??
$\epsilon > 0$

$$\Pr[U_S(F) > \epsilon] = \Pr[\exists f \in F, A_S(f) - A_D(f) > \epsilon] \le \sum_{f \in F} \Pr[A_S(f) - A_D(f) > \epsilon] = \sum_{f \in F} \Pr[\frac{1}{k}\sum_{i=1}^{k} f(s_i) - A_D(f) > \epsilon].$$

?

$$\Pr[\frac{1}{k}\sum_{i=1}^{k} f(s_i) - A_D(f) > \epsilon] \le \exp\left(-\frac{2k\epsilon^2}{m^2}\right).$$

$$\Pr[U_S(F) > \epsilon] \le |F| \exp\left(-\frac{2k\epsilon^2}{m^2}\right).$$

$$\Pr[\sup_{f \in F}[A_D(f) - A_S(f)] > \epsilon] \le |F| \exp\left(-\frac{2k\epsilon^2}{m^2}\right).$$

$$\Pr[\sup_{f \in F}|A_S(f) - A_D(f)| > \epsilon] \le 2|F| \exp\left(-\frac{2k\epsilon^2}{m^2}\right).$$

$\delta > 0$
$1 - \delta$

$$\sup_{f \in F}|A_S(f) - A_D(f)| \le m\sqrt{\frac{\log(2|F|) + \log(1/\delta)}{2k}}.$$

??
??
??
?
s_1
s_2
$\ldots$
s_k
D
$f : D^k \to [0, +\infty]$
c
$g : D^{k-1} \to R$
s_1
$\ldots$
s_{j-1}
s_{j+1}
$\ldots$
$s_k \in D$

$$0 \le f(s_1, \cdots, s_k) - g(s_1, \cdots, s_{j-1}, s_{j+1}, \cdots, s_k) \le c,$$

$$\sum_{j=1}^{k}[f(s_1, \cdots, s_k) - g(s_1, \cdots, s_{j-1}, s_{j+1}, \cdots, s_k)] \le f(s_1, \cdots, s_k).$$

?
?
$Z = f(s_1, \cdots, s_k)$
c
$t \le Z$

$$\Pr[Z - Z \ge t] \le \exp\left(-\frac{t^2}{2cZ}\right).$$

$t > Z$
s_1
s_2
$\ldots$
s_k

$$\Pr[R_S(F) \ge R_S(F) + t] \le \exp\left(-\frac{kt^2}{4mR_S(F)}\right),$$

s_1
s_2
$\ldots$
s_k
$R_S(F)$
$c = 2m/k$

$$Z = R_S(F) =_\sigma \sup_{f \in F}\left[\frac{2}{k}\sum_{i=1}^{k} \sigma_i f(s_i)\right],$$