Transition State Tools for ReaxFF

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Chapter 1

Transition State Tools for ReaxFF

1.1 List of abbreviations

CG Conjugate Gradient

FIRE Fast Inertial Relaxation Engine

L-BFGS Limited-memory Broyen-Fletcher-Goldfarb-Shanno

NEB Nudged Elastic BandNEB Nudged Elastic Band

NEB-TR Nudged Elastic Band method with the removal of translational

and rotational degrees of freedom

 $\mathbf{QM} \qquad \qquad \mathrm{Quick\text{-}Min}$

SD Steepest Descents

1.2 General

Table 1.1: Overview of the existing control tags for the transition state tools.

Control tag	Value	Description
	0	Normal MD-run (Default)
	1	Energy minimisation
imetho	2	MD-minimisation
	3	Vibrational analysis
	4	Quasi-Newton optimisation
	5	Transition State Tools for ReaxFF
	0	Nudged Elastic Band calculation (Default)
	1	Energy minimisation (Available optimiser: FIRE,
runNeb		CG, L-BFGS, QM, SD)
	2	Energy minimisation and subsequent frequency analysis
	3	Frequency analysis
	4	Dimer search
	5	Lanczos method

1.3 Nudged Elastic Band

For the execution of a Nudged Elastic Band (NEB) calculation, the text files control, geo, and ffield are required. Depending on the selected control tag imgInt, the geo file must contain the endpoints and the respective N intermediate images (imgInt=0, imgNeb=N), the initial and final structure (imgInt=1 or imgInt=4) or the endpoints and one intermediate image (imgInt=2 or ImgInt=5). If the optimisation of the endpoints is activated before the NEB calculation, the energy minimisation criteria optEmi, endEmi, and maxEmi (see Table 1.3) are applied.

Table 1.2: Overview of the existing control tags for the NEB module of the transition state tools.

Control tag	Value	Description
imgNeb	int	Number of NEB images between initial and final state (Default: 0)
imgInt	0	No image interpolation (all images must be included in the geo) (Default)
	1	Linear interpolation of the images between the two end points (geo file must hold two structures)
	2	Linear interpolation of the images between the initial, transition, and final state (geo file must hold three structures)
	4	Plain output of the linear image interpolation between the two endpoints into the fort.90 file (without an NEB calculation)
	5	Plain output of the linear image interpolation between initial, transiton and final state into the fort.90 file (without an NEB calculation)
imgNeb	int	Number of NEB images between initial and final state (Default: 0)
imgInt	0 — 1	Use the climbing image algorithm for the NEB calculation: 0 - Off (default), 1 - On
dnudge	0	Disable double nudging during the NEB calculation (Default)
Ü	1	Enable modified double nudging during the NEB calculation (smoothly turns off the double nudging as the NEB converges)
	2	Enable double nudging during the NEB calculation (original approach by Trygubenko and Wales [1])
redRTr	0 — 1	Use the NEB-TR algorithm for the NEB calculation: 0 - Off (Default), 1 - On
maxNeb	int	Number of maximal NEB steps (Default: 5000)
spring	real	Spring constant between the images in $kcal \text{ mol}^{-1} \text{ Å}^{-2}$ (Default: $5.0 \text{ kcal mol}^{-1} \text{ Å}^{-2}$)
optNeb	int	Set the optimiser for the NEB calculation: 1 - CG, 2 - FIRE (Default), 3 - L-BFGS, 4 - QM, 5 - SD
opeNeb	0 — 1	Optimise the endpoints before the NEB calculation: 0 - Off, 1 - On
endNeb	real	Convergence criterion for the NEB calculation in kcal $\text{mol}^{-1} \text{ Å}^{-1}$ (Default: $1.0 \text{ kcal mol}^{-1} \text{ Å}^{-1}$)

1.4 Structure optimisation

Table 1.3: Overview of the structure optimisation control tags within the transition state tools for ReaxFF.

Control tag	Value	Description
optEmi	int	Set the optimiser for the energy minimisation: 1 - CG, 2 - FIRE (Default), 3 - L-BFGS, 4 - QM, 5 - SD
endEmi	real	Convergence criterion for the energy minimisation in kcal mol^{-1} Å ⁻¹ (Default: $1.0 \text{ kcal mol}^{-1}$ Å ⁻¹)
maxEmi	int	Maximal energy minimisation steps (Default: 5000)
convcr	0 — 1	Satisfaction of the convergence criterion is based on 0 - $\mathbf{F}_{\text{total}}^{1/2}$, 1 - $\mathbf{F}_{\text{max,atom}}$ (Default)
itraj	0 — 1	Output of the optimisation trajectory: 0 - Off (Default), 1 - On
frtraj	int	Frequency of the optimisation trajectory output (Default: 1)

Within the transition state tools for ReaxFF, it can be specified whether the atom's respective coordinate(s) are allowed to change during the structural optimisation. To this end, the keyword CONSTRAINTS has to be added to the header of the geo file:

CONSTRAINTS [At1 At2 flagX flagY flagZ]

All atoms with indices between At1 and At2 are optimised according to the specified flags. Setting flag values of 0 or 1 disables or allows the optimisation of the specified atoms along the respective spatial directions. Setting all flags to 0 is equivalent to the use of the FIXATOMS keyword. All non-specified atoms are assumed to be free of optimisation constraints (all flags are set to 1).

```
XTLGRF 200
DESCRP A1100
REMARK A1100_with_Au_constraint
FIXATOMS 1 8
CONSTRAINTS 13 13 0 0 1
CRYSTX 5.72756 5.72756
FORMAT ATOM (a6,1x,i5,1x,a5,1
                                                                       13.75000
                                                                                            90.00000
                                                                                                                  90.00000
                                  0.00000
0.00000
0.00000
       HETATM
                                                                                                                                     A1
A1
A1
A1
A1
A1
A1
A1
A1
       HETATM
HETATM
                                 A1
A1
A1
A1
A1
A1
A1
A1
       HETATM
                                                                                                                                                       0.00000
       HETATM
HETATM
HETATM
                                                                         1.43189
4.29567
1.43189
                                                                                                                6.02500
6.02500
6.02500
                                                                                                                                                       0.00000
0.00000
0.00000
                                                                                            1.43189
1.43189
12
13
14
15
16
17
18
19
                                                                                             4.29567
       HETATM
                                                                         4.29567
                                                                                             4.29567
                                                                                                                6.02500
                                                                                                                                                       0.00000
                                                                         0.00000
2.86378
0.00000
       HETATM
HETATM
                                                                                             0.00000
                                                                                                                8.05000
                                                                                                                                                       0.00000
                                                                                             0.00000
       HETATM
HETATM
                           ^{1\,1}_{1\,2}
                                                                                             2.86378
2.86378
                                                                                                                8.05000 \\ 8.05000
                                                                                                                                                       0.00000 \\ 0.00000
       HETATM
END
                                                                         1.43189
                                                                                             1.43189
                                                                                                                9.75000
```

Listing 1.1: Example for using the CONSTRAINTS keyword. Due to the FIX-

ATOMS keyword, the atoms 1 to 8 are kept in their bulk position. The gold atom (atom index 13) will only be relaxed in the z-direction.

1.5 Optimiser settings

Table 1.4: Overview of the control tags for the respective force-based optimisers of the transition state tools.

Control tag	Value	Description
maxmov	real	Maximum allowed step size for translation (Default: 0.2)
fdsOpt	real	Finite difference step size (CG, L-BFGS(line)) (Default: 0.005)
dtOpt	real	Finite difference step size (FIRE, QM) (Default: 0.005)
memOpt	int	Number of memorised steps for the construction of the inverse Hessian matrix (L-BFGS) (Default: 20)
aicOpt	0 — 1	Initial inverse curvature is determined automatically (L-BFGS): 0 - Off, 1 - On (Default)
ivcOpt	real	Initial inverse curvature for the construction of the inverse Hessian matrix (L-BFGS) (Default: 0.01)
damOpt	real	Damping Parameter to control the variation of the step size (L-BFGS) (Default: 1.1)
flmOpt	0 — 1	L-BFGS approach to determine the new atomic positions: 0 - L-BFGS(Hess) (Default), 1 - L-BFGS(Line)
mdtOpt	real	Define the maximum allowed dynamic time step (Default: 1.0)
idtOpt	real	Factor to increase the dynamic time step (FIRE) (Default: 1.1)
ddtOpt	real	Factor to decrease the dynamic time step (FIRE) (Default: 0.5)
falOpt	real	Parameter to adjust the velocity damping (FIRE) (Default: 0.1)
dalOpt	real	Factor to decrease α (FIRE) (Default: 0.99)
fnmOpt	int	Minimum number of steps before modifying α and the dynamical time step (FIRE) (Default: 5)
qmfOpt	0 — 1	Force the velocity in the direction of the force (FIRE): 0 - Off (Default), 1 - On
salOpt	real	Factor to control the step size (SD) (Default: 0.01)

1.6 Frequency analysis

Table 1.5: Overview of the control tags for the vibrational analysis module of the transition state tools for ReaxFF.

Control tag	Value	Description
ifrMet	int	Method to perform the vibration analysis: 0 - finite difference method [2] (Default), 1 - modified approach by Frederiksen <i>et al.</i> [3]
infree	0	Single displacement per atom and cartesian coordinate (backwards) Single displacement per atom and cartesian coordinate (forward)
	2	Central difference displacements per atom and cartesian coordinate (Default)
	4	Four displacements per atom and cartesian coordinate
freAll	0	The vibration modes are calculated according to the FREQANALYSIS keyword in the geo file
	1	Calculate the vibrational modes for all non-fixed atoms (specified via the FIXATOMS keyword)
	2	Calculate the vibrational modes for all atoms (Default)
ifrMet	real	Magnitude of atom displacements in Å (for $freAll \neq 0$) (Default: in 0.01 Å)

The calculation of the vibration modes can be controlled via the FREQ-ANALYSIS keyword in the header of the geo file:

FREQANALYSIS [At1 At2 disX disY disZ]

The frequency analysis is carried out for all atoms in the range of At1 and At2 using disX, disY, and disZ as displacement values (in \mathring{A}) in the respective spatial directions.

```
XTLGRF 200
     DESCRP Al100
REMARK frequency_analysis
FIXATOMS 1 8
FREQANALYSIS 13 13 0.05 0.05 0.05
                        13.75000
                                                                       90.00000
                                                                                       90.00000
                                                                                                        90.00000
     CRYSTX
                      5.72756
                                       5.72756
                                                                       x, a5, 3 f10.5
0.00000 4
                                                                                                     , i 2 , 1 x , f 8
Al 0 0
Al 0 0
     FORMAT ATOM
                                                       , a3 , 1 x , a1
0.00000
                                                                                                                       00000
                                                        2.86378
     HETATM
                                                                       0.00000
                                                                                      4.00000
                                                                                                                    0.00000
10
     HETATM
                      3
                                                        0.00000
                                                                       2.86378
                                                                                      4.00000
                                                                                                       A1
                                                                                                            0
                                                                                                                    0.00000
11
12
13
     HETATM
HETATM
                                                        2.86378
1.43189
                                                                                                      A1
A1
A1
                                                                                                                    0.00000
                                                                       2.86378
                                                                                      4.00000
                      5
6
7
8
9
     HETATM
                                                        4.29567
                                                                       1.43189
                                                                                      6.02500
                                                                                                                    0.00000
     HETATM
                                                        1.43189
                                                                       4.29567
                                                                                      6.02500
                                                                                                      A1
A1
A1
A1
A1
                                                                                                                    0.00000
14 \\ 15 \\ 16 \\ 17
                                                        4.29567
     HETATM
HETATM
                                                                       4.29567
                                                                                      6.02500
8.05000
                                                                                                                    0.00000
                     10
11
12
     HETATM
                                                        2.86378
                                                                       0.00000
                                                                                      8.05000
                                                                                                            0
                                                                                                                    0.00000
     HETATM
HETATM
                                                                                                                    0.00000
                                                        0.00000
                                                                       2.86378
                                                                                      8.05000
                                                                                      8.05000
9.75000
     HETATM
END
                                                        1.43189
                                                                       1.43189
                                                                                                                    0.00000
```

Listing 1.2: Example for using the FREQANALYSIS keyword. Here, the vibration modes are calculated for the Au adatom (atom index 13) using an atomic displacement of $0.05\,\text{Å}$, respectively.

1.7 Dimer search and Lanczos method

In order to perform a saddle point search (Dimer search and Lanczos method), the text files geo, control and ffield are required. Also, the initial lowest modes are needed at the beginning of a saddle point search. If the control tag iSMode=0 is set, the 3N initial directions along the dimer for the N atoms are taken from the modes.in file. The line sequence corresponds to the atom order from the geo file. The three columns correspond to the atoms' initial direction along the respective coordinate. Also, the geo file has to provide the initial structure for the saddle point search).

```
XTLGRF 200
DESCRP image_004
REMARK image_004
     FIXATOMS
CRYSTX
FORMAT ATOM
HETATM
                      5.76999
                                     5.76999
                                                    26.12000
                                                                    90.00000
                         1 Au
2 A"
                                                                                HETATM
                                                     4.32749
                                                                    1.44250
                                                                                10.00000
                                                                                                 Au
                                                                                                          0
                                                                                                              0.00000
                                                     1.44250
4.32749
                                                                                 10.00000
                                                                                                              0.00000
0.00000
     HETATM
                       Au
                                                                      32749
     HETATM
                                                     0.00000
                                                                   0.00000
                                                                                 12.04000
                                                                                                 Au
Au
Au
Au
Au
Au
Au
                                                                                                              0.00000
                                                                                                              0.00000
0.00000
0.00000
     HETATM
                                                     2 88500
                                                                    0 00000
                                                                                 12.04000
13
14
15
     HETATM
HETATM
                                                                   2.88500
2.88500
                                                                                 12.04000
12.04000
12.04000
                                                     2.88500
     HETATM
                                                     1.41837
                                                                    1.44250
                                                                                 14.16715
                                                                                                          0
                                                                                                              0.00000
                    10
11
12
                                                                                 14.16585
16
17
18
19
     HETATM
                                                     4 32155
                                                                      44250
                                                                                                              0.00000
     HETATM
                                                     4.31938
                                                                    4.32749
                                                                                 14.16674
                                                                                                              0.00000
                    13
14
15
                                                                                                 Au
Au
Au
     HETATM
                                                     5.72280
                                                                    5.76232
                                                                                 16.34443
                                                                                                          0
                                                                                                              0.00000
     HETATM
HETATM
                                                     2.85003
5.72280
                                                                   0.00860
2.89267
                                                                                 16.26047
16.34443
                                                                                                              0.00000
     HETATM
                                                     2.85003
                                                                                                              0.00000
     HETATM
                                                                    4.32750
                                                                                 18.55147
     END
```

Listing 1.3: The geo file contains the initial structure for the saddle point search, whereby the atom sequence has to match the line entries from the modes in file.

To restart a calculation, the resulting new modes.out file from the previous run can be used as the new modes. in file.

Alternatively, the entries of the modes in file can be generated automatically by

```
0.000000
                                                         0.000000
                                0.000000
      0.000000
                               0.000000
                                                         0.000000
      0.000000
0.000000
0.000000
                               0.000000
0.000000
0.000000
                                                         0.000000
      0.000000
                               0.000000
                                                         0.000000
      0.000000
                               0.000000
                                                         0.000000
      0.052585
                                                        -0.002857
                               0.000000
      0.038785
                               0.000000
                                                         0.001094
11
12
13
      0.051217
0.040670
                               0.000000
                                                        -0.001034
-0.004286
0.001824
-0.039971
      0.143408
                               0.008420
      0.106233
                               -0.006748
                                                         0.027843
                               -0.006748
-0.008420
0.006748
                                                         0.027843
-0.039971
0.027843
0.036505
      0.143408
       -0.959900
                                 0.000000
```

Listing 1.4: The modes in file contains an Nx3 matrix with the initial directions along the dimer in the respective spatial directions for the N atoms of the system.

setting iSMode=1. Here, a target structure must be provided as a second entry in the geo file.

```
XTLGRF 200
DESCRP Initial_structure
REMARK Saddle point search
FIXATOMS 1 8 8 5.76999 5.76
(a6,1x,i5,1)
                                        \begin{smallmatrix} & & & & & & & \\ 6999 & & 5.76999 & 26.12000 \\ (a6,1x,i5,1x,a5,1x,a3,1x,a1 \\ 44250 & & & & \\ & & & & & \\ \end{smallmatrix} 
                                                                                                       90.00000
                                                                                                                               90.00000
                                                                                                                                                       90.00000
                                                                                                       x, a5, 3 f10
1.44250
                                                                                                                           .5,1x,a5,i3
                                                                                                                                                    , i 2 , 1 x , f8 . 5 )
Au 0 0 0.00000
        HETATM
                                    Au
                                                                                  4.32749
                                                                                                        1.44250
                                                                                                                            10.00000
                                                                                                                                                     0.00000
        HETATM
HETATM
                                                                                                        4.32749
                                                                                                                           10.00000
                                                                                                                                                                         0.00000
\begin{array}{c} 10 \\ 11 \\ 12 \\ 13 \\ 14 \\ 15 \\ 16 \\ 17 \\ 18 \\ 22 \\ 23 \\ 24 \\ 22 \\ 23 \\ 24 \\ 25 \\ 27 \\ 28 \\ 29 \\ 30 \\ 33 \\ 34 \\ 35 \\ 36 \\ 37 \\ 38 \\ 40 \\ 41 \\ 42 \\ 44 \\ 45 \\ 46 \\ 47 \\ 48 \\ 49 \\ \end{array}
        HETATM
                                                                                  0.00000
                                                                                                       0.00000
                                                                                                                           12.04000
                                                                                                                                                                         0.00000
        HETATM
                                                                                  2.88500
                                                                                                        0.00000
                                                                                                                            12.04000
                                                                                                                                                                         0.00000
        HETATM
HETATM
                                                                                                        2.88500
2.88500
                                                                                                                           12.04000
12.04000
12.04000
                                                                                  2.88500
                                                                                                                                                                         0.00000
                                                                                                                           14.16715
14.16585
14.16875
14.16674
        HETATM
                                                                                  1.41837
                                                                                                        1.44250
                                                                                                                                                                  0
0
0
0
0
0
0
                                                                                                                                                                         0.00000
        HETATM
HETATM
HETATM
                               10
11
12
                                                                                 4.32155
1.42036
4.31938
5.72280
                                                                                                        1.44250
4.32749
4.32749
                                                                                                                                                                         0.00000
                                                                                                                                                                         0.00000
                               13
14
15
16
        HETATM
                                                                                                        5.76232
                                                                                                                            16.34443
                                                                                                                                                                         0.00000
        HETATM
HETATM
HETATM
                                                                                  2.85003
5.72280
2.85003
                                                                                                       0.00860
2.89267
2.87640
                                                                                                                            16.26047
                                                                                                                                                                         0.00000
        HETATM
                                                                                  3.20078
                                                                                                        4.32750
                                                                                                                           18.55147
                                                                                                                                                                         0.00000
        END
        XTLGRF 200
DESCRP Target_structure
REMARK Automatic modes.in generation
FIXATOMS 1 8
                                 5.76999
                                                                               26.12000
                                                        5.76999
                                                                                                       90.00000
                                                                                                                               90.00000
                                                                                                                                                       90.00000
        CRYSTX
FORMAT ATOM
                                1 Au 1.44250
2 Au 4.32749
                                                                                                    ,1x, a5, 3 f10.5, 1x, a5, i3
1.44250 10.00000
                                                                                                                                                    , i 2 , 1 x , f8 . 5 )
Au 0 0 0.00000
Au 0 0 0.00000
        HETATM
HETATM
                                    Au
                                                                                                        1.44250
                                                                                                                            10.00000
                                3
4
5
        HETATM
                                                                                  1.44250
                                                                                                        4.32749
                                                                                                                            10.00000
                                                                                                                                                     Au
                                                                                                                                                                         0.00000
        HETATM
HETATM
                                                                                                                                                                         0.00000
                                                                                  4.32749
                                                                                                        4 32749
                                                                                                                            10.00000
                                                                                  0.00000
                               6
7
8
9
        HETATM
                                                                                  2.88500 \\ 0.00000
                                                                                                        0.00000
                                                                                                                           12.04000
                                                                                                                                                                         0.00000
        HETATM
                                                                                                        2.88500
                                                                                                                           12.04000
                                                                                                                                                                         0.00000
        HETATM
HETATM
HETATM
                                                                                                                           12.04000
12.04000
14.16621
14.16621
                                                                                                                                                                         0.00000
0.00000
0.00000
                                                                                 2.88500 \\ 1.43567
                                                                                                        2.88500
                                                                                  4.33431
                                                                                                        1.44250
                               11
12
13
14
                                                                                  1.43721
4.33276
5.76998
        HETATM
                                                                                                        4.32749
                                                                                                                            14.16734
                                                                                                                                                                         0.00000
        HETATM
HETATM
                                                                                                                            14.16734
16.33128
                                                                                                                                                                         0.00000
                                                                                  2.88498
5.76998
2.88498
2.88498
        HETATM
                                                                                                        0.00638
                                                                                                                            16.26963
                                                                                                                                                                         0.00000
        HETATM
HETATM
HETATM
                               15
16
17
                                                                                                                           16.26963
16.26963
18.56348
                                                                                                                                                                         0.00000
0.00000
0.00000
                                                                                                        2.88990
                                                                                                        2.87862
4.32750
```

Listing 1.5: The geo file has to contain the initial structure for the saddle point search and the target structure as an additional entry for the automatic generation of the modes.in file.

In the absence of a target structure and modes.in, the initial lowest modes may be generated randomly $(iSMode{=}2)$. However, this increases the chance of failure in the saddle point search or results in unexpected behaviour.

Table 1.6: Overview of the saddle point search control tags within the transition $\frac{1}{2}$

state tools for ReaxFF.

Control tag	Value	Description
optSad	int	Set the optimiser for the saddle point search: 1 - CG (Default, recommended), 2 - FIRE, 3 - L-BFGS, 4 - QM, 5 - SD
endSad	real	Convergence criterion for the saddle point search in kcal mol^{-1} Å $^{-1}$ (Default: $1.0 \text{ kcal mol}^{-1}$ Å $^{-1}$)
maxSad	int	Maximal saddle point search steps (Default: 5000)
iSMode	0 1 2	Read initial lowest modes from modes.in file (Default) Generate initial lowest modes from input structures Create random initial lowest modes (not recommended)
disSad	0 — 1	Perform an initial displacement before starting the saddle point search: 0 - Off (Default), 1 - On
disFac	real	Scaling factor for initial displacement (Default: 0.01)
intSad	0 — 1	Interpolate the initial structure for the saddle point search from two given end point structures: 0 - Off (Default), 1 - On
DRoMax	int	Set the maximum number of rotational steps per translational step (Dimer search) (Default: 1)
DdrSep	real	Separation of the dimers in Å (Dimer search) (Default: 0.001Å)
DFNMin	real	Threshold for the rotational force in kcal mol^{-1} Å ⁻¹ below which the dimer is not rotated (Dimer search) (Default: $0.23 \text{ kcal mol}^{-1}$ Å ⁻¹)
DFNMax	real	Threshold for the rotational force in kcal mol^{-1} Å ⁻¹ below which dimer rotation stops. At least one rotation iteration is carried out if the rotational force ranges between DFNMin and DFNMax. (Dimer search) (Default: $2.3 \mathrm{kcal}\mathrm{mol}^{-1}$ Å ⁻¹)
lanTol	real	Tolerance for eigenvalue convergence (Lanczos) (Default: 0.01)
lanSdr	real	Finite difference step length in Å (Lanczos) (Default: 0.001Å)
lanSnl	int	Maximum size of Lanczos matrix (Lanczos) (Default: 20)
lanQLc	real	Convergence criteria for QL algorithm (Lanczos) (Default: 10^{130})
lanQLs	real	Implicit shift in QL-routine (Lanczos) (Default: 0.0001)

Bibliography

- [1] S. A. Trygubenko, D. J. Wales, The Journal of Chemical Physics 2004, 120, 2082–2094.
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- [3] T. Frederiksen, M. Paulsson, M. Brandbyge, A.-P. Jauho, *Phys. Rev. B* **2007**, *75*, 205413.