3. Portfolio Sorting

Contents

1 Learning objective:

- **Reading list:** Menkhoff, Sarno, Schmeling, and Schrimpf (2011), Lecture notes
- Knowledge and understanding of: Portfolio sorting based on characteristics and covariances, the construction of zero-cost risk factors, and their implications for market efficiency
- Discuss and conduct a portfolio sort using individual assets, evaluate the resulting portfolio returns, construct long-short factors, evaluate their returns, and reflect on the implications

2 Suggested presentation 10 min

1. Empirical

Part I
Short version

3 Portfolio sorting & risk factors

3.1 Introduction

- Portfolio sorting key ingredient of many asset pricing studies
- Strengths:
  - No a-priori assumptions
  - Can help in discovering new relationships, both linear and non-linear
  - High flexibility
- Drawbacks
  - High flexibility makes data snooping easy
  - Leads to an abundance of proposed factors which might not hold out-of-sample

3.2 Univariate sort

Starting point with one sorting factor.
Calculation in multiple steps:

1. Compute breakpoints, $\mathcal{B}_{k,t}$
   (a) Percentiles
   (b) Flexibility in: Number of breakpoints, percentiles used
2. Portfolio formation:
   (a) $P_{k,t} = \{i | \mathcal{B}_{k-1,t} \leq F_{i,t} \leq \mathcal{B}_{k,t}\}$
3. Compute returns
   (a) Equal weighted
      i. Average of returns
      ii. Equal weights on all stocks
   (b) Value weighted
      i. Returns weighted to market value
      ii. Downplays the importance of small and illiquid stocks
      iii. Vastly increases performance with stocks

iv. $r_{k,t} = \frac{\sum_{i=1}^{N_{k,t}} ME_{i,t-1} * r_{i,t}}{\sum_{i=1}^{N_{k,t}} ME_{i,t-1}}$

4. Examine returns
   (a) Goal: Identify reliable cross-sectional relation between sorting variable and future asset returns
   (b) Compute descriptive statistics portfolio excess returns and long-short portfolio
      i. Look for monotonic relations in average returns
      ii. Can identify relationships - even nonlinear ones!
   (c) Control for otehr risk factors

3.3 Bivariate sorts

3.3.1 Independent

- Now sort on two factors with each their breakpoints ⇒ exaggerating researchers' flexibility
- Portfolios formed in same way as before, but from the intersection between sorting variable conditions
- $P_{k,j,t} = \left\{i | \mathcal{B}^1_{k-1,t} \leq F^1_{i,t} \leq \mathcal{B}^1_{k,t}\right\} \cap \left\{i | \mathcal{B}^2_{j-1,t} \leq F^2_{i,t} \leq \mathcal{B}^2_{j,t}\right\}$
- Now we effectively get a matrix of portfolios where the ordering doesn't matter

3.3.2 Dependent

- First factor becomes control variable, breakpoints for second factor calculated within first factor portfolios
- Amend equation: $P_{k,j,t} = \left\{i | \mathcal{B}^1_{k-1,t} \leq F^1_{i,t} \leq \mathcal{B}^1_{k,t}\right\} \cap \left\{i | \mathcal{B}^2_{k,j-1,t} \leq F^2_{i,t} \leq \mathcal{B}^2_{k,j,t}\right\}$
- Ordering is now important!

3.4 Ang et al

Example of use ⇒ Ang et al
Uses portfolio sorting within risk to find two interesting results:

- Stock with high loadings to innovations in market volatility have low average returns
  - Makes economic sense, as these are hedge assets
- Idiosyncratic volatility negatively predicts returns
  - Puzzling as you'd expect a premium for taking on this risk
  - Low risk anomaly

3.5 Factor zoo

- High flexibility also allows for a lot of leeway from researchers, which can be good an bad
- Indications when lookaing at t-statistics that there seems to be some kind of p-hacking or publication bias going on
- Two sides, claiming that factors are just p-hacks and statistical bias, or they are real, motivated and indications of mispricings

3.5.1 MacLean & Pontiff

Three periods

1. In-sample
2. Out of sample
3. Post publication

Three interpretations:

1. Statistical bias $\Rightarrow$ excess return disappears OOS
2. Risk $\Rightarrow$ excess returns same in all periods $\Rightarrow$ compensation for risk
3. Mispricing $\Rightarrow$ excess returns disappear/diminish post-publication

Main finding:
Mispricing

3.5.2 Linnainmaa & Roberts

Three periods

1. Pre-sample
2. In sample
3. Out of sample

Three interpretations:

1. Data snooping $\Rightarrow$ only in-sample
2. Risk $\Rightarrow$ all periods
3. Mispricing $\Rightarrow$ pre and in-sample

Part II
Long version

4 Introduction

5 Portfolio sorting & risk factors

5.1 Introduction

Portfolio sorting is a key ingredient of many asset pricing studies, as portfolios provide a way to examine the effect of various risk factors. The method has several advantages, such as not requiring any a priori assumptions about cross-sectional relationships between sorting variables and expected returns, it is highly flexible, and it is possible to discovery new relations, both linear and nonlinear, by the use. At the same time, the methods have severe drawbacks in terms of limitations in the number of portfolios that can be worked with, and the high flexibility allows for data-snooping and similar which has led to an abundance of papers on various proposed factors.

- Portfolios is applied for exploring and identifying cross-sectional relationships between asset characteristics and future returns.
- Constructing portfolios based on one or more sorting variables believed to have information about predictable variation in the cross-section of future return.
- Allows data to speak for itself.
- Drawbacks is that the researcher can only control for a limited set of control when examining a cross-sectional relationship.
- A lot of choices is left for the researcher which can give rise to concerns about data snooping.
- A sorting can be a characteristic such as size or value, but can also be economically motivated risk factor such as macroeconomic risk.

5.2 Univariate portfolio sort

A univariate portfolio sort only considers a single sorting variable. This is the most simple portfolio sorting method.

We consider a sorting varible $F_{i,t}$for security i at time t, where we wish to study the cross-sectional relationship between the factor $F_{i,t}$ and the future returns to individiual assets.

5.2.1 Step 1: Calculate breakpoints

Use the following formula

$$\mathcal{B}_{k,t} = \text{Percentile}_{p_k}\left(F_{i,t}\right)$$

where p_k denotes the k'th percentile of the values of $F_{i,t}$ across all available assets and $\mathcal{B}$ is the k'th breakpoint.

The researcher need to make economically motivated and well-argued choices for the breakpoints. Fama and French (1993) uses the $30th$ and $70th$ percentiles when constructing size and value factors.

5.2.2 Step 2: Portfolio formation

We can identify all securities i that belong to the k'th portfolio at time t as the set of securities with values of $F_{i,t}$ that satisfy the relation

$$P_{k,t} = \{i|\mathcal{B}_{k-1,t} \leq F_{i,t} \leq \mathcal{B}_{k,t}\}$$

Using the chosen percentile, the universe of assets are divided into the different portfolios.

5.2.3 Step 3: Computing portfolio returns

When having allocated the assets into their respective portfolios, we need to construct the portfolio returns. We often distinguish between portfolios with equal-weighted and portfolios with value-weighted returns.
Equal weighted:

$$r_{k,t} = \frac{1}{N_{k,t}} \sum_{i=1}^{N_{k,t}} r_{i,t}$$

Value weighted:

$$r_{k,t} = \frac{\sum_{i=1}^{N_{k,t}} ME_{i,t-1} * r_{i,t}}{\sum_{i=1}^{N_{k,t}} ME_{i,t-1}}$$

Value weighting can be beneficial as it avoids assigning too large weights to small and illiquid assets that are hard and expensive to trade. The use of value-weighted returns in empirical analyses are often viewed as more representative of the returns that an investor would have realized by implementing a given strategy.

Finally, it is common to compute returns of a zero-cost long-short portfolio that takes a long position in the last portfolio and a short position in the first portfolio (spread portfolio). The average return to the traded long-short factor equals its risk price in a cross-sectional asset pricing exercise if the factor is priced.

5.2.4 Step 4: Examining portfolio returns

We want to determine whether there is a reliable cross-sectional relation between the sorting variable and future asset returns.
To do this,

- Compute descriptive statistics for the portfolio excess returns and the long-short portfolio (mean portfolio returns, std. deviations, skewness and so on).
- Test whether portfolio returns are significantly different from zero.
- Look for monotonic relationships in the average returns between the first and the last portfolio. Patton and Timmermann (2010) provide a test to test for monotonicity in portfolio returns/factor exposure.
- Last, we examine whether the observed patterns survive when controlling for known risk factors from the asset pricing literature such as size, value and momentum.

Preferably, we'd also like to test whether the return pattern survives when controlling for other known risk factors from the asset pricing litterature, but it can be troublesome to get all this information.

5.3 Bivariate portfolio sorts

Simple extension where portfolios are based on two sorting variables instead of one, as we commonly see in popular asset pricing literature. The only difference is in how we construct the breakpoints and thereby the portfolios, with two possibilities: Independent and dependent double sorts.

5.3.1 Independent double sort

The independent double sort builds portfolios by sorting on two variables independently. The independence of the sorts implies that the ordering of the sorting variables is inconsequential. It makes no difference which variable is considered the first and which is considered the second.

We create n_{p_1} groups based on the first variable and n_{p_2} groups based on the second variable to get a total of $n_{p_1} * n_{p_2}$ portfolios.
Breakpoints are defined almost as before with

$$\mathcal{B}^1_{k,t} = \text{Percentile}_{p_k}\left(F^1_{i,t}\right)$$

$$\mathcal{B}^2_{j,t} = \text{Percentile}_{p_j}\left(F^2_{i,t}\right)$$

And then one defines the portfolios as the intersection of the groups, based on the two sorting variables

$$P_{k,j,t} = \left\{ i | \mathcal{B}^1_{k-1,t} \leq F^1_{i,t} \leq \mathcal{B}^1_{k,t} \right\} \cap \left\{ i | \mathcal{B}^2_{j-1,t} \leq F^2_{i,t} \leq \mathcal{B}^2_{j,t} \right\}$$

A classic example is the 25 size and book-to-market-equity portfolios constructed in Fama and French (1992, 1993), where the number of stocks in the different portfolios varies greatly.

5.3.2 Dependent double sort

Here we use the first variable as a control variable, and the breakpoints for the second sorting variable are then formed within each group of the first sorting variable. We construct the first n_{p_1} groups as in the independent sort, and then define the breakpoint for the second sorting variable as:

$$\mathcal{B}^2_{k,j,t} = \text{Percentile}_{p_j} \left(F^2_{i,t} | \mathcal{B}^1_{k-1,t} \leq F^1_{i,t} \leq \mathcal{B}^1_{k,t} \right)$$

The portfolios are then again defined as the intersection of the conditional groups. Here it should be noted that the ordering of the sorting variables will now be critically important and can lead to variances in results if changed.

$$P_{k,j,t} = \left\{ i | \mathcal{B}^1_{k-1,t} \leq F^1_{i,t} \leq \mathcal{B}^1_{k,t} \right\} \cap \left\{ i | \mathcal{B}^2_{k,j-1,t} \leq F^2_{i,t} \leq \mathcal{B}^2_{k,j,t} \right\}$$

A dependent portfolio sort is therefore useful when the objective is to understand the relation between $F^2_{i,t}$ and future returns conditional on $F^1_{i,t}$.

5.4 Literature within portfolio sorting

Some interesting literature within this field is that of Ang et al who investigate the relation between idiosyncratic volatility and returns, and find two things:

1. That stocks with high sensitivities to innovations in aggregate volatility have low average returns, which is consistent with risk-based asset pricing, in terms of high loadings on market volatility serving as a hedge asset, since volatilities are high in bad times.
2. That idiosyncratic volatility negatively predicts returns, which is puzzling as one would expect a premium for taking on more "risk" through volatility. This "belongs" to the larger literature on low risk anomalies.

6 Empirical illustration

Momentum factor is constructed from independent double sort on size and momentum resulting in 6 portfolios (Carhart, 1997). The sorting is done by dividing into two size groups using the size median as breakpoint. There are formed 3 momentum groups using the $30th$ and the $70th$ percentile. The intersections provide us with six portfolios: Big Losers (BL), Big Neutrals (BN), Big Winners (BW), Small Losers (SL), Small Neutrals (SN), and Small Winners (SW). The momentum factor is then constructed from the six portfolios as

$$MOM = \frac{1}{2}[SW + BW] - \frac{1}{2}[SL + BL]$$

The factor is designed to capture the risk of being a winner relative to being a loser stock while keeping the average size the same.
Jegadeesh and Titman (1993) which introduced this buy-winners and sell-loosers strategy/factor also talked about market efficiency. Namely if the abnormal returns of the long-short portfolio is not explained by differences in systematic risk or exposures to other risk factors it is a sign of market inefficiency.

7 Currencies

- Strategies that borrow in low interest rate currencies and invest in high interest rate currencies are called "carry trades."
- According to uncovered interest parity (UIP), if investors are risk neutral and form expectations rationally, exchange rate changes will eliminate any gain arising from the differential in interest rates across countries.
- The idea of Menkhoff et al. (2012) is to perform a univariate sort of currencies based on interest rate and group them into quintile portfolios.

- From these a long-short portfolio or carry trade portfolio is constructed where we sell low interest rate currencies and buy high interest rate currencies.
- They find that the long-short portfolio delivers a significant positive return.
- Menkhoff et al. (2012) further examine whether global exchange volatility can explain the cross-section of portfolios sorted on interest rate.
- The findings of the paper is that high-interest rate currencies have a negative relation to aggregated FX vol and, hence, deliver a negative return due to increases in volatility.
- The profitability of the carry trade is, hence, compensation for risk.

7.0.1 McLean and Pontiff (2016)

Der ligger noter på denne artikel inde i readings-overleaf
Look a the behavior of returns over three periods to determine if cross-sectional return predictability persists post-publication.

1. The original study's sample period
2. Out-of-sample period, before publication
3. Post-publication period

With the following expectations:

1. Statistical bias $\Rightarrow$ predictability disappears out-of-sample.
2. Rational risk $\Rightarrow$ predictability is the same in-sample, OOS, and post-publication
3. Mispricing $\Rightarrow$ pricing should disappear or decay post-publication, dependent on whether it is costly to engage in arbitrage or not.

Main findings: mispricing, which investors learn about after publication, thus reducing the anomaly. Lucky for asset pricing.

$$
\begin{aligned}
\text{anomaly }_{it} &= a + b_1 \text{ in-sample index} \\
&\quad + \text{ post }_{it} \times \big(b_4 \text{ in-sample index } b_{-i,t} + b_5 \text{ post-sample index }_{-i,t} + b_3 \text{ post }_{it}
\end{aligned}
$$

7.0.2 Linnainmaa and Roberts (2018)

Der ligger noter på denne artikel inde i readings-overleaf
Similarly looks over three periods to determine if accounting-based anomalies persist out of sample.

1. In sample
2. Pre sample
3. Post-sample

Again distinguishing between three possible explanations:

1. Unmodeled risk $\Rightarrow$ Multidimensional stock risk, with misspecified models
2. Mispricing $\Rightarrow$ Investor irrationality and limits to arbitrage cause anomaly returns.
3. Data-snooping $\Rightarrow$ Artifacts of chance error

Their main conclusion is that most anomaly returns are decidedly an in-sample phenomenon, indicating that they are false and based on data-snooping. Does so by looking at a regression

7.0.3 Solutions?

Robustness checks, control factors (perhaps two-pass with LASSO).

7.1 Risk factors (factor zoo)

The ease of use and the high flexibility for the researchers in terms of portfolio sorting has led to an abundance of identified risk factors. The debate goes on whether these risk factors are real, which is argued for in terms of economical motivation and using robustness checks to state that these are mispricings, against a "darker" side claiming that factors are results of data-mining, p-hacking and chance results.
Looking at the distribution of t-stats we see that there might be indications of publication bias and p-hacking as t-statistics conveniently assemble at a point where findings are statistically significant. Several authors investigate these in different ways.

It should be noted that the financial literature points in both directions with regards to whether factors are true or false.

- False:
  - Hou et al. (2020): re-evaluate 452 anomalies and find that mitigating for microcaps using NYSE breakpoints and value-weighted returns leads to a failure rate of 65% using a 1.96 cutoff.
  - Tian (2020) runs a data-mining experiment. Findings suggest that the threshold of factor model success needs to be raised
- True:
  - Jacobs and Müller (2020) extend the work of McLean and Pontiff (2016) by investigating 241 anomalies in 39 countries. Their results similar point to mispricing rather than data mining.
  - Calluzzo et al. (2019) show that there is an increase in anomaly-related trading when information about the anomalies is readily available through academic publications.