

1 Flow equation

$$\partial_t \Gamma_k \sim \partial_t \{ \bar{q} (Z_{q,k} i \gamma_\mu q_\mu + m_f) q \}$$

(1)

$$\begin{aligned}
\eta_q^\sigma &= -\frac{1}{Z_{q,k}} \frac{1}{p^2} \tilde{\partial}_t \left(\frac{1}{4} \text{tr} \left(i \vec{p} \cdot \vec{\gamma} \right. \right. \\
&\quad \left. \left. \text{---} \overbrace{\hspace{1cm}}^{\sigma} \text{---} \right) \Big|_p \right) \\
&= -\frac{1}{Z_{q,k}} \frac{1}{p^2} \tilde{\Delta} \left(\frac{1}{Z_{\phi,k} Z_{q,k}} \frac{h_k^2}{4} (\vec{p} \cdot \vec{q}_F) \bar{G}_k^q(q) \bar{G}_k^\sigma(q-p) \right) \\
&= -\frac{1}{Z_{\phi,k} Z_{q,k}^2} \frac{1}{p^2} \frac{h_k^2}{4} \tilde{\Delta} \left((\tilde{\partial}_t \vec{q}_F) \cdot \vec{p} \bar{G}_k^q(q) \bar{G}_k^\sigma(q-p) + (\vec{q}_F \cdot \vec{p}) \tilde{\partial}_t \bar{G}_k^q(q) \bar{G}_k^\sigma(q-p) \right. \\
&\quad \left. + (\overrightarrow{q-p})_F \cdot \vec{p} \bar{G}_k^q(q-p) \tilde{\partial}_t \bar{G}_k^\sigma(q) \right)
\end{aligned} \tag{2}$$

$$(\tilde{\partial}_t q_F) \cdot \vec{p} = \vec{q}(\tilde{\partial}_t r_F) \cdot \vec{p} = \vec{q} \cdot \vec{p} \frac{1}{Z_{q,k}} \partial_t (Z_{q,k} r_F) = [(1 - \eta_q)x^{-\frac{1}{2}} + \eta_q] \theta(1 - x) \vec{q} \cdot \vec{p} \quad (3)$$

$$\tilde{\partial}_t \bar{G}_k^q(q) = -2k^2 (\bar{G}_k^q(q))^2 [(1 - \eta_q) + \eta_q x^{\frac{1}{2}}] \theta(1 - x) \quad (4)$$

$$\tilde{\partial}_t \bar{G}_k^\sigma(q) = -k^2 (\bar{G}_k^\sigma(q))^2 [(2 - \eta_\phi) + \eta_\phi x] \theta(1 - x) \quad (5)$$

$$\begin{aligned}
& T \sum_n \int \frac{d^3 q}{(2\pi)^3} (\tilde{\partial}_t \vec{q}_F) \cdot \vec{p} \tilde{G}_k^q(q) \tilde{G}_k^\sigma(q-p) \\
&= T \sum_n \int \frac{d^3 q}{(2\pi)^3} [(1-\eta_q)x^{-\frac{1}{2}} + \eta_q] \theta(1-x) (\vec{q} \cdot \vec{p}) \frac{1}{k^4} \tilde{G}_k^q(q) \tilde{G}_k^\sigma(q-p) \\
&= \int \frac{d^3 q}{(2\pi)^3} [(1-\eta_q)x^{-\frac{1}{2}} + \eta_q] \theta(1-x) (\vec{q} \cdot \vec{p}) \frac{1}{k^3} \frac{T}{k} \sum_n \tilde{G}_k^q(q) \tilde{G}_k^\sigma(q-p)
\end{aligned} \tag{6}$$

here we note that

$$\begin{aligned}
& \frac{T}{k} \sum_n \tilde{G}_k^q(q) \tilde{G}_k^\phi(q-p) \\
&= \mathcal{F}1 \mathcal{B}1(m_q; m_{\phi, q-p})
\end{aligned} \tag{7}$$

then

$$\begin{aligned}
above &= \int \frac{d^3 q}{(2\pi)^3} [(1-\eta_q)x^{-\frac{1}{2}} + \eta_q] \theta(1-x) (qp \cos \theta) \frac{1}{k^3} \mathcal{F}1 \mathcal{B}1(m_q; m_{\phi, q-p}) \\
&= \frac{p}{(2\pi)^2} \int_0^\infty q^2 dq \int_{-1}^1 d \cos \theta \frac{1}{k^3} [(1-\eta_q)x^{-\frac{1}{2}} + \eta_q] \theta(1-x) q \cos \theta \mathcal{F}1 \mathcal{B}1(m_q; m_{\phi, q-p}) \\
&= \frac{p}{(2\pi)^2} \int_0^\infty q^3 dq \frac{1}{k^3} [(1-\eta_q)x^{-\frac{1}{2}} + \eta_q] \theta(1-x) \int_{-1}^1 d \cos \theta \cos \theta \mathcal{F}1 \mathcal{B}1(m_q; m_{\phi, q-p}) \\
&= \frac{kp}{2(2\pi)^2} \int_0^1 x dx [(1-\eta_q)x^{-\frac{1}{2}} + \eta_q] \int_{-1}^1 d \cos \theta \cos \theta \mathcal{F}1 \mathcal{B}1(m_q; m_{\phi, q-p}) \\
&= \frac{kp}{2(2\pi)^2} \int_0^1 dx [(1-\eta_q)x^{\frac{1}{2}} + \eta_q x] \int_{-1}^1 d \cos \theta \cos \theta \mathcal{F}1 \mathcal{B}1(m_q; m_{\phi, q-p})
\end{aligned} \tag{8}$$