A Conditional Capital Shortfall Measure of Systematic Risk

Group 9

Members: PENG Shiyu, YANG Rui, ZHOU Luyun, CHEN Yeying, ZHI Haiqing

The financial crisis in 2007-2008 is a full-blown international banking crisis and leads to a great recession. One of the lessons we learned from the crisis is that a capital shortfall is dangerous for a firm, its bondholders and also for the whole economy.

Therefore, we try to use an empirical methodology called SRISK to measure the degree of undercapitalization a financial firm would experience, conditional on severe distress in the entire system.

1. Definition of Systemic Risk Measurement

1.1 Capital Shortfall and Prolonged Market Decline

The variable we introduce to measure the distress of a financial firm is capital shortfall, which is defined as the capital reserves that the firm needs to hold minus the its equity (Acharya, Engle, Richardson, 2012). The formula for firm i on time t is $CS_{it} = k(D_{it} + E_{it}) - E_{it}$, where k *is* prudential capital fraction, E_{it} is market value of equity, and D_{it} is book value of debt. In particular, we set k to 8%.

For prolonged market decline, which is also taken as systemic event, we define it as $\{R_{mt+1:t+h} < C\}$, where $R_{mt+1:t+h}$ is multiperiod arithmetic market return between period $t+1$ and $t+h$ (Brownlees, 2017). In our work, we choose time horizon h as 1 quarter and threshold C as -10%.

1.2 SRISK

SRISK for single firm is defined as $\mathrm{SRISK}_{it} = E_t(CS_{it+h}|R_{mt+1:t+h} < C)$. To compute it, we further assume that when there is a systemic event, debt cannot be renegotiated, $E_t(D_{it+h}|R_{mt+1:t+h} < C) = D_{it}$. We also introduce a variable called long-run marginal expected shortfall (LRMES). Formally, $\mathrm{LRMES_{it}} = -E_t(R_{it+1:t+h}|R_{mt+1:t+h} < C)$, where $R_{it+1:t+h}$ is multiperiod arithmetic firm equity return between t+1 and t+h. Then SRISK can be rearrange into $SRISK_{it} = E_{it}[k\, LVG_{it} + (1-k)LRMES_{it} - 1]$, where LVG_{it} denotes the quasi leverage ratio $(E_{it} + D_{it})/E_{it}$.

As for the whole financial entity with N firms, $SRISK_t = \sum_{i=1}^{N} max\,(SRISK_{it},\, 0)$. Here we don't consider negative SRISK as it means capital surplus.

2. Computation of SRISK

2.1 GARCH-DCC Method

The computation of our ideal measure-SRISK requires us to find an efficient estimator for LRMES. In order to specify this model for the market and firm returns, we first construct LRMES estimations with a GARCH-DCC model. Logarithmic returns of firms and market are defined as $r_{it} = \log(1 + R_{it})$ and $r_{mt} = \log(1 + R_{mt})$ respectively and we assume the return pair follows the distribution with zero mean and time varying covariance conditional on information set available on time $t-1$

$$\begin{bmatrix} r_{it} \\ r_{mt} \end{bmatrix} | \mathcal{F}_t \sim D\left(\mathbf{0}, \begin{bmatrix} \sigma_{it}^2 & \rho_{it}\sigma_{it}\sigma_{mt} \\ \rho_{it}\sigma_{it}\sigma_{mt} & \sigma_{mt}^2 \end{bmatrix}\right)$$

Since this approach needs volatility and correlation specification, we exploit GJR-GARCH volatility model and the standard DCC correlation model (Glosten, Jaganathan, and Runkle 1993, Rabemananjara, and Zakoïan 1993, Engle 2002). The model specification are as follows.

GAR-GARCH: $\sigma_{it}^2 = \omega_{\mathrm{v_i}} + \alpha_{\mathrm{v}_i} r_{it-1}^2 + \gamma_{v_i} r_{it-1}^2 \mathrm{I}_{\mathrm{it}-1}^{-} + \beta \mathrm{v_i} \sigma_{it-1}^2,$

$$\sigma_{mt}^2 = \omega_{\mathrm{v_m}} + \alpha_{\mathrm{v}_m} r_{mt-1}^2 + \gamma_{v_m} r_{mt-1}^2 \mathrm{I}_{\mathrm{mt}-1}^{-} + \beta \mathrm{v_m} \sigma_{mt-1}^2$$

with $\mathrm{I}_{\mathrm{it}}^{-} = 1 \; if \; r_{it} < 0$ and $\mathrm{I}_{\mathrm{mt}}^{-} = 1 \; if \; r_{mt} < 0$

Standard DCC correlation model:

$$\epsilon_{it} = \frac{r_{it}}{\sigma_{it}}, \epsilon_{mt} = \frac{r_{mt}}{\sigma_{mt}}, \qquad Cor\begin{pmatrix} \epsilon_{it} \\ \epsilon_{mt} \end{pmatrix} = R_t = \begin{bmatrix} 1 & \rho_{it} \\ \rho_{it} & 1 \end{bmatrix} = diag(Q_{it})^{-\frac{1}{2}} Q_{it} diag(Q_{it})^{-\frac{1}{2}},$$

$$Q_{it} = (1 - \alpha_{Ci} - \beta_{Ci})S_i + \alpha_{Ci} \begin{bmatrix} \epsilon_{it-} \\ \epsilon_{mt-1} \end{bmatrix} \begin{bmatrix} \epsilon_{it-1} \\ \epsilon_{mt-1} \end{bmatrix}' + \beta_{Ci} Q_{it-1}$$

where Q_{it} is the pseudo correlation matrix and S_i is unconditional correlation matrix of the firm and market adjusted returns.

So far, we cannot obtain the closed formula of this model, but we can implement a simulation toward the LRMES estimation. The steps are as follows.

Firstly, construct the GARCH-DCC standardized innovations, which follow the distribution with zero mean and unit variance. Notably, they are cross-sectionally and serially uncorrelated.

$$\epsilon_{mt} = \frac{r_{mt}}{\sigma_{mt}}, \xi_{it} = \left(\frac{r_{it}}{\sigma_{it}} - \rho_{it}\frac{r_{mt}}{\sigma_{mt}}\right) / \sqrt{1 - \rho_{it}^2}$$

Then construct S pseudo samples of GARCH–DCC innovations from T+1 to T+h, and use them as inputs to deliver S pseudo samples of GARCH-DCC logarithmic returns. The multiperiod arithmetic firm and market return are computed respectively.

$$R^s_{iT+1:T+h} = \exp\left\{\sum_{t=1}^{h} r_{iT+t}\right\} - 1 \quad R^s_{mT+1:T+h} = \exp\left\{\sum_{t=1}^{h} r_{mT+t}\right\} - 1$$

Finally, take advantage of Monte Carlo simulation to compute LRMES

$$LRMES_{iT} = -\frac{\sum_{s=1}^{S} R^s_{iT+1:T+h} I\{R^s_{mT+1:T+h} < C\}}{\sum_{s=1}^{S} I\{R^s_{mT+1:T+h} < C}$$

2.2 Copula

In static normal modal, the firm and market returns follow static bivariate normal distribution with mean zero. Market and firm volatilities and correlation are from GJR-GARCH-DCC model, then we calculate LRMES as follow:

$$\begin{aligned}\mathrm{LRMES}^{stat}_{it} &= -\sqrt{h}\beta_i \mathrm{E}(r_{m\,t+1} | r_{m\,t+1} < \mathrm{c}) \\ &= -\exp\left\{\frac{h}{2}(\beta^2 {\sigma_m}^2 + (1-\rho^2){\sigma_i}^2)\right\} \frac{\Phi\left(\frac{\beta \log(1+C)}{\sqrt{h}\beta\sigma_m} - h\beta^2{\sigma_m}^2\right)}{\frac{1}{2} + \frac{1}{2}\mathrm{erf}\left(\frac{\beta\log(1+C)}{\sqrt{2}\sqrt{h}\beta\sigma_m}\right)} + 1\end{aligned}$$

where $\beta_i = \rho_i \frac{\sigma_i}{\sigma_m}, \mathrm{c} = \frac{\log(1+C)}{\sqrt{h}}, \mathrm{erf(x)} = 2\,\Phi(\sqrt{2}x) - 1,\ \Phi$ is cumulative function of standard normal.

In Dynamic Rotated Gumbel copula model, we assume the marginal distributions F_{mt} and F_{it} follow ARMA (0,0), GJR-GARCH model:

$$r_t = \sigma_t \varepsilon_t,\ \varepsilon_t \sim t_v,\ \sigma_t^2 = \omega + \alpha r_{t-1}^2 + \gamma r_{t-1}^2 I_{t-1}^- + \beta \sigma_{t-1}^2$$

In this copula, range of parameter δ_t is $[1, \infty)$. Evolution function (1) makes sure parameter within range. The lag-delta captures persistence in dependence of parameter, average term captures how far from comonotonicity of the data.

$$\delta_t = 1 + (\omega + \alpha \frac{1}{10} \sum_{\tau=1}^{10} |u_{mt} - u_{it}| + \beta \delta_{t-1})^2 \qquad (1)$$

In Dynamic Normal copula, marginal distributions stay the same. Evolution function (2) makes sure parameter δ_t within (-1,1). Average term captures any variation of the dependence.

$$\delta_t = \varphi\left(\omega + \alpha \frac{1}{10} \sum_{\tau=1}^{10} \Phi^{-1}(u_{m\tau}) \Phi^{-1}(u_{i\tau}) + \beta \delta_{t-1}\right), where\ \varphi(x) = \frac{1-e^{-x}}{1+e^{-x}} \qquad (2)$$

3. Data and Results

3.1 Data Selection

Our model analysis based on the panel data of US stocks between Jan 1, 2000 and

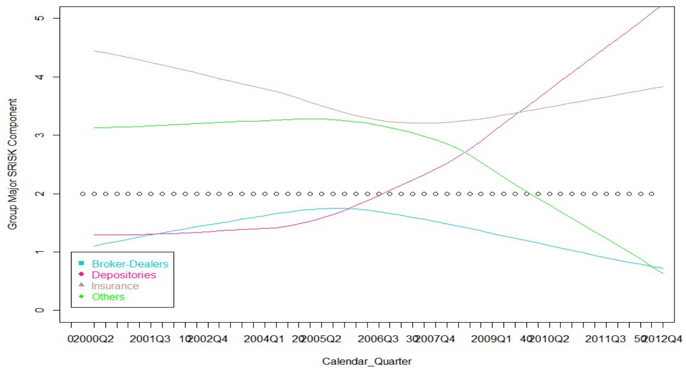
Dec 31, 2012. We selected 76 firms from the database CRSP and COMPUTSTAT, the market capitalization of which exceed 5 billion USD as of the end of 2007. Meanwhile, we also obtained the daily log returns and the daily CRSP market value-weighted index return as the proxy for our market return index. The total 76 firms are categorized into four groups: Depositories, Insurance, Broker-Dealers, and Others.

3.2 SRISK% Ranking Results by GARCH-DCC

Calendar_	SRISK	firm_name:	group	Calendar	SRISK	firm_name:	group
2005Q1	78369677	FNM	Others	2006Q1	360267680	UNH	Insurance
2005Q1	60157539	JPM	Deposito	2006Q1	207716559	AET	Insurance
2005Q1	41833013	FRE	Others	2006Q1	194778195	FNM	Others
2005Q1	32280009	HIG	Insuranc	2006Q1	186532254	FITB	Others
2005Q1	26220136	PRU	Insuranc	2006Q1	144865659	GS	Broker-Dealers
2005Q1	21823019	LEH	Broker-D	2006Q1	119875242	MET	Insurance
2005Q1	18856578	HCBK	Deposito	2006Q1	115719119	PRU	Insurance
2005Q1	16807455	PGR	Insuranc	2006Q1	114729701	LEH	Broker-Dealers
2005Q1	14876911	GS	Broker-D	2006Q1	100346740	FRE	Others
2005Q1	13819516	MET	Insuranc	2006Q1	91168306	AMTD	Others
Calendar_	SRISK	firm_name:	group	Calendar	SRISK	firm_name:	group
2007Q1	72496189	FNM	Others	2008Q1	67959671	FNM	Others
2007Q1	54678318	JPM	Deposito	2008Q1	67511408	LEH	Broker-Dealers
2007Q1	52313515	FRE	Others	2008Q1	62522707	FRE	Others
2007Q1	46323888	GS	Broker-D	2008Q1	61062049	GS	Broker-Dealers
2007Q1	18582308	LEH	Broker-D	2008Q1	59417066	JPM	Depositories
2007Q1	15214029	RF	Deposito	2008Q1	50391158	BAC	Depositories
2007Q1	10549943	HIG	Insuranc	2008Q1	16439244	HIG	Insurance
2007Q1	5005940.2	MET	Insuranc	2008Q1	11862931	BSC	Broker-Dealers
2007Q1	3665260	CIT	Others	2008Q1	11631203	CIT	Others
2007Q1	1905633.1	FITB	Others	2008Q1	11301450	PRU	Insurance

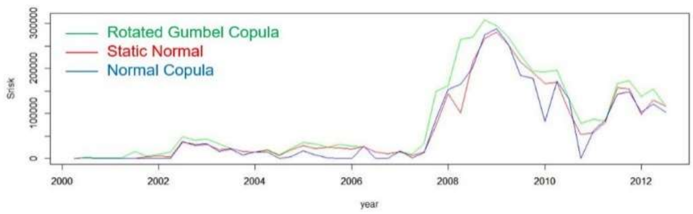
Calendar	SRISK	firm_name:	group	Calendar_	SRISK	firm_name:	group
2009Q1	167194680	BAC	Deposito	2010Q1	500189897	BAC	Depositories
2009Q1	120705408	JPM	Deposito	2010Q1	269205742	FNM	Others
2009Q1	76185103	FRE	Others	2010Q1	194349997	WFC	Depositories
2009Q1	75562268	WFC	Deposito	2010Q1	192798330	FRE	Others
2009Q1	75043431	FNM	Others	2010Q1	192662784	JPM	Depositories
2009Q1	46357209	GS	Broker-D	2010Q1	166003625	BK	Depositories
2009Q1	31689643	MET	Insuranc	2010Q1	115656435	STI	Depositories
2009Q1	30849706	PRU	Insuranc	2010Q1	115503263	CME	Others
2009Q1	21904754	HIG	Insuranc	2010Q1	106078820	COF	Others
2009Q1	11483590	STI	Deposito	2010Q1	105257353	AMTD	Others
Calendar	SRISK	firm_name:	group	Calendar_	SRISK	firm_name:	group
2011Q1	100962772	BAC	Deposito	2012Q1	552005929	BAC	Depositories
2011Q1	47850093	JPM	Deposito	2012Q1	299514790	JPM	Depositories
2011Q1	42955187	PRU	Insuranc	2012Q1	186707803	MET	Insurance
2011Q1	42593704	MET	Insuranc	2012Q1	173953053	GS	Broker-Dealers
2011Q1	35889373	HIG	Insuranc	2012Q1	173438214	WFC	Depositories
2011Q1	14415322	GS	Broker-D	2012Q1	122257593	UNH	Insurance
2011Q1	12386860	BK	Deposito	2012Q1	68609194	AFL	Insurance
2011Q1	10079472	COF	Others	2012Q1	60047706	FITB	Others
2011Q1	9933683.2	STI	Deposito	2012Q1	57458273	USB	Depositories
2011Q1	9052545.7	AFL	Insuranc	2012Q1	49920831	BK	Depositories

As we can see from the comparison, except the firm selection difference, it performs consistently based on different algorithms, and we find the group contribution has a significant time trend.



Depositories group experience a fast increase in risk contribution, while Others decrease sharply for less contribution by FNM and FRE. Insurance and Broker-Dealers groups performs steadily and constantly, however, due to the small group size, Broker-Dealers group actually stays as a more significant part of the systematic risk estimation.

3.3 SRISK Results by Copula



As above graph, SRISKs from normal copula and static normal are smaller than SRISK from Dynamic Rotated Gumbel copula. It may because normal distribution cannot capture heavy tail well. The shapes of the plots are similar.

Pr(<F)	PREDICTOR		
Dependent	Static-Normal	Static-Normal	Rotated-Gumbel
Static-Normal		0.7003	4.648e-05 (***)
Normal-Copula	0.2235		0.03947 (*)
Rotated-Gumbel	0.4425	0.6902	

Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

As Granger causality test result in above table, only SRISK by Rotated Gumbel can be significant predictor of the other two, so it is most timely.

4. Conclusion

In our project, we aim to build a measure to assess a financial firm’s contribution to systematic risk level. Due to sample and technique limitations, we however obtain somewhat different result compared to the thesis, which may owe to the assumption relaxation for thesis model in the aspect of firm selection, sequential time period, model specification and LRMES simulation. GARCH-DCC model can capture some useful information through SRISK and SRISK% rankings, while copula models perform more consistently with thesis results. We also make our new contribution to identifying the group contribution to whole systematic risk.

5. Reference:

[1] Lehar, A. 2005. Measuring systemic risk: A risk management approach. Journal of Banking & Finance 29:2577–603.

[2] Patton, Andrew J. 2001. Modelling Time-Varying Exchange Rate Dependence Using the Conditional Copula.

[3] Jordà, O., M. Schularick, and A. M. Taylor 2013. When credit bites back. Journal of Money, Credit and Banking 45:3–28.

[4] Christian Brownlees and Robert F. Engle 2017. SRISK: A Conditional Capital Shortfall Measure of Systemic Risk, The Review of Financial Studies, Volume 30, Issue 1, 1 January 2017, Pages 48–79.