RESONATOR-IN-A-LOOP FILTER BANKS
– Based on a Lerner Grouping of Outputs –

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Abstract

A frequency-sampling Fourier Transform can be efficiently and robustly realized by a resonator-in-a-loop filter bank with uniformly spaced resonators. If the resonator outputs are grouped using a Lerner weighting, where alternating signs are used for each resonator output, then filter banks with greatly reduced side-lobes are obtained at the expense of wider passbands. Further, filters obtained by grouping the real Fourier Transform outputs are linear phase and have exactly 90° phase shift differences compared to those obtained by a Lerner grouping of the imaginary outputs. This allows for the realization of very accurate Hilbert Transform bandpass filters.

1. Introduction

Uniformly-spaced filter-banks are used in a variety of applications such as phased-array processing[1], sub-band coding[2][4], multi-carrier data transmission[3], signal-characterization[4], and frequency-domain adaptive filters [5]. Historically, there have been three major approaches to realizing uniformly-spaced filter banks; those based on spectral transforms such as the Fourier Transform [6], those based on using a tree of half-band filters [7], and those based on modulating a lowpass filter [8][9][10]. Filters based on the first approach are not easily programmed using general purpose DSPs due to large data storage and manipulation requirements. More importantly, they have very poor side-lobe performance, which causes large amounts of channel cross-talk. Filter-banks based on the latter approaches can be designed to have better side-lobe performance, but are computationally complex especially in applications where there is no decimation following the filters.

The proposed realization method for filter banks is an extension of spectral transform methods to get greatly improved side-lobe performance. In addition, a very efficient resonator-in-a-loop architecture for realizing the algorithm is described. This architecture has a complexity that is linear with respect to the number of channels, and is very insensitive to round-off errors or finite-arithmetic accuracy.

It is well know that the real outputs of Fourier transforms realize uniformly spaced filter banks [6], albeit ones that have very poor side-lobe performance. It is possible to combine adjacent outputs to realize fewer filters, but ones with greatly improved side-lobe performance. One very efficient method of combining outputs is similar to what has been used for *Lerner Filters* [11]. These filter design methods were originally proposed for realizing continuous-time filter-banks having almost linear-phase band-pass outputs with good stopband performance [11]. In [11], each bandpass filter was realized by a weighted sum of adjacent parallel second-order biquadratic filters. The weighting coefficients were ± 1 for adjacent resonators, except ± 0.5 was used for the bandpass-edge resonators [12]. The + signs where used for all the odd biquads, whereas the - signs where used for all the even biquads, or vice-versa. These filter realization techniques where extended to the digital domain using the matched-z transform [13], and the impulse-invariant transform [14]. In the proposed filter-bank, a Lerner grouping is used on the real (and/or imaginary) outputs of the Fourier Transform. In this case, uniformly-spaced filter banks result that have a number of features. These include:

1. Each filter-bank output has exactly unity gain at the frequencies of the outputs that were grouped with ± 1, and has transmission zeros at the frequencies of the Fourier Transform outputs that were not used in the particular bandpass output.
2. Each filter-bank output has exactly linear phase. (It might be mentioned that the real part of the Fourier Transform outputs do not have exactly linear phase – the imaginary parts of the Fourier Transform outputs do have linear phase).
3. The side-lobes of the filter outputs not only are much smaller than the side-lobes of the original Fourier Transform outputs, but the stopband attenuation increases the further the frequencies differ from the passband frequencies.
4. Under some minimal constraints, the sum of all the filter bank outputs realized with Lerner groupings of real Fourier-Transform outputs is an exact delay. This is important in adaptive equaliztion applications.
5. Filter outputs realized by using Lerner weightings of the imaginary Fourier-Transform outputs have exactly a 90° phase-shift difference compared to outputs realized using similar Lerner weightings of the corresponding real Fourier-Transform outputs. In addition,

the magnitude of their transfer functions are very similar (although not identical). These filters can be very efficient in realizing Hilbert Transforms with good accuracies over moderate to wide bandwidths.

6. The Lerner grouped outputs have fairly flat passband responses. If the outputs of adjacent Lerner bandpass filters are summed, then the transfer function of the resulting filter also has a very flat but wider bandpass response.

Although the proposed filter banks can be realized using a variety of fast algorithms for calculating Fourier Transforms, a resonator-in-a-loop architecture is very efficient, robust, and easy to realize using custom ICs or DSPs, for applications where the outputs are not decimated. Even when the outputs are decimated, the proposed filter-bank architectures are reasonable alternatives for filter-banks of 64 or less channels.

2. Resonator-in-a-Loop Filter Banks

The resonator-in-a-loop filter-bank is composed of a number of parallel undamped digital resonators with a common feedback around all of them. The resonators can be either complex first-order filters or undamped second-order biquads having real coefficients. This architecture has been independently arrived at by a number of researchers (including the authors [15][16]). For example, [17][18] is based on observer theory, whereas [19][20] is based on an adaptive filter approach. The resonator-in-a-loop filter bank has the unique characteristic that the transfer function from the input to the i'th resonator output is exactly unity at the frequency of the i'th resonator and is exactly zero at the frequencies of all other resonators.

If the resonant frequencies of first-order complex resonators are uniformly spaced around the unit circuit, and the gain constants of the resonators are appropriately chosen, then the resonator outputs are identical to the outputs of a *sliding* Fourier Transform, similar to the outputs of frequency-sampling structures[21], but much less sensitive to finite-arithmetic effects. Indeed, in [17] it was shown that digital filters based on this filter-bank have orthogonal internal-states, provide minimum round-off noise for fixed-point implementations, and suppress zero-input limit cycles under magnitude truncations. When a resonator-in-a-loop architecture is used to realize a sliding Fourier Transform, any finite arithmetic inaccuracies leave poles at the origin which are only partially cancelled, as opposed to the frequency-sampling realizations where finite-arithmetic errors leave uncancelled poles on the unit circle.

Since the resonator pole frequencies are uniformly spaced around the unit circuit, and there are resonators at unity and $f_c/2$, then there are complex conjugate resonator pairs that can be combined into undamped second-order biquads having real coefficients; this results in increased efficiencies. (The resonators at unity and $f_c/2$ already have real coefficients and remain as first order resonators.) Each biquad has two outputs, one for the real-part output, and one for the imaginary-part output. The real-part outputs are summed together and then 'fed-back' and subtracted from the input signal. The resulting structure is shown in Fig. 1. There are a variety of ways

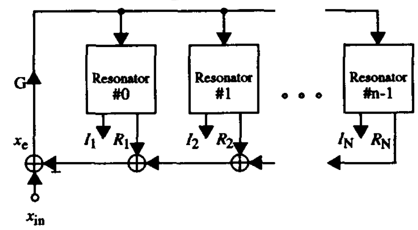


Fig. 1. A resonator-in-a-loop filter bank.

that each biquad can be realized. These include direct-form realizations [17] and LDI-based realizations [16], as only two possibilities. The adjacent real outputs (or alternatively the imaginary outputs) can then be combined with Lerner weightings to realize efficient bandpass filters, uniformly spaced around the unit-circle, that have very good stop-band performance.

3. Filter-Banks with Lerner Weighted Outputs

In the proposed filter-bank, the outputs of a few adjacent resonators are added with alternating signs to realize each Lerner band-pass output. The more resonator outputs are included in each filter-bank output, the wider the passband will be. Normally, the resonators at the passband edges are weighted with a $\pm 1/2$ rather than a ± 1. This gives a better transition-band performance. In addition, the transition-band outputs are shared in the weightings for adjacent band-pass outputs [12]. For good stopband performance (or equivalently small side-lobes), the total weighting in each band-pass output of all the resonator outputs having even indices must equal the total weighting of all the resonator outputs having odd indices. A possible weighting scheme which results in this constraint is shown in Fig. 2, where R_i designates the real

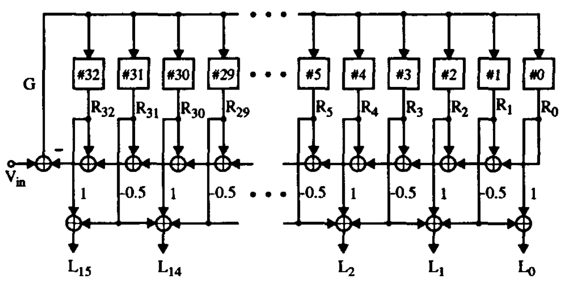


Fig. 2. Three-resonator Lerner groupings.

output of the i'th biquad. Note that for this choice of weightings, the first and last Lerner outputs of the filter-bank use only two resonator outputs, whereas all other Lerner outputs have a weighted sum of three resonator outputs. This is because the resonators at dc and $f_c/2$ are only first-order filters, and thus, an adjacent biquad with a weighting of 1/2 will cancel their side-lobes. Fig. 3 shows the transfer function of L_1 of Fig. 2, along with the transfer function of R_2. The passband is about twice as

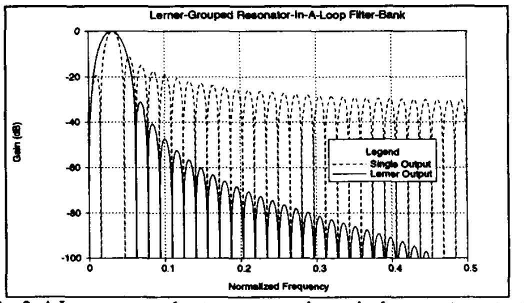


Fig. 3. A Lerner grouped output compared to a single resonator output.

wide, whereas the stop-band attenuation is greatly improved, especially the further one gets from the passband. Alternatively, the order could have been doubled for the Lerner-grouped filter. This would result in an equal width passband and a further improved stopband at the expense of greater complexity. There are many other possible Lerner weighting schemes that result in small side-lobes.

The filter-bank outputs based on a Lerner weighting of resonator-in-a-loop outputs have exactly linear phase. This was only approximately the case for other digital filter banks based on Lerner weightings[13][14]. In addition, the group delays of all the filter-bank outputs (whether real or imaginary) are constant and exactly equal. Filter-banks based on grouping the real-outputs of the resonators have transfer functions with exactly 90° phase differences, as compared to grouping the imaginary parts of the corresponding resonators. Furthermore, filter-banks with Lerner-grouped real outputs have almost the same magnitude response as compared to filter-banks where the corresponding imaginary outputs have been grouped. In addition, if all the Lerner outputs based on weighting the real outputs of the biquads are added together, and if in the sum, the total contribution from each real resonator output has magnitude unity and alternating sign, then the transfer function from the filter input to the sum of the filter-bank outputs is exactly a delay of N/2 (where the filter-bank order is N). This is not the case for the sum of all the imaginary outputs, as the first-order resonators at dc and $f_c/2$ have no imaginary outputs and, thus, the transfer function to the sum of the imaginary outputs has approximately unity magnitude at most frequencies, but goes to zero at dc and $f_c/2$. Proofs of these properties have been derived and will be given at the conference.

4. Applications

The proposed filter-bank can be used in applications similar to where other filter banks are used. Many of these applications will be described at the conference. An obvious application is signal characterization where an in-band signal must be detected with large out-of-band interference. In many signal characterization applications, the relatively flat passbands of the Lerner outputs are important. The superior side-lobe performance of the proposed filter-banks can also be very important in these applications.

Another promising application is in systems requiring Hilbert Transforms, such as QAM and single-side-band communication systems. The 90° phase-shift difference between Lerner outputs consisting of grouping the real resonator outputs, as opposed to Lerner outputs where the imaginary resonator outputs have been grouped, means that the outputs of the two different systems are Hilbert Transforms of each other. Furthermore, since both the real and the imaginary filters are obtained using the same structure but with different outputs, the realization is very efficient. Shown in Fig. 4, with an ex-

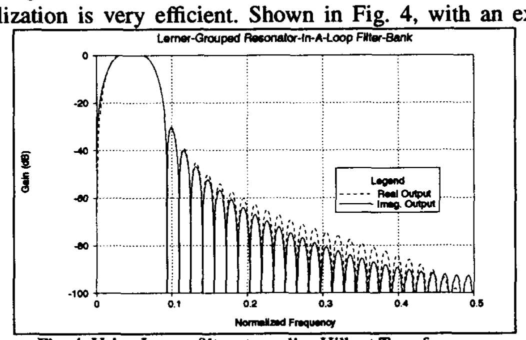


Fig. 4. Using Lerner filters to realize Hilbert Transforms.

panded view of the passband in Fig. 5, are the transfer functions of the real and the imaginary Lerner outputs, where five resonator outputs have been grouped to realize each bandpass filter. The phases of the two different filters are shown in Fig. 6. It can been seen that there is exactly 90° phase difference, and almost the same amplitude response, which is quite flat in the passband. As another example, Fig. 6 shows the transfer function resulting from grouping all of the real and imaginary resonator outputs into wide-band Lerner bandpass filters. It can been seen that a very wide-band Hilbert Transform results with almost flat amplitude response, especially at frequencies not close to dc and $f_c/2$.

Another possible application is in programmable amplitude and phase equalizers.The Lerner grouped filters are bandpass functions with a constant delay (i.e. the

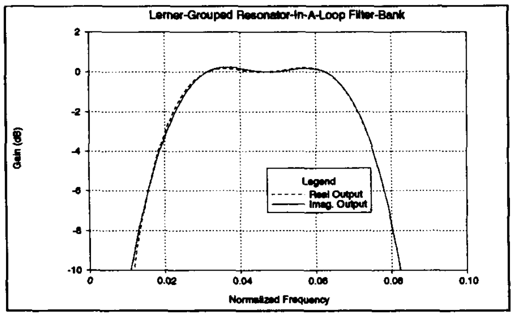


Fig. 5. An expanded view of the pass-band of Fig. 4.

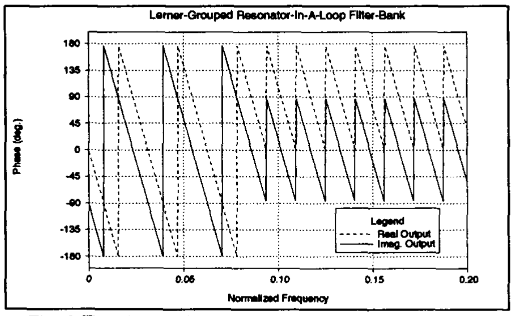
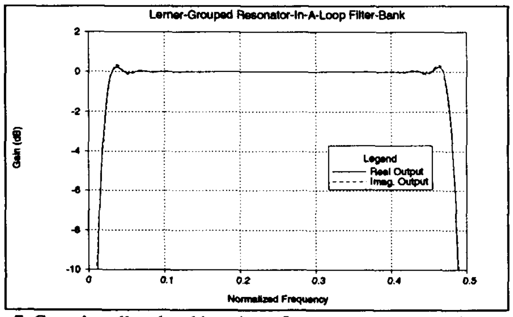


Fig. 6. The phases of the real and imaginary Lerner outputs.



'ig. 7. Grouping all real and imaginary Lerner outputs except those at dc and fc/2.

group delay). At the frequencies of the resonators, the phase and amplitude is trivially determined exactly by taking a weighted sum of the real and imaginary Lerner outputs. Due to the small side-lobes, the errors at frequencies other than the resonant frequencies of the biquads are small. A closely related application is frequency-domain adaptive filters where a weighted sum of the real and imaginary Lerner outputs are used to adapt to the desired phase and magnitude in each frequency subband. Small side-lobes are important in this application to keep the adaptation algorithm from trying to minimize large but out-of-band interference. Rather, the adaptation of each Lerner band-pass output is only sensitive to signals around the passband frequencies.

The proposed filter bank can also be useful in multi-- carrier data transmission systems and maximally decimated sub-band coding. These and other application will be discussed at the conference, with simulations given of many of them.

References

[1] A.B. Baggeroer, "Sonar Signal Processing," in *Applications of Digital Signal Processing*, ed. Alan V. Oppenheim, pp. 332-438, Prentice Hall, 1978.

[2] T. Parsons, *Voice and Speech Processing*, sect. 9-5, Filter-Bank Coders, pp. 246-256, McGraw Hill, 1987.

[3] John Bingham, "Multicarrier Modulation for Data Transmission: An Idea Whose Time Has Come," *IEEE Communications Magazine*, pp. 5-14, May 1990.

[4] R. Crochiere and L. Rabiner, *Multirate Digital Signal Processing*, Chapt. 7: "Multirate Techniques in Filter Banks, Spectrum Analyzers, and Synthesizers," pp. 289-400, Prentice Hall, 1983.

[5] E. Ferrara, Jr., "Frequency-Domain Adaptive Filtering," in *Adaptive Filters*, ed. C. Cowan and P. Grant, pp. 145-179, Prentice Hall, 1985.

[6] L. Rabiner and B. Gold, *Theory and Application of Digital Signal Processing*, Chapt. 6: "Spectral Analysis using the FFT," pp. 386-388., Prentice Hall, 1975.

[7] M.J.T. SMith and T.P. Parnwell III, "A Procedure for Designing Exact-Reconstruction Filter-Banks for Tree-Structured Subband Coders," *IEEE Int. Conf. on Acoustics, Speech, and Signal Processing*, pp. 27.1.1-27.1.4, San Diego, CA, March 1984.

[8] P.P. Vaidyanathan, "Multirate Digital Filters, Filter Banks, Polyphase Networks, and Applications: A Tutorial," *Proceedings of the IEEE*, Vol. 78, No. 1, pp. 56-93, Jan. 1990.

[9] H. Malvar, "Modulated QMF Filter Banks with Perfect Reconstruction,", *Electronics Letters*, vol. 26, No. 13, pp. 906-907, Jun. 1990.

[10] M. Vetterli, and D. Gall, "Perfect Reconstruction FIR Filter Banks: Some Properties and Factorizations," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, Vol. 37, No. 7, pp. 1057-1071, July 1989.

[11] R.M. Lerner, "Band-Pass Filters with Linear Phase," *Proceedings of the IEEE*, Vol. 52, pp. 249-268, March 1964.

[12] P.R. Drouilhet Jr., L.M. Goodman, "Pole-Shared Linear-Phase Band-Pass Filter Bank," *Proceedings of the IEEE*, Vol. 54, pp. 701-703, April 1966.

[13] J.S. Chang, Y.C. Tong, "A Pole Sharing Technique for Linear-Phase Switched-Capacitor Filter-Banks," *IEEE Trans. on Circuits and Systems*, Vol. CAS-37, pp. 1465-1479, Dec. 1990.

[14] G. Doblinger, "An Efficient Algorithm for Uniform and Nonuniform Digital Filter Banks," *1991 Int. Symp. on Circuits and Systems*, Singapore, pp. 646-649, June 1991.

[15] M. Padmanabhan and K. Martin, "The Spectral Analysis of Short Data-Segments Using an IIR Adaptive Filter", *1991 Int. Conf. on Acoustics, Speech, and Signal Processing*, Toronto, pp. 3237-3240, May 1991.

[16] K. Martin and M. Padmanabhan, "ResonatorI-Based Filter-Banks for Frequency-Domain Applications," accepted for publication in *IEEE Trans. Circuits and Systems*.

[17] Gabor Peceli, "Resonator Based Digital Filters", *IEEE Trans. Ckts. and Syst.*, Vol. CAS-36, pp. 156-159, Jan. 1989.

[18] Gabor Peceli, "Structurally Passive Resonator Based Digital Filters", *IEEE Intl. Symp. on Circuits and Systems*, pp. 1421-1424, 1988.

[19] W.F. McGee, "Frequency Interpolation Filter Bank," *IEEE Intl. Symp. on Circuits and Systems*, pp. 1563-1566, 1989.

[20] W.F. McGee, "Fundamental Relations Between LMS Spectrum Analyzer and Recursive Least Squares Estimation", *IEEE Trans. Circuits and Systems*, vol CAS-36, pp. 156-159, Jan. 1989.

[21] R. Bitmead and B. Anderson, "Adaptive Frequency-Sampling Filters," *IEEE Trans. on Cirucits and Systems*, Vol. CAS-28, No. 6, June 1981.