

0.1 定义

现将介子波函数重整化系数 $Z_{\Sigma,k}$ 分开,我们定义的介子波函数重整化

$$Z_{\Sigma,k} \equiv \begin{pmatrix} Z_{\phi_L,k} & Z_{\phi_L,k} & Z_{\phi_S,k} \\ Z_{\phi_L,k} & Z_{\phi_L,k} & Z_{\phi_S,k} \\ Z_{\phi_S,k} & Z_{\phi_S,k} & Z_{\phi_S,k} \end{pmatrix} \quad (1)$$

所以介子场

$$\bar{\phi}_i = Z_{\phi_L,k}^{\frac{1}{2}} \phi_i, \quad \text{for } i = 1, 2, 3 \quad (2)$$

$$\bar{\phi}_i = Z_{\phi_S,k}^{\frac{1}{2}} \phi_i, \quad \text{for } i = 4, 5, 6, 7 \quad (3)$$

$$\bar{\phi}_L = Z_{\phi_L,k}^{\frac{1}{2}} \phi_L, \quad (4)$$

$$\bar{\phi}_S = Z_{\phi_S,k}^{\frac{1}{2}} \phi_S, \quad (5)$$

其中 $\phi = \pi/\sigma$ 表示赝标/标量场, 这里 L, S 与08基之间存在转动关系:

$$\begin{pmatrix} \phi_L \\ \phi_S \end{pmatrix} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & \sqrt{2} \\ -\sqrt{2} & 1 \end{pmatrix} \begin{pmatrix} \phi_8 \\ \phi_0 \end{pmatrix} \quad (6)$$

介子的质量由Hessian矩阵来计算:

$$H_{ij} = \frac{\partial^2 \tilde{U}_k}{\partial \phi_i \partial \phi_j} \quad (7)$$

对Hessian矩阵进行重整化:

$$\bar{H}_{ij} = \frac{\partial^2 \bar{\tilde{U}}_k}{\partial \bar{\phi}_i \partial \bar{\phi}_j} \quad (8)$$

即

$$\bar{H}_{ii} = \frac{H_{ii}}{Z_{\phi_L}}, \quad \text{for } i = 1, 2, 3, L \quad (9)$$

$$\bar{H}_{ii} = \frac{H_{ii}}{Z_{\phi_S}}, \quad \text{for } i = 4, 5, 6, 7, S \quad (10)$$

$$\bar{H}_{LS} = \frac{H_{LS}}{Z_{\phi_L}^{1/2} Z_{\phi_S}^{1/2}} \quad (11)$$

Hessian矩阵在 LS 基下存在非零的非对角元 \bar{H}_{LS} 和 \bar{H}_{SL} ，所以介子质量

$$\bar{m}_{f0}^2 = \cos^2 \varphi_s \bar{H}_{s,SS} + \sin^2 \varphi_s \bar{H}_{s,LL} - 2 \sin \varphi_s \cos \varphi_s \bar{H}_{s,LS} \quad (12)$$

$$\bar{m}_{\sigma}^2 = \sin^2 \varphi_s \bar{H}_{s,SS} + \cos^2 \varphi_s \bar{H}_{s,LL} + 2 \sin \varphi_s \cos \varphi_s \bar{H}_{s,LS} \quad (13)$$

$$\bar{m}_{a0}^2 = \bar{H}_{s,11} \quad (14)$$

$$\bar{m}_{\kappa}^2 = \bar{H}_{s,55} \quad (15)$$

$$\bar{m}_{\eta}^2 = \cos^2 \varphi_p \bar{H}_{p,SS} + \sin^2 \varphi_p \bar{H}_{p,LL} - 2 \sin \varphi_p \cos \varphi_p \bar{H}_{p,LS} \quad (16)$$

$$\bar{m}_{\eta'}^2 = \sin^2 \varphi_p \bar{H}_{p,SS} + \cos^2 \varphi_p \bar{H}_{p,LL} + 2 \sin \varphi_p \cos \varphi_p \bar{H}_{p,LS} \quad (17)$$

$$\bar{m}_{\pi}^2 = \bar{H}_{p,11} \quad (18)$$

$$\bar{m}_K^2 = \bar{H}_{p,55} \quad (19)$$

其中混合角

$$\varphi_{s/p} = \frac{1}{2} \arctan \left(\frac{2\bar{H}_{s/p,LS}}{\bar{H}_{s/p,SS} - \bar{H}_{s/p,LL}} \right) \quad (20)$$

对混合化简一下

$$\bar{m}_{f0/\eta}^2 = \frac{\bar{H}_{s/p,LL} + \bar{H}_{s/p,SS} + \sqrt{(\bar{H}_{s/p,LL} - \bar{H}_{s/p,SS})^2 + 4\bar{H}_{s/p,LS}^2}}{2} \quad (21)$$

$$\bar{m}_{\sigma/\eta'}^2 = \frac{\bar{H}_{s/p,LL} + \bar{H}_{s/p,SS} - \sqrt{(\bar{H}_{s/p,LL} - \bar{H}_{s/p,SS})^2 + 4\bar{H}_{s/p,LS}^2}}{2} \quad (22)$$

这种方案下 $\bar{m}_{f0/\eta}^2$ 和 $\bar{m}_{\sigma/\eta'}^2$ 无法写成简单的 $\bar{m}^2 = \frac{m^2}{Z_\phi}$,或者说混合的 Z_ϕ 比较复杂,不能由 Z_{ϕ_L}, Z_{ϕ_S} 简单表示。这使得规制函数前面的波函数重整化系数难以写出,需要近似。