Modeling and Control of a Quadrotor Motor

Topic of interest: Quadrotor

Specifications

Dimensions (Overall): 1.88 Inches (H) x 15.89 Inches (W) x 8.61 Inches (D)

Weight: .18 Pounds

Suggested Age: 8 Years and Up

Battery Capacity: 500 (mAh)

Indoor/Outdoor: Outdoor

Assembly Details: No Assembly Required

Connection Type: USB-C

Maximum Speed: 4.5 Miles per Hour

CPSC Choking Hazard Warnings: Choking_hazard_small_parts

Video Recording Resolution: No Built-In Video Recorder

Wireless Technology: Radio Frequency

Control Type: Remote Control

Maximum Running Time: 6 Minutes

Skill Level: Beginner

Maximum Operating Range: 50 Meter

Battery: Lithium Ion

MAE 3260 Final Group Work: Exploring a System of Interest

Proposal

Title: Modeling and Control of a Quadrotor Maneuver

Topic of Interest: Quadrotor

Abstract: Our final group work investigates the dynamics and control of a quadrotor as a representative multi-input/output system. Our group chose a quadrotor because quadrotors are widely used in inspection, hobbying, delivery, and war tasks, where precise and safe maneuvering is critical. We aim to derive dynamic models through obtaining ODEs, transfer functions, and state space models for key aspects(pitching, yawing, and throttling). By using these models, we will analyze the open-loop and closed-loop behaviors with step response and Bode plot through MATLAB. Our ultimate goal is to combine the course concepts and utilize them in our derivation of the system dynamics of the quadrotor.

Students/Roles:

Student	**Task/Role**
Name of student	1-3 sentences for what the student plans to work on, what skills in the class they will use/build upon, what they expect the final result to look like
Angus Chang	Create a Block diagram of the quadrotor and controller. Open/close loop analysis for different types of maneuvers (stopping, hovering, cruise speed/altitude).
Jeffrey Wang	Develop a state space model of a rotor. This should include most of the common parameters such as the moment of inertia of the rotor, the angular velocity of the rotor, and perhaps the thrust generated as a result.
Sam Kuzmishin	Find transfer function(s) for the rotor. Some transfer functions to potentially find include voltage to rotor speed, rotor speed to thrust, and input torque to roll, and pitch. This will build on my ability to model systems, as well as my ability to mathematically manipulate equations.
Huxley Holcombe	Design and tune feedback controller for quadrotor using transfer function and state-space model. Simulate and evaluate performance using MATLAB.

These roles changed heavily after performing the dissection and finding aspects of the practical quadrotor that we did not predict seeing from the proposal. Thus the written sections are more findings based than theory based.

Data Collection – Rotor Speed

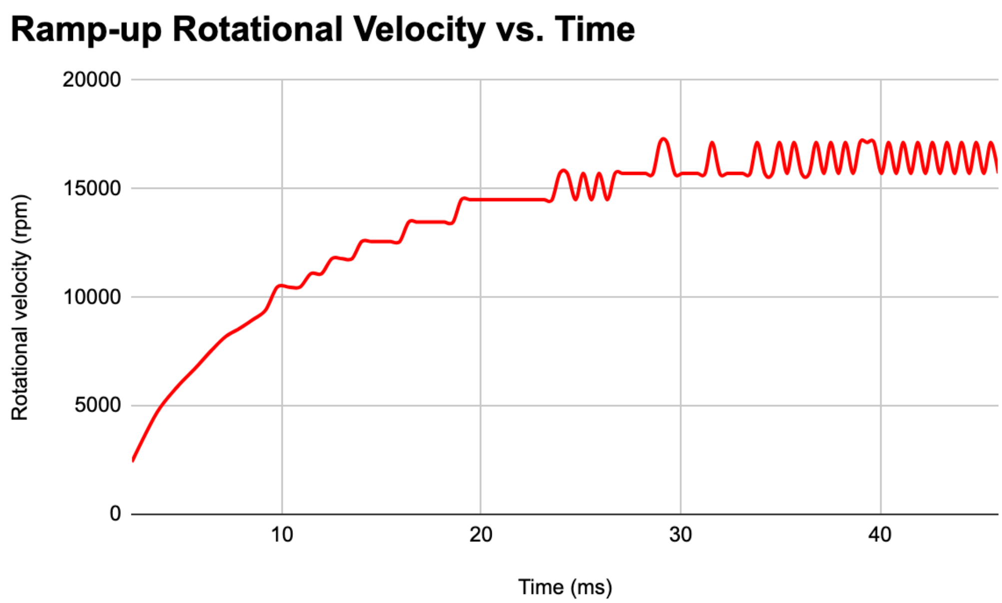
Sam Kuzmishin

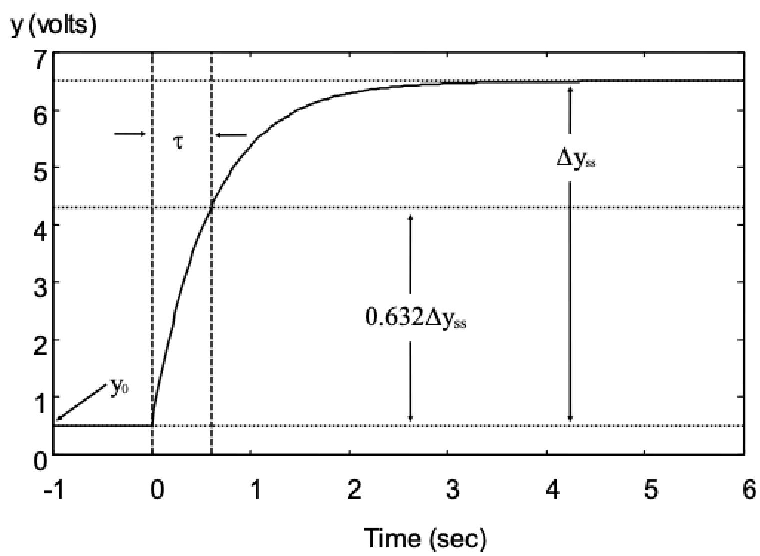
Now that we have established a connection between time and voltage in our quadcopter's motors, it is crucial to model rotational speed as a function of time. By using time as an intermediate variable, we can thus find the relationship between voltage and the rotational speed of a rotor. Lastly, we can connect this to thrust, allowing our flight controller to determine the forces acting on the drone depending on the voltage the flight computer is outputting. The predictable nature between voltage and thrust is the crux of the functionality of a drone, making accurate assessment of rotor speed crucially important.

To collect our data, we utilized a high-speed camera shooting at 3,142 frames per second and recorded our rotor during a ramp up. We then watched the video back frame by frame and took note of how many frames it took for a blade to make a full rotation. To make it easier to keep track of the blades, we used a reflective marker to color the tip of an airfoil, allowing us to see a flash every spin. With this data, we used a spreadsheet to calculate time per spin, and finally the rpm of our rotor.

Rotation #	# of Frames	Frame Number	Time(ms)	Time for Rotation (ms)	RPM
1	78	78	2.482495226	2.482495226	2416.923077
2	40	118	3.755569701	1.273074475	4713
3	32	150	4.774029281	1.01845958	5891.25
4	28	178	5.665181413	0.8911521324	6732.857143
5	25	203	6.46085296	0.7956715468	7540.8
6	23	226	7.192870783	0.732017823	8196.521739
7	22	248	7.893061744	0.7001909612	8569.090909
8	21	269	8.561425843	0.6683640993	8977.142857
9	20	289	9.197963081	0.6365372374	9426
10	18	307	9.770846595	0.5728835137	10473.33333
11	18	325	10.34373011	0.5728835137	10473.33333
12	18	343	10.91661362	0.5728835137	10473.33333
13	17	360	11.45767027	0.5410566518	11089.41176
14	17	377	11.99872693	0.5410566518	11089.41176

Our rotational speed begins to stagnate at roughly 17,000 rotations per minute, which happens 28.96 milliseconds after the ramp-up begins. Graphed below is our rpm as a function of time. We can see that it is very similar to the graph of voltage versus time found from the oscilloscope.





We reach Y_{ss}, which is 17138 RPM at approximately 29 milliseconds. From here, we can multiply this value by .632, and see at what time that value intersects the line. This will give us our time constant. 17138 x 0.63 = 10797. Interpreting this data on our graph, we can see this occurs at roughly 12ms, giving us a time constant of .012 seconds for prop ramp up.

Data Analysis – Block Diagram

Angus Chang

Our block diagram consists of both **current** and **speed**’s 1st ODE’s:

Current:

$$L(di/dt) =- Ri - K_b w + v$$

L = Winding inductance

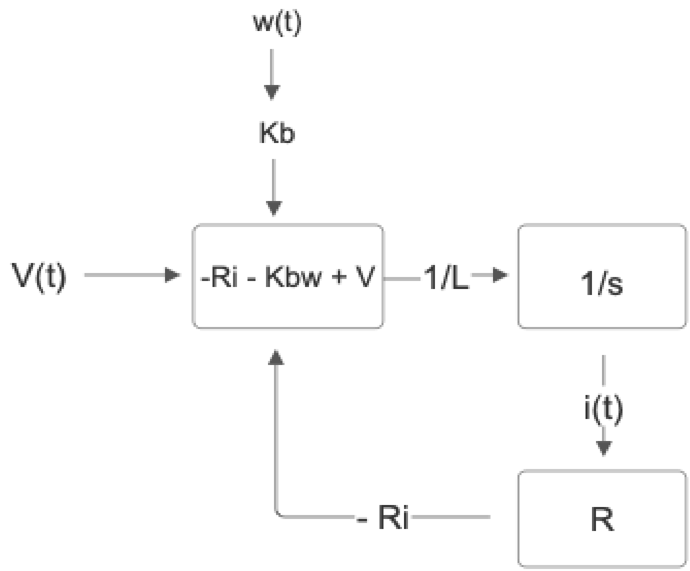
i = Armature current

R = Winding Resistance

K_b = Back EMF constant

w = Rotor angular speed

v = Input voltage



Speed:

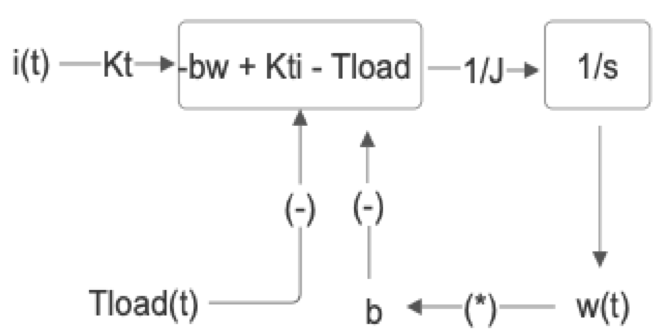
$$J(dw/dt) = -bw + k_t i - \tau_{load}$$

J = Rotor Inertia

b =Viscous friction coefficient

K_t =Torque constant

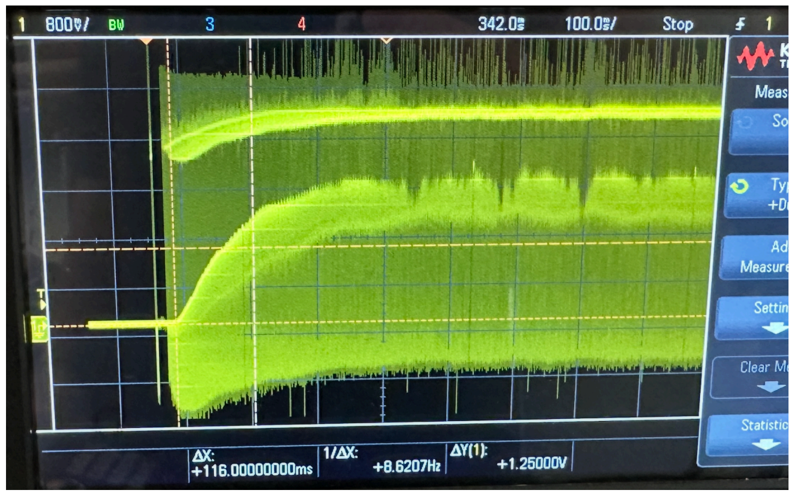
π_{load} = Aerodynamic load torque (prop)



Analyzing the Voltage Data from the Oscilloscope

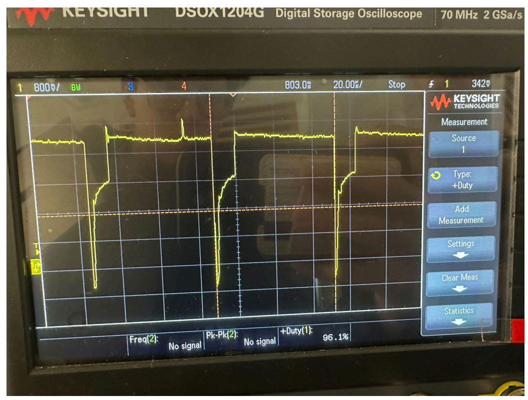
Jeffrey Wang

We were able to measure the ramp up voltage of the motor by partially stripping away the two wires leading into the small DC motor from the flight controller. The voltage we found was extremely interesting, and did not show a clear voltage ramp up line, instead appearing as concentrations of voltage within a wide range of voltages. However, we were able to pick out what appeared to be a voltage curve that resembled a first order open loop system that we developed for Lab 1. This makes sense physically when compared to the closed loop system, as typical ESCs do not back measure the rpm of the rotor, and in turn cannot create a closed loop with just the motor. The closed loop response of the quadcopter comes from the flight controller gyro rather than the motor and ESC. Measuring the values relevant to this curve, we found a y0 of -10mV and a Δy=1.98V. For the steady state value, we measured the middle average of the darker shaded curve. From this, we calculated a time constant τ of 116ms for the response.

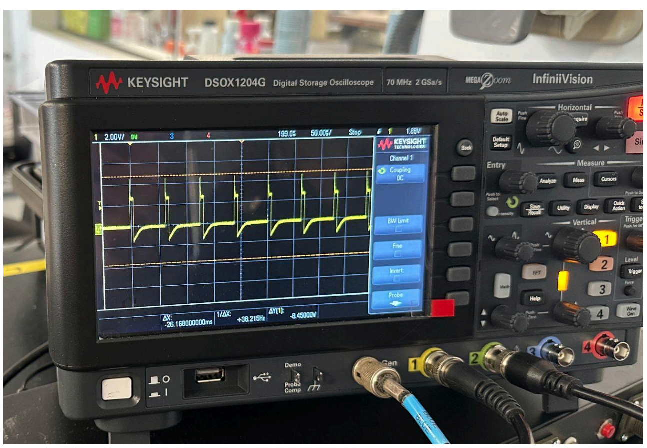


Our τ is largely defined by the moment of inertia I of the propeller and gears that it is driving. We can use this to find a 10-90% rise time of about 0.255s, which is much higher than the 29ms response found from the high speed camera. This value is far too high to be plausible for a quadcopter as the feedback loop would be far too slow to control flight requirements. A change in the measured line could probably bring τ down by 20-30% to .179-0.2s, but this would still be far from the expected value. There is a gear ratio of 2:1 between the motor and prop, but this would only affect the steady state values, not the speed of response. There was likely something inconsistent between the two tests to cause this, as the flight controller is responsible for the power each motor gets to stabilize, and the tilting of the drone and intended response of the drone was likely different in either test.

We can compare the response to the open loop differential equation found in lab: $\tau\dot{y} + y = Ku$. To find the gain K, we can compare the $\Delta y/\Delta u$, 1.98V/.96V (found from the duty cycle), and find K=2.0625. From the transfer function $G(s) = \frac{Y(s)}{U(s)} = \frac{K}{\tau s+1}$, we can create a governing transfer function of $G(s) = \frac{2.0625}{.116s+1}$.



We determined that the initially noisy signal was due to a pulse width modulation (PWM) signal that the flight controller sends to the motor to control the rotation speed. The PWM signal rapidly turns on and off the signal to vary the duty cycle. The average of the voltage is what is seen by the DC motor, and is the solid line that we tracked earlier to measure voltage changes. It likely oscillates between 0V and 2V, which are the bounds of that signal that we were seeing in the oscilloscope when zoomed out. However, we zoomed into the signal to see the individual duty cycles.



Before the ramp up, we measured a duty cycle of 0%. Halfway through the ramp up, the duty cycle measured 38%, and 96% at the steady state voltage.

Further Tasks to Fully Model the Quadcopter System

Huxley Holcombe

While significant progress has been made in characterizing the electrical and mechanical response of the quadrotor's motors, a complete model of the aircraft requires several additional layers of analysis. The collected data on voltage signal behavior and the corresponding angular velocity behaviour of the propeller have led us to first order current and speed ODEs. These have defined the foundation of the motor and controller model, but a quadrotor is a much more complicated system whose motion is a combination of the interaction of aerodynamics, rigid-body dynamics, sensor feedback, and control architecture. This section outlines some additional steps needed to develop a more complete model of the quadrotor system. This will also include how they build off of our existing model of inputs and outputs.

Modeling the aerodynamics:

Now that we have an idea of the signal to angular velocity of the motor and controller system, the result of the motor rotation must be analysed and modeled. The motor rotation leads to a thrust force which follows a non-linear relation to the angular velocity of the rotor.

Standard rotor theory gives a model for the thrust of a propeller as:

$T = 2\rho A v^2$ (where A is rotor area, ρ is air density, and v is induced velocity)

Given the constant nature of the values 2, A and ρ, these terms can be simplified to one single thrust constant giving us the equation:

$T = k_T v^2$

Further more, induced velocity is proportional to ω (angular velocity of the rotor). Thus, the thrust equation can be written as a function of ω given the additional inclusion of the proportional nature of induced velocity and angular velocity into the thrust constant. The resulting equation is:

$$T = k_T \omega^2$$

This gives us an equation relating the square of angular velocity with the thrust by the proportion of a thrust constant. Now, in order to determine k_T (thrust constant) additional tests would need to be conducted. The thrust generated from the motor during ramp up needs to be analysed. This could be measured against time, just as the motor signal and rotor speed data was taken, or against one of the other pieces of data, namely the motor voltage signal or the rotor speed. Doing so would allow us to connect the signal from the motor controller to the output thrust of the motor, taking us one step closer to completely modeling the system. Such added analysis would add an extra block to our block diagram.

Further analysis of the dynamics resulting from motor rotation would flesh out an even more complete and accurate model of the system. For example, rotor torque resulting from angular acceleration as well as from aerodynamics drag torque are a result of propeller rotation and are a feature used for yaw control.

Additional important aspects to fully model the system include but are not limited to:

Aerodynamic thrust model - Thrust, drag, motor torque

Modeling of sensors and feedback paths - Gyro model, accelerometer model, noise behavior

Rigid body dynamics - Complete 6-DOF rigid body dynamics

Control architecture model - Control mechanisms, loop analysis

The steps outlined in this section offer a path from the component level measurements done in our analysis to a fully functioning dynamics model of a quadcopter.