

1 Flow equation

$$\partial_t \Gamma_k \sim \partial_t \{\bar{q} m_f q\}$$

(1)

$$\partial_t(m_f(p))^\sigma = -\tilde{\partial}_t\left(\frac{1}{4}\text{tr}\left(\begin{array}{c} \text{---} \text{---} \text{---} \text{---} \text{---} \\ \text{---} \text{---} \text{---} \text{---} \text{---} \end{array}\right)\bigg|_p\right) \quad (2)$$

$$= -\tilde{\partial}_t \mathbb{J} \left(\frac{1}{Z_{\phi,k} Z_{q,k}} \frac{h_k^2}{4} \bar{m}_f \bar{G}_k^q(q) \bar{G}_k^\sigma(q-p) \right)$$

$$= -\frac{1}{Z_{\phi,k}Z_{q,k}}\frac{h_k^2}{4}\bar{m}_f\sum\left(\tilde{\partial}_t\bar{G}_k^q(q)\bar{G}_k^\sigma(q-p)+\bar{G}_k^q(q-p)\tilde{\partial}_t\bar{G}_k^\sigma(q)\right)$$

$$\tilde{\partial}_t \bar{G}_k^q(q) = -2k^2 (\bar{G}_k^q(q))^2 [(1 - \eta_q) + \eta_q x^{\frac{1}{2}}] \theta(1 - x) \quad (3)$$

$$\tilde{\partial}_t \bar{G}_k^\sigma(q) = -k^2 (\bar{G}_k^\sigma(q))^2 [(2 - \eta_\phi) + \eta_\phi x] \theta(1 - x) \quad (4)$$

$$\begin{aligned}
& T \sum_n \int \frac{d^3 q}{(2\pi)^3} (\tilde{\partial}_t \tilde{G}_k^q(q)) \tilde{G}_k^\sigma(q-p) \\
&= T \sum_n \int \frac{d^3 q}{(2\pi)^3} (-2) \frac{1}{k^2} (\tilde{G}_k^q(q))^2 [(1-\eta_q) + \eta_q x^{\frac{1}{2}}] \theta(1-x) \frac{1}{k^2} \tilde{G}_k^\sigma(q-p) \\
&= -\frac{2}{k^4} \frac{1}{(2\pi)^3} \int_0^\infty q^2 dq \int_{-1}^1 d\cos\theta (2\pi) [(1-\eta_q) + \eta_q x^{\frac{1}{2}}] \theta(1-x) T \sum_n \tilde{G}_k^q(q))^2 \tilde{G}_k^\sigma(q-p) \\
&= -\frac{1}{(2\pi)^2} \int_0^1 x^{\frac{1}{2}} [(1-\eta_q) + \eta_q x^{\frac{1}{2}}] dx \int_{-1}^1 d\cos\theta \frac{T}{k} \sum_n \tilde{G}_k^q(q))^2 \tilde{G}_k^\sigma(q-p)
\end{aligned} \tag{5}$$

here we note that

$$\begin{aligned}\mathcal{F}2\mathcal{B}1p(m_q; m_\phi) &\equiv \frac{T}{k} \sum_n \tilde{G}_k^q(q)^2 \tilde{G}_k^\phi(q-p) \\ &= \mathcal{F}2\mathcal{B}1(m_q; m_{\phi, q-p})\end{aligned}\tag{6}$$

Then

$$above = -\frac{1}{(2\pi)^2} \int_0^1 x^{\frac{1}{2}} [(1-\eta_q) + \eta_q x^{\frac{1}{2}}] dx \int_{-1}^1 d\cos\theta \mathcal{F}2\mathcal{B}1(m_q; m_{\sigma, q-p})\tag{7}$$

$$\begin{aligned}T \sum_n \int \frac{d^3q}{(2\pi)^3} \tilde{G}_k^q(q-p) \tilde{\partial}_t \tilde{G}_k^\sigma(q) \\ = -\frac{1}{2(2\pi)^2} \int_0^1 x^{\frac{1}{2}} [(2-\eta_\phi) + \eta_\phi x] dx \int_{-1}^1 d\cos\theta \mathcal{F}1\mathcal{B}2(m_{q, q-p}; m_\sigma)\end{aligned}\tag{8}$$

Then

$$\begin{aligned}\partial_t(\bar{m}_f(p))^\sigma &= \frac{\bar{h}_k^2 \bar{m}_{f,k}}{8(2\pi)^2} \left(2 \int_0^1 x^{\frac{1}{2}} [(1-\eta_q) + \eta_q x^{\frac{1}{2}}] dx \int_{-1}^1 d\cos\theta \mathcal{F}2\mathcal{B}1(m_q; m_{\sigma, q-p}) \right. \\ &\quad \left. + \int_0^1 x^{\frac{1}{2}} [(2-\eta_\phi) + \eta_\phi x] dx \int_{-1}^1 d\cos\theta \mathcal{F}1\mathcal{B}2(m_{q, q-p}; m_\sigma) \right)\end{aligned}\tag{9}$$

similar

$$\begin{aligned}\partial_t(\bar{m}_f(p))^\pi &= \frac{3\bar{h}_k^2 \bar{m}_{f,k}}{8(2\pi)^2} \left(2 \int_0^1 x^{\frac{1}{2}} [(1-\eta_q) + \eta_q x^{\frac{1}{2}}] dx \int_{-1}^1 d\cos\theta \mathcal{F}2\mathcal{B}1(m_q; m_{\pi, q-p}) \right. \\ &\quad \left. + \int_0^1 x^{\frac{1}{2}} [(2-\eta_\phi) + \eta_\phi x] dx \int_{-1}^1 d\cos\theta \mathcal{F}1\mathcal{B}2(m_{q, q-p}; m_\pi) \right)\end{aligned}\tag{10}$$

$$\begin{aligned}\partial_t(\bar{m}_f(p))^A &= -\frac{3\bar{m}_{f,k} \bar{g}_k^2 C_2(N_c)}{2(2\pi)^2} \left(2 \int_0^1 x^{\frac{1}{2}} [(1-\eta_q) + \eta_q x^{\frac{1}{2}}] dx \int_{-1}^1 d\cos\theta \mathcal{F}2\mathcal{B}1(m_q; m_{A, q-p}) \right. \\ &\quad \left. + \int_0^1 x^{\frac{1}{2}} [(2-\eta_A) + \eta_A x] dx \int_{-1}^1 d\cos\theta \mathcal{F}1\mathcal{B}2(m_{q, q-p}; m_A) \right)\end{aligned}\tag{11}$$

$$\partial_t(\bar{m}_f(p)) = \partial_t(\bar{m}_f(p))^\sigma + \partial_t(\bar{m}_f(p))^\pi + \partial_t(\bar{m}_f(p))^A\tag{12}$$

$$\begin{aligned}\bar{m}_f(p) &= \bar{m}_f(p)\Big|_\Lambda + \int_0^{\ln(IR/\Lambda)} (\partial_t(\bar{m}_f(p))) dt = 0 + \int_\Lambda^{IR} (\partial_t(\bar{m}_f(p))) \frac{dk}{k} \\ &= -\int_{IR}^\Lambda \frac{\partial_t(\bar{m}_f(p))}{k} dk\end{aligned}\tag{13}$$