Solutions to 1.2 Exercises

Shiyu Wang(2022141500089)

September 8, 2025

1 exercise 1.2

Exercise 1: **Problem:**
Prove that for any constant C, the function $x(t) = Ce^{-3t} + 2t + 1$ is a solution of the ODE:

$$\frac{dx}{dt} + 3x = 6t + 5$$

and find the solution satisfying the initial condition $x(0) = 3$.

Solution:
First, we compute the derivative of $x(t)$:

$$\frac{dx}{dt} = -3Ce^{-3t} + 2.$$

Now, we substitute $x(t)$ and $\frac{dx}{dt}$ into the left-hand side of the ODE:

$$\frac{dx}{dt} + 3x = (-3Ce^{-3t} + 2) + 3(Ce^{-3t} + 2t + 1).$$

Simplifying this expression, we get:

$$-3Ce^{-3t} + 2 + 3Ce^{-3t} + 6t + 3 = 6t + 5.$$

Thus, the left-hand side equals the right-hand side of the ODE, confirming that $x(t)$ is indeed a solution.

Next, we apply the initial condition $x(0) = 3$:

$$x(0) = Ce^0 + 2(0) + 1 = C + 1 = 3.$$

Solving for C, we find $C = 2$. Therefore, the specific solution satisfying the initial condition is:

$$x(t) = 2e^{-3t} + 2t + 1.$$

Exercise 2: **Problem:**

Solution: