1 Effective Action

$$
\begin{aligned}
\Gamma_k = \int_x \Big\{ & \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + Z_c (\partial_\mu \bar{c}^a) D^{ab}_\mu c^b + \frac{1}{2\xi} (\partial_\mu A^a_\mu)^2 \\
& + \frac{1}{2} \int_p A^a_\mu(-p) (\Gamma^{(2)ab}_{AA\,\mu\nu} - Z_A \Pi^\perp_{\mu\nu} \delta^{ab} p^2) A^b_\nu(p) \\
& + \bar{q} [Z_q (\gamma_\mu D_\mu - \gamma_0 (\hat{\mu} + igA_0)] q - \lambda_q \sum_{a=0}^{8} [(\bar{q} T_a q)^2 + (\bar{q} i\gamma_5 T_a q)^2] \\
& + \bar{q} h_k^{1/2} \cdot \Sigma_5 \cdot h_k^{1/2} q + tr\big(Z^{1/2}_{\Sigma,k} \cdot \partial_\mu \Sigma \cdot Z^{1/2}_{\Sigma,k} \cdot \partial_\mu \Sigma^\dagger \big) + \tilde{U}_k(\Sigma, \Sigma^\dagger) + V_{glue}(L, \bar{L}) \Big\}
\end{aligned}
\tag{1}
$$

here, the meson field :

$$
\Sigma = T^a (\sigma^a + i\pi^a) \,. \quad (a = 0, 1, ..., 8) \tag{2}
$$

and

$$
\Sigma_5 = T^a (\sigma^a + i\gamma_5 \pi^a) \,. \quad (a = 0, 1, ..., 8) \tag{3}
$$

with $T^a = \lambda^a/2 (a = 1, ..., 8)$ and $T^0 = \frac{1}{\sqrt{2N_f}} \mathbb{I}_{N_f \times N_f}$ are generators of $SU(N_f = 3)$. σ^a and π^a mean the scalar and pseudoscalar fields, respectively. The physical meson can be written obviously:

$$
\Sigma = \frac{1}{2} \begin{pmatrix} a^0_0 + \sigma_L + i\pi^0 + i\eta_L & \sqrt{2}(a^+_0 + i\pi^+) & \sqrt{2}(\kappa^+ + iK^+) \\ \sqrt{2}(a^-_0 + i\pi^-) & -a^0_0 + \sigma_L - i\pi^0 + i\eta_L & \sqrt{2}(\kappa^0 + iK^0) \\ \sqrt{2}(\kappa^- + iK^-) & \sqrt{2}(\bar{\kappa}^0 + i\bar{K}^0) & \sqrt{2}(\sigma_S + i\eta_S) \end{pmatrix} \tag{4}
$$

the meson effective potential can be devided into three parts

$$
\tilde{U}_k(\Sigma) = U_k(\rho_1, \tilde{\rho}_2) - c_A \xi - c_L \sigma_L - c_S \sigma_S \,, \tag{5}
$$

here $U_k(\rho_1, \tilde{\rho}_2)$ is an arbitrary function of chiral symmetry invariant variables $\rho_1, \tilde{\rho}_2$. $c_A \xi$ is Kobayashi-Maskawa-'t Hooft trem which breaks $U_A(1)$ symmetry. The last two terms of Eq.(5) are linear sigma terms, which break the chiral symmetry.

$$
\rho_1 = \mathrm{tr}(\Sigma \cdot \Sigma^\dagger) \,, \tag{6}
$$

$$
\tilde{\rho}_2 = \mathrm{tr}\Big(\Sigma \cdot \Sigma^\dagger - \frac{1}{3} \rho_1 \, \mathbb{I}_{3\times 3} \Big)^2 \,. \tag{7}
$$

The Yukawa coupling

$$h_k = \begin{pmatrix} h_{l,k} & 0 & 0 \\ 0 & h_{l,k} & 0 \\ 0 & 0 & h_{s,k} \end{pmatrix} \tag{8}$$

and meson and quark wave function renormalization

$$Z_{\sigma,k} = \begin{pmatrix} Z_{\phi_l,k} & 0 & 0 \\ 0 & Z_{\phi_l,k} & 0 \\ 0 & 0 & Z_{\phi_s,k} \end{pmatrix} \qquad Z_{q,k} = \begin{pmatrix} Z_{l,k} & 0 & 0 \\ 0 & Z_{l,k} & 0 \\ 0 & 0 & Z_{s,k} \end{pmatrix} \tag{9}$$

At present, we assume $Z_{\sigma,k} = Z_{\pi,k}$ and $Z_{q,k} = Z_{l,k}$.

2 Flow Equations

The the Wetterich equation with dynamical hadronisation reads

$$\partial_t \Gamma_k[\Phi] + \int \langle \partial_t \hat{\phi}_{k,i} \rangle \left(\frac{\delta \Gamma_k[\Phi]}{\delta \phi_i} + j_\sigma \delta_{i\sigma} \right) = \frac{1}{2} \mathrm{Tr}(G_k[\Phi] \partial_t R_k) + \mathrm{Tr}\left(G_{\phi \Phi_j}[\Phi] \frac{\delta \langle \partial_t \hat{\phi}_{k,i} \rangle}{\delta \Phi_j} R_\phi \right) \tag{10}$$

we assume

$$\begin{aligned} \langle \partial_t \hat{\phi}_k \rangle = \dot{A}_{l,k}[(\bar{q} T_a q) + (\bar{q} i \gamma_5 T_a q)] + \dot{A}_{s,k}[(\bar{q} T_b q) + (\bar{q} i \gamma_5 T_b q)] + \dot{B}_k \Sigma, \\ \text{for} \quad a = L, 1, \cdot 3, b = 4, \cdot 7, S \end{aligned} \tag{11}$$

here T^L, T^S are given in Appendix **??** As pointed out in ref [] , we choose $\dot{B}_k = 0$. By taking the derivative of of each side of Eq. (10)

$$\frac{\overrightarrow{\delta}}{\delta(\bar{q} T^a q)} (Eq.(10)) \frac{\overleftarrow{\delta}}{\delta(\bar{q} T^a q)}, \tag{12}$$

we get

$$-\partial_t \lambda_q + \dot{A} h_k = -\mathrm{Flow}^{(4)}_{(\bar{q} T^a q)(\bar{q} T^a q)} \tag{13}$$

with the condication

$$\lambda_q \equiv 0, \quad \forall k \tag{14}$$

we get the renormalised hadronisation function

$$\dot{\bar{A}} = -\frac{1}{\bar{h}_k}\overline{\text{Flow}}^{(4)}_{(\bar{q}T^a q)(\bar{q}T^a q)} \tag{15}$$

we split the expression

$$\dot{\bar{A}}_{l,k} = -\frac{1}{\bar{h}_{l,k}}\overline{\text{Flow}}^{(4)}_{(\bar{q}T^L q)(\bar{q}T^L q)} \tag{16}$$

$$\dot{\bar{A}}_{s,k} = -\frac{1}{\bar{h}_{s,k}}\overline{\text{Flow}}^{(4)}_{(\bar{q}T^S q)(\bar{q}T^S q)} \tag{17}$$

And to calculate the yukawa flow equation:

$$\frac{\delta}{\delta\sigma^a}\frac{\delta}{\delta(\bar{q}T^a q)}(Eq.(10)) \quad a = L/S \tag{18}$$

we get

$$\partial\bar{h}_{l,k} = \left(\eta_{l,k} + \frac{1}{2}\eta_{\phi,k}\right) - \frac{\delta^2\bar{\tilde{U}}(\Sigma)}{(\delta\bar{\sigma}_L)^2}\dot{\bar{A}}_{l,k} + \overline{\text{Flow}}^{(3)}_{(\bar{q}T^L q)\sigma_L} \tag{19}$$

$$\partial\bar{h}_{s,k} = \left(\eta_{s,k} + \frac{1}{2}\eta_{\phi,k}\right) - \frac{\delta^2\bar{\tilde{U}}(\Sigma)}{(\delta\bar{\sigma}_S)^2}\dot{\bar{A}}_{s,k} + \overline{\text{Flow}}^{(3)}_{(\bar{q}T^S q)\sigma_S} \tag{20}$$

A simpler way given in []

$$\frac{1}{\sigma^a}\frac{\delta}{\delta(\bar{q}T^a q)}(Eq.(10)) \quad a = L/S \tag{21}$$

and we get

$$\partial\bar{h}_{l,k} = \left(\eta_{l,k} + \frac{1}{2}\eta_{\phi,k}\right) - \frac{1}{\bar{\sigma}_L}\frac{\delta\bar{\tilde{U}}(\Sigma)}{\delta\bar{\sigma}_L}\dot{\bar{A}}_{l,k} + \frac{1}{\bar{\sigma}_L}\text{Re}\overline{\text{Flow}}^{(2)}_{(\bar{q}T^L q)} \tag{22}$$

$$\partial\bar{h}_{s,k} = \left(\eta_{s,k} + \frac{1}{2}\eta_{\phi,k}\right) - \frac{1}{\bar{\sigma}_S}\frac{\delta\bar{\tilde{U}}(\Sigma)}{\delta\bar{\sigma}_S}\dot{\bar{A}}_{s,k} + \frac{1}{\bar{\sigma}_S}\text{Re}\overline{\text{Flow}}^{(2)}_{(\bar{q}T^S q)} \tag{23}$$

the next step is to calculate the $\overline{\text{Flow}}$ terms.

3 Result

Pressure:

$$\frac{p}{T^4} = \frac{U(0,0) - U((T,\mu)}{T^4} \tag{24}$$

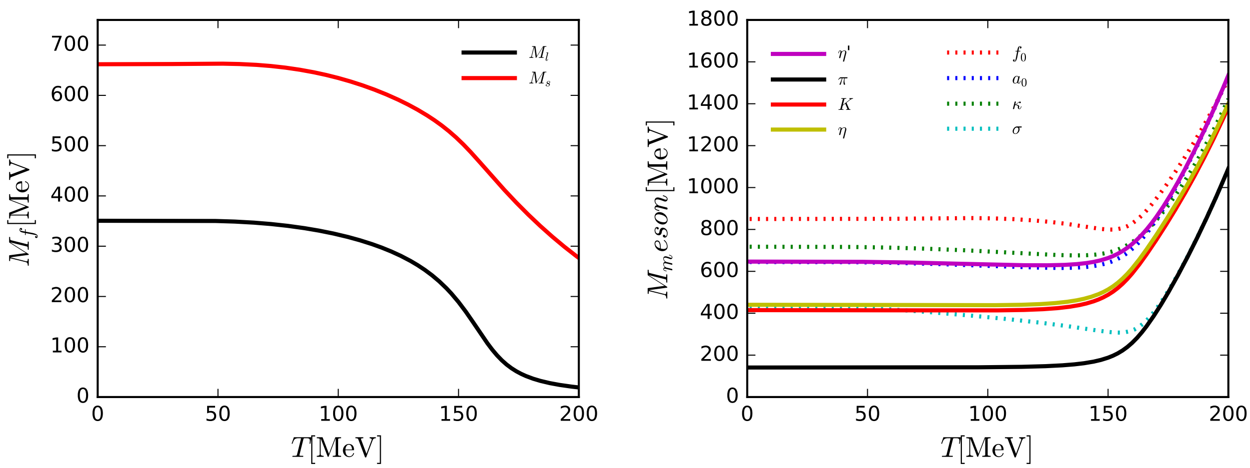

Figure 1: Quark and meson masses as functions of temperature with js/jl=17.

and n-th order cumulats

$$\chi_n^B = \frac{\partial^n}{\partial(\mu_B/T)^n}\frac{p}{T^4} \tag{25}$$

4 Appendix.A

The meson masses can be obtained by Hessian matrix:

$$H_{p,LL} = \frac{c_A\sigma_S}{\sqrt{2}} + \frac{1}{6}U^{(0,1)}(\sigma_L^2 - 2\sigma_S^2) + U^{(1,0)} \tag{26}$$

$$H_{p,LS} = \frac{c_A\sigma_L}{\sqrt{2}} \tag{27}$$

$$H_{p,SS} = U^{(1,0)} - \frac{1}{3}U^{(0,1)}(\sigma_L^2 - 2\sigma_S^2) \tag{28}$$

$$H_{p,11} = -\frac{c_A\sigma_S}{\sqrt{2}} + \frac{1}{6}U^{(0,1)}(\sigma_L^2 - 2\sigma_S^2) + U^{(1,0)} \tag{29}$$

$$H_{p,44} = -\frac{c_A\sigma_L}{2} + U^{(1,0)} + \frac{1}{6}U^{(0,1)}(\sigma_L^2 - 3\sqrt{2}\sigma_L\sigma_S + 4\sigma_S^2) \tag{30}$$

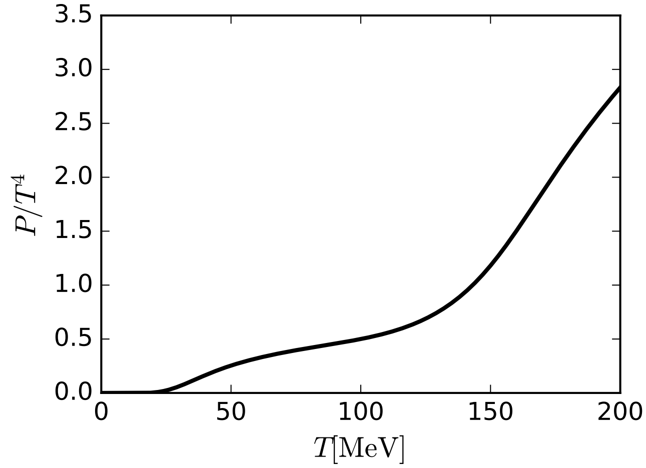

Figure 2: pressure

$$H_{s,LL} = -\frac{c_A\sigma_S}{\sqrt{2}} + U^{(1,0)} + U^{(2,0)}\sigma_L^2 + \frac{1}{6}U^{(0,1)}(3\sigma_L^2 - 2\sigma_S^2) \tag{31}$$
$$+\frac{1}{36}\sigma_L^2(\sigma_L^2 - 2\sigma_S^2)(U^{(0,2)}(\sigma_L^2 - 2\sigma_S^2) + 12U^{(1,1)})$$
$$H_{s,LS} = -\frac{c_A\sigma_L}{\sqrt{2}} + U^{(2,0)}\sigma_L\sigma_S - \frac{2}{3}U^{(0,1)}\sigma_L\sigma_S \tag{32}$$
$$-\frac{1}{18}U^{(0,2)}\sigma_L\sigma_S(\sigma_L^2 - 2\sigma_S^2)^2 - \frac{1}{6}U^{(1,1)}\sigma_L\sigma_S(\sigma_L^2 - 2\sigma_S^2)$$
$$H_{s,SS} = U^{(1,0)} + U^{(2,0)}\sigma_S^2 - \frac{1}{3}U^{(0,1)}(\sigma_L^2 - 6\sigma_S^2) \tag{33}$$
$$+\frac{1}{9}\sigma_S^2(-6U^{(1,1)}(\sigma_L^2 - 2\sigma_S^2) + U^{(0,2)}(\sigma_L^2 - 2\sigma_S^2)^2)$$
$$H_{s,11} = \frac{c_A\sigma_S}{\sqrt{2}} + U^{(1,0)} + \frac{1}{6}U^{(0,1)}(7\sigma_L^2 - 2\sigma_S^2) \tag{34}$$
$$H_{s,44} = \frac{c_A\sigma_L}{2} + U^{(1,0)} + \frac{1}{6}U^{(0,1)}(\sigma_L^2 + 3\sqrt{2}\sigma_L\sigma_S + 4\sigma_S^2) \tag{35}$$

The coefficients in Eq.(23 are given as

$$\frac{1}{\sigma_L}\frac{\delta\tilde{U}(\Sigma)}{\delta\sigma_L} = U^{(1,0)} - \frac{c_A\sigma_S}{\sqrt{2}} + \frac{1}{6}U^{(0,1)}(\sigma_L^2 - 2\sigma_S^2) \tag{36}$$
$$\frac{1}{\sigma_S}\frac{\delta\tilde{U}(\Sigma)}{\delta\sigma_L} = U^{(1,0)} - \frac{ck\sigma_L^2}{2\sqrt{2}\sigma_S} - \frac{1}{3}U^{(0,1)}(\sigma_L^2 - 2\sigma_S^2) \tag{37}$$

One interesting thing is that $\frac{1}{\sigma_L}\frac{\delta\tilde{U}(\Sigma)}{\delta\sigma_L} = m_\pi^2$.

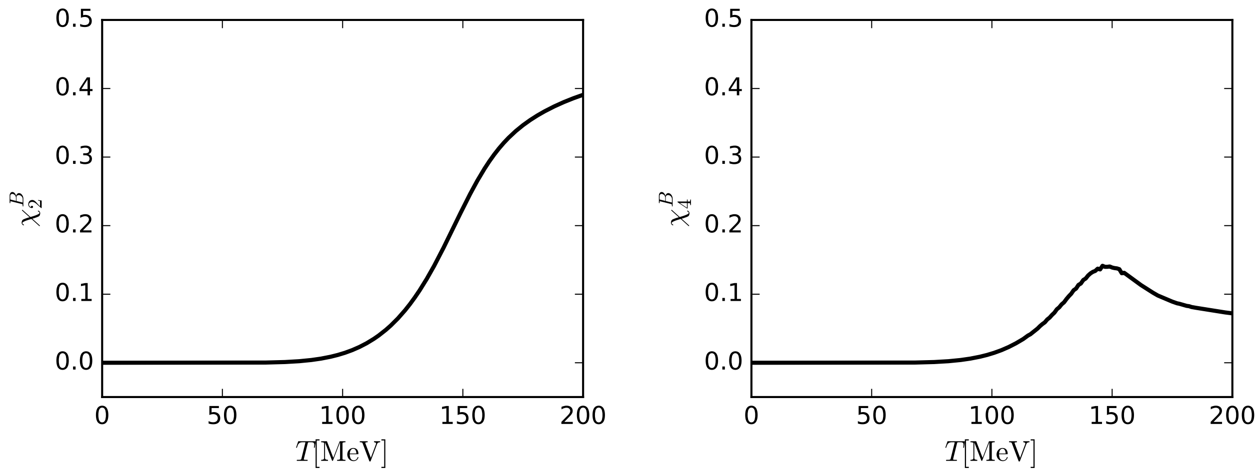

Figure 3: cumulats,$T_{glue} = 250MeV, \alpha = 0.57$,

the three-point meson vertex are defined as

$$\lambda_{\phi_i,\phi_j,\phi_l,k} = \frac{\partial^3 U_k(\Sigma)}{\partial\phi_i, \partial\phi_j, \partial\phi_l}\bigg|_{\phi_0} \tag{38}$$

because we assume

$$Z_\phi = Z_{\pi^+} \tag{39}$$

we choose the three-point meson vertex involve one π^+, which are given as

$$\begin{aligned}\lambda_{\pi^+\pi^- f_0,k} = &\frac{1}{36}\{18\sqrt{2}c_A \sin\phi_S \\ &+ 2\sin\phi_S\sigma_S[12U^{(0,1)} - 18U^{(2,0)} + (\sigma_L^2 - 2\sigma_S^2)(3U^{(1,1)} + U^{(0,2)}\sigma_L^2 - 2U^{(0,2)}\sigma_S^2)] \\ &+ \cos\phi_S\sigma_L[12U^{(0,1)} + 36U^{(2,0)} + (\sigma_L^2 - 2\sigma_S^2)(12U^{(1,1)} + U^{(0,2)}\sigma_L^2 - 2U^{(0,2)}\sigma_S^2)]\}\end{aligned} \tag{40}$$

$$\begin{aligned}\lambda_{\pi^+\pi^- \sigma,k} = &\frac{1}{36}\{-18\sqrt{2}c_A \cos\phi_S \\ &- 2\cos\phi_S\sigma_S[12U^{(0,1)} - 18U^{(2,0)} + (\sigma_L^2 - 2\sigma_S^2)(3U^{(1,1)} + U^{(0,2)}\sigma_L^2 - 2U^{(0,2)}\sigma_S^2)] \\ &+ \sin\phi_S\sigma_L[12U^{(0,1)} + 36U^{(2,0)} + (\sigma_L^2 - 2\sigma_S^2)(12U^{(1,1)} + U^{(0,2)}\sigma_L^2 - 2U^{(0,2)}\sigma_S^2)]\}\end{aligned} \tag{41}$$

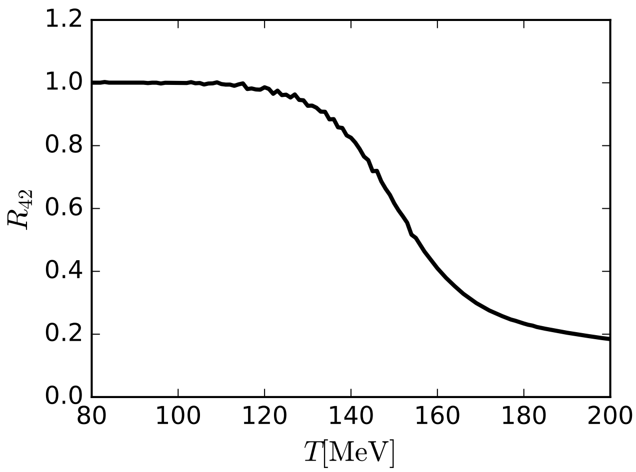

Figure 4: ratio of cumulats

$$\lambda_{\pi^+ a_0^- \eta,k} = \sin\phi_P \frac{c_A}{\sqrt{2}} + \cos\phi_P U^{(0,1)} \sigma_L \tag{42}$$

$$\lambda_{\pi^+ a_0^- \eta',k} = -\cos\phi_P \frac{ck}{\sqrt{2}} + \sin\phi_P U^{(0,1)} \sigma_L \tag{43}$$

$$\lambda_{\pi^+ \kappa^- K^0,k} = \lambda_{\pi^+ K^- \kappa^0,k} = \frac{c_A}{\sqrt{2}} + U^{(0,1)} \sigma_S \tag{44}$$

5 Appendix.B

As we defined

$$\begin{aligned} \rho_1 &= \frac{1}{2}(\sigma_l^2 + \sigma_s^2) \\ \rho_2 &= \frac{1}{24}(\sigma_l^2 - 2\sigma_s^2)^2 \end{aligned} \tag{45}$$

Note that

$$\sigma_l < \sqrt{2}\sigma_s \tag{46}$$

then

$$\begin{aligned} 2\rho_1 &= \sigma_l^2 + \sigma_s^2 \\ -2\sqrt{6\rho_2} &= \sigma_l^2 - 2\sigma_s^2 \end{aligned} \tag{47}$$

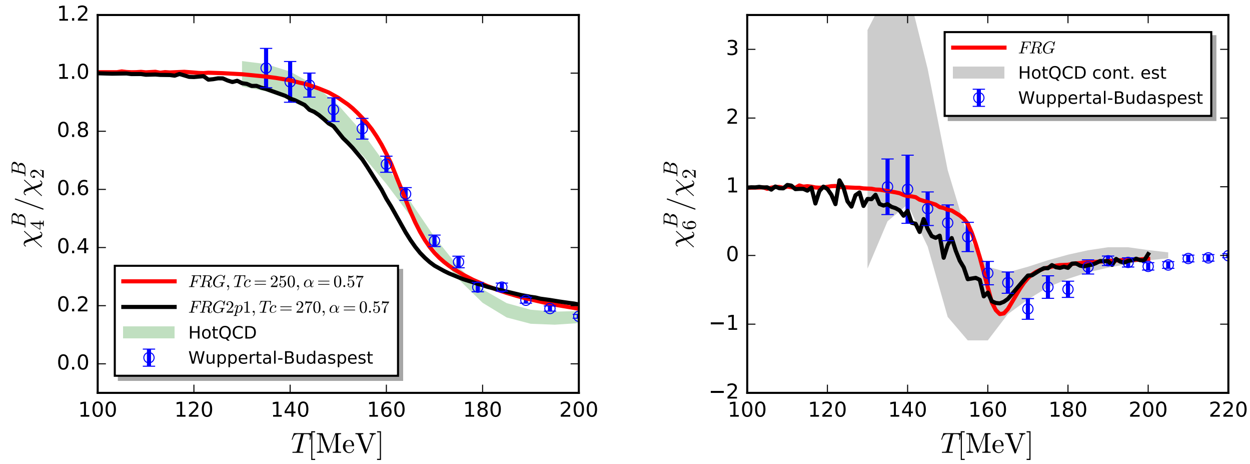

Figure 5: ratio of cumulats

then

$$\sigma_l^2 = \frac{2}{3}(2\rho_1 - \sqrt{6\rho_2})$$
$$\sigma_s^2 = \frac{2}{3}(\rho_1 + \sqrt{6\rho_2}) \tag{48}$$

so

$$\frac{\partial \sigma_l^2}{\partial \rho_1} = \frac{4}{3} \quad \frac{\partial \sigma_l^2}{\partial \rho_2} = -\frac{\sqrt{6}}{3}\rho_2^{-1/2}$$
$$\frac{\partial \sigma_s^2}{\partial \rho_1} = \frac{2}{3} \quad \frac{\partial \sigma_s^2}{\partial \rho_2} = \frac{\sqrt{6}}{3}\rho_2^{-1/2} \tag{49}$$

so we get

$$\frac{\partial \sigma_l^2}{\partial \rho_2} = -\frac{\partial \sigma_s^2}{\partial \rho_2} \tag{50}$$

and

$$\frac{\partial^2 \sigma_l^2}{\partial \rho_2^2} = -\frac{\partial^2 \sigma_s^2}{\partial \rho_2^2} = \frac{\sqrt{6}}{6}\rho_2^{-3/2} \tag{51}$$

The quark masses are given as

$$m_l^2 = h^2\frac{\sigma_l^2}{4} \quad m_s^2 = h^2\frac{\sigma_s^2}{2} \tag{52}$$

therefore

$$\frac{\partial m_l^2}{\partial \rho_2} = -\frac{1}{2}\frac{\partial m_s^2}{\partial \rho_2} = -\frac{\sqrt{6}}{12}h^2\rho_2^{-1/2}$$
$$\frac{\partial^2 m_l^2}{\partial \rho_2^2} = -\frac{1}{2}\frac{\partial^2 m_s^2}{\partial \rho_2^2} = \frac{\sqrt{6}}{24}h^2\rho_2^{-3/2} \tag{53}$$

The quark loop function

$$l^{(f)} = \frac{1}{3}\Big(1-\frac{\eta_q}{4}\Big)\frac{1}{\sqrt{1+\bar{m}_f^2}}(1-n_f(E+\mu)-n_f(E-\mu)) \tag{54}$$

here $\bar{m}_f$ is dimensionless mass.

Then the light and strange quarks part of the the potential flow:

$$\partial_t U = -4N_c\frac{k^4}{4\pi^2}\Big[2l^{(f)}(\bar{m}_l^2)+l^{(f)}(\bar{m}_s^2)\Big] \tag{55}$$

For simplify,we only consider the square brackets above:

$$A_{qk} = 2l^{(f)}(\bar{m}_l^2)+l^{(f)}(\bar{m}_s^2) \tag{56}$$

and

$$\begin{aligned}\frac{\partial A_{qk}}{\partial\rho_2} &= 2\frac{\partial l^{(f)}(\bar{m}_l^2)}{\partial\bar{m}_l^2}\frac{\partial\bar{m}_l^2}{\partial\rho_2}+\frac{\partial l^{(f)}(\bar{m}_s^2)}{\partial\bar{m}_s^2}\frac{\partial\bar{m}_s^2}{\partial\rho_2}\\ &= 2\frac{\partial\bar{m}_l^2}{\partial\rho_2}\Big(\frac{\partial l^{(f)}(\bar{m}_l^2)}{\partial\bar{m}_l^2}-\frac{\partial l^{(f)}(\bar{m}_s^2)}{\partial\bar{m}_s^2}\Big)\end{aligned} \tag{57}$$

We consider $T=0$ case:

$$l^{(f)} = \frac{1}{3}\Big(1-\frac{\eta_q}{4}\Big)\frac{1}{\sqrt{1+\bar{m}_f^2}} \tag{58}$$

then

$$\begin{aligned}\frac{\partial A_{qk}}{\partial\rho_2} &= 2\frac{\partial\bar{m}_l^2}{\partial\rho_2}\Big(\frac{\partial l^{(f)}(\bar{m}_l^2)}{\partial\bar{m}_l^2}-\frac{\partial l^{(f)}(\bar{m}_s^2)}{\partial\bar{m}_s^2}\Big)\\ &= -\frac{\partial\bar{m}_l^2}{\partial\rho_2}\frac{1}{3}\Big(1-\frac{\eta_q}{4}\Big)\Big((1+\bar{m}_l^2)^{-3/2}-(1+\bar{m}_s^2)^{-3/2}\Big)\end{aligned} \tag{59}$$

Because $\bar{m}_q^2 \ll 1, \bar{m}_l^2 \sim 5\times 10^{-16}$ at $k=\Lambda$, we use Taylor expansion

$$\begin{aligned}\frac{\partial A_{qk}}{\partial\rho_2} &= -\frac{1}{3}\Big(1-\frac{\eta_q}{4}\Big)\frac{\partial\bar{m}_l^2}{\partial\rho_2}\Big((1-\frac{3}{2}\bar{m}_l^2+\frac{15}{8}\bar{m}_l^4\cdots)-(1-\frac{3}{2}\bar{m}_s^2+\frac{15}{8}\bar{m}_s^4+\cdots)\Big)\\ &= -\frac{1}{3}\Big(1-\frac{\eta_q}{4}\Big)\Big(-\frac{\sqrt{6}}{12}\frac{h^2}{k^2}\rho_2^{-1/2}\Big)\Big(-\frac{3}{2}(\bar{m}_l^2-\bar{m}_s^2)+\frac{15}{8}(\bar{m}_l^4-\bar{m}_s^4)+\cdots\Big)\\ &= -\frac{\sqrt{6}}{24}\Big(1-\frac{\eta_q}{4}\Big)\Big(\frac{h^2}{k^2}\rho_2^{-1/2}\Big)\Big((\bar{m}_l^2-\bar{m}_s^2)-\frac{5}{4}(\bar{m}_l^4-\bar{m}_s^4)+\frac{35}{24}(\bar{m}_l^6-\bar{m}_s^6)\cdots\Big)\end{aligned} \tag{60}$$

If we consider second order derivative, it will be worse:

$$
\begin{aligned}
\frac{\partial}{\partial\rho_2}\Big(\frac{\partial A_{qk}}{\partial\rho_2}\Big) &= 2\Big[\frac{\partial^2\bar{m}_l^2}{\partial\rho_2^2}\Big(\frac{\partial l^{(f)}(\bar{m}_l^2)}{\partial\bar{m}_l^2} - \frac{\partial l^{(f)}(\bar{m}_s^2)}{\partial\bar{m}_s^2}\Big) + \frac{\partial\bar{m}_l^2}{\partial\rho_2}\Big(\frac{\partial\bar{m}_l^2}{\partial\rho_2}\frac{\partial^2 l^{(f)}(\bar{m}_l^2)}{\partial(\bar{m}_l^2)^2} - \frac{\partial\bar{m}_s^2}{\partial\rho_2}\frac{\partial^2 l^{(f)}(\bar{m}_s^2)}{\partial(\bar{m}_s^2)^2}\Big)\Big]\\
&= 2\Big[\frac{\partial^2\bar{m}_l^2}{\partial\rho_2^2}\Big(\frac{\partial l^{(f)}(\bar{m}_l^2)}{\partial\bar{m}_l^2} - \frac{\partial l^{(f)}(\bar{m}_s^2)}{\partial\bar{m}_s^2}\Big) + \Big(\frac{\partial\bar{m}_l^2}{\partial\rho_2}\Big)^2\Big(\frac{\partial^2 l^{(f)}(\bar{m}_l^2)}{\partial(\bar{m}_l^2)^2} + 2\frac{\partial^2 l^{(f)}(\bar{m}_s^2)}{\partial(\bar{m}_s^2)^2}\Big)\Big]\\
&= \frac{2}{3}\Big(1-\frac{\eta_q}{4}\Big)\Big[-\frac{\sqrt{6}}{48}\frac{h^2}{k^2}\rho_2^{-3/2}\big((1+\bar{m}_l^2)^{-3/2} - (1+\bar{m}_s^2)^{-3/2}\big)\\
&+ \frac{1}{32}\frac{h^4}{k^4}\rho_2^{-1}\big((1+\bar{m}_l^2)^{-5/2} + 2(1+\bar{m}_s^2)^{-5/2}\big)\Big]\\
&= \frac{1}{24}\Big(1-\frac{\eta_q}{4}\Big)\frac{h^2}{k^2}\rho_2^{-3/2}\Big[-\frac{\sqrt{6}}{3}\big((1-\frac{3}{2}\bar{m}_l^2 + \frac{15}{8}\bar{m}_l^4 + \cdots) - (1-\frac{3}{2}\bar{m}_s^2 + \frac{15}{8}\bar{m}_s^4\bar{m}_l^4 + \cdots)\big)\\
&+ \frac{1}{2}\frac{h^2}{k^2}\rho_2^{1/2}\big((1-\frac{5}{2}\bar{m}_l^2 + \cdots) + 2(1-\frac{5}{2}\bar{m}_s^2 + \cdots)\big)\Big]\\
&= \frac{1}{24}\Big(1-\frac{\eta_q}{4}\Big)\frac{h^2}{k^2}\rho_2^{-3/2}\Big[-\frac{\sqrt{6}}{3}\big((1-\frac{3}{2}\bar{m}_l^2 + \frac{15}{8}\bar{m}_l^4 - \frac{35}{16}\bar{m}_l^6 + \cdots)\\
&- (1-\frac{3}{2}\bar{m}_s^2 + \frac{15}{8}\bar{m}_s^4 - \frac{35}{16}\bar{m}_s^6 + \cdots)\big)\\
&+ \frac{1}{2}\frac{2}{\sqrt{6}}(\bar{m}_s^2 - \bar{m}_l^2)\big((1-\frac{5}{2}\bar{m}_l^2 + \frac{35}{8}\bar{m}_l^4 + \cdots) + 2(1-\frac{5}{2}\bar{m}_s^2 + \frac{35}{8}\bar{m}_s^4 + \cdots)\big)\Big]
\end{aligned}
\tag{61}
$$

Obviously, the leading-order equal zero, and the next-leading-order is also vanished, and

$$
\begin{aligned}
\frac{\partial}{\partial\rho_2}\Big(\frac{\partial A_{qk}}{\partial\rho_2}\Big) &= \frac{1}{24}\Big(1-\frac{\eta_q}{4}\Big)\frac{h^2}{k^2}\rho_2^{-3/2}\Big[-\frac{5\sqrt{6}}{8}(\bar{m}_l^4 - \bar{m}_s^4) + \frac{5\sqrt{6}}{12}(\bar{m}_l^2 - \bar{m}_s^2)(\bar{m}_l^2 + 2\bar{m}_s^2) + \mathscr{O}(\bar{m}_f^6)\Big]\\
&= \frac{5\sqrt{6}}{96}\Big(1-\frac{\eta_q}{4}\Big)\frac{h^2}{k^2}\rho_2^{-3/2}\Big[-\frac{1}{2}(\bar{m}_l^4 - \bar{m}_s^4) + \frac{1}{3}(\bar{m}_l^2 - \bar{m}_s^2)(\bar{m}_l^2 + 2\bar{m}_s^2)\\
&+ \frac{7}{12}(\bar{m}_l^6 - \bar{m}_s^6) - \frac{7}{12}(\bar{m}_l^2 - \bar{m}_s^2)(\bar{m}_l^4 + 2\bar{m}_s^4) + \cdots\Big]
\end{aligned}
\tag{62}
$$

As calculated above, the leading-order of $\partial A_{qk}/\partial\rho_2$ and the leading-order and next leading-order of $\partial^2 A_{qk}/(\partial\rho_2)^2$ are canceled out. This will cause numerical problems and we introduce a scalar $\Lambda_2 \sim 2GeV$. Above the scalar Λ_2, Taylor expansion of quark loop function at zero temperature are employed.

6 Appendix.C

The Kobayashi-Maskawa-'t Hooft coupling $\bar{c}_A$, should decrease at high scalar and high temperatures. However, if we keep c_A a constant, and

$$\bar{c}_A = \frac{c_A}{Z_\phi^{3/2}} \tag{63}$$

$\bar{c}_A$ will increase with scalar and temperarure. One scheme is given in ref [**?**], which assume that $\bar{c}_A$ is a constant. However, this scheme cause a very sharp phase transitions. We assume c_A is a infrared enhancement function

$$c_A = c_{A,IR} \frac{1}{e^{\frac{k-k_{cut}}{\Delta_k}} + 1} \tag{64}$$

here $k_{cut} = 1GeV$ and $\Delta_k = 20MeV$. And $\bar{c}_A$ still dressed as $\bar{c}_A = c_A / Z_\phi^{3/2}$.

7 Appendix.D

As we known that

$$T_0 = \frac{1}{\sqrt{6}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad T_8 = \frac{1}{2\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \tag{65}$$

We have

$$\Sigma_0 = T_0\sigma_0 + T_8\sigma_8 = \begin{pmatrix} \frac{1}{\sqrt{6}}\sigma_0 + \frac{1}{2\sqrt{3}}\sigma_8 & 0 & 0 \\ 0 & \frac{1}{\sqrt{6}}\sigma_0 + \frac{1}{2\sqrt{3}}\sigma_8 & 0 \\ 0 & 0 & \frac{1}{\sqrt{6}}\sigma_0 - \frac{1}{\sqrt{3}}\sigma_8 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} \sigma_L & 0 & 0 \\ 0 & \sigma_L & 0 \\ 0 & 0 & \sqrt{2}\sigma_S \end{pmatrix} = T_L\sigma_L + T_S\sigma_S \tag{66}$$

so

$$T_L = \frac{1}{2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad T_S = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{67}$$

8 Appendix.E:mesons anomalous dimension

As assumed above,

$$\eta_\phi = \eta_{\pi^+} \tag{68}$$

and we get

$$\begin{aligned}\eta_\phi(p_0=0,|\mathbf{p}|=0/k) =& \frac{\bar{Z}_\phi(0,k)}{Z_\phi(0,0/k)}\Bigg\{\frac{1}{3\pi^2k^2}\big[\bar{\lambda}^2_{\pi^+\pi^- f_0}\mathscr{B}\mathscr{B}^{(\pi,f_0)}_{(2,2)} + \lambda^2_{\pi^+\pi^-\sigma}\mathscr{B}\mathscr{B}^{(\pi,\sigma)}_{(2,2)} + \lambda^2_{\pi^+a_0^-\eta}\mathscr{B}\mathscr{B}^{(a_0,\eta)}_{(2,2)} \\ &+ \lambda^2_{\pi^+a_0^-\eta'}\mathscr{B}\mathscr{B}^{(a_0,\eta')}_{(2,2)} + (\lambda^2_{\pi^+\kappa^-K^0} + \lambda^2_{\pi^+K^-\kappa^0})\mathscr{B}\mathscr{B}^{(K,\kappa)}_{(2,2)}\big] + quark(l)\Bigg\}\end{aligned} \tag{69}$$