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Financial frictions and investment: requiem in Q ☆

Russell Cooper [a,*] and João Ejarque [b]

[a] *Department of Economics, University of Texas at Austin, USA*
[b] *Institute of Economics, University of Copenhagen, Studiestraede 6, 1455 Copenhagen K, Denmark*

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Abstract

Understanding the nature of financial frictions faced by firms is relevant for both monetary and fiscal policy experiments. Empirical investment studies commonly find that proxies for firms' internal funds are significant as explanatory variables, particularly in the Q-theory based regression framework. These findings are often interpreted as evidence of financial frictions. This paper investigates that inference by specifying and estimating a class of dynamic optimization models where imperfectly competitive firms face financial constraints. Market power induces the principal link between investment and internal funds. We find no evidence to support the argument that capital market imperfections contribute to the relationship between investment and profitability.

1. Motivation

Leading models of investment behavior generate a linear relationship between a firm's investment rate and Tobin's Q, defined as the expected value of the firm relative to its capital stock. Empirical studies based upon this relationship typically find that investment is not very responsive to variations in Q and, in contrast to the prediction of the theory, proxies for firms' internal funds are significant as explanatory variables. This sensitivity of investment rates to internal funds is taken as a sign of capital market imperfections. But

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* Corresponding author.
E-mail addresses: rcooper@bu.edu (R. Cooper), joao.ejarque@econ.ku.dk (J. Ejarque).

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there are reasons to be cautious about this conclusion as there is no model with borrowing constraints which supports the inference drawn from the Q-theory based empirical results.[1]

In this paper, we ask whether the implications of models with financial frictions are consistent with the empirical evidence on investment based upon Q-theory.[2] We find that market power, *not* capital market imperfections, induces the principal link between investment and internal funds.

The presentation of this argument starts from a discussion of Q-theory investment regressions (hereafter, Q-regressions). We then restate the findings of Cooper and Ejarque (2001) using a dynamic optimization framework in which firms have market power but do not face any financial constraints.[3] Adding financial frictions to the model explored in Cooper and Ejarque (2001) allows us to assess the relative contributions of market power and financial frictions. Though the borrowing restrictions are imposed rather than derived from first principles, these specifications capture salient features of financial frictions.

We present evidence from two investment models with financial frictions that limit capital accumulation. In the first case, firms are required to finance all investment expenditures from current profit flows. This extreme setting bars firms from borrowing and also prohibits any retained earnings. The second case allows firms access to capital markets but at a cost. Here we analyze the implications of both fixed and marginal costs of capital market access, as in Gomes (2001) and Whited (2002). For these models, parameters of the firm's optimization problem are estimated using a simulated method of moments approach.

We evaluate these models in terms of their ability to match key characteristics of the data. The moments we highlight are chosen both for their prominence in the literature as well as their informativeness about the underlying structural parameters. Accordingly, the moments include Q-regression results partly because these coefficients have held center stage in the investment literature and partly because they capture the responsiveness of investment to both fundamentals and cash flow. We also use evidence on the average value of Q, the serial correlation of investment rates and the variance of profit rates to identify key parameters of the firm's optimization problem.

This methodology allows us to determine whether adding these financial frictions to a model with market power improves our ability to match observed investment behavior, as summarized by these moments.[4] Our specification allows for both market power and financial frictions. We find that adding financial frictions *does not* improve the fit of the model relative to a model with market power alone. This finding is complementary to

[1] Erickson and Whited (2000) and Gomes (2001) argue that the significance of internal funds in investment regressions may reflect measurement error. Further, Cooper and Ejarque (2001) argue that if firms have market power, then investment may depend on financial variables even in the absence of capital market imperfections. Abel and Eberly (2002) reach a similar conclusion in a model without adjustment costs. Gilchrist and Himmelberg (1998) present a model with both market power and financial frictions but do not make the complete link back to the Q-theory evidence.

[2] Our analysis does not consider results based upon Euler equation estimation. See Whited (1998) for a thorough review of that methodology and findings.

[3] Throughout, we rely on the results reported in Cooper and Ejarque (2001) as a starting point for the analysis. Those results are being extended during the revision of this paper.

[4] In particular, the specification without financial frictions explored in Cooper and Ejarque (2001) is unable to match observed serial correlation in investment rates.

that of Gomes (2001), though here the parameters are estimated using Q-regressions and additional moments so that the model matches relevant aspects of investment behavior.

2. Market power and Q-regressions: CE revisited

We start with a brief review of the approach and findings of Cooper and Ejarque (2001). This provides a framework for interpreting the Q-regression results. These findings set the stage for an evaluation of restrictions on firm borrowing.

2.1. Optimization problem

Cooper and Ejarque (2001) study the capital accumulation decision of a firm with market power.[5] The value function for the firm $V(K, A)$ solves

$$V(K,A) = \max_{K'} \pi(K,A) - p\big(K' - K(1-\delta)\big) - C(K',K) + \beta E_{A'|A} V(K',A') \quad (1)$$

for all (K, A), where K is the current capital stock at the firm and A is a measure of profitability. The function $\pi(K, A)$ is a reduced form profit function obtained from the firm's optimization over freely adjustable factors of production.[6] Thus the shock to the profit function, A, reflects variations in technology, input prices and demand. For simplicity, we assume that the price of new capital goods is constant and that the firm discounts at a constant rate. The function $C(K', K)$ represents the firm's costs of adjusting its capital stock.[7]

The firm chooses tomorrow's capital (K') using its conditional expectations of future profitability, A' and given the current capital stock. The policy function is denoted by $K' = h(K, A)$. It satisfies

$$C_{K'}(K',K) + p = \beta E_{A'|A} V_{K'}(K',A'), \quad (2)$$

where subscripts on the functions denote partial derivatives.[8] The right side of this expression is termed "marginal Q" and denoted by q. Using (1), this expression can be simplified to an Euler equation

$$C_{K'}(K',K) + p = \beta\big\{E_{A'|A}\pi_K(K',A') + p(1-\delta) - C_{K'}(K'',K')\big\}. \quad (3)$$

[5] Alternatively, as in (Gomes, 2001), one could consider decreasing returns to scale in the production function.

[6] Let $p = Sy^{-\eta}$ be the demand curve where S is a demand shock and η is the inverse of the demand elasticity. Let $y = \tilde{A}K^{\phi}l^{(1-\phi)}$ be the production function. Maximization of profit over the flexible factor, l, leads to a reduced form profit function, $\pi(K, A) = AK^{\alpha}$, where A includes shocks to productivity as well as variations in factor prices and in demand. The exponent on capital is $\phi(\eta - 1)/((1-\phi)(1-\eta) - 1)$.

[7] The exact nature of this adjustment cost function remains an active research topic, though, from plant-level observations, there is clearly a need for some form of adjustment cost to dampen the response of investment to fundamentals. This point is discussed in Cooper and Haltiwanger (2000). In contrast, neither Gomes (2001) nor Abel and Eberly (2002) has adjustment costs.

[8] The functions $\pi(K, A)$ and $C(K', K)$ are continuous and differentiable. The capital state space is bounded and β is less than one. So there will exist a solution to (1).

Hayashi (1982) assumes that $\pi(K, A)$ is proportional to K, and that the cost of adjustment function is quadratic:

$$C(K', K) = \frac{\gamma}{2}\left(\frac{K' - (1-\delta)K}{K}\right)^2 K. \tag{4}$$

With this specification, $V(K, A)$ is proportional to K so that marginal Q equals $V(K, A)/K$, a term that is called "average Q" and denoted as $\bar{q}(A)$.[9] Using (4) in (2) along with the result that marginal and average Q are equal implies

$$\frac{I}{K} = \frac{1}{\gamma}\left[\beta E_{A'|A}\bar{q}(A') - p\right], \tag{5}$$

where $E_{A'|A}\bar{q}(A')$ is the expected value of average Q in the next period and I is gross investment in the current period. Theory thus predicts a very specific investment equation for the Q-theory models: the investment rate depends only on the discounted expected value of average Q.[10]

2.2. Q-theory based estimation results

Letting it denote the period t observation for firm i, tests of Q-theory on panel data are frequently conducted using an empirical specification along the lines of

$$(I_{it}/K_{it}) = a_{i0} + a_1 E\bar{q}_{it+1} + a_2(\pi_{it}/K_{it}) + \varepsilon_{it}, \tag{6}$$

where (I_{it}/K_{it}) and (π_{it}/K_{it}) are the investment and profit rates at firm i in period t, respectively.[11] This empirical specification is different from (5) as it includes both a fixed effect, a_{i0}, and an error term, ε_{it}. These are introduced into the theory model by generalizing the quadratic cost of adjustment function to include a fixed effect and a stochastic element:

$$C_i(K', K, \varepsilon) = \frac{\gamma}{2}\left(\frac{K' - (1-\delta)K}{K} - a_{i0} - \varepsilon\right)^2 K. \tag{7}$$

Alternatively, the error term in (6) could be interpreted as measurement error and this is the approach taken here as we pursue the implications of misspecification caused by the substitution of average for marginal Q.[12]

Q-theory has two important implications for the regression model. First, from (5), a_1, the coefficient on expected average Q, estimates β/γ. Second, (π_{it}/K_{it}) is introduced into the empirical model as a test of the theory: Q-theory implies that the coefficient on the profit rate, a_2, should be zero.

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[9] Here $\bar{q}$ will depend on A. See the related discussion in (Lucas and Prescott, 1971) where the cost of adjustment is part of the capital transition equation: $k' = kz(i/k)$.

[10] There is a one-period delay associated with the delivery and installation of new capital. In some applications, new investment is assumed to be immediately productive so that the appropriate measure of average Q is the current one.

[11] Gilchrist and Himmelberg (1995) use the ratio of profits to capital as their measure of cash flow. Fazzari et al. (1988) use a measure of cash flow equal to income minus interest and taxes plus noncash deductions from income (depreciation allowances and amortization).

[12] Our model will not include a shock to the cost of adjustment.

Table 1
Q regression results and other moments

Study	a_1	a_2	mean ($\bar{q}$)	std($\bar{q}$)	std(I/K)	std(π/K)	sc(I/K)
GH95	0.03	0.242	2.95	2.28	0.132	0.257	0.4
$\text{FHP88}_{(\text{low})}$	0.0008	0.46	3.8	n.a.	0.17	0.2	n.a.
$\text{FHP88}_{(\text{high})}$	0.002	0.23	1.6	n.a.	0.06	0.06	n.a.
G01	0.06	0.14	1.56	n.a.	0.139	n.a.	0.239
CHH94	0.019	0.164	n.a.	n.a.	n.a.	n.a.	n.a.

Note. Gilchrist and Himmelberg (1995) (GH95), Tables 2 and 6, sc(I/K) from Charles Himmelberg; Fazzari et al. (1988) (FHP88), Table 4, Class 1 (low dividend); FHP88, Table 4, Class 3 (high dividend); Gomes (2001) (G01), Tables 3 and 6; Cummins et al. (1994) (CHH94), Table 3. I/K is the investment rate, $\bar{q}$ is average q, π/K is either the cash flow rate or profit rate, std(x) is the standard deviation of x, sc(x) is the serial correlation of x.

Table 1 provides a summary of the evidence from regressions based on (6). Here we report estimates of a_1 and a_2 as well as other moments from these studies.[13]

The results obtained from various studies based upon (6) have been troubling to the investment literature.[14] First, the estimates indicate that investment is relatively insensitive to variations in average Q. Hayashi (1982) reports $a_1 = 0.0423$. Abel and Blanchard (1986) obtain insignificant coefficients for contemporaneous average Q. Fazzari et al. (1988) obtain extremely low coefficients (for example, $a_1 = 0.0065$ in one of their specifications). Table 3 of Cummins et al. (1994) summarizes their findings for various Q-based regressions. Once again, the responsiveness of investment to variations in Q is quite small. Gilchrist and Himmelberg (1995) obtain an estimate for a_1 of 0.033.

The Q-theory model interprets this low value of a_1 as a relatively high value of the adjustment cost parameter since $a_1 = \beta/\gamma$ in that model. For example, using the Hayashi (1982) estimate, the marginal adjustment cost (evaluated at the steady state) is about 45% of the steady state capital stock.

Second, profits or other financial variables matter for investment: a_2 is significantly greater than zero in many studies. Fazzari et al. (1988) report significant effects of cash flow on investment for *all* types of firms though firms with higher dividend/income ratios have smaller cash flow coefficients. Cummins et al. (1994) find significant cash flow effects in their investment regressions as do Gilchrist and Himmelberg (1995).

The significance of a_2 also implies that interpreting a_1 as the inverse of γ is not appropriate. Once a_2 is significant, which contradicts the null of Q-theory, then using a_1 to infer adjustment costs is not valid.[15]

The common interpretation of these findings is that cash flow matters for investment due to the presence of financial frictions. In good times, profits are high and there is sufficient cash flow to finance investment purchases. To the extent that these constraints pertain more

[13] To be careful, there are differences over these studies with regard to the estimation of (6), particularly with respect to OLS vs. GMM. Cummins et al. (1994) compare these methods of estimation in their Table 3 and report that a_1 varies from 0.02 to 0.06 while a_2 varies between 0.15 and 0.35.

[14] In fact, the view that these models “fail empirically” is commonly held. See Erickson and Whited (2000) and Gomes (2001) for recent critiques of this literature.

[15] This point is taken up in more detail in Cooper and Ejarque (2001).

to small than large firms, the sensitivity of investment to cash flow will be higher for that set of firms.

There are important implications of these results for monetary and fiscal policy. Bernanke and Gertler (1995) argue that investment is relatively insensitive to variations in the cost of capital so that the conventional effects of monetary policy, acting through interest rate movements, are not large. But, the credit channel, reflecting frictions in capital markets, provides a source of amplification and propagation of monetary policy. This logic corresponds neatly with the estimates of a_1 and a_2: large adjustment costs imply little sensitivity of investment to average Q and investment depends on financial factors. A parallel point can be made for fiscal interventions: the effects of variations in tax rates may impact investment through variations in the user cost of capital or through cash flow.[16] Based on the estimates of (6), the cash-flow effects may be substantial.

2.3. An alternative interpretation: the unconstrained case

Cooper and Ejarque (2001) provide an alternative perspective on these findings.[17] They argue that the apparent failure of Q-theory may stem from misspecification of the firm's optimization problem as it ignores market power. Lindenberg and Ross (1981) argue that high values of average Q reflect market power. For the Gilchrist and Himmelberg (1995) sample, which we rely on extensively, average Q was about 3.

Suppose that the profit function is given by

$$\pi(K, A) = AK^{\alpha}. \tag{8}$$

Clearly $\alpha \neq 1$ violates the conditions for the equality of average and marginal Q. As a consequence, average and marginal Q diverge so that the use of $\bar{q}_{it}$ in the Q-regression induces measurement error which may be positively correlated with profits.[18]

Cooper and Ejarque (2001) study (1) along with (4) and (8). As the capital accumulation is not linked in any way to financial variables, we term this the *unconstrained* case.

Estimates of the parameters characterizing the technology and the stochastic process for the shocks are obtained following the methodology of indirect inference procedures described by Gourieroux and Monfort (1996) and Gourieroux et al. (1993). This is a version of simulated method of moments. The structural parameters, denoted Θ, are chosen to minimize the distance between moments generated by the data and those calculated from the simulated data. Specifically,

$$\min_{\Theta} J(\Theta) = \big(\Psi^d - \Psi^s(\Theta)\big)' W \big(\Psi^d - \Psi^s(\Theta)\big), \tag{9}$$

[16] See Cummins et al. (1994) for discussions of the implications of credit market frictions for the conduct of fiscal policy.

[17] This argument complements the measurement error perspective of Erickson and Whited (2000) and Gomes (2001).

[18] Cooper and Ejarque (2001) do not characterize this measurement error analytically but use a simulated environment to understand its implications. As the value function is strictly concave, Hayashi (1982) demonstrates that in this case marginal Q is always less than average Q.

where Ψ^d are moments calculated from the data, $\Psi^s(\Theta)$ are moments calculated from a simulation and W is a weighting matrix.

Cooper and Ejarque (2001) use the regression results, $a_1 = 0.03$ and $a_2 = 0.24$, of Gilchrist and Himmelberg (1995) as representative of the empirical investment literature based upon Q-theory. These two moments are supplemented by three other moments reported by Gilchrist and Himmelberg (1995): the serial correlation of investment rates (0.4), the standard deviation of profit rates (0.25) and the average value of average Q (3).[19] So, $\Psi^d = [0.03,\ 0.24,\ 0.4,\ 0.25,\ 3.0]$.

These moments were chosen for two reasons. First, the coefficients from the Q-regressions are center stage in many discussions of the empirical investment literature. In particular, the a_2 parameter captures the covariance between financial variables and real investment decisions, conditional on average Q. Further, the mean of average Q is informative about market power and the serial correlation of investment rates helps to determine the magnitude of adjustment costs: as γ increases, so does this serial correlation.

Second, these moments are quite informative for determining the structural parameters. In simulation, one can see that the moments are quite responsive to variations in the parameters.[20]

One important issue is unobserved heterogeneity.[21] Our model does not include any unobserved structural heterogeneity. Instead of introducing this into the model, we have chosen to eliminate this from the moments. This is achieved by looking at the Q-regressions with their fixed effects. As argued earlier, there is a structural counterpart to this fixed effect obtained by introducing a firm-specific constant in the cost of adjustment function. Still there is an issue here as the other moments we look at do not control for firm-specific effects. From our experimentation, it seems that only the serial correlation in investment rates is sensitive to the introduction of unobserved heterogeneity in the cost of adjustment function.[22]

Cooper and Ejarque (2001) set some parameters at levels assumed in Gilchrist and Himmelberg (1995): $\delta = 0.15$ and $\beta = 0.95$. The vector of parameters, $\Theta \equiv (\alpha, \gamma, \rho, \sigma)$ are left to be estimated where ρ, σ are the parameters of the stochastic process for the firm-specific shocks to A.[23]

The vector of simulated moments, $\Psi^s(\Theta)$, is obtained by the following algorithm:

(1) given Θ, solve (1) along with (4) and (8) using value function iteration;

[19] The average value of average Q and the standard deviation of the profit rate (measured as cash flow) come from Table 6 in Gilchrist and Himmelberg (1995). The serial correlation of the investment rate comes directly from Charles Himmelberg and we are grateful to him for supplying this calculation.

[20] This sensitivity underlies the low values of the standard errors reported in Cooper and Ejarque (2001).

[21] We are grateful to Sam Kortum for pushing this point.

[22] We used the parameter estimates for the unconstrained case and introduced a fixed effect as in (7).

[23] Cooper and Ejarque (2001) decompose A into common and firm-specific components and use the estimates of Cooper and Haltiwanger (2000) for the common component of A. The methods outlined in (Tauchen, 1986) are used to create a discrete state space representation of the process for any (ρ, σ), where ρ is the serial correlation of the shock and σ is the standard deviation of the innovation.

Table 2
(a) Structural parameter estimates by model

Model	Structural parameter estimates (Θ)			
	α	γ	ρ	σ
Unconstrained	0.699	0.1647	0.111	0.857
	(0.01)	(0.017)	(0.007)	(0.029)
Internal	0.6967	0.2307	0.1053	0.8382
	(0.01)	(0.02)	(0.01)	(0.03)

(b) Moments by model

Model	Reduced-form coefficients and moments					
	a_1	a_2	$\mathrm{sc}(I/K)$	$\mathrm{std}(\pi/K)$	$\bar{q}$	$J(\Theta)$
GH95	0.03	0.24	0.4	0.25	3.0	n.a.
Unconstrained	0.045	0.24	0.040	0.250	2.96	40.39
Internal	0.046	0.233	0.064	0.248	2.98	42.36

(2) the resulting policy functions are used to create a panel data set;[24]
(3) $\Psi^s(\Theta)$ is obtained from the panel.[25]

Using this procedure to construct $\Psi^s(\Theta)$, (9) is solved by simulation.

The second row of Table 2(a) presents the estimates of structural parameters and standard errors.[26] Table 2(b) presents the actual and simulated moments. In Table 2(b) and throughout, GH95 refers to Gilchrist and Himmelberg (1995), $\mathrm{sc}(I/K)$ indicates the serial correlation of the investment rate, $\mathrm{std}(\pi/K)$ indicates the standard deviation of the profit rate, and $\bar{q}$ denotes average Q.

The policy function for the estimated model is shown in Fig. 1 along with the firm's total resources, defined as profits plus undepreciated capital, available to the firm. These resources are sufficient to finance investment except for the extreme states in which both profitability shock and the capital stock are low. That is, $K'(A,K) \leqslant \pi(A,K) + (1-\delta)K$ except for low values of (A,K).

From Table 2(b), the model, with its four parameters, does a good job of matching four of the five moments though $J(\Theta)$ is large. A problem is the low level of predicted serial correlation in plant-level investment rates relative to the observed serial correlation.[27]

[24] As the moments from Gilchrist and Himmelberg (1995) come from a panel, those studied in here and in Cooper and Ejarque (2001) do as well, assuming 1000 firms and 50 years of data.

[25] Gilchrist and Himmelberg (1995) first differentiate their data and then use an IV procedure to estimate the parameters. We have no unobserved heterogeneity so that taking first differences is not necessary. Experimenting with different treatment of year effects is warranted.

[26] These parameters yield a slightly better fit to the moments than the ones reported in Cooper and Ejarque (2001) though qualitatively there are no substantial differences. The computation of standard errors follow the description in Chapter 4 of Gourieroux and Monfort (1996). The procedure for calculating W is discussed by Cooper and Ejarque (2001).

[27] This can be remedied by a larger value of γ. The W matrix puts little weight on matching the serial correlation of investment.

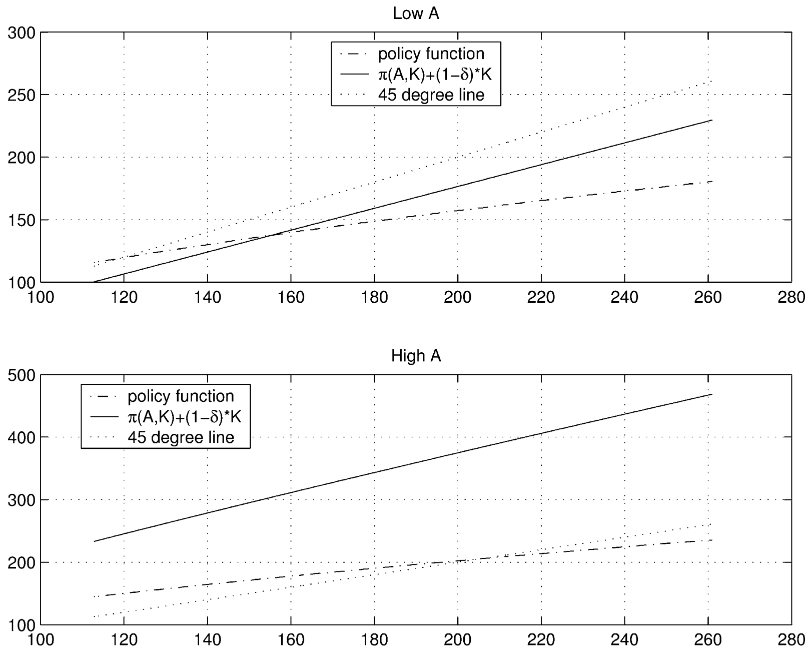

Fig. 1. Unconstrained policy functions.

The estimated structural parameters are not at odds with other studies. The estimated curvature of the profit function of 0.699 implies a markup of about 15%. Moreover, the estimate of γ is not identified from the regression coefficient on $\bar{q}$: $\gamma = 0.165$ is far from the inverse of the coefficient on $\bar{q}$ (about 4). Evidently, relatively small adjustment costs can lead to small values of a_1 and thus the *incorrect* inference of large adjustment costs.

The model with imperfect competition thus succeeds in reproducing two key facts from the Q-based investment literature: investment is relatively insensitive to variations in average Q but is quite responsive to variations in profit rates. Importantly, these results arise without introducing borrowing restrictions. Instead, the response of investment to cash flow indicates the additional information, relative to average Q, contained in this variable for forecasting future profitability. *Thus, Q-theory based investment regression results are not necessarily evidence of financial frictions and any inferences about large cash flow effects may be misplaced.*

Cooper and Ejarque (2001) also argue that it is possible to reproduce results on sample splits, small vs. large firms, using their approach. Interestingly, the key difference in parameters between small and large firms is the serial correlation of the shocks rather than differences in market power.[28]

[28] Other sample splits based upon the payment of dividends and other financial indicators are not possible in that model.

3. Costly external finance

Based in part on the significance of profit rates (or other measures of liquid assets) in investment regressions such as (6), investment is viewed as being sensitive to financial variables due to the presence of financial frictions. To study this formally, we consider a series of capital accumulation models with capital market restrictions. We study these models jointly with a strictly concave profit function.[29] In this way, our structure nests both the market power and capital market imperfection views. Thus, we can study whether the results of Cooper and Ejarque (2001) simply reflected the omission of borrowing restrictions.

Specifically, we ask whether models with these frictions can improve upon the ability of the model with market power to explain the results from the investment literature. In particular, does the introduction of these frictions help improve the ability of the model to fit some moments, such as the serial correlation of investment rates? Formally, does the value of $J(\Theta)$ in (9) fall when financial frictions are introduced into the model? Finally, does the introduction of financial frictions change the parameter estimates in any interesting ways?

3.1. Cash flow constrained investment: the internal finance case

Here we study the case in which investment must be financed out of current profit flows: $I \leqslant \pi(A, K)$.[30] We term this the *internal finance* case, an extreme alternative to the unconstrained case studied in the previous section.

A first question is whether this constraint ever binds. Given the market power, firms may have sufficient internal funds to finance their investment expenditures. Using the estimates reported for the unconstrained case, investment expenditures exceed profits in about 21% of the observations in a simulated sample, largely in the states where the realization of the profitability shock is low. In this simulation, about 80% of total investment was funded internally. Interestingly, this accords with observations: Gomes (2001) cites evidence that about 80% of all financing is internal and Fazzari et al. (1988) report that internal finance generates about 70% of the investment expenses of US manufacturing firms.

Table 2 presents our parameters estimates for the internal finance case (row labelled *internal*). The estimation routine followed the steps outlined above except that the constraint $I \leqslant \pi(A, K)$ was added to the dynamic optimization problem.[31]

Figure 2 illustrates the policy function for this case along with profits. Here we see that investment is constrained by profits, for small K, particularly for low A. Recall from Fig. 1 that investment exceeded profits in these states.

[29] Gilchrist and Himmelberg (1998) also consider a model with both market power and financial frictions and find evidence of both. Additionally there is evidence of both frictions from Euler equation studies, summarized by Whited (1998).

[30] In our model, $\pi(A, K)$ is a measure of cash flow: revenues minus the costs of inputs. We can also subtract adjustment costs. In practice, this difference is not important to our results.

[31] In effect, we solved (1) along with (4) and (8) with a large penalty if the internal financing constraint was violated.

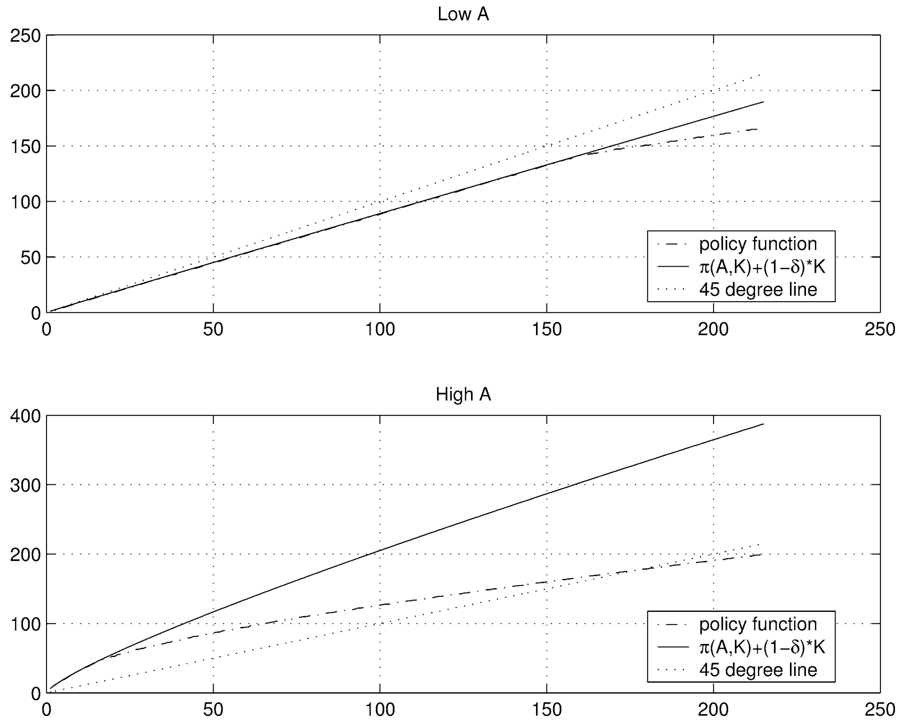

Fig. 2. Internal finance policy and profit functions.

Figure 3 illustrates the implications of the constraint by displaying policy functions for the unconstrained and internal finance cases. The policy function in the internal finance case lies below the unconstrained policy function. This partly reflects the binding constraint. It also appears that future capital is more sensitive to variations in (A, K) in the internal finance case.

The parameter estimates reported in Table 2 are quite close to those obtained from the unconstrained case. Market power remains important for these results. The estimation nests the case of $\alpha = 1$ but the model is better able to match the observations at a level of $\alpha < 1$. From Table 2, the model is tightly estimated and α is clearly below unity.[32]

The fit of the model, $J(\Theta)$, in the internal finance case exceeds that reported for the unconstrained case. So, introducing this constraint does not improve the ability of the model to match these particular moments.[33] There does seem to be some extra serial correlation in investment induced by the borrowing restrictions.

32 Though the precision of the approximation is reduced as α tends to 1.

33 The results for this case are sensitive to the approximation to the state space. The estimation was done using simulated annealing and a grid of 300 points. Once the estimation was completed, a single simulation with 1500 points was performed and those results are reported in the table. This includes the $J(\Theta)$ statistics. For the smaller state space, $J(\Theta)$ was slightly lower than that in the unconstrained case.

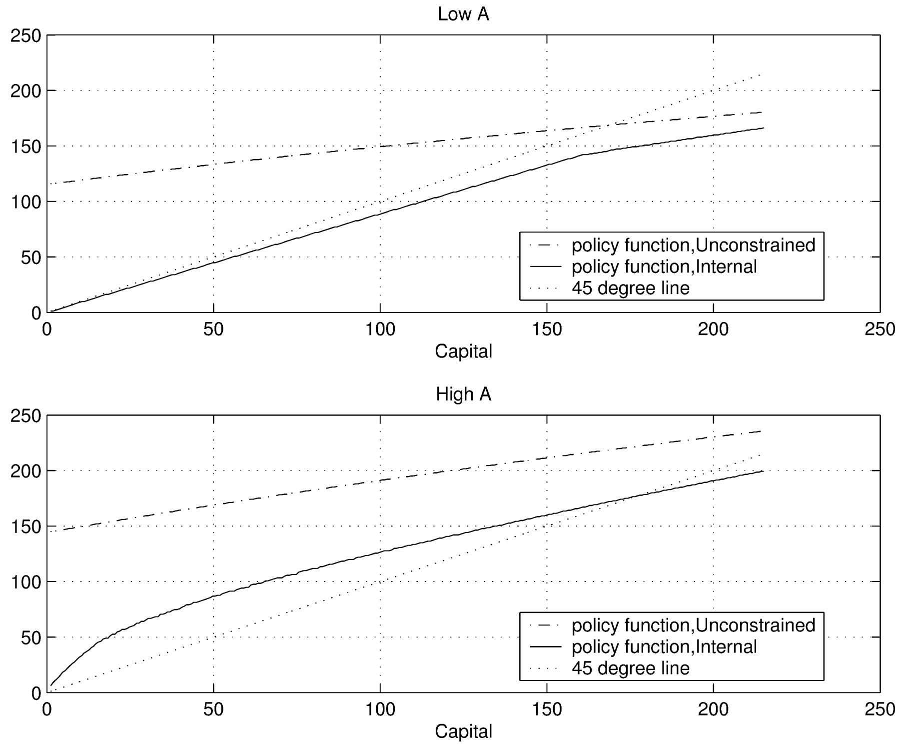

Fig. 3. Internal finance and unconstrained policy functions.

In fact, as a matter of theory, some non-homotheticity, such as that induced by market power, is necessary for a_2 to be significant in (6). If $\alpha = 1$, then marginal and average Q are equal even with the introduction of a restriction that $I \leqslant \pi(A, K)$. So, as noted previously by Chirinko (1993, 1997), average Q will completely summarize the return to investment even in the presence of financial frictions.[34] Or, as discussed below, there may be restrictions on investment that are not homothetic.[35]

One of the problems with this specification is that the firm is not allowed to borrow *and* is not allowed to lend either. Thus, the only way to "save" during periods of high profits is to accumulate capital. Yet, in the presence of adjustment costs, this may be inefficient. We return below to a model in which the firm can lend.

[34] Essentially K' is proportional to K when the constraint binds and when it does not. Therefore, $V(A, K)$ is proportional to K.

[35] This point is also relevant for the sample splits conducted by many authors. The fact that large and small firms behave differently implies that some type of non-constant returns to scale must be present in the model.

3.2. Costly external finance

Both the unconstrained and internal finance cases are extreme in that firms either have free access to capital markets or that access is prohibitively expensive. Here we study a more plausible model in which the access is costly but not prohibitive, as in (Gomes, 2001) and (Whited, 2002). Our analysis differs from these papers in that we will estimate the parameters of our dynamic optimization problem in this costly external finance environment. We term this the *costly* case.

Let $\phi(E) = \phi_0 + \phi_1 \times E$ be the cost of external finance where E measures the extent of external financing and is given by $E \equiv I - \pi(A, K)$. Gomes (2001) assumes $\phi(E) = 0.08 + 0.028 \times E$ and the fixed cost relative to the average capital stock is 0.1%. Whited (2002) assumes $\phi(e) = 0.04 + 0.0264 \times E$. This specification is intended to capture a variety of costs of going to financial markets to raise capital. This would include the fixed and variable costs of public stock offerings, costs of monitoring the firm and the discounted present value of any premia associated with external debt and equity finance.[36]

This specification is also promising from the perspective of matching observations on investment. The investment literature stresses non-linearities in the response of investment to fundamentals. Further, there is evidence of both bursts of investment as well as inactivity in capital adjustment. This is summarized by Cooper et al. (1999) at the plant level and by Whited (2002) for firms. Many researchers use this evidence as a basis for introducing non-convex adjustment costs and/or irreversibilities into the firm's (plant's) dynamic optimization problem. But, perhaps, these non-convexities actually reflect fixed costs of capital market participation. If so, this would be a useful step towards uncovering the source of lumpy investment activity.

Further, Barnett and Sakellaris (1999) and Abel and Eberly (1999) allow for investment rates to depend non-linearly on average Q. They find evidence of non-linearity: investment is less responsive to average Q for high values of average Q.[37] Perhaps this non-linearity is induced by the capital market frictions.

With this cost of capital market participation, the firm's dynamic optimization problem is more complicated. It includes a decision about whether to incur the cost of external financing along with a decision on investment. The optimization problem is

$$V(K, A) = \max\{V^e(K, A), V^i(K, A)\} \tag{10}$$

where

$$\begin{aligned} V^e(K, A) = \max_{K' > (\pi(K,A)+(1-\delta)K)} & \pi(K, A) - p\big(K' - K(1-\delta)\big) - C(K', K) - \phi_0 \\ & - \phi_1\big(p\big(K' - K(1-\delta)\big) - \pi(K, A)\big) \\ & + \beta E_{A'|A} V(K', A') \end{aligned} \tag{11}$$

[36] Undoubtedly a more complete model would distinguish these sources of external finance. On this point, see the discussion by Fazzari et al. (1988) and Tirole (2001).

[37] Barnett and Sakellaris (1999) also report a convex response for very low values of average Q.

Table 3

(a) Structural parameter estimates: costly external finance

Model	Structural parameter estimates (Θ)				
	α	γ	ρ	σ	$\tilde{\phi}_0$
Costly	0.6956 (0.01)	0.1331 (0.04)	0.0976 (0.02)	0.8932 (0.03)	0 (0.05)

(b) Moments by model

Model	Reduced-form coefficients and moments						
	a_1	a_2	sc(I/K)	std(π/K)	$\bar{q}$	Ext. frac.	$J(\Theta)$
Moments	0.03	0.24	0.4	0.25	3.0	0.25	n.a.
Costly	0.0388	0.2358	0.0158	0.2614	2.9413	0.2236	68.27

and

$$V^i(K,A) = \max_{K' \leqslant (\pi(K,A)+(1-\delta)K)} \pi(K,A) - p\big(K' - K(1-\delta)\big)C(K',K) \\ + \beta E_{A'|A} V(K',A') \tag{12}$$

for all (K,A). In this problem, the superscripts "e" and "i" refer to external and internal finance, respectively.

Table 3 reports the estimation results and standard errors for this case (row labelled *costly*), including an estimate of the fixed cost of capital market access relative to the mean capital stock, denoted $\tilde{\phi}_0$. The estimation focuses on $\tilde{\phi}_0$, setting $\phi_1 = 0$. We return to allowing $\phi_1 \neq 0$ below.

The estimation procedure is the same as that described in the unconstrained case.[38] As indicated in Table 3(b), we added a new moment for this estimation: the mean fraction of investment expenditures financed externally. This moment was taken from the discussion by Fazzari et al. (1988) and Gomes (2001) and set at 0.25.[39]

The parameter estimates indicate that once again market power plays an important role: the estimate of α is far below unity. *Strikingly, there seems to be no support for the presence of costs of external finance: the estimated value of* $\tilde{\phi}_0 = 0$. Given the presence of market power, there is apparently no gain in terms of matching moments from the model with capital market frictions.[40] The standard errors are relatively small indicating that the parameters are tightly estimated. By simulation, increases in $\tilde{\phi}_0$ to even 1% of the average capital stock reduced the serial correlation of investment rates and also reduced participation in capital markets. Thus, this variation clearly reduced the fit of the model.

[38] The estimation here is a bit trickier as a large state space is needed due to the non-convexity induced by the fixed cost. The space used for these estimates had 600 elements. Enlarging the state space to 800 did not have a large effect on the moments. The weighting matrix was computed using the same procedure as outlined by Cooper and Ejarque (2001). We start with a diagonal matrix to obtain initial structural parameter estimates. We then simulate a sequence of panels, calculate moments for each panel and use this to create a variance–covariance matrix. The estimation is then repeated using this new estimate of W.

[39] For the estimation, this moment is measured as the fraction of total investment which is financed externally.

[40] Given the estimate of $\tilde{\phi}_0 = 0$, the same optimization problem studied in Section 2.3 was estimated here. There is a modest difference in results as we have the additional moment of external financing.

In both cases, the estimated models succeed in matching all but the serial correlation of investment. It is not the case that the model is incapable of creating highly serially correlated investment. In fact, a high value of γ will suffice.[41]

The result of $\tilde{\phi}_0 = 0$ implies that financial market frictions are not the source of the lumpiness reported in the investment literature. Nor is this the source of investment nonlinearities described above. This supports the view that the nonlinearities in the capital adjustment process reflect non-convexities in adjustment costs at the plant level.[42]

We have extended the analysis to allow $\phi_1 \neq 0$. To emphasize a point made earlier, this is an additional constant cost of (externally financed) investment that would (we conjecture) be largely captured by average Q, particularly if $\alpha = 1$ and $\phi_0 = 0$. We estimated the model allowing $\phi_1 \neq 0$ but keeping other parameters at the estimated values reported in Table 3. Allowing $\phi_1 \in [0, 0.10]$ did not improve the fit of the model as measured by $J(\Theta)$.

4. Some extensions

In this section we briefly describe extensions of the analysis.[43] These extensions are partly motivated by missing elements in the theoretical model and also the empirical failings of the existing model, in particular the low predicted serial correlation of investment rates relative to observation.

4.1. Borrowing/lending

The above analysis is restrictive in that access to capital markets means only the ability to borrow to buy capital. This forces a link between capital market participation and investment. Yet firms may want to bunch borrowing but spread out capital accumulation due to convex adjustment costs. Related, retained earnings are an important part of firms' optimization that is absent in the above formulations. Thus, the value of capital market participation may be unreasonably low in the previous model.

Consider the following optimization problem at the firm level.

$$V(K, A, B, D) = \max\left\{V^e(K, A, B, D), V^i(K, A, B, D)\right\},$$

where

$$V^i(K, A, B, D) = \max_{K', B', D' \leqslant D} d + \beta E_{A'|A} V(K', A', B', D'),$$
$$V^e(K, A, B, D) = \max_{K', B', D' \geqslant 0} d - \phi_0 + \beta E_{A'|A} V(K', A', B', D'),$$

[41] For the early round of estimates with a diagonal weighting matrix, we estimated $\gamma = 1.48$ and were able to generate a serial correlation of 0.445 but both a_1 and a_2 were far from the estimates in the literature. With the weighting matrix created from the simulated panels, there is less weight put on matching the serial correlation and relative to these other moments.

[42] This of course opens up a next step; the integration of non-convex adjustment costs and financial frictions.

[43] Interesting progress on these issues is reported by Bayraktar et al. (2003). They find support for both non-convex adjustment costs and financial constraints in a model estimated using an indirect inference procedure.

for all $(K, A, B \geqslant 0, D \geqslant 0)$ where $d = \pi(A, K) + R^l B + D' - I - R^b D - B' - C(K', K)$.

In this problem, the superscripts "e" and "i" refer to external and internal financing, respectively. The objective is to maximize the expected discounted value of dividends, denoted d and defined as the difference between the sources (profits plus interest income) and uses (capital investment, financial investment and payments on debt outstanding) by the firm. In the expression for d, R^l is the rate of interest on the firm's assets (B) and R^b is the rate of interest on its debt (D). We impose $d \geqslant 0$ so that firms cannot borrow from the households without incurring the cost of external finance.

If the firm chooses external finance, then, as in the previous case, it incurs a cost of accessing these markets again denoted by ϕ_0. We assume $R^b > R^l$ so that the firm will not have an incentive to go to the external markets once and save the amount it borrows until it is needed for investment.[44]

If the firm chooses internal finance, then investment is again financed internally. But, the firm can pay for capital goods from retained earning and/or from the flow of profits. There is a constraint imposed:

$$B' + pK' \leqslant \pi(K, A) + (1 + r)B + pK(1 - \delta). \tag{13}$$

The value of future financial and capital assets cannot exceed the flow of profits plus the value of existing financial assets plus the value of the capital stock. In other words, if the firm does not go to capital markets, dividends must be non-negative.

There are additional constraints in the internal financing regime which depend on the sign of B. If $B \geqslant 0$, then $B' \geqslant 0$. In this case, the firm can have investment expenditures in excess of profits simply by reducing its (positive) financial assets. If $B < 0$, so that the firm is already in debt, then $B' \geqslant B$. So, the firm cannot increase its indebtedness without going to the capital market.

There is an interesting asymmetry here. If a firm is in debt, then it must resort to external markets in order to finance a burst of investment that exceeds its profit flow. But, if a firm has financial assets $(B > 0)$, then investment can be financed out of internal funds and reserves without incurring the costs of external finance.

A final consideration is to guarantee that the firm does not have an incentive to "take the money and run." If the firm makes this decision at the start of a period given the state, then the appropriate constraint is

$$V(A, K, B) \geqslant 0 \quad \text{for all } (A, K, B). \tag{14}$$

The firm receives the value $V(A, K, B)$ if it remains in operation. Else, the firm can exit, avoid any obligations but is unable to recover the flow of profits from its operations. This acts as a borrowing restriction on the firm as it would never be allowed to accumulate so much debt that it would prefer to renege on its obligations. In equilibrium, there are no defaults. In fact, adding this constraint to our problem explicitly is not necessary: (14) is implied by the requirement that dividends are non-negative.

Finally, it is useful to relate this optimization to the one specified earlier in which there were no financial assets. Suppose that there are no costs of external finance and

[44] In a financial contracting model, the gap between the borrowing and lending rates might depend on other state variables of the firm.

no constraint that dividends must be non-negative. Then, $V(A, K, B) = V(A, K, 0) + (1 + r)B$ where $V(A, K, 0)$ is the value of the firm from (1). Given the assumption that $\beta(1 + r) = 1$, the firm is indifferent with regards to its financial assets and the choice of physical capital is independent of B.

The model is very difficult to estimate due to the additional state variable and the need for a fine state space. Further developments along this line seems warranted in order to more fully integrate capital market and investment decisions at the firm level.

4.2. Non-convex adjustment costs

As noted earlier, there is evidence of lumpy capital adjustment at the plant level. The estimate of a zero cost of capital market access implies that costly capital market access is not the source of lumpy investment.

But, perhaps the effect goes in the opposite direction? That is, it might be that non-convexities in adjustment costs at the plant level or firm level may generate large variations in the demand for capital, i.e. investment spikes, which may not be easily financed out of profit flows. Put differently, the smoothing of capital adjustment due to assumed quadratic adjustment costs reduces the importance of external finance.

The natural approach is to modify (1) to include a non-convex adjustment cost in $C(K', K)$.[45] In addition, one would allow $\phi_0 \neq 0$ so that both costs of adjustment capital and financial market participation were present. The key would be the identification of these two non-convex adjustment costs. Observations on investment bursts and capital market participation would be ideal for this exercise.[46]

5. Conclusions

This paper considers the question of whether investment is sensitive to financial factors, perhaps reflecting financial frictions. This is an important question since, among other things, it bears on the design of fiscal and monetary policy.

Many researchers interpret the statistical significance of cash flow in Q-based investment regressions as reflecting capital market imperfections. Recent papers have called this into question and have argued that similar results can arise even in the absence of borrowing restrictions.

This paper has taken the next step by considering whether an estimated model with borrowing restrictions can match the Q-theory based investment models and other statistics. In doing so, the model nests both imperfect competition and financial frictions.

The results indicate that adding financial frictions to a model of firm investment behavior with imperfect competition does not increase the ability of the model to match observations. To put this into context, start, as we did in this presentation, with the basic

[45] Cooper and Haltiwanger (2000) present various forms of non-convex adjustment costs along with estimates at the plant-level.

[46] For their estimation, Bayraktar et al. (2003) use a reduced form non-linear regression in which investment rates are “explained” by profitability shocks and also financial variables.

Q-model. That specification would be unable to match the response of investment to cash flow, given average Q. Add to that specification model market power. We have seen that this additional feature substantially improves the fit of the model. The final step adds a fixed cost of access to capital markets. We estimate this fixed cost at zero. Thus, based on this evidence, we conclude that the Q-based investment regressions essentially reflect market power and not capital market imperfections.

The paper finished with suggestions for future work. These are partly motivated by missing considerations in the theoretical model and by the inability of the model to match some observations, such as the serial correlation of investment rates and observed lumpiness of investment, at least at the plant level.[47]

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[47] Interestingly, the serial correlation of investment rates is much lower at the plant-level than the 0.40 estimate used for the firm level data.

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