Small Side-Lobe Filter Design for Multitone Data-Communication Applications

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***Abstract*—An approach for realizing filter banks having improved side-lobe performance compared to approaches such as those based on inverse Fourier transforms (IFT's), especially for greater frequency differences from the passband frequencies, is presented. The approach is based on using a weighted-sum of near-adjacent IFT filters to realize the individual channel-bank filters, but with constraints added that results in significantly improved stopband performance while still achieving small reconstruction errors. The proposed channel banks are suitable for realizing multitone digital data communication systems, such as Asymmetric Digital Subscriber Line (ADSL) systems, where stopband performance is critical. Under the conditions of maximal decimation, the reconstruction is not perfect, but aliasing errors are small enough to be negligible in practical communication systems. For some cases, the filter coefficients can be determined exactly without using optimization. Given the frequency-weighting coefficients reported herein, near-optimal multirate filter banks may be designed *exactly* without optimization for all even n.**

***Index Terms*—Data communications, filters, multirate.**

I. Introduction

The use of IFT's and FT's for multitone data communication channels is well established [3]–[5]. These filter banks are capable of perfect reconstruction with maximal decimation assuming a perfect memoryless communication channel. Unfortunately, given practical communication channels, the poor side-lobe (or, equivalently, stop-band) performance on the order of only -13 dB (for the sidelobe closest to the main lobe) is a serious limitation. Reasons for the stopband performance being of paramount importance in multitone communication applications include the following.

1) If filters have good stopbands, then narrow-band interference degrades only one (or a few) channels, and has minimal effect on most channels. This is not true for IFT-based systems.
2) At high frequencies, the frequency response of telephone wires is greatly attenuated. Good containment means data being transmitted on low-frequency channels does not "leak" into high-frequency channels given improper crosstalk cancellation and equalization.
3) In other applications, where filter banks are used as equalizers, large sidelobes can cause the equalization of one frequency band to seriously degrade the equalization of other bands substantially separated in frequency.

For these and other reasons, researchers have been investigating the use of alternative filter banks having smaller side lobes [6], [7]. Better stopband performance has been shown in [7], where filters based on the use of wavelet transforms are described, to affect improved performance of Asymmetric Digital Subscriber Line (ADSL) systems. The filters reported here offer improved stopband performance compared to those reported in [7] for similar orders.

Not only is stopband performance paramount for data communication applications, a constant amplitude stopband is not desirable; a continually decreasing stopband is preferable in order to maximize interference rejection from external sources and minimize noise leakage into attenuated channels. This guarantees that narrow-band interference from sources such as AM radio signals affects only a few nearby channels and that widely separated channels are unaffected even though their signals may be greatly attenuated by channel loss. It also minimizes errors due to improper equalization and echo cancellation for widely separated and perhaps attenuated channels. In order to obtain improved stopband performance while still achieving maximal decimation, it is required to have filters with larger orders than the number of channels; filters having this property are often called overlapped filters [6]. Assuming the number of channels is determined by the particular application and is not a design choice, then inherent penalties for the improved sidelobe performance are greater latency and computational complexity. In systems where these penalties can be tolerated, the advantages of the improved performance often substantially outweigh the disadvantages, as has been discussed in [7] for channel banks based on wavelet transforms. Alternatively, for a given filter order and latency, having fewer channels with significantly-improved stopband performance can often be a preferable engineering tradeoff.

Most of these alternative filter banks, described to date, have been designed using optimization to have almost-perfect reconstruction under the case of maximal decimation and stopbands having near equal amplitude loss throughout the stopband frequencies. For multitone data communication systems, perfect reconstruction is not only not necessary, but is undesirable if its achievement results in degradation of stopband performance; what is necessary is that errors due to filter crosstalk and intersymbol interference are much smaller than the distance between symbols in a particular channel, and also smaller than unavoidable noise introduced both by nonperfect channel equalization and echo cancellation, external noise sources, and A/D and D/A converter nonidealities.

In this brief, examples of our proposed approach are presented for applications where the order of the filters are 3, 4, 6, and 8 times greater than the number of channels. For the first two cases, exact design procedures are given. *The proposed filters have more attenuation the greater the separation between a filter's stopband frequency and a filter's passband frequency.* When the filters are used for data communication applications having maximal decimation, the filters are not perfectly distortionless; there are errors from both intersymbol interference and crosstalk aliasing, but these errors, resulting in SNR's of -46, -68, -92, and -115 dB, for the examples described (*irrespective of the filter order*), are negligible in most practical multitone communication systems and more than justified by the greatly improved side-lobe performance. The proposed filters have linear phase.[1]

The filters are based on previous research on ***Lerner filters*** [8]–[10], where the outputs of a number of adjacent (or near-adjacent) filter channels obtained using IFT's or FT's were linearly combined with the constraint that the sum of the weighting coefficients is zero and the signs alternate for coefficients used for adjacent filters. This constraint results in the greatly improved side-lobe performance, as has also been discussed in [11]. *The previous research is extended to show how these filters can be used in multirate applications having near-perfect reconstruction under the case of maximal decimation.*

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[1] It is possible that further-improved stopband attenuation could result if the linear-phase requirement was relaxed. This can only be ascertained with additional research.

Once the weighting coefficients have been found (they are given in a later section), then exact design techniques can be used for all even n, where n is the order of the IFT.[2]

An example Matlab script is included in the Appendix for calculating the channel-bank filter coefficients for the filters described herein.[3] This allows many of the details regarding obtaining the filter coefficients to be given more concisely since the rigorous calculations are easily ascertained by examining the code.

In a multitone communication system, there are many issues that are important besides the determination of the coefficients of the channel filters. These include issues such as equalization, crosstalk cancellation, cyclic prefixes, synchronization, etc. The successful resolution of these issues, although equally important, is a separate issue from the design of the channel-filter coefficients and is not dealt with in this brief. Furthermore, the brief is primarily concerned with the design of the low-pass prototype filter. This can then be frequency shifted to realize the other filters of the bank; the code for this is included in the Matlab script. The complete bank can then be realized using either FFT, polyphase, or alternative architectures. is also is a separate issue and not dealt with here. The proposed filters can be used in many different types of filter banks such as *modified complex-modulated filter banks* (MDFT) and *cosine-modulated pseudo-QMF filter banks* [1], [2]; simulations are described for the latter type.

Although the proposed filters are intended primarily for multitone data communication applications, they are expected to be useful for other applications that use maximally decimated filter banks and where error sources, such as those due to the finite truncation of signals, are unavoidable. For example, the designs having the filter order equal to 6 and 8 times the number of channels could be useful for audio applications.

II. Using Lerner Filters for MultiTone Applications—The Prototype Filter

A simplified system, where IFT and FT channel banks are used for multitone applications, is shown in Fig. 1. The transmitted data are taken in parallel once each frame. These data are transformed using an IFT, perhaps added to by a cyclic preface, transmitted over the channel serially, separated from the cyclic preface, and then inverse transformed using an FT. When transforming the input data, each row of the IFT is equivalent to the coefficients of an FIR filter. At the receive end, the rows of the FT are equivalent to the coefficients of "matched" FIR filters where the complex conjugates of the coefficients of the transmit filters are taken in reverse order in time.[4] In a channel bank based on Lerner filters, adjacent IFT outputs are added together in a linear combination to form each transmit filter.[5] The coefficients are constrained such that the signs of adjacent coefficients alternate in sign and, in addition, the sum of the weighting coefficients equal zero. *This constraint results in the excellent stopband performance.* It is equivalent to windowing in the time domain. There are many possible choices of weighting

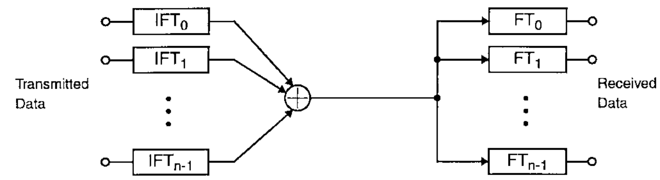

Fig. 1. A simplified diagram of a multitone data communication channel.

coefficients. For example, if the ith filter is taken as

$$T_i(\omega) = \mathrm{IFT}_i(\omega) - \tfrac{1}{2}[\mathrm{IFT}_{i-1}(\omega) + \mathrm{IFT}_{i+1}(\omega)] \tag{1}$$

for n filters (assuming an n-point IFT is used), then it is easy to show that the operation is equivalent to a Hanning window [11]. This is not a good choice for multitone channel-bank applications as it results in considerable distortion due to aliasing. However, other choices have been found to be acceptable if one doesn't mind having fewer channels than the order of the IFT. This case is commonly denoted as overlapped orthogonal transforms [6]. In particular, solutions are described for the cases where n, the order of the original IFT, is greater than the number of filter banks by factors of 3, 4, 6, and 8. Solutions for other cases are similar.

A. Filter Banks with $n/3$ Channels

In this example, the order of the IFT linearly combined filters is three times the number of channels; the order of the resulting filters is $n-1$.[6] Aliasing errors occur, but they are small enough to be unimportant when 5 bits or less is transmitted per real symbol, which is commonly the case in multitone communication systems. The prototype filter is realized by a linear combination of five adjacent IFT filters.[7] The number of frequency-weighting coefficients used, in general, is equal to $(2n/M)-1$. The $i/3$th transmit filter is given by

$$\begin{aligned} T_{i/3}(\omega) = {} & k_2 \mathrm{IFT}_{i-2}(\omega) + k_1 \mathrm{IFT}_{i-1}(\omega) + k_0 \mathrm{IFT}_i(\omega) \\ & + k_1 \mathrm{IFT}_{i+1}(\omega) + k_2 \mathrm{IFT}_{i+2}(\omega) \end{aligned} \tag{2}$$

where

$$\mathrm{IFT}_i(\omega) = \frac{1}{n} \sum_{k=0}^{n} e^{j2\pi ik/n} e^{-j\omega Tk} \tag{3}$$

under the constraint

$$k_2 + k_1 + k_0 + k_1 + k_2 = 0. \tag{4}$$

Therefore, the kth coefficient of the $i/3$ transmit filter is given by

$$\begin{aligned} t_{k,\, i/3} = \frac{1}{n}\Big(& k_0 e^{j(2\pi ki/n)} + k_1\{e^{j[2\pi k(i-1)/n]} + e^{j[2\pi k(i+1)/n]}\} \\ & + k_2\{e^{j[2\pi k(i-2)/n]} + e^{j[2\pi k(i+2)/n]}\}\Big). \end{aligned} \tag{5}$$

For the prototype filter at dc, this simplifies to

$$t_{k,0} = \frac{1}{n}\left[k_0 + 2k_1 \cos\left(\frac{2\pi k}{n}\right) + 2k_2 \cos\left(\frac{4\pi k}{n}\right)\right]. \tag{6}$$

In channel-bank applications, the filters are frequency shifted to be symmetric about the real axis, and then the filters (actually their coefficients) are multiplied alternately by $e^{\pm j\pi/4}$. The details of these steps are most easily ascertained from the Matlab script in the Appendix. The resulting filters are conjugate symmetric about the real axis. The coefficients of the receive filters, $R_{i/3}(\omega)$, are taken as the complex conjugates of the transmit-filter coefficients in reverse order.

[2] The resulting filters have order $n-1$.

[3] The Matlab script may also be obtained via anonymous ftp from ~ftp/pub/software/martin/multi_flt.m in ftp.eecg.toronto.edu.

[4] This is similar to the Carrierless Amplitude/Phase Modulation System (CAP) [13], which is also used in ADSL, where the receive filters are matched filters. Indeed CAP can be considered a single-channel filter, whereas IFT-based systems have the number of channels equal to the filter length. The proposed approach is between these two extreme cases.

[5] In an actual realization, the individual IFT filters would not be realized, rather the linear combination would be used to find the coefficients of the transmit filters. Once the coefficients have been determined, any of the popular multirate realization architectures, such as those based on polyphase or FFT techniques, are possibilities.

[6] The first coefficients of the IFT filters are all 1, and therefore the weighted sum is always equal to zero which drops the filter order by 1.

[7] The indices are taken modulo n. Thus, for example, $\mathrm{IFT}_{-1} = \mathrm{IFT}_{n-1}$.

The satisfying of the constraint given by (4) is what results in the very good stopband performance, especially as the frequencies get further away from the passband. Basically, this constraint causes the side lobes of the IFT component channels to approximately cancel, and this cancellation is more accurate the further the frequency is from the passband frequency, which is preferable in data communication applications.

Additional conditions are required to ensure the cascade of the transmit and receive filters approximately satisfy the Nyquist criteria *(which represents the major contribution of this brief.)* That is, $T_{i/3}(\omega)R_{i/3}(\omega)$ must satisfy the Nyquist criteria where $R_{i/3}(\omega)$ is the matched receive filter corresponding to the ith transmit filter. Since the transmit and receive filters are each composed of linear combinations of IFT and FT filters which are orthogonal, then at the center frequency of the ith IFT filter, the response is due to terms containing that filter (i.e., IFT_i and FT_i), only; other filters (i.e., IFT_h and FT_h for $h \neq i$) have no effect. To be more exact, the center frequency of IFT_i is given by

$$\omega_i = \frac{2\pi i}{nT}. \tag{7}$$

The transfer function of the ith filter to center frequency ω_h is easily shown to be

$$\mathrm{IFT}_i(\omega_h) = \frac{1}{n}\sum_{k=0}^{n} e^{j2\pi k(i-h)/n} \tag{8}$$

which is zero for $h \neq i$ and one otherwise. Similar relationships hold for FT_i. Therefore,

$$\mathrm{IFT}_i(\omega_i)\mathrm{FT}_h(\omega_i) = 0 \tag{9}$$

for $h \neq i$ and one otherwise.

To achieve exactly unity gain for the sum of the cascade of all transmit and receive filters at the IFT center frequencies, and near-unity gain at other frequencies, we need

$$\sum_{j=0}^{M} T_j(\omega)R_j(\omega) = 1 \tag{10}$$

at the center frequencies of the IFT_i for all i. For example, consider $T_{i/3}(\omega)R_{i/3}(\omega) + T_{(i+3)/3}(\omega)R_{(i+3)/3}(\omega)$. At the center frequency of IFT_i, there is no overlap between $T_{i/3}(\omega)R_{i/3}(\omega)$ and $T_{(i+3)/3}(\omega)R_{(i+3)/3}(\omega)$, or for that matter with any other term in (10), and the gain is k_0^2. The same is true at the center frequency of IFT_{i+3}. However, at the center frequency of IFT_{i+1}, there is overlap. The gain at this frequency is $k_1^2 + k_2^2$. The first term is from the weighting coefficient of IFT_{i+1} in $T_{i/3}(\omega)R_{i/3}(\omega)$ and the second term is from the weighting coefficient of IFT_{i+1} in $T_{(i+3)/3}(\omega)R_{(i+3)/3}(\omega)$. No other terms from (10) contribute at this frequency. The gain is the same at the center frequency of IFT_{i+2}. Thus, the necessary and sufficient conditions to satisfy (10) at the center frequencies of all IFT filters are

$$\begin{aligned} k_0 &= 1 \\ k_1^2 + k_2^2 &= 1. \end{aligned} \tag{11}$$

Solving the quadratic that results from (4) and (11) results in

$$\begin{aligned} k_1 &= -0.911438 \\ k_2 &= 0.411\,438. \end{aligned} \tag{12}$$

The amplitude responses of filters 30–35 for a 128-channel filter bank ($n = 384$) are shown in Fig. 3(a). The overlap between next to adjacent channels is adequate for most data communication applications where no more than 5 bits are transmitted per each real symbol.[8]

B. Filter Banks with $n/4$ Channels

If we allow for a greater overlap between transmitted blocks, the aliasing errors can be considerably reduced compared to the previous example. In this example, a linear combination of seven adjacent IFT outputs are added to realize each transmit filter. The number of filters realized is 1/4 the order of the IFT for this example. We now need

$$k_3 + k_2 + k_1 + k_0 + k_1 + k_2 + k_3 = 0 \tag{13}$$

for good stopband performance and

$$\begin{aligned} k_0 &= 1 \\ k_1^2 + k_3^2 &= 1 \\ 2k_2^2 &= 1 \end{aligned} \tag{14}$$

to approximately meet the Nyquist criteria. The second condition of (14) results from the overlap of adjacent channels at IFT_{i+1} and IFT_{i+3}. The third condition of (14) results from the overlap of adjacent channels at IFT_{i+2}. Solving the equations in (14) results in a quadratic function for k_1 and k_3. The exact solution to this quadratic function is

$$\begin{aligned} k_1 &= -0.9719598 \\ k_2 &= 0.707\,106\,8 \\ k_3 &= -0.235\,147\,0. \end{aligned} \tag{15}$$

This solution holds irrespective of the order chosen for the IFT and FT, again for n even. The resulting filters are again of order $n-1$. Example transfer functions for $n = 512$ (i.e., 128 filters) for channels 30–35 are shown in Fig. 2(a). The scale and the channels displayed correspond to those displayed in [7, Fig. 3(b)] which is also shown in Fig. 2(b) to facilitate comparison. The excellent stopband performance and superiority of the proposed filters are evident. There is no crosstalk between adjacent filters in the example filter bank due to the 90° phasing differences between them[9] [12]. The crosstalk between next-to-adjacent filter banks is due to the overlap in the stopbands. From Fig. 2, it is seen that next-to-adjacent channels have minimum stopband attenuation of approximately −40 dB at frequencies halfway between their center frequencies compared to about −35 dB for Fig. 3(b) of [7] (which is the overlap of the skirts of the transition regions, not the first side lobes). More importantly, the improved rejection the further one gets from the stopband, which is often denoted containment, is very evident. Extensive simulations in [7] indicate a strong correlation between this containment and communication system performance in "real" environments. *The crosstalk and stopband overlap of the proposed filters are independent of the filter order and only dependent on the ratio of the number of filters to the order of the filters.* It is speculated and, more so, proposed, that this is in general true for any filter bank having linear-phase FIR filters.

It is only necessary to calculate the coefficients of the prototype filter (i.e., T_0) given the appropriate weighting coefficients listed in Table I, and the coefficients of all other filters of the transmit channel banks are easily found without optimization by frequency

[8]The number of bits transmitted per real symbol is normally limited by intersymbol interference introduced by the transmission-channel frequency response, and by noise introduced primarily by external interference and A/D and D/A converter resolution. Inherent thermal noise is typically not the major limitation.

[9]To the best of the author's knowledge, this fact has not been explicitly stated in the literature. It is inherent in equation (8.1.30) of [1], and was explained to the author by W. McGee [12].

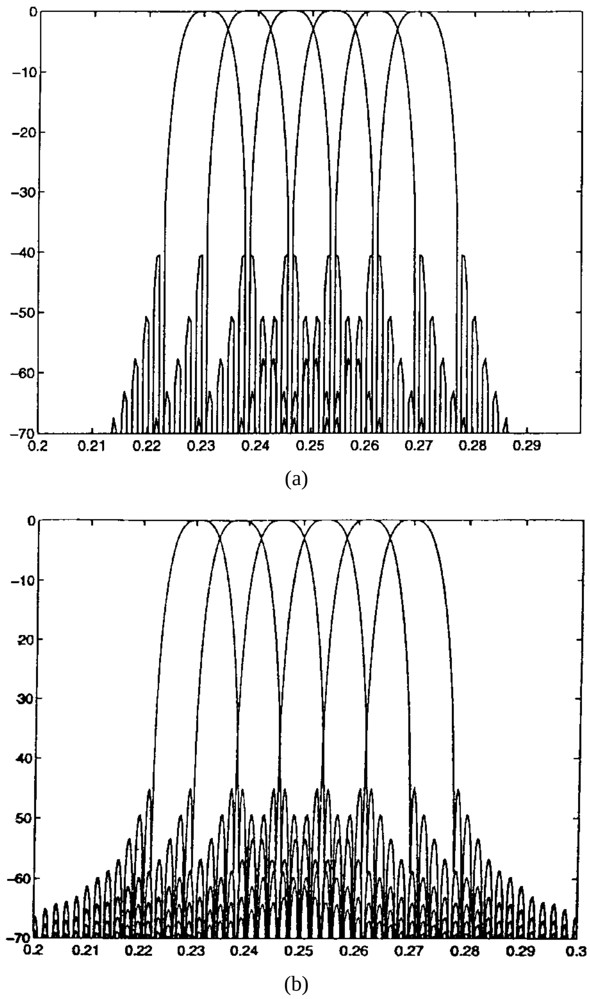

Fig. 2. (a) Example transfer functions (in dB) for Lerner filters for $n = 512$ (i.e., 128 complex channels or 64 real channels) and (b) copy of Fig. 3(b) of [7].

translating the coefficients of the prototype filter and then multiplying all filters by an appropriate power of j (i.e., $\sqrt{-1}$) to eliminate aliasing between adjacent filters (see [1] and [2]). The coefficients of the receive filters are simply the coefficients of the corresponding transmit filters in reverse order and conjugated. That is, the receive filters are "matched filters" to the transmit filters.

C. Filter Banks with Fewer than $n/4$ Channels

Exact solutions for the case when there are fewer than $n/4$ channels have not been found. However, a problem formulation where optimization is used to derive the weighting coefficients has been found. This formulation is described for the special case where there are $n/8$ channels. The solution for $n/6$ channels has also been found, but the solution will not be described as it is very similar and simpler than the case where there are $n/8$ channels; however, the weighting coefficients and simulation results for this case will be reported as well.

The optimization formulation is much simpler than many previously reported formulations, and a single optimization suffices for all even values of n. The aliasing errors that occur are so small to almost certainly be less than errors caused by any other nonidealities, such as finite-arithmetic accuracy or intersymbol interference due to nonideal channels, in actual applications. In this example, the problem is formulated to guarantee the sum of the weighting coefficients is zero and that the overall gain of the sum of all channels is exactly unity at the center frequencies of the IFT filters. These constraints guarantee good stopband performance and minimal aliasing. *These constraints also reduce the problem formulation to finding values for only two unknown coefficients irrespective of the filter order.*[10] Indeed, the optimization can be done for a very low-order case with high numerical accuracy, and the resulting solution is valid for all even orders. Contrast this with a more standard formulation where every filter coefficient is unknown, and the optimization must be redone for every different value of n. *For $\boldsymbol{n}$ on the order of 512 to 1024, these traditional approaches are computationally complex.*

The formulation is based on minimizing the impulse response of selected samples of the convolution of the prototype transmit and receive baseband filters. At the same time, the crosstalk is also minimized by minimizing particular samples of the crosstalk to the next to adjacent receive filter.[11] In both cases, the impulse response samples selected to be minimized are chosen every $n/8$ samples *after* the center sample[12] for our example. The weighting coefficients chosen to be optimized were k_1 and k_2, since they have the greatest magnitude, except for k_0 which is always normalized to unity. Given values for these coefficients, found from optimization, the values for the other k_i, where $i = 0, \cdots, 7$ are found exactly. Some of the details of the formulation follow.

1) Initialize k_1 and k_2. Reasonable starting values are $k_1 = -0.98$ and $k_2 = 0.95$.
2) Calculate the other coefficients given k_1 and k_2, which guarantee minimal aliasing, intersymbol interference using the following exact formulas:

$$\begin{aligned} k_0 &= 1 \\ k_4 &= \sqrt{2} \\ k_6 &= \sqrt{1 - k_2^2} \\ k_7 &= -\sqrt{1 - k_1^2} \\ K &= -k_0/2 + k_1 + k_2 + k_4 + k_6 + k_8 \\ b &= K \\ c &= (K^2 - 1)/2 \\ k_3 &= -b/2 - \sqrt{b^2/4 - c} \\ k_5 &= -\sqrt{1 - k_3^2}. \end{aligned} \tag{16}$$

3) Calculate the prototype filter using the weighting coefficients just found for the linear combination of the 15 IFT filters centered around dc.
4) Calculate the next-to-adjacent transmit filter by forming an inner product with the appropriate frequency-translation vector. The corresponding receive filters are found by reversing the orders and taking complex conjugates of the coefficients.
5) Calculate the convolution of the dc transmit filter with its corresponding receive filter. This first impulse response is used to minimize intersymbol interference. Calculate a second impulse response by convolving the dc prototype transmit filter with a next-to-adjacent receive filter. This second impulse response is used to minimize crosstalk.

[10] For the case $M = n/6$, only one independent coefficient is adapted.

[11] It doesn't matter which next-to-adjacent filter is chosen as they are guaranteed complex-conjugate symmetric.

[12] Since the prototype filter being designed is guaranteed linear-phase with symmetric real coefficients only, it is not necessary to minimize selected samples *before* the center sample.

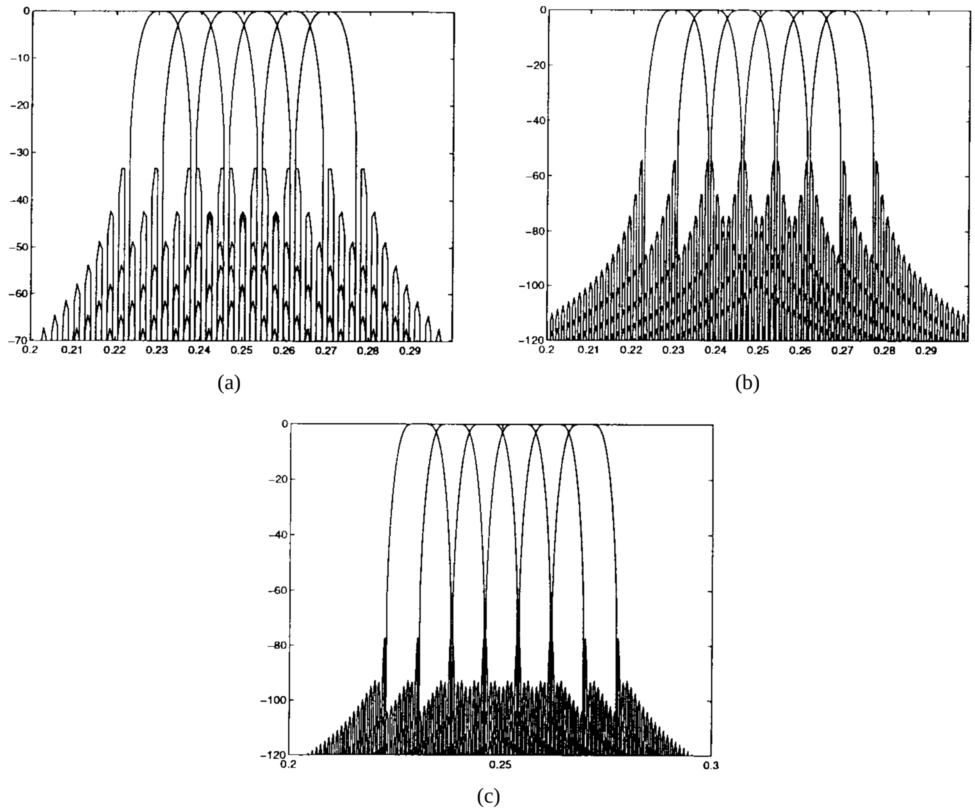

Fig. 3. Five adjacent transmit filters for (a) $n = 384$, (b) $n = 768$, and (c) $n = 1024$. In each case, there are 128 complex channels or, equivalently, 64 real channels.

6) Adapt k_1 and k_2 to minimize selected values of the appropriate impulse responses.
7) Go back to 2.

This formulation was solved using the fsolve() routine from Matlab[13] for $n = 32$ (chosen somewhat arbitrarily). The resulting values for the coefficients are given in Table I. The time required for the optimization on a Pentium Pro running Linux was instantaneous compared to a human's time constant (i.e., 10 ms). Also included in Table I are coefficients found from a similar optimization for the case of $M = n/6$ channels, as well as the results of the two previous examples with $M = n/3$ and $M = n/4$ for completeness. The transfer functions of a number of adjacent transmit filters (i.e., filters 30–35) for the case $n = 1024$ (i.e., 128 complex filters—64 real filters) are shown in Fig. 3(c). Note that different scales used for the y-axes in order to illustrate more fully the details of the excellent stopband performance when M is much less than n. Included in Fig. 3(b) are the transfer functions of five adjacent filters for the case of $n = 768$ and $M = n/6$ channels. The error in the magnitude transfer function of the normalized sum of the cascade of the transmit and receive filters (i.e., the distortion function [1], [2]) was found to be less than $\pm 3.5 \times 10^{-5}$ different from unity for $M = n/8$. This level of distortion is inconsequential in most practical multirate filter-bank applications.

III. Maximally Decimated Examples

An example of using the proposed baseband filters for a data communications example is described. In this example, the prototype filter is frequency shifted by multiples of $\pm(2i+1)(\pi/M)$ resulting in M complex filters, where $M = n/3$, $n/4$, $n/6$, or $n/8$ for our examples. Adjacent filters next have their coefficients multiplied by $e^{\pm j(\pi/4)}$ in order to eliminate crosstalk between adjacent channels. If it is desired that the transmission channel be real, then the positive and negative filters can be combined as they are complex conjugates of each other resulting in $M/2$ real filters, each of which is a "cosine-modulated" version of the prototype filter [before the $e^{\pm j(\pi/4)}$ multiplications]. In our simulation, this was not done; complex channels were assumed. This example is similar to that described in [2, Section 7.4] (except Fliege defines M to be the number of resulting real filters, which is $M/2$ for us). The architecture of this channel bank was originally explained to the author by McGee

TABLE I
The Frequency-Weighting Coefficients Proposed for the Filters. Given These Coefficients, the Design of the Filter Banks is Exact for All Even n

Coefficient	Value $M = n/3$	Value $M = n/4$	Value $M = n/6$	Value $M = n/8$
k_0	+1.000000	+1.0000000	+1.0000000	+1.0000000
k_1	-0.91143783	-0.97195983	-0.99722723	-0.99988389
k_2	+0.41143783	+0.70710681	+0.94136732	+0.99315513
k_3		-0.23514695	-0.70710681	-0.92708081
k_4			+0.3373834	+0.70710681
k_5			-0.07441672	-0.37486154
k_6				+0.11680273
k_7				-0.01523841

[13] Matlab is a trademark of The MathWorks. The source code for this example may be obtained from martin@eecg.toronto.edu.

TABLE II
ERRORS DUE TO ALIASING OF MAXIMALLY DECIMATED FILTER BANKS BASED ON PROPOSED PROTOTYPE FILTERS

M	n	Maximum of Error	Standard-Deviation of Error	SNR	Equivalent A/D Conv. Accuracy
n/3	384	1.3e-2	3.4e-3	46 dB	7 bits
n/4	512	8.0e-4	2.7e-4	68 dB	11 bits
n/6	768	5.2e-5	1.7e-5	92 dB	15 bits
n/8	1024	5.5e-6	1.3e-6	115 dB	19 bits

[12]. This modulation scheme results in all filters having equal widths as opposed to DFT transmission schemes where the filters at dc and half the sampling frequency have narrower widths (although the proposed prototype filters are suitable for these schemes as well). The case of real transmission channels corresponds to the same real input being used for positive and negative frequency filters of the complex channel case. This cuts the input data rate by 1/2, but since only real signals are being transmitted over the channel rather than complex pairs, the transmission data rate is also cut by two; both the real and the complex filter case result in maximal data transmission rates over the corresponding channel. In Matlab simulations, where the only imperfections modeled were the errors due to self interference and crosstalk, a uniformly distributed random real input signal between ± 1 was applied to all analysis filters every $M/2$ samples and the output of each synthesis filter at the receive end was sampled at the same rate. The standard deviation of the errors between the input and output signals are listed in Table II. Also listed is the maximum error (when $2n$ real symbols were transmitted), the signal-to-noise ratios for the cases of $M = n/3, n/4, n/6$, and $n/8$, and the number of bits of an A/D converter that would have approximately the same SNR. Since most multitone data communication systems are limited to no more than 5 bits per real symbol, we see that the filters are adequate with the higher-order filters having aliasing errors that would be completely negligible. To place some of these results in perspective, example 2 in [14] corresponds to $M = n/8$. The reported aliasing errors were -96 dB compared to total errors worst case of -105 dB and total errors rms of -115 dB for the proposed filters.

The peak-to-rms ratio of the channel signal obtained from simulations was less than 4. This should be compared to the peak-to-rms ratio of IFT-based systems which is approximately 6. Although simulations alone don't prove that the proposed filter banks have superior peak-to-rms ratios, they are strong supporting evidence. If proven true, it is significant; at the transmit side of a multitone system, this would mean the voltage levels and therefore the power dissipation of the line drivers, the major source of power dissipation, can be cut. The reduced power-supply voltages would also allow for faster IC technologies to be used since transistor break-down voltages need not be as large. At the receive side, the accuracy requirements of the A/D converter, echo cancellation, and channel equalization are also reduced.

IV. CONCLUSION

Designs for prototype filters intended for multitone applications have been given. The designs are based on a linear combination of near-adjacent IFT filters with appropriate constraints. Given the coefficients listed herein, filter banks may be designed exactly for all even n. The filters described emphasize stopband attenuation as opposed to achieving perfect reconstruction, although reconstruction errors are small.

APPENDIX

The following is Matlab©[14] script for calculating the filter coefficients for the proposed filters.

```
function [T, R] = multi_flt(n, K)
% function [T, R] = multi_flt(n, K) is used to find the coefficients
% of the transmit and receive channel bank filters. n is the filter
% order. K is the ratio of the filter order to the number of channels.
% The rows of T are the coefficients of the transmit filters. The rows
% of R are the coefficients of the receive filters.
% The transmit filters are scaled by 1/(Kn) and the receive filters
% are scaled by 1 for transmultiplexor applications. For unity-gain
% at the center change the scale factors to 1/n for both filters.
% For real channels, just take the real part of the first M/2 rows of
% the transmit filters.
if ((K == 3)|(K == 4)|(K == 6)|(K == 8))
  M = n/K;
else
  error('The ratio of filt. order to the number of filt. must
    be 3, 4, 6 or 8');
end
if ((n <= 16)|(rem(n, 2) ~= 0))
  error('The filter order must even and at least 16.');
end
% Construct the prototype filter
P = proto(n, K);
% Get space for the coefficients for all transmit filters
T = zeros(M, n);
R = zeros(M, n);
T(1, :) = P;
% We next translate the prototype filter from dc by
% 2*pi/(n/(M/2)).
% This will eventually result in conjugate symmetry about d.c.
% and no filters at d.c. or Fs/2.
freq_offset = 2*pi/(2*M);
W = diag(exp(j*freq_offset). ^ (0:n - 1));
T(1, :) = T(1, :)*W;
% Next we find the other filters which are separated by 2*pi/M
freq_diff = 2*pi/M;
W = diag(exp(j*freq_diff). ^ (0:n - 1));
for i = 2:M
  T(i, :) = T(i - 1, :)*W;
end
% Next we multiply all coefficients of adjacent filters by a plus or
% minus 45 deg. phase offset.
% The j difference between filters is to eliminate cross-talk between
% adjacent filters.
% The phase offset is added so that there will be conjugate
% symmetry about dc.
T = exp(j*pi/4).*diag((j). ^ (0:n/K - 1))*T;
R(:, 1:n) = conj(T(:, n: - 1:1));
T = T./sqrt(K*n);
% The function proto(n, K) is used to get the prototype filter
function P = proto(n, K)
phi = (2*pi/n).*(0:n - 1);
switch K
  case 3
P = 1 - 0.91143783*2.* cos(phi) + 0.41143783*2.* cos(2.*phi);
  case 4
P = 1 - 0.97195983*2.* cos(phi) + 0.70710681*2.* cos(2.*phi) ...
-0.23514695*2.* cos(3.*phi);
```

[14] Matlab is a trademark of The MathWorks.

```
  case 6
P = 1 - 0.99722723*2.*cos(phi) + 0.94136732*2.*...
cos(2.*phi) - 0.70710678*2.*cos(3.*phi) + 0.337383417*2.*...
cos(4.*phi) - 0.07441672*2.*cos(5.*phi);
  case 8
P = 1 - 0.99988389*2.*cos(phi) + 0.99315513*2.*...
cos(2.*phi) - 0.92708081*2.*cos(3.*phi) + 0.707106781*2.*...
cos(4.*phi) - 0.37486154*2.*cos(5.*phi) + 0.11680273*2.*...
cos(6.*phi) - 0.01523841*2.*cos(7.*phi);
end
```


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The author would like to acknowledge the help of W. McGee who patiently explained the mechanisms whereby aliasing was minimized or eliminated in the various filter bank approaches.

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An Efficient Approach for the Design of Nearly Perfect-Reconstruction QMF Banks

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***Abstract*—In this brief, an approach for the design of a class of quadrature mirror filter (QMF) banks is presented based on a combination of IIR analysis filters and FIR synthesis filters. By examining the phase property of the all-pass based analysis filter, a condition on the synthesis part for a perfect reconstruction is established, making it possible to realize a nearly perfect-reconstruction (PR) filter bank by using a nonlinear-phase FIR synthesis filter. The distinct feature of the proposed mixed IIR/FIR filter bank is that it enjoys both a small reconstruction error and a low design and implementation complexity.**

***Index Terms*—Digital filters, filter banks, quadrative mirror filters.**

I. INTRODUCTION

Two-channel quadrature mirror filter (QMF) banks have found significant applications in many fields, such as subband coding for speech and image signals [1]–[6], [13], [14]. Many design algorithms have been developed to obtain a perfect or nearly perfect reconstruction filter bank. However, most of these essentially focus on the design of FIR filter banks and, therefore, the resulting analysis and synthesis filters require a large number coefficients to meet the requirement of certain magnitude specifications. From the point of view of decreasing the number of coefficients, the IIR QMF bank is more efficient. A broad class of IIR QM filters have been extensively studied, resulting in very efficient all-pass based realization architectures [7]–[10]. This class of filters can be used to implement two-channel IIR QMF banks that are stable, causal, and free of aliasing error and amplitude distortion. However, the phase distortion in these banks is unavoidable. Some researchers have introduced anticausal IIR filters so as to obtain a PR IIR filter bank [5]. But this scheme usually leads to a filter synthesis that is considerably complex, since the implementation of an anticausal filter requires the segmentation and reversion of the input signal sequence in the time domain. In order to avoid the problem of synthesizing an anticausal filter, one may consider an IIR synthesis filter that has such a frequency response that the overall bank can approximate a PR performance. However, designing an IIR filter that has to meet both magnitude and phase specifications simultaneously is generally difficult, since the cost function is rather complex and the stability of the filter has to be considered.

In this brief, we consider a mixed realization scheme for QMF banks. By using the all-pass based IIR filter to implement the analysis part of the bank and by approximating an FIR filter that can efficiently offset the nonlinear phase of the analysis filter for the synthesis part, a class of nearly PR QMF bank is developed with an objective to reduce the overall design and implementation complexity.

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