Shiyu Zhang MA 677 HW2

Q1 ~~y~~ y can be 1, 2, 3 ⋯ 8

$E(y) = 1\times\frac{1}{42} + 2\times\frac{2}{42} + 3\times\frac{6}{42} + 4\times\frac{4}{42} + 6\times\frac{6}{42}$
$+ 7\times\frac{7}{42} + 8\times\frac{16}{42} = 6$

Q2 $E(xy) = \int_0^1\int_0^x xy\,f(x,y)\,dx\,dy$
$= \int_0^1 x\int_0^x 12y^3\,dy\,dx$
$= \int_0^1 3x - x^4\,dx = \frac{1}{2}$

Q3 $E(X_1) = E(X_2) = E(X_3) = \frac{1}{2}$
$E(X_1^2) = E(X_2^2) = E(X_3^2) = \int_0^1 x^2\,dx = \frac{1}{3}$
$E(X_1X_2) = \frac{1}{4} = E(X_2X_3)$
$E[(X_1 - 2X_2 + X_3)^2] = E(X_1^2 + 4X_2^2 + X_3^2 - 4X_1X_2 - 4X_2X_3 + 2X_1X_3)$
$= \frac{1}{2}$

Q4 $E(y) = \int_0^{+\infty} \varphi(x)f(x)\,dx = \int_0^{\infty} e^{\frac{3}{4}x}\cdot e^{-x}\,dx = -4\int_0^{\infty} e^{-\frac{x}{4}}\,d(-\frac{x}{4})$
$= 4$

Q5 $E(Y) = E(2X^2+1) = 2E(X^2)+1 = \cancel{2\times6}\ 31.33$

Q6 $E(X) = \int_0^1 x f(x)\,dx = \int_0^1 x\cdot 2(1-x)\,dx = \frac{1}{3}$
$E(X^2) = \int_0^1 x^2 f(x)\,dx = \int_0^1 x^2\cdot 2(1-x)\,dx = \frac{1}{6}$
$E(Y^2) = E(4X^2+4X+1) = 4\times\frac{1}{3} + 4\times\frac{1}{6} + 1 = 3$

Q7. $E[(aX+b)^n] = E\left(\sum_{i=0}^{n}\binom{n}{i}(aX)^{n-i}\cdot b^i\right]$
$= \sum_{i=0}^{n} E\left[\binom{n}{i}(aX)^{n-i}\cdot b^i\right]$
$= \sum_{i=0}^{n}\binom{n}{i}a^{n-i}\cdot b^i\,E(X^{n-i})$

Q8 $n = X + Y$ $E(X) = np$

$E(X-Y) = E(X-(n-X)) = 2np - n$

So, when $n = 20$ $p = 5\%$ $E(X-Y) = 20 \times (-0.9) = -18$