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HW1

1.

a) $\left(\frac{7}{8}\right)^{16} = 0.1181$

b) $\binom{16}{1}\frac{1}{8} \cdot \binom{16}{15}\frac{7}{8} = 0.2699$

c) $\binom{16}{2}\frac{1}{8} \cdot \binom{16}{14}\left(\frac{7}{8}\right) = 0.2891$

d) $1 - \left[\left(\frac{7}{8}\right)^{16} + \binom{16}{1}\frac{1}{8} \cdot \binom{16}{15}\frac{7}{8} + \binom{16}{2}\frac{1}{8} \cdot \binom{16}{14}\left(\frac{7}{8}\right) = 0.3229\right.$

2 a) $Z_1 = \frac{135-145}{22} = -0.4545$

$Z_2 = \frac{155-145}{22} = 0.4545$

According to the normal distribution table

$A_1 = 0.3264$

$A_2 = 0.6736$

$P = A_2 - A_1 = 34.72\%$

b) $t = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$ $\quad Z_1 = \frac{155-145}{\frac{22}{\sqrt{16}}} = 1.82$ $\quad A_1 = 0.9656$

$Z_2 = \frac{135-145}{\frac{22}{\sqrt{16}}} = -1.82$ $\quad A_2 = 0.0344$

$P = 0.9656 - 0.0344 = 93.12\%$

c) $Z_1 = \frac{155-145}{\frac{22}{\sqrt{32}}} = 2.5712$ $\quad A_1 = 0.9949$

$Z_2 = \frac{135-145}{\frac{22}{\sqrt{32}}} = -2.5712$ $\quad A_2 = 0.0051$

$P = 0.9949 - 0.0051 = 98.98\%$

3. Let $\cancel{R}H$ = Right Hand

F = Right foot

$$P(H) = \frac{2012+142}{2391} = \frac{718}{797}$$

$$P(F) = \frac{2012+121}{2391} = \frac{711}{797}$$

$$P(H \cap F) = \frac{2012}{2391} = \frac{2012}{2391} \approx 0.84$$

$$P(H) \cdot P(F) = \frac{718}{797} \cdot \frac{711}{797} \approx 0.80.$$

So, they are not independent

Exercize 2.1

a. $S = \{(B,B), \underline{(B,G)}, \underline{(G,B)}\}$ * He has at least a boy

so, $P = \frac{2}{3}$

b. $S = \{B, \underline{G}\}$ * The other child is either boy or girl.

so, $P = \frac{1}{2}$

2.2

a. G = defendant is guilty

B = defendant has the crime blood type

so, $P(G|B)$ is the defendant is guilty with crime blood type.

$P(G|B) = \frac{P(G \cap B)}{P(B)}$ which is way smaller than

$P(G) = 99\%$

so the claim is logically ~~[illegible]~~ incorrect

b. The 8000 people with the blood type are not equally likely ~~to~~ in this crime, so it is incorrect to say he is 1/8000 chance to do it.

2.4.

Let P = Your test is positive

D = You actually have the disease

Given $P(D) = \frac{1}{10000}$

$$P(P|D) = \frac{99}{100} = \frac{P(D \cap P)}{P(D)}$$

then

$$P(D|P) = \frac{P(D \cap T)}{P(P)}$$

$$= \frac{P(P|D) \cdot P(D)}{P(P) = P(T \cap D) + P(T \cap \bar{D})} \leftarrow \text{known}$$

$$= \frac{P(T|D) \cdot P(D)}{P(T|D)P(D) + P(T|\bar{D})P(\bar{D})}$$

$$= \frac{\frac{99}{100} \cdot \frac{1}{10000}}{\frac{99}{100} \cdot \frac{1}{10000} + \frac{1}{100} \cdot \frac{9999}{10000}} = 0.98\%$$

EX. 2.5

Let A = the prize is behind door 1

B = " " " " " " " 2

C = " " " " " " " 3

H = the host opens door 3

$P(A) = P(B) = P(C) = \frac{1}{3}$

$P(H|A) = \frac{1}{2}$, $P(H|B) = 1$, $P(H|C) = 0$

$$P(A|H) = \frac{P(H|A)P(A)}{P(H)}$$

$$= \frac{P(H|A)\cdot P(A)}{P(H|A)\cdot P(A) + P(H|B)\cdot P(B) + P(H|C)\cdot P(C)}$$

$$= \frac{\frac{1}{3}\cdot\frac{1}{2}}{\frac{1}{2}\cdot\frac{1}{3} + \frac{1}{3}\cdot 1 + 0}$$

~~So~~ $= \frac{1}{3}$

so, there is $\frac{2}{3}$ chance at door 2, we should change to it

Ex. 2.12

$$I(X,Y) = \Sigma_{x,y} \log \frac{P(x,y)}{P(x)P(y)}$$

$$H(X) = -\sum_{x=1}^{K} P(X=k) \log P(X=k)$$

$$= -\Sigma_x P(x) \log P(x)$$

$$H(X|Y) = \Sigma_y P(y) H(X|Y=y)$$

$$= \Sigma_y P(y)\left(-\Sigma_x P(x) \log P(x) \mid Y=y\right)$$

$$= -\Sigma_{y,x} P(x,y) \log \frac{P(x,y)}{P(y)}$$

$$H(X) - H(X|Y) = -\Sigma_x P(x) \log P(x) + \Sigma_{y,x} P(x,y) \log \frac{P(x,y)}{P(y)}$$

$$= -\Sigma_y P(y) \Sigma_x P(x) \log P(x) + \Sigma_{yx} P(x,y) \log \frac{P(x,y)}{P(y)}$$

$$= -\Sigma_{yx} P(x,y) \log P(x) + \Sigma_y \Sigma_x P(x,y) \log \frac{P(x,y)}{P(y)}$$

$$= \Sigma_{x,y} \cancel{\log}\, P(x,y) \log \frac{P(x,y)}{P(x)P(y)}$$

$$= I(x,y)$$