Homework #3

1.Make a Python program of the final alg.

Make a Python program of the final algorithm and find the solution

- **Step 2-3: Plug the derivatives into the algorithm**

$$\frac{\partial E}{\partial w_0} = 4w_1 + 6w_0 - 6$$

$$\frac{\partial E}{\partial w_1} = 4w_1 + 4w_0 - 6$$

Randomly choose an initial solution, $w_0^0\ w_1^0$

$t = 0$

Repeat

$$w_0^{t+1} = w_0^t - \eta \frac{\partial E}{\partial w_0}\bigg|_{w_0 = w_0^t, w_1 = w_1^t}$$

$$w_1^{t+1} = w_1^t - \eta \frac{\partial E}{\partial w_1}\bigg|_{w_0 = w_0^t, w_1 = w_1^t}$$

$t = t + 1$

Until stopping condition is satisfied

- **Final algorithm**

Randomly choose an initial solution, $w_0^0\ w_1^0$

$t = 0$

Repeat

$$w_0^{t+1} = w_0^t - \eta(4w_1^t + 6w_0^t - 6)$$

$$w_1^{t+1} = w_1^t - \eta(4w_1^t + 4w_0^t - 6)$$

$t = t + 1$

Until stopping condition is satisfied

2.Make Python programs

You have four samples as follows: D = {(x, t)| (-1, 1), (0, 1), (1, 1), (1, 0) }

a) Your model is $f(x) = w_1 x + w_0$. You have to find w_1 and w_0 so that $f(x)$ best fits the samples. Make a quadratic function to optimize. Find the solution by GDM.

b) Your model is $f(x) = w_1 \cos \pi x + w_0$. You have to find w_1 and w_0 so that $f(x)$ best fits the samples. Make a quadratic function to optimize. Find the solution by GDM.

3. Gradient

You have four samples as follows: D = {(x, t)| (-1, 1), (0, 1), (1, 1), (1, 0) }. The error function is defined as follows:

$$E = \sum_{(\mathbf{x},t)\in Data} \left(t - f(\mathbf{x})\right)^2$$

a) Your model is $f(x) = w_2 x + \cos w_1\, x + w_0$. What is the gradient of E at $w_0 = 1, w_1 = 1, w_2 = 1$. Use the following equation.

$$\left.\frac{\partial E}{\partial w_1}\right|_{w_0=w_0^k, w_1=w_1^k} = \sum_{(\mathbf{x}_i,t_i)\in Data} \left.\frac{\partial E_i}{\partial w_j}\right|_{w_0=w_0^k, w_1=w_1^k}$$

b) Your model is $f(x) = w_2 x + \cos w_1\, x + w_0$. What is the gradient of E at $w_0 = 2\; w_1 = 2, w_2 = 2$. Use the following equation.

$$\left.\frac{\partial E}{\partial w_1}\right|_{w_0=w_0^k, w_1=w_1^k} = \sum_{(\mathbf{x}_i,t_i)\in Data} \left.\frac{\partial E_i}{\partial w_j}\right|_{w_0=w_0^k, w_1=w_1^k}$$

4. Python program for new algorithm

You have four samples as follows: D = {(x, t)| (-1, 1), (0, 1), (1, 1), (1, 0) }. The error function is defined as follows:

$$E = \sum_{(\mathbf{x},t)\in Data} \left(t - f(\mathbf{x})\right)^2$$

a) Your model is $f(x) = w_2 x + \cos w_1\, x + w_0$. Find the best-fit $f(x)$. Use the new algorithm.

Randomly choose an initial solution, $w_0^0\ w_1^0$

$t = 0$

Repeat

$g_0^t = 0;\ g_1^t = 0$

for all $(\mathbf{x}_i, t_i) \in Data$

$$g_0^t = g_0^t + \left.\frac{\partial E_i}{\partial w_0}\right|_{w_0=w_0^t, w_1=w_1^t}$$

$$g_1^t = g_1^t + \left.\frac{\partial E_i}{\partial w_0}\right|_{w_0=w_0^t, w_1=w_1^t}$$

$w_0^{t+1} = w_0^t - \eta g_0^t$

$w_1^{t+1} = w_1^t - \eta g_1^t$

$t = t + 1$

Until stopping condition is satisfied