1 Effective Action

$$\begin{aligned}\Gamma_k = \int_x \Big\{ & \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + Z_c (\partial_\mu \bar{c}^a) D^{ab}_\mu c^b + \frac{1}{2\xi} (\partial_\mu A^a_\mu)^2 \\ & + \frac{1}{2} \int_p A^a_\mu(-p) (\Gamma^{(2)ab}_{AA\,\mu\nu} - Z_A \Pi^\perp_{\mu\nu} \delta^{ab} p^2) A^b_\nu(p) \\ & + \bar{q}[Z_q(\gamma_\mu D_\mu - \gamma_0(\hat{\mu} + igA_0)] q - \lambda_q \sum_{a=0}^{8} [(\bar{q} T_a q)^2 + (\bar{q} i\gamma_5 T_a q)^2] \\ & + \bar{q} h_k^{1/2} \cdot \Sigma_5 \cdot h_k^{1/2} q + tr\big(Z^{1/2}_{\Sigma,k} \cdot \partial_\mu \Sigma \cdot Z^{1/2}_{\Sigma,k} \cdot \partial_\mu \Sigma^\dagger\big) + \tilde{U}_k(\Sigma, \Sigma^\dagger) + V_{glue}(L, \bar{L}) \Big\}\end{aligned} \tag{1}$$

here, the meson field :

$$\Sigma = T^a(\sigma^a + i\pi^a). \quad (a = 0, 1, ..., 8) \tag{2}$$

and

$$\Sigma_5 = T^a(\sigma^a + i\gamma_5 \pi^a). \quad (a = 0, 1, ..., 8) \tag{3}$$

with $T^a = \lambda^a/2 (a = 1, ..., 8)$ and $T^0 = \frac{1}{\sqrt{2N_f}} \mathbb{I}_{N_f \times N_f}$ are generators of $SU(N_f = 3)$. σ^a and π^a mean the scalar and pseudoscalar fields, respectively. The physical meson can be written obviously:

$$\Sigma = \frac{1}{2} \begin{pmatrix} a_0^0 + \sigma_L + i\pi^0 + i\eta_L & \sqrt{2}(a_0^+ + i\pi^+) & \sqrt{2}(\kappa^+ + iK^+) \\ \sqrt{2}(a_0^- + i\pi^-) & -a_0^0 + \sigma_L - i\pi^0 + i\eta_L & \sqrt{2}(\kappa^0 + iK^0) \\ \sqrt{2}(\kappa^- + iK^-) & \sqrt{2}(\bar{\kappa}^0 + i\bar{K}^0) & \sqrt{2}(\sigma_S + i\eta_S) \end{pmatrix} \tag{4}$$

the meson effective potential can be devided into three parts

$$\tilde{U}_k(\Sigma) = U_k(\rho_1, \rho_2) - c_A \xi - c_L \sigma_L - c_S \sigma_S, \tag{5}$$

here $U_k(\rho_1, \rho_2)$ is an arbitrary function of chiral symmetry invariant variables ρ_1, ρ_2. $c_A \xi$ is Kobayashi-Maskawa-'t Hooft trem which breaks $U_A(1)$ symmetry. The last two terms of Eq.(5) are linear sigma terms, which break the chiral symmetry. The ρ_1, ρ_2 are defined as:

$$\rho_1 = \mathrm{tr}(\Sigma \cdot \Sigma^\dagger) \tag{6}$$

$$\rho_2 = \sqrt{6 \cdot \mathrm{tr}\Big(\Sigma \cdot \Sigma^\dagger - \frac{1}{3} \rho_1 \mathbb{I}_{3\times 3}\Big)^2} \tag{7}$$

And the effective potential is Taylor expaned as

$$U_k(\rho_1,\rho_2)=\sum_{i,j=0}^{N}\frac{\lambda_{ij,k}}{i!j!}(\rho_1-\kappa_1)^i(\rho_2-\kappa_2)^j \tag{8}$$

Here, we choose the expansion order $N=5$.

On the vacuum expectation value,ρ_1,ρ_2 are given as:

$$\rho_1=\frac{1}{2}(\sigma_l^2+\sigma_s^2) \tag{9}$$
$$\rho_2=\frac{1}{2}(2\sigma_s^2-\sigma_l^2) \tag{10}$$

The Yukawa coupling

$$h_k=\begin{pmatrix} h_{l,k} & 0 & 0 \\ 0 & h_{l,k} & 0 \\ 0 & 0 & h_{s,k} \end{pmatrix} \tag{11}$$

and meson and quark wave function renormalization

$$Z_{\sigma,k}=\begin{pmatrix} Z_{\phi_l,k} & 0 & 0 \\ 0 & Z_{\phi_l,k} & 0 \\ 0 & 0 & Z_{\phi_s,k} \end{pmatrix} \qquad Z_{q,k}=\begin{pmatrix} Z_{l,k} & 0 & 0 \\ 0 & Z_{l,k} & 0 \\ 0 & 0 & Z_{s,k} \end{pmatrix} \tag{12}$$

At present, we assume $Z_{\sigma,k}=Z_{\pi,k}$.

The quark masses are given as (Appx(5)):

$$m_l=\frac{h_{l,k}}{2}\sigma_l \tag{13}$$
$$m_s=\frac{h_{s,k}}{\sqrt{2}}\sigma_s \tag{14}$$

2 Flow Equations

The the Wetterich equation with dynamical hadronisation reads

$$\partial_t\Gamma_k[\Phi]+\int\langle\partial_t\hat{\phi}_{k,i}\rangle\left(\frac{\delta\Gamma_k[\Phi]}{\delta\phi_i}+c_\sigma\delta_{i\sigma}\right)=\frac{1}{2}\mathrm{Tr}(G_k[\Phi]\partial_t R_k)+\mathrm{Tr}\left(G_{\phi\Phi_j}[\Phi]\frac{\delta\langle\partial_t\hat{\phi}_{k,i}\rangle}{\delta\Phi_j}R_\phi\right) \tag{15}$$

we assume

$$\langle \partial_t \hat{\phi}_k \rangle = \dot{A}_{l,k}[(\bar{q}T_a q) + (\bar{q}i\gamma_5 T_a q)] + \dot{A}_{s,k}[(\bar{q}T_b q) + (\bar{q}i\gamma_5 T_b q)] + \dot{B}_k \Sigma, \qquad (16)$$
$$\text{for} \quad a = L, 1, \cdots 3, b = 4, \cdots 7, S$$

here T^L, T^S are given in Appendix 7 As pointed out in ref [] , we choose $\dot{B}_k = 0$. By taking the derivative of of each side of Eq. (15)

$$\frac{\overrightarrow{\delta}}{\delta(\bar{q}T^a q)}(Eq.(15))\frac{\overleftarrow{\delta}}{\delta(\bar{q}T^a q)}, \qquad (17)$$

we get

$$-\partial_t \lambda_q + \dot{A}h_k = -\text{Flow}^{(4)}_{(\bar{q}T^a q)(\bar{q}T^a q)} \qquad (18)$$

with the condication

$$\lambda_q \equiv 0, \quad \forall k \qquad (19)$$

we get the renormalised hadronisation function

$$\dot{\bar{A}} = -\frac{1}{\bar{h}_k}\overline{\text{Flow}}^{(4)}_{(\bar{q}T^a q)(\bar{q}T^a q)} \qquad (20)$$

we split the expression

$$\dot{\bar{A}}_{l,k} = -\frac{1}{\bar{h}_{l,k}}\overline{\text{Flow}}^{(4)}_{(\bar{q}T^L q)(\bar{q}T^L q)} \qquad (21)$$
$$\dot{\bar{A}}_{s,k} = -\frac{1}{\bar{h}_{s,k}}\overline{\text{Flow}}^{(4)}_{(\bar{q}T^S q)(\bar{q}T^S q)} \qquad (22)$$

And to calculate the yukawa flow equation:

$$\frac{\delta}{\delta \sigma^a}\frac{\delta}{\delta(\bar{q}T^a q)}(Eq.(15)) \quad a = L/S \qquad (23)$$

we get

$$\partial \bar{h}_{l,k} = \left(\eta_{l,k} + \frac{1}{2}\eta_{\phi,k}\right) - \frac{\delta^2 \bar{\tilde{U}}(\Sigma)}{(\delta \bar{\sigma}_L)^2}\dot{\bar{A}}_{l,k} + \overline{\text{Flow}}^{(3)}_{(\bar{q}T^L q)\sigma_L} \qquad (24)$$
$$\partial \bar{h}_{s,k} = \left(\eta_{s,k} + \frac{1}{2}\eta_{\phi,k}\right) - \frac{\delta^2 \bar{\tilde{U}}(\Sigma)}{(\delta \bar{\sigma}_S)^2}\dot{\bar{A}}_{s,k} + \overline{\text{Flow}}^{(3)}_{(\bar{q}T^S q)\sigma_S} \qquad (25)$$

A simpler way given in []

$$\frac{1}{\sigma^a}\frac{\delta}{\delta(\bar{q}T^a q)}(Eq.(15)) \quad a = L/S \tag{26}$$

and we get

$$\partial\bar{h}_{l,k} = \left(\eta_{l,k} + \frac{1}{2}\eta_{\phi,k}\right) - \frac{1}{\bar{\sigma}_L}\frac{\delta\bar{\tilde{U}}(\Sigma)}{\delta\bar{\sigma}_L}\dot{\bar{A}}_{l,k} + \frac{1}{\bar{\sigma}_L}\mathrm{Re}\overline{\mathrm{Flow}}^{(2)}_{(\bar{q}T^L q)} \tag{27}$$

$$\partial\bar{h}_{s,k} = \left(\eta_{s,k} + \frac{1}{2}\eta_{\phi,k}\right) - \frac{1}{\bar{\sigma}_S}\frac{\delta\bar{\tilde{U}}(\Sigma)}{\delta\bar{\sigma}_S}\dot{\bar{A}}_{s,k} + \frac{1}{\bar{\sigma}_S}\mathrm{Re}\overline{\mathrm{Flow}}^{(2)}_{(\bar{q}T^S q)} \tag{28}$$

the next step is to calculate the $\overline{\mathrm{Flow}}$ terms.

3 Result

Pressure:

$$\Omega = U(\sigma_l, \sigma_s) + V_{glue}(L, \bar{L}) \tag{29}$$

$$\frac{p}{T^4} = \frac{\Omega(0,0) - \Omega((T,\mu)}{T^4} \tag{30}$$

$$\partial_t U = \tag{31}$$

and n-th order cumulats

$$\chi_n^B = \frac{\partial^n}{\partial(\mu_B/T)^n}\frac{p}{T^4} \tag{32}$$

4 Appendix.A

The meson masses can be obtained by Hessian matrix:

$$H_{p,LL} =\frac{c_A\sigma_S}{\sqrt{2}}+U^{(1,0)}-U^{(0,1)} \tag{33}$$

$$H_{p,LS} =\frac{c_A\sigma_L}{\sqrt{2}} \tag{34}$$

$$H_{p,SS} =U^{(1,0)}+2U^{(0,1)} \tag{35}$$

$$H_{p,11} =-\frac{c_A\sigma_S}{\sqrt{2}}+U^{(1,0)}-U^{(0,1)} \tag{36}$$

$$H_{p,44} =-\frac{c_A\sigma_L}{2}+U^{(1,0)}+\frac{\sigma_L^2-3\sqrt{2}\sigma_L\sigma_S+4\sigma_S^2}{2\sigma_S^2-\sigma_L^2}U^{(0,1)} \tag{37}$$

$$H_{s,LL} =-\frac{c_A\sigma_S}{\sqrt{2}}+U^{(1,0)}-U^{(0,1)}+(U^{(2,0)}-2U^{(1,1)}+U^{(0,2)})\sigma_L^2 \tag{38}$$

$$H_{s,LS} =-\frac{c_A\sigma_L}{\sqrt{2}}+(U^{(2,0)}+U^{(1,1)}-2U^{(0,2)})\sigma_L\sigma_S \tag{39}$$

$$H_{s,SS} =U^{(1,0)}+2U^{(0,1)}+(4U^{(2,0)}+4U^{(1,1)}+U^{(2,0)})\sigma_S^2 \tag{40}$$

$$H_{s,11} =\frac{c_A\sigma_S}{\sqrt{2}}+U^{(1,0)}+\frac{7\sigma_L^2-2\sigma_S^2}{2\sigma_S^2-\sigma_L^2}U^{(0,1)} \tag{41}$$

$$H_{s,44} =\frac{c_A\sigma_L}{2}+U^{(1,0)}+\frac{\sigma_L^2+3\sqrt{2}\sigma_L\sigma_S+4\sigma_S^2}{2\sigma_S^2-\sigma_L^2}U^{(0,1)} \tag{42}$$

Because the nonvanishing nondiagonal element $H_{s/p,LS}$ we introduce the mixing angles between LS and physical basis:

$$\begin{pmatrix} f_0 \\ \sigma \end{pmatrix} = \begin{pmatrix} \cos\varphi_s & -\sin\varphi_s \\ \sin\varphi_s & \cos\varphi_s \end{pmatrix} \begin{pmatrix} \sigma_L \\ \sigma_S \end{pmatrix}, \tag{43}$$

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\varphi_p & -\sin\varphi_p \\ \sin\varphi_p & \cos\varphi_p \end{pmatrix} \begin{pmatrix} \eta_L \\ \eta_S \end{pmatrix}. \tag{44}$$

here

$$\varphi_{s/p} = \frac{1}{2}arctan\left(\frac{2H_{s/p,LS}}{H_{s/p,SS}-H_{s/p,LL}}\right) \tag{45}$$

so the square of meson mass are given as

$$m_{f_0}^2 = \cos^2\varphi_s H_{s,SS} + \sin^2\varphi_s H_{s,LL} - 2\sin\varphi_s\cos\varphi_s H_{s,LS} \tag{46}$$

$$m_{\sigma}^2 = \sin^2\varphi_s H_{s,SS} + \cos^2\varphi_s H_{s,LL} + 2\sin\varphi_s\cos\varphi_s H_{s,LS} \tag{47}$$

$$m_{a_0}^2 = H_{s,11} \tag{48}$$

$$m_{\kappa}^2 = H_{s,44} \tag{49}$$

$$m_{\eta}^2 = \cos^2\varphi_p H_{p,SS} + \sin^2\varphi_p H_{p,LL} - 2\sin\varphi_p\cos\varphi_p H_{p,LS} \tag{50}$$

$$m_{\eta'}^2 = \sin^2\varphi_p H_{p,SS} + \cos^2\varphi_p H_{p,LL} + 2\sin\varphi_p\cos\varphi_p H_{p,LS} \tag{51}$$

$$m_{\pi}^2 = H_{p,11} \tag{52}$$

$$m_{K}^2 = H_{p,44} \tag{53}$$

We can simplify them as

$$m_{f_0/\eta}^2 = \frac{H_{s/p,LL} + H_{s/p,SS}}{2} + \sqrt{(H_{s/p,LL} - H_{s/p,SS})^2 + 4H_{s/p,LS}^2} \tag{54}$$

$$m_{\sigma/\eta'}^2 = \frac{H_{s/p,LL} + H_{s/p,SS}}{2} - \sqrt{(H_{s/p,LL} - H_{s/p,SS})^2 + 4H_{s/p,LS}^2} \tag{55}$$

And the diagonal element of meson field become:

$$\Sigma_{(1,1)} = \frac{1}{2}(a_0^0 + \cos\varphi_s f_0 + \sin\varphi_s \sigma + i\pi^0 + i\cos\varphi_p \eta + i\sin\varphi_p \eta') \tag{56}$$

$$\Sigma_{(2,2)} = \frac{1}{2}(-a_0^0 + \cos\varphi_s f_0 + \sin\varphi_s \sigma - i\pi^0 + i\cos\varphi_p \eta + i\sin\varphi_p \eta') \tag{57}$$

$$\Sigma_{(3,3)} = \frac{1}{\sqrt{2}}(-\sin\varphi_s f_0 + \cos\varphi_s - i\sin\varphi_p \eta + i\cos\varphi_p \eta') \tag{58}$$

The coefficients in Eq.(28) are given as

$$\frac{1}{\sigma_L}\frac{\delta\tilde{U}(\Sigma)}{\delta\sigma_L} = -\frac{c_A\sigma_S}{\sqrt{2}} + U^{(1,0)} - U^{(0,1)} \tag{59}$$

$$\frac{1}{\sigma_S}\frac{\delta\tilde{U}(\Sigma)}{\delta\sigma_L} = -\frac{c_A\sigma_L^2}{2\sqrt{2}\sigma_S} + U^{(1,0)} + 2U^{(0,1)} \tag{60}$$

One interesting thing is that $\frac{1}{\sigma_L}\frac{\delta\tilde{U}(\Sigma)}{\delta\sigma_L} = m_\pi^2$.

the three-point meson vertex are defined as

$$\lambda_{\phi_i,\phi_j,\phi_l,k} = \left.\frac{\partial^3 U_k(\Sigma)}{\partial\phi_i,\partial\phi_j,\partial\phi_l}\right|_{\phi_0} \tag{61}$$

because we assume

$$Z_\phi = Z_{\pi^+} \tag{62}$$

we choose the three-point meson vertex involve one π^+, which are given as

$$\lambda_{\pi^+\pi^- f_0,k} = \frac{c_A}{\sqrt{2}}\sin\phi_S + \cos\phi_S(U^{(0,2)} - 2U^{(1,1)} + U^{(2,0)})\sigma_L + \sin\phi_S(2U^{(0,2)} - U^{(1,1)} - U^{(2,0)})\sigma_S \tag{63}$$

$$\lambda_{\pi^+\pi^-\sigma,k} = -\frac{c_A}{\sqrt{2}}\cos\phi_S + \sin\phi_S(U^{(0,2)} - 2U^{(1,1)} + U^{(2,0)})\sigma_L - \cos\phi_S(2U^{(0,2)} - U^{(1,1)} - U^{(2,0)})\sigma_S \tag{64}$$

$$\lambda_{\pi^+ a_0^- \eta,k} = \sin\phi_P\frac{c_A}{\sqrt{2}} + 6\cos\phi_P U^{(0,1)}\frac{\sigma_L}{2\sigma_S - \sigma_L} \tag{65}$$

$$\lambda_{\pi^+ a_0^- \eta',k} = -\cos\phi_P\frac{c_A}{\sqrt{2}} + 6\sin\phi_P U^{(0,1)}\frac{\sigma_L}{2\sigma_S - \sigma_L} \tag{66}$$

$$\lambda_{\pi^+\kappa^- K^0,k} = \lambda_{\pi^+ K^-\kappa^0,k} = \frac{c_A}{\sqrt{2}} + 6U^{(0,1)}\frac{\sigma_S}{2\sigma_S - \sigma_L} \tag{67}$$

5 Appendix.B

The quark masses are given as

$$M_q = \begin{pmatrix} m_{l,k} & 0 & 0 \\ 0 & m_{l,k} & 0 \\ 0 & 0 & m_{s,k} \end{pmatrix} = \frac{\overrightarrow{\delta}}{\delta\bar{q}}\Gamma_k\frac{\overleftarrow{\delta}}{\delta q}\bigg|_{\Sigma=\Sigma_0} \tag{68}$$

with the condication Eq.(19),

$$M_q = \frac{\overrightarrow{\delta}}{\delta\bar{q}}\bar{q}h^{\frac{1}{2}}\Sigma_0 h^{\frac{1}{2}}q\frac{\overleftarrow{\delta}}{\delta q} = \begin{pmatrix} \frac{h_{l,k}}{2}\sigma_L & 0 & 0 \\ 0 & \frac{h_{l,k}}{2}\sigma_l & 0 \\ 0 & 0 & \frac{h_{s,k}}{\sqrt{2}}\sigma_s \end{pmatrix} \tag{69}$$

which are shown in Eq.(13).

The meson quark vertex are given by

$$V_{\bar{q}q\phi_i} = \frac{\delta}{\delta\phi_i}\frac{\overrightarrow{\delta}}{\delta\bar{q}}\Gamma_k\frac{\overleftarrow{\delta}}{\delta q} \tag{70}$$

here, we force on u and s quark anomalous dimension and Yukawa coupling, so meson quark vertex which are used:

$$V_{\bar{u}u\phi_i} = \frac{\delta}{\delta\phi_i} h_l \Sigma_{5(1,1)} \quad V_{\bar{u}d\phi_i} = \frac{\delta}{\delta\phi_i} h_l \Sigma_{5(1,2)} \quad V_{\bar{u}s\phi_i} = \frac{\delta}{\delta\phi_i} h_l^{1/2} h_s^{1/2} \Sigma_{5(1,3)} \tag{71}$$

$$V_{\bar{s}u\phi_i} = \frac{\delta}{\delta\phi_i} h_l^{1/2} h_s^{1/2} \Sigma_{5(3,1)} \quad V_{\bar{s}d\phi_i} = \frac{\delta}{\delta\phi_i} h_l^{1/2} h_s^{1/2} \Sigma_{5(3,2)} \quad V_{\bar{s}s\phi_i} = \frac{\delta}{\delta\phi_i} h_s \Sigma_{5(3,3)} \tag{72}$$

then we get

$$V_{\bar{u}uf_0} = h_l \cos\varphi_s/2 \quad V_{\bar{u}u\sigma} = h_l \sin\varphi_s/2 \quad V_{\bar{u}ua_0} = h_l/2 \tag{73}$$

6 Appendix.C

The Kobayashi-Maskawa-'t Hooft coupling $\bar{c}_A$, should decrease at high scalar and high temperatures. However, if we keep c_A a constant, and

$$\bar{c}_A = \frac{c_A}{Z_\phi^{3/2}} \tag{74}$$

$\bar{c}_A$ will increase with scalar and temperarure. One scheme is given in ref [**?**], which assume that $\bar{c}_A$ is a constant. However, this scheme cause a very sharp phase transitions. We assume c_A is a infrared enhancement function

$$c_A = c_{A,IR} \frac{1}{e^{\frac{k-k_{cut}}{\Delta_k}} + 1} \tag{75}$$

here $k_{cut} = 1GeV$ and $\Delta_k = 20MeV$. And $\bar{c}_A$ still dressed as $\bar{c}_A = c_A/Z_\phi^{3/2}$.

7 Appendix.D

As we known that

$$T_0 = \frac{1}{\sqrt{6}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad T_8 = \frac{1}{2\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \tag{76}$$

We have

$$\Sigma_0 = T_0\sigma_0 + T_8\sigma_8 = \begin{pmatrix} \frac{1}{\sqrt{6}}\sigma_0 + \frac{1}{2\sqrt{3}}\sigma_8 & 0 & 0 \\ 0 & \frac{1}{\sqrt{6}}\sigma_0 + \frac{1}{2\sqrt{3}}\sigma_8 & 0 \\ 0 & 0 & \frac{1}{\sqrt{6}}\sigma_0 - \frac{1}{\sqrt{3}}\sigma_8 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} \sigma_L & 0 & 0 \\ 0 & \sigma_L & 0 \\ 0 & 0 & \sqrt{2}\sigma_S \end{pmatrix} = T_L\sigma_L + T_S\sigma_S \tag{77}$$

so

$$T_L = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad T_S = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{78}$$

8 Appendix.E:mesons anomalous dimension

As assumed above,

$$\eta_\phi = \eta_{\pi^+} \tag{79}$$

and we get

$$\begin{aligned} \eta_\phi(0) =& \frac{\bar{Z}_\phi}{Z_\phi(0)}\Big\{ \frac{1}{3\pi^2 k^2}\big[\bar{\lambda}^2_{\pi^+\pi^- f_0}\mathscr{B}\mathscr{B}^{(\pi,f_0)}_{(2,2)} + \lambda^2_{\pi^+\pi^-\sigma}\mathscr{B}\mathscr{B}^{(\pi,\sigma)}_{(2,2)} + \lambda^2_{\pi^+ a_0^- \eta}\mathscr{B}\mathscr{B}^{(a_0,\eta)}_{(2,2)} \\ &+ \lambda^2_{\pi^+ a_0^- \eta'}\mathscr{B}\mathscr{B}^{(a_0,\eta')}_{(2,2)} + (\lambda^2_{\pi^+\kappa^- K^0} + \lambda^2_{\pi^+ K^- \kappa^0})\mathscr{B}\mathscr{B}^{(K,\kappa)}_{(2,2)}\big] \\ &+ \frac{N_c h^2_{l,k}}{6\pi^2}\big[(2\eta_{l,k} - 3)\mathscr{F}_{(2)}(\tilde{m}^2_{l,k};T,\mu_q) - 4(\eta_{l,k} - 2)\mathscr{F}_{(3)}(\tilde{m}^2_{l,k};T,\mu_q)\big]\Big\} \end{aligned} \tag{80}$$

$$
\begin{aligned}
\eta_\phi(0,k) =& \frac{1}{3\pi^2 k^2}\big[\bar{\lambda}^2_{\pi^+\pi^- f_0}\mathscr{B}\mathscr{B}^{(\pi,f_0)}_{(2,2)} + \lambda^2_{\pi^+\pi^-\sigma}\mathscr{B}\mathscr{B}^{(\pi,\sigma)}_{(2,2)} + \lambda^2_{\pi^+ a_0^- \eta}\mathscr{B}\mathscr{B}^{(a_0,\eta)}_{(2,2)} \\
&+ \lambda^2_{\pi^+ a_0^- \eta'}\mathscr{B}\mathscr{B}^{(a_0,\eta')}_{(2,2)} + (\lambda^2_{\pi^+\kappa^- K^0} + \lambda^2_{\pi^+ K^-\kappa^0})\mathscr{B}\mathscr{B}^{(K,\kappa)}_{(2,2)}\big] \qquad (81)\\
&- \frac{N_c}{\pi^2}\bar{h}_k^2 \int_0^1 dx \Big[(1-\eta_{l,k})\sqrt{x} + \eta_{l,k} x\Big] \\
&\times \int_{-1}^1 d\cos\theta \Bigg\{ \Bigg[\Big(\mathscr{F}\mathscr{F}_{(1,1)}(\tilde{m}^2_{l,k},\tilde{m}^2_{l,k}) - \mathscr{F}_{(2)}(\tilde{m}^2_{l,k}) \Big) \\
&- \Big(\mathscr{F}\mathscr{F}_{(2,1)}(\tilde{m}^2_{l,k},\tilde{m}^2_{l,k}) - \mathscr{F}_{(3)}(\tilde{m}^2_{l,k}) \Big) \Bigg] \\
&+ \Bigg[\Big(\sqrt{x} - \cos\theta\Big)\Big(1 + r_F(x')\Big)\mathscr{F}\mathscr{F}_{(2,1)}(\tilde{m}^2_{l,k},\tilde{m}^2_{l,k}) \\
&- \mathscr{F}_{(3)}(\tilde{m}^2_{l,k}) \Bigg] - \frac{1}{2}\Bigg[\Big(\sqrt{x} - \cos\theta\Big)\Big(1 + r_F(x')\Big) \\
&\times \mathscr{F}\mathscr{F}_{(1,1)}(\tilde{m}^2_{l,k},\tilde{m}^2_{l,k}) - \mathscr{F}_{(2)}(\tilde{m}^2_{l,k}) \Bigg] \Bigg\}, \qquad (82)
\end{aligned}
$$

9 Appendix.F:quarks anomalous dimension

10 Appendix.G:Yukawa coupling

$$
\begin{aligned}
\partial_t \bar{h}_l =& (\eta_l + \frac{1}{2}\eta_\phi)\bar{h}_l + \frac{1}{8\pi^2}\bar{h}_l^3\big[3\mathscr{L}^{(l,a_0)}_{(1,1)} - 3\mathscr{L}^{(l,\pi)}_{(1,1)} + \cos^2\varphi_s \mathscr{L}^{(l,f_0)}_{(1,1)} \\
&- \cos^2\varphi_p \mathscr{L}^{(l,\eta)}_{(1,1)} + \sin^2\varphi_s \mathscr{L}^{(l,\sigma)}_{(1,1)} - \sin^2\varphi_p \mathscr{L}^{(l,\eta')}_{(1,1)}\big] \\
&- \frac{3}{2\pi^2}\frac{N_c^2-1}{2N_c} g_{\bar{l}Al}\bar{h}_l \mathscr{L}^{(l,0)}_{(1,1)} \qquad (83)\\
\partial_t \bar{h}_s =& (\eta_s + \frac{1}{2}\eta_\phi)\bar{h}_l s + \frac{1}{8\pi^2} 2\bar{h}_s^3\big[\sin^2\varphi_s \mathscr{L}^{(s,f_0)}_{(1,1)} - \sin^2\varphi_p \mathscr{L}^{(s,\eta)}_{(1,1)} \\
&+ \cos^2\varphi_s \mathscr{L}^{(s,\sigma)}_{(1,1)} - \cos^2\varphi_p \mathscr{L}^{(s,\eta')}_{(1,1)}\big] \\
&- \frac{3}{2\pi^2}\frac{N_c^2-1}{2N_c} g_{\bar{s}As}\bar{h}_s \mathscr{L}^{(s,0)}_{(1,1)} \qquad (84)
\end{aligned}
$$

	1	2	3	4	5
$\bar{a}_i$	-44.14	151.4	-90.0677	2.77173	3.56403
$\bar{b}_i$	-0.32665	-82.9823	3.0	5.85559	
$\bar{c}_i$	-50.7961	114.038	-89.4596	3.08718	6.72812
$\bar{d}_i$	27.0885	-56.0859	71.2225	2.9715	6.61433

11 Appendix.H: Polyakov-loop potential

We empoly the parameterization Polyakov-loop potential from [**?**], that is

$$\bar{V}_{\text{glue-Haar}} = -\frac{\bar{a}(T)}{2}\bar{L}L + \bar{b}(T)\ln M_H(L,\bar{L}) + \frac{\bar{c}(T)}{2}(L^3+\bar{L}^3) + \bar{d}(T)(\bar{L}L)^2 \tag{85}$$

with

$$M_H(L,\bar{L}) = 1 - 6\bar{L}L + 4(L^3+\bar{L}^3) - 3(\bar{L}L)^2\,. \tag{86}$$

for $x \in \{\bar{a},\bar{c},\bar{d}\}$

$$x(T) = \frac{x_1 + x_2/(t+1) + x_3/(t+1)^2}{1 + x_4/(t+1) + x_5/(t+1)^2} \tag{87}$$

and

$$\bar{b}(T) = \bar{b}_1(t+1)^{-\bar{b}_4}(1-e^{\bar{b}_2/(t+1)^{\bar{b}_3}}) \tag{88}$$

the coefficients are list in table

The reduced temperature of QCD:

$$t_{\text{YM}} \to \alpha\, t_{\text{glue}}, \quad t_{\text{glue}} \equiv (T - T_c^{\text{glue}})/T_c^{\text{glue}} \tag{89}$$

here we choose

$$t_{\text{glue}} = 250[\text{MeV}], \quad \alpha = 0.42 \tag{90}$$