1. 认为精细结构常数随时间演化，若采取runaway dilaton模型：

$$\frac{\Delta\alpha}{\alpha}(z) = -\gamma ln(1+z) \tag{1}$$

2. FRB河外色散量的理论值：

$$DM_{ext}^{th}(z) \equiv DM_{host}(z) + DM_{IGM}(z) \tag{2}$$

- 宿主星系：

$$DM_{host}(z) = \frac{DM_{host,0}}{(1+z)} f(\alpha, z)$$

若不考虑精细结构常数，$f(\alpha, z) = 1$ (host 1)；考虑精细结构常数时，$f(\alpha, z) = -\gamma(1+z)+1$, (host 2)

- 星系间介质：

$$DM(z) = \int_0^z \frac{dz'}{H(z')} \frac{cn_e(z')}{(1+z')^2} (\frac{\Delta\alpha(z')}{\alpha_0} + 1)$$

这是考虑了精细结构常数，对这一部分的色散量的贡献是 $\frac{\Delta\alpha(z')}{\alpha_0} + 1$。经过推导，这一部分也可以写作$1 - \gamma ln(1+z)$

3. FRB的色散量自关联功率谱：

$$\begin{aligned}
C_\ell^{\mathrm{IGM,IGM}} &= \int \mathrm{d}z W_{\mathrm{DM,IGM}}^2(z) \frac{H(z)}{\chi^2(z)} b_{\mathrm{b}}^2 P_{\mathrm{m}} \left(\frac{\ell + 1/2}{\chi(z)}, z \right), \\
C_\ell^{\mathrm{IGM,host}} &= 2 \int \mathrm{d}z W_{\mathrm{DM,IGM}}(z) W_{\mathrm{DM,host}}(z) \frac{H(z)}{\chi^2(z)} \times b_{\mathrm{FRB}} b_{\mathrm{b}} P_{\mathrm{m}} \left(\frac{\ell + 1/2}{\chi(z)}, z \right), \\
C_\ell^{\mathrm{host,host}} &= \int \mathrm{d}z W_{\mathrm{DM,host}}^2(z) \frac{H(z)}{\chi^2(z)} b_{\mathrm{FRB}}^2 P_{\mathrm{m}} \left(\frac{\ell + 1/2}{\chi(z)}, z \right)
\end{aligned} \tag{3}$$

$$\begin{aligned}
W_{\mathrm{DM,IGM}}(z) &= \left(1 - \frac{1}{2}Y\right) f_{\mathrm{IGM}}(z) \frac{\bar{\rho}_{\mathrm{b},0}}{m_{\mathrm{p}}} \frac{(1+z)}{H(z)} \int_z^\infty n(z) \mathrm{d}z \times (1 - \gamma ln(1+z)) \\
W_{\mathrm{DM,host}}(z) &= \frac{\mathrm{DM_{host}}(z)}{(1+z)} n(z) f(\alpha, z),
\end{aligned} \tag{4}$$

4. FRB的色散量noise功率谱：

$$N_\ell^{\mathrm{DM}} = \sqrt{\frac{1}{(2\ell+1) f_{\mathrm{sky}}}} \left[C_\ell^{\mathrm{DM}} + N_\ell^{\mathrm{host}} \right] \tag{5}$$

与FRB的个数有关，假设FRB个数范围100-10^6，f_{sky}=0.8，宿主星系色散量弥散$\sigma_{\mathrm{host},0} = 30\ pc/cm^3$：

$$N_\ell^{\mathrm{host}} = 4\pi f_{\mathrm{sky}} \sigma_{\mathrm{host}}^2 / N$$

$$\sigma_{host} = \sigma_{host,0} \int_0^z 1 - \gamma ln(1+z) dz$$

5. 信噪比

$$\mathrm{S/N} \equiv \sum_{\ell=2}^{\ell_{\max}} C_\ell^{\mathrm{DM}} / N_\ell^{\mathrm{DM}}$$

Likelihood:

$$\chi^2 = \left(\hat{C}_\ell^{\mathrm{DM,obs}} - C_\ell^{\mathrm{DM,th}}\right) \delta_{\ell,\ell'} \left(N_\ell^{\mathrm{DM}}\right)^2 \left(\hat{C}_{\ell'}^{\mathrm{DM,obbs}} - C_{\ell'}^{\mathrm{DM,th}}\right)^{\mathrm{T}} \tag{6}$$

参数：$\gamma, f_{IGM}, DM_{host,0}$