^{(K,\kappa)}] \quad (79)
\end{aligned}$$

$$\begin{aligned}
& - \frac{N_c \bar{h}_k^2}{\pi^2} \int_0^1 dx \left[(1 - \eta_{l,k}) \sqrt{x} + \eta_{l,k} x \right] \\
& \times \int_{-1}^1 d\cos\theta \left\{ \left[\left(\mathcal{F} \mathcal{F}_{(1,1)}(\tilde{m}_{l,k}^2, \tilde{m}_{l,k}^2) - \mathcal{F}_{(2)}(\tilde{m}_{l,k}^2) \right) \right. \right. \\
& - \left. \left(\mathcal{F} \mathcal{F}_{(2,1)}(\tilde{m}_{l,k}^2, \tilde{m}_{l,k}^2) - \mathcal{F}_{(3)}(\tilde{m}_{l,k}^2) \right) \right] \\
& + \left[\left(\sqrt{x} - \cos\theta \right) \left(1 + r_F(x') \right) \mathcal{F} \mathcal{F}_{(2,1)}(\tilde{m}_{l,k}^2, \tilde{m}_{l,k}^2) \right. \\
& - \left. \mathcal{F}_{(3)}(\tilde{m}_{l,k}^2) \right] - \frac{1}{2} \left[\left(\sqrt{x} - \cos\theta \right) \left(1 + r_F(x') \right) \right. \\
& \left. \left. \times \mathcal{F} \mathcal{F}_{(1,1)}(\tilde{m}_{l,k}^2, \tilde{m}_{l,k}^2) - \mathcal{F}_{(2)}(\tilde{m}_{l,k}^2) \right] \right\}, \quad (80)
\end{aligned}$$

9 Appendix.F:quarks anomalous dimension

10 Appendix.G:Yukawa coupling

$$\begin{aligned}
\partial_l \bar{h}_l = & (\eta_l + \frac{1}{2} \eta_\phi) \bar{h}_l + \frac{1}{8\pi^2} \bar{h}_l^3 [3\mathcal{L}_{(1,1)}^{(l,a_0)} - 3\mathcal{L}_{(1,1)}^{(l,\pi)} + \cos^2 \varphi_s \mathcal{L}_{(1,1)}^{(l,f_0)} \\
& - \cos^2 \varphi_p \mathcal{L}_{(1,1)}^{(l,\eta)} + \sin^2 \varphi_s \mathcal{L}_{(1,1)}^{(l,\sigma)} - \sin^2 \varphi_p \mathcal{L}_{(1,1)}^{(l,\eta')}] \\
& - \frac{3}{2\pi^2} \frac{N_c^2 - 1}{2N_c} g_{\bar{l}Al} \bar{h}_l \mathcal{L}_{(1,1)}^{(l,0)}
\end{aligned} \tag{81}$$

$$\begin{aligned}
\partial_l \bar{h}_s = & (\eta_s + \frac{1}{2} \eta_\phi) \bar{h}_s + \frac{1}{8\pi^2} 2\bar{h}_s^3 [\sin^2 \varphi_s \mathcal{L}_{(1,1)}^{(s,f_0)} - \sin^2 \varphi_p \mathcal{L}_{(1,1)}^{(s,\eta)} \\
& + \cos^2 \varphi_s \mathcal{L}_{(1,1)}^{(s,\sigma)} - \cos^2 \varphi_p \mathcal{L}_{(1,1)}^{(s,\eta')}] \\
& - \frac{3}{2\pi^2} \frac{N_c^2 - 1}{2N_c} g_{\bar{s}As} \bar{h}_s \mathcal{L}_{(1,1)}^{(s,0)}
\end{aligned} \tag{82}$$