

Earnings-based borrowing constraints and pecuniary externalities*

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Abstract

Prices in financial constraints give rise to pecuniary externalities, which means that policy can improve market outcomes in which households and firms do not internalize the effects of their decisions on prices. This paper examines the pecuniary externalities that arise from earnings-based borrowing constraints, which are common for US firms. While asset-based collateral constraints typically result in ‘over-borrowing’ relative to the social optimum, we show that earnings-based constraints lead to ‘under-borrowing.’ The reason is that higher input prices decrease earnings and thus tighten earnings-based borrowing constraints. In particular, borrowing decisions today are suboptimal when firms do not internalize that they impact future equilibrium wages and thereby change future borrowing limits. Across a range of settings that are motivated by recent microeconomic evidence, optimal macroprudential policy is shown to depend critically on the specific form of financial constraints.

JEL Codes: D62, E32, E44, G28.

Keywords: Financial frictions; Pecuniary externalities; Collateral constraints; Earnings-based borrowing constraints; Macroprudential policy.

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1 Introduction

Should policy-makers intervene in financial markets? If so, why and how? This paper studies pecuniary externalities that arise when prices affect financial constraints but firms and households do not internalize the consequences of their choices on these prices. Our central contribution is to examine how empirically observed credit limits, in which firms' input prices and their credit access are linked, induce sub-optimal borrowing decisions. Our findings highlight that optimal macroprudential policy depends on the specific form of financial constraints.

Recent research distinguishes two types of credit constraints faced by US companies: asset-based and earnings-based constraints ([Lian and Ma, 2020](#), [Drechsel, 2020](#)). An asset-based borrowing limit ties credit access to the value of an asset, such as a building or machine. With earnings-based constraints, the ability to obtain funds is linked to the borrower's earnings, usually measured before interest, taxes, depreciation and amortization (EBITDA). The normative implications of asset-based constraints have been studied comprehensively (see e.g. [Jeanne and Korinek, 2010](#), [Dávila and Korinek, 2018](#)).¹ However, there is still a limited understanding of the pecuniary externalities that arise from earnings-based credit constraints.²

The contribution of this paper is to advance this understanding. Following the findings of recent empirical studies, we examine specific forms of credit limits observed in US corporate loan contracts, such as debt-to-earnings constraints, earnings-to-interest (or interest coverage) constraints, and earnings-based working capital constraints. At the heart of our analysis is a macroeconomic model, inspired by [Dávila and Korinek \(2018\)](#), that we use to study how pecuniary externalities operate through different types of borrowing constraints. In a two-agent three-period structure with capital, an intratemporal production input (labor), and financial asset markets, we first characterize the competitive equilibrium and planner solution for a general formulation of credit constraints, in which various prices and quantities may enter the constraint. We then specialize this general formulation along various dimensions to study the welfare effects of the specific constraints that have been the focus of recent empirical

¹Asset-based constraints have also been studied extensively from a positive point of view, going back at least to the seminal work of [Kiyotaki and Moore \(1997\)](#).

²There are a few important exceptions in which earnings do play a role in credit constraints, such as in the influential work of [Bianchi \(2016\)](#). Typically, when firms' earnings or revenues enter financial constraints in existing studies, this occurs through the presence of working capital. We provide a detailed literature review with further explanations below, and study the interrelation between what we define as earnings-based borrowing constraints and traditional working capital constraints.

work. Our introduction of labor markets to the model is crucial to study earnings-based constraints, with wages being a key price that affects firms' costs and thereby earnings. Modeling labor markets also brings additional challenges in determining the sign of pecuniary externalities, something that the literature has generally pointed out as a difficulty. One of the insights of this paper is that a key challenge with labor is that both supply and demand respond within a given period to changes in aggregate net worth. This is different in most models with collateral constraints, where capital supply at the beginning of a given period is usually fixed so the analysis of price movements can focus on demand responses. We explore additional theoretical conditions on wage determination that allow signing the relevant externalities.³

Our main findings are the following. First, an earnings-based credit constraint, in which the borrower's debt-to-earnings ratio is restricted by a maximum value, leads to 'under-borrowing' from a welfare point of view. The intuition is that when borrowing increases in the current period, borrower net worth will be lower next period. Under the relevant conditions in our model, this reduction in borrower net worth leads real wages to fall next period. A lower real wage means lower costs and higher earnings for firms, which through the earnings-based borrowing constraint allows for more credit. However, when firms borrow today they do not take into account this positive impact of their decisions today on the future borrowing limit through wages. Therefore firms borrow a smaller amount in the current period than what a social planner would implement as a constrained efficient allocation, that is, they under-borrow.

Second, we clarify that this result is the opposite to what holds under an asset-based constraint. With that constraint, the resale value of the borrower's capital serves as collateral. As with real wages, lower borrower net worth leads to lower capital prices next period, but in a collateral constraint this tightens rather than relaxes the future constraint. Borrowers in the current period do not internalize this negative effect of future debt access. They 'over-borrow' relative to the social optimum, in line with previous findings in the literature. Essentially, in an earnings-based credit constraint the wage bill enters with the opposite sign to how the value of capital enters in an asset-based constraint. When real wages and the price of capital respond with the same sign to changes in borrower net worth, then the implications for under- vs. over-borrowing are the opposite for the two constraint types.

³Building on [Dávila and Korinek \(2018\)](#), we disentangle different channels through which externalities occur, labeled *distributive effects* and *constraint effects*. As we explain in the main text, our exposition analyzes the constraint effects, those that operate directly through the credit limit. We focus on whether these effects lead to 'over-borrowing' or 'under-borrowing.' We do not characterize 'over-investment' or 'under-investment' effects, which generally cannot be signed.

Third, we find that an interest coverage constraint leads to either over-borrowing or under-borrowing. Interest coverage constraints are also linked to firms' earnings, but impose a minimum on the ratio of earnings to interest expenses, rather limiting the debt-to-earnings ratio. They are frequently observed for US companies, as emphasized by [Greenwald \(2019\)](#). The intuition we provide for their ambiguous normative implications is that interest coverage constraints feature the same pecuniary externality through wages as the one described in our first result, but the presence of interest expenses in the constraint give rise to an opposing force. Interest rates are inversely related to bond prices, so the interest coverage constraint links higher bond prices with looser credit constraints, for a given level of earnings. Therefore, when bond prices move in the same direction as the price of capital in response borrower net worth changes, then the presence of interest payments affects the constraint in the same direction as the price of capital in an asset-based constraint. As a consequence, from welfare point of view an interest coverage constraint can be interpreted as a mixture between an asset-based and earnings-based constraint, with pecuniary externalities operating through both wages and interest rates in opposite directions. This paper is the first to uncover this property of interest coverage constraints.

Finally, we study several extensions. First, we examine a setting with working capital constraints ([Bianchi and Mendoza, 2010](#); [Jermann and Quadrini, 2012](#); [Bianchi, 2016](#)). We find that when firms need to pre-finance wages and in addition face earnings-based limits on credit, the pecuniary externality through wages is magnified, so the under-borrowing effect becomes stronger. We also clarify differences between our mechanism and those in [Bianchi and Mendoza \(2010\)](#) and [Bianchi \(2016\)](#). Second, we explore how sticky wages would alter our conclusions. Sticky wages respond less to changes in aggregate borrower net worth so the pecuniary externality in an earnings-based constraints is less pronounced. However, wage rigidities give rise to distinct additional externalities, such as aggregate demand externalities ([Farhi and Werning, 2016](#)), which we discuss in the context of our results. Third, while we study a closed economy setting with endogenous interest rates, we connect our findings to the important macroprudential policy considerations in small open economies ([Mendoza, 2006, 2010, Bianchi, 2011](#)). Finally, in our setting the price of consumption goods is normalized to one and the prices of capital, labor and debt are expressed in relative terms. We discuss implications of relaxing this assumption.

Our results have consequences for the design of an effective regulatory system. Collateral constraints imply that credit can be excessive, and that policy-makers should

tame credit booms with macroprudential tools. However, macroprudential policy guided solely by a collateral mechanism could be counterproductive in credit markets where earnings-based borrowing constraints are dominant. The evidence motivating our analysis is based on nonfinancial companies, so the regulation of corporate credit is where our results are likely to be most critical. Collateral constraints, on the other hand, are likely a dominant force in household mortgage markets, where real estate serves as collateral, or in trade between financial institutions, where financial assets are pledged in repurchase agreements. This paper makes the case for studying carefully which pecuniary externalities are critical in which types of credit markets, and shows that the distinction between asset and input prices in credit constraint is of first-order importance for normative analysis.

Contribution to the literature. Our work contributes to three strands of macroeconomic research. The first strand studies pecuniary externalities with financial constraints.⁴ We build on the framework of [Dávila and Korinek \(2018\)](#). The main difference in our setting is that we introduce labor markets as well as additional types of financial constraints. The introduction of labor markets provides new challenges in signing the externalities of interest, and a contribution of this paper is to explore relevant model restrictions. Our insight that higher wages tighten financial constraints is complementary to a related mechanism in [Bianchi \(2016\)](#), where firms face working capital and equity constraints, and do not internalize that when they hire workers, wages increase, which in turn tightens other firms' equity constraints.⁵ A few other studies consider income-related rather than asset-based credit constraints in normative analysis, for example [Bianchi \(2011\)](#) where tradable and nontradable income restrict the economy's external position. [Benigno et al. \(2013\)](#) and [Schmitt-Grohé and Uribe \(2020\)](#) note the possibility of under-borrowing, both in an open economy context and through channels that are different from ours. In [Benigno et al. \(2013\)](#), higher wage income relaxes rather than tightens the borrowing constraint faced by a representative household. In [Schmitt-Grohé and Uribe \(2020\)](#) under-borrowing is a result of precautionary savings in the face of self-fulfilling crises. [Fazio \(2021\)](#) proposes

⁴Important contributions include, but are not limited to, [Greenwald and Stiglitz \(1986\)](#), [Gromb and Vayanos \(2002\)](#), [Mendoza \(2006, 2010\)](#), [Lorenzoni \(2008\)](#), [Jeanne and Korinek \(2010\)](#), [Korinek \(2011\)](#), [Bianchi \(2011\)](#), [Stein \(2012\)](#), [Benigno et al. \(2013\)](#), [Bianchi \(2016\)](#), [Bianchi and Mendoza \(2018\)](#).

⁵The externality in [Bianchi \(2016\)](#) works through higher labor demand having a contemporaneous negative effect on other firms' dividend constraints. In our framework, the pecuniary externality and resulting under-borrowing effect arise from firms' current borrowing exerting a positive effect on future credit limits through borrower net worth. Our setting also features more general preferences and we characterize the role of both labor supply and labor demand for pecuniary externalities.

a framework with earnings-based constraints on firms to study the implications of a credit crunch at the zero lower bound (ZLB) on interest rates. What distinguishes our paper from all of the above is that we analyze a variety of credit constraints that is observed in microeconomic data, allowing us to understand important subtleties in their policy implications. For example, no existing study considers the different normative consequences between debt-to-earnings and interest coverage constraints. Another contribution that distinguishes our paper from the literature is that we characterize a setting in which pecuniary externalities operate through both labor demand and labor supply, and show that this complicates signing the externalities.⁶ Finally, a related paper is [Ottonello, Perez, and Varraso \(2019\)](#) who focus on the timing of collateral constraints and show that conclusions can change depending on whether current or future prices of collateral affect credit access. We instead focus on different variables entering the constraint, going beyond asset-based constraints.⁷

The second strand of research that our work relates to studies aggregate demand externalities ([Schmitt-Grohé and Uribe, 2016](#); [Farhi and Werning, 2016](#); [Korinek and Simsek, 2016](#)). Like our paper, this literature emphasizes externalities for which wage setting is important. However, these aggregate demand externalities do not work through financial constraints, but through the combination of nominal wage rigidities and other constraints, such as the ZLB or a fixed nominal exchange rate. [Wolf \(2020\)](#) studies pecuniary externalities that arise from wage rigidities, but are independent of both financial constraints and aggregate demand channels.

The third strand of research we contribute to provides our empirical background. Recent studies, in particular [Lian and Ma \(2020\)](#) and [Drechsel \(2020\)](#), highlight the distinction between asset-based and earnings-based constraints, but do not consider normative implications. We survey this literature in [Section 2.2](#).

Structure of the paper. [Section 2](#) previews the intuition behind the main insights of this paper, and provides the empirical motivation. [Section 3](#) presents the model. [Section 4](#) carries out the efficiency analysis in the model and presents the main results. [Section 5](#) explores several extensions. [Section 6](#) concludes.

⁶[Bianchi and Mendoza \(2010\)](#), [Bianchi \(2016\)](#), and [Fazio \(2021\)](#) all focus on [Greenwood, Hercowitz, and Huffman \(1988\)](#) (GHH) preferences which eliminate wealth effects on labor supply. In our model this would lead to externalities only operating through labor demand.

⁷We also analyze the timing for the earnings-based constraint, and find that the presence of under-borrowing effects are not sensitive to whether current or future earnings enter the constraint.

2 Main intuition and empirical background

This section previews the main insights of our paper, by illustrating some key economic relationships with minimal formality. It also provides our empirical motivation, by drawing on recent studies of corporate borrowing constraints.

2.1 Financial constraints, prices, and pecuniary externalities

Consider a generic financial constraint faced by an economic agent

$$\Phi(x', z, \tilde{z}) \geq 0 \quad (1)$$

where x' is the net position in a financial asset (negative values of x' indicate borrowing, positive values saving). The $'$ -notation indicates that the choice is made in the current period, with repayment in the next period. z is a vector of endogenous variables chosen by the agent, and \tilde{z} is a vector of endogenous or exogenous variables that the agents takes as given. z and \tilde{z} may contain past, current and future expected realizations of variables. Φ is some function. When prices are included in \tilde{z} , and these prices are affected by the agent's choices in equilibrium, then pecuniary externalities arise: agents do not internalize that their choices move prices in (1).

The *direction* of these price movements is critical for the normative implications of financial constraints. Consider the widely studied version of (1) in which the borrowing agent pledges collateral. Let k' be the choice of capital and q its price. We define $z = k'$ and $\tilde{z} = q$ and $\Phi(x', k', q) = x' + \phi q k'$, which gives

$$x' \geq -\phi q k' \quad (2)$$

where ϕ is a parameter and $0 < \phi < 1$. This constraint imposes that borrowing cannot exceed the amount of collateral $\phi q k'$, a fraction of the market value of capital. Importantly, the agent chooses x' and k' , but takes q as given. q is a market price and a function of the economy's aggregate state variables $q = q(X, K)$, where capitalized letters denote aggregate states. Aggregate states are taken as given by the economic agents, that is, they do not internalize how their individual choice of say x' influences X' and thereby moves prices in the following period.

Now suppose the equilibrium response of q to an increase in aggregate borrower net worth is positive and loosens the financial constraint. The fact that agents do not internalize this equilibrium effect is a source of inefficiency. Borrowing by

an individual agent today reduces future aggregate net worth of borrowers in the economy, which decreases future capital prices and thus tightens future borrowing limits. Not internalizing this pecuniary externality, agents over-borrow today relative to the social optimum.

One of our key insights is that there are financial constraints in which \tilde{z} contains prices other than that of collateral, and that the equilibrium movements of these prices may have the opposite effect on credit constraints. The leading example we point to is when a firm's debt access is limited by its earnings. Formally, (1) is written with $z = [y \ell]$ and $\tilde{z} = w$ and $\Phi(x', [y \ell], w) = x' + \tilde{\phi}(y - w\ell)$:

$$x' \geq -\tilde{\phi}(y - w\ell) \quad (3)$$

y is the firm's output, ℓ is the input choice (labor), and w is the input price (wage). $\tilde{\phi} > 0$ is a parameter. y is related to input ℓ through a production function. The difference between sales and input costs $y - w\ell$ defines the firms' earnings (EBITDA), which restricts debt access. Wages depend on the aggregate state variables, $w = w(X, K)$. If wages also respond positively to an increase in aggregate borrower net worth, the way the price of capital does, then the pecuniary externality from w in the constraint based on earnings has the opposite effect of that from q in the collateral constraint.⁸ While q enters with negative sign in (2) (it positively affects debt space), w enters with positive sign in (3) (it negatively affects debt space).⁹ When q and w respond with the same sign to changes in aggregate state variables, then these price changes move the two borrowing limits in the opposite direction.

The earnings-based constraint therefore leads to under-borrowing: individual firms do not internalize that borrowing reduces future aggregate borrower net worth, which lowers wages and relaxes credit constraints in the future. They thus borrow less today than what is socially desired. We characterize this effect more generally in our full theoretical analysis, after reviewing the recent microeconomic evidence on firms' borrowing constraints.

⁸In our full model, we discuss the formal conditions that need to hold for wages to indeed positively respond to an increase in sector-wide net worth.

⁹In the case of household rather than firm debt, wages could relax rather than tighten constraints, as for example in [Benigno et al. \(2013\)](#). Our focus is most applicable to firm credit.

2.2 Evidence for earnings-based vs. asset-based credit

There is mounting microeconomic evidence in favor of (3) being a relevant constraint for firms. Earnings-based borrowing constraints can arise through debt covenants, which are legal provisions that link debt access to earnings indicators, but also through credit rating methods or through bankruptcy procedures in which recovered debt payments are calculated based on earnings (EBITDA).¹⁰ Table 1 summarizes the findings of three recent papers.

Table 1: DIFFERENT BORROWING CONSTRAINTS: EVIDENCE AND CONSEQUENCES

Study	Asset-based		Earnings-based	
	Debt-to-Earnings		Interest coverage	
Lian and Ma (2020)	Prevalence: 20% (classification procedure; several data sources, including hand-collected data)		Prevalence: 80%	
	Strong sensitivity of corporate borrowing to changes in EBITDA; low sensitivity of corporate borrowing to changes in real estate values (regression analysis, natural experiment based on accounting rule change)			
	Financial amplification (fire sale) effects (Structural model)	Financial amplification dynamics mitigated		
Drechsel (2020)	> 61% of loan debt has earnings-related covenants (Dealscan)			
	Model response of debt to investment shocks \neq empirical response (Structural model, macro data, Compustat-Dealscan)	Model response of debt to investment shocks = empirical response		
	Markups countercyclical (New Keynesian model, macro data)	Markups procyclical		
Greenwald (2019)	Most firms with interest coverage covenants (Compustat-Dealscan)		> 80% of firms which have any loan covenants	
	Weak response to monetary policy State dependence based on level of interest rats (Structural model, Compustat-Dealscan)	Strong response to monetary policy		

Notes: Summary of findings in the literature on earnings-based borrowing constraints and their contrast with asset-based constraints. Both the evidence for these constraints and findings on their consequences are included. In brackets the method and data sources are stated for each result.

Lian and Ma (2020) develop a procedure to classify corporate debt contracts in into primarily asset-based or earnings-based. Combining a variety of data sources, they find that only 20% of US firm credit is asset-based, while 80% is earnings-based. Motivated by this evidence, their paper investigates the marginal effects of changes in different

¹⁰See Chava and Roberts (2008) for a study on debt covenants. Lian and Ma (2020) explain in detail how creditor claims in the event of bankruptcy are calculated in different types of debt contracts. Drechsel (2020) discusses additional implicit links between earnings and debt access.

firm-level variables on borrowing and investment, and find that changes in EBITDA have a strong effects while changes in real estate values have a limited impact. [Lian and Ma \(2020\)](#) also discuss that an earnings-based constraint can insulate firms from fire sale dynamics. While not examined in a normative context, this effect also works through prices in the constraints.¹¹

[Drechsel \(2020\)](#) studies how earnings-based borrowing constraints affect the transmission of macroeconomic shocks. In a theoretical model, investment-specific shocks lower the value of collateral but raise earnings, and should therefore allow for more borrowing with an earnings-based constraint but less borrowing with an asset-based constraint. The empirical dynamics in both macro and firm-level data are in line with the predictions that hold with earnings-based constraints. Furthermore, [Drechsel \(2020\)](#) studies the implications of earnings-based constraints for the behavior of price markups in New Keynesian models, and shows that the constraints affects fundamental macroeconomic stabilization tradeoffs.

[Greenwald \(2019\)](#) studies constraints in which interest payments are restricted by earnings. Such interest coverage constraints often appear alongside the earnings-to-debt restrictions emphasized by [Lian and Ma \(2020\)](#) and [Drechsel \(2020\)](#). Using a combination of model and data, [Greenwald \(2019\)](#) shows that interest coverage constraints amplify changes in monetary policy. The simultaneity with other constraints makes the transmission dependent on the level of interest rates.

While the papers reviewed in Table 1 use evidence from public companies, a very recent study using supervisory data by [Caglio, Darst, and Kalemli-Özcan \(2021\)](#) shows that earnings-based are prevalent for private small and medium-sized companies (SMEs). For SMEs, these constraints are shown to come in the form of bank credit that is secured by accounts receivable and blanket liens.

Taken together, this review of the literature makes clear that microeconomic evidence on earnings-based borrowing constraints is ample and growing, and that their consequences are being explored in various directions. Importantly, while these studies exclusively examine positive consequences at the firm-level and the macroeconomic level, this paper focuses on the normative implications. We do so for the three types of constraints shown in Table 1, and explore several extensions.

¹¹The price of capital drives financial amplification, as in the seminal work of [Kiyotaki and Moore \(1997\)](#), while amplification is muted with earnings. As shown by [Dávila and Korinek \(2018\)](#), amplification effects are not necessary or sufficient for inefficiencies to arise. We study the normative consequences of earnings-based and asset-based constraint and thereby provide a novel angle on how these credit limits may affect regulatory policy.

3 Model

Our model is based on [Dávila and Korinek \(2018\)](#) [henceforth ‘DK18’]. We make two distinct contributions to their framework. First, we generalize it to feature a market for intratemporal production inputs (labor). Second, we allow for a number of additional types of credit constraints. In combination, these two novelties enable us to examine in particular the pecuniary externalities that operate through input prices (wages) in earnings-based credit constraints.

3.1 Economic environment

There are three discrete time periods $t = 0, 1, 2$. The economy is populated by a unit measure of both borrowers and lenders, denoted by index $i \in \{b, l\}$. The state of nature is realized at date $t = 1$ and is denoted by $\theta \in \Theta$.

Preferences. Agent type i derives utility from consumption $c_t^i \geq 0$ and labor $\ell_{st}^i \geq 0$ according to the time separable utility function

$$U^i = \mathbb{E}_0 \left[\sum_{t=0}^2 \beta^t u^i(c_t^i, \ell_{st}^i) \right] \quad (4)$$

where $u^i(\cdot, \cdot)$ is strictly increasing and weakly concave in consumption, and strictly decreasing and weakly convex in labor. While we think of variable $\ell \geq 0$ as labor, the model is general enough to think of it as any intratemporal production input that can be produced by incurring a utility cost. We set $u^i(c_0^i, \ell_{s0}^i) = u^i(c_0^i)$ as there is no production and input choice at date $t = 0$.

Endowments and production technology. There are consumption goods and capital goods. $e_t^{i,\theta}$ is the endowment of consumption goods agent i receives at date $t = 1, 2$ given state θ . Time-0 endowments are denoted by e_0^i . At date $t = 0$, agents can invest $h^i(k_1^i)$ units of consumption good to produce k_1^i units of date-1 capital goods.¹² The functions $h^i(\cdot)$ are increasing and convex and satisfy $h^i(0) = 0$. k_1^i can be used for the production of consumption goods in period $t = 1$ and be carried over for production in period $t = 2$. $k_2^{i,\theta}$ denotes the amount of capital that agent i carries from date 1 to 2. Capital fully depreciates after date 2. To produce consumption goods in $t \geq 1$, agent i

¹²Note that $k_1^{i,\theta} = k_1^i$ since it is chosen in $t = 0$, thus not conditional on the state of nature θ .

employs both capital and labor to produce $F^i(k_t^{i,\theta}, \ell_{dt}^{i,\theta})$ units of the consumption good. $\ell_{dt}^{i,\theta}$ is labor demanded by agent i at date t . The production functions $F^i(\cdot, \cdot)$ are strictly increasing and weakly concave in each argument and satisfy $F^i(0, 0) = 0$. They are allowed to be different across agents $i \in \{b, l\}$.

Market structure. At date $t = 0$, agents trade state-contingent assets that pay 1 unit of the consumption good in period $t = 1$ and state θ . $x_1^{i,\theta}$ denotes the date-0 state- θ purchases by agent i and m_1^θ is the corresponding asset price, taken as given by the agent. Agent i spends $\int_{\theta \in \Theta} m_1^\theta x_1^{i,\theta} d\theta$ in total on these securities. Without further uncertainty between $t = 1$ and $t = 2$, agents trade non-contingent one-period bonds $x_2^{i,\theta}$ at time $t = 1$ at price m_2^θ . There is a competitive labor market.¹³ Wages at date $t \geq 1$ and state θ are denoted by w_t^θ .¹⁴ There is also a market to trade capital at a price q^θ at date 1 after production has taken place. There is no trading of capital at date 2 because of the full depreciation. Taken together, the budget constraints of agent $i \in \{b, l\}$ are

$$c_0^i + h^i(k_1^i) + \int_{\theta \in \Theta} m_1^\theta x_1^{i,\theta} d\theta = e_0^i \quad (5)$$

$$c_1^{i,\theta} + q^\theta \Delta k_2^{i,\theta} + m_2^\theta x_2^{i,\theta} = e_1^{i,\theta} + x_1^{i,\theta} + F^i(k_1^i, \ell_{d1}^{i,\theta}) - w_1^\theta \ell_{d1}^{i,\theta} + w_1^\theta \ell_{s1}^{i,\theta}, \quad \forall \theta \quad (6)$$

$$c_2^{i,\theta} = e_2^{i,\theta} + x_2^{i,\theta} + F^i(k_2^{i,\theta}, \ell_{d2}^{i,\theta}) - w_2^\theta \ell_{d2}^{i,\theta} + w_2^\theta \ell_{s2}^{i,\theta}, \quad \forall \theta \quad (7)$$

where $\Delta k_2^{i,\theta} \equiv k_2^{i,\theta} - k_1^i$. Recall that the state of nature θ materializes in $t = 1$ so there is one set of choices made in the initial period (in expectation of the possible states occurring in the future), whereas choices in the subsequent two period are made conditional on the realized state of nature.

Financial constraints. We assume that there are constraints on the holdings of securities between periods $t = 0$ and $t = 1$, as well as between periods $t = 1$ and $t = 2$. At date $t = 0$, borrowers' holdings of $x_1^b = \{x_1^{b,\theta}\}_{\theta \in \Omega}$ are subject to a constraint

$$\Phi_1^b(x_1^b, k_1^b) \geq 0 \quad (8)$$

¹³The role of market power and rigid wages in labor markets are discussed in Section 5.

¹⁴We assume that both borrower and lender demand and supply labor from and to each other. Especially in the context of the earnings-based borrowing constraint, it is appealing to think of the borrower as a firm and the lender as a household (see Drechsel, 2020). In this case, one would exogenously restrict the borrower to demanding labor and the lender to supplying it. As we will discuss below, this nested version of the model features fewer sources of externalities, but those externalities that remain present still generate the same qualitative results.

At date $t = 1$, borrowers' holdings of $x_2^{b,\theta}$ are subject to a state-dependent constraint

$$\Phi_2^{b,\theta}(x_2^{b,\theta}, k_2^{b,\theta}, \{\ell_{dt}^{b,\theta}, \ell_{st}^{b,\theta}\}_{t=1}^2; q^\theta, w_1^\theta, w_2^\theta, m_2^\theta) \geq 0, \forall \theta \quad (9)$$

This is a general formulation of a financial constraint in this economy, in which *any quantities and prices that are not predetermined at the beginning of period $t = 1$* may restrict access to credit for the borrower. This includes capital and labor, as well as capital prices, wages and asset prices. Section 4 studies the efficiency properties of various types of credit constraints that are special cases of (9). We assume $\Phi_1^l(\cdot) = \Phi_2^{l,\theta}(\cdot) = 0$, that is, lenders are financially unconstrained.

3.2 Decentralized equilibrium

A decentralized equilibrium is defined by the set of real allocations $\{c_0^i, c_1^{i,\theta}, c_2^{i,\theta}, k_1^i, k_2^{i,\theta}, \ell_{d1}^{i,\theta}, \ell_{d2}^{i,\theta}, \ell_{s1}^{i,\theta}, \ell_{s2}^{i,\theta}\}_{i \in \{b,l\}, \theta \in \Theta}$, asset allocations $\{x_1^{i,\theta}, x_2^{i,\theta}\}_{i \in \{b,l\}, \theta \in \Theta}$, and prices $\{q^\theta, w_1^\theta, w_2^\theta, m_1^\theta, m_2^\theta\}_{\theta \in \Theta}$, such that agents solve their optimization problems and markets clear. The market clearing conditions are given by

$$\sum_i [c_0^i + h^i(k_1^i)] \leq \sum_i e_0^i \quad (10)$$

$$\sum_i c_t^{i,\theta} \leq \sum_i [e_t^i + F^i(k_t^{i,\theta}, \ell_{dt}^{i,\theta})], \quad t = 1, 2, \forall \theta \quad (11)$$

$$\sum_i k_2^{i,\theta} \leq \sum_i k_1^i, \quad \forall \theta \quad (12)$$

$$\sum_i \ell_{dt}^{i,\theta} = \sum_i \ell_{st}^{i,\theta}, \quad t = 1, 2, \forall \theta \quad (13)$$

$$\sum_i x_t^{i,\theta} = 0, \quad t = 1, 2, \forall \theta \quad (14)$$

Solution for periods 2 and 1. The solution for the decentralized equilibrium can be obtained via backward induction. Optimal choices at time $t = 2$ are purely intratemporal decisions on consumption and labor supply and demand. Asset positions are settled. In $t = 1$, two sets of variables fully characterize the state of the economy. The first is the holdings of capital by both agents k_1^i . The second one is agents' net worth $n_1^{i,\theta} \equiv e_1^{i,\theta} + x_1^{i,\theta}$.¹⁵ Since agents take aggregate states as

¹⁵DK18 include production output as part of net worth. In our model, the quantity $F^i(k_1^i, \ell_{d1}^{i,\theta})$ is not predetermined because of the labor choice that happens during period $t = 1$. We therefore do not include it as part of the state variable $n_1^{i,\theta}$. We have formally verified that this change would not alter any of the

given it is helpful to distinguish individual states $\{n_1^{b,\theta}, n_1^{l,\theta}, k_1^b, k_1^l\}$ from aggregate states $\{N_1^{b,\theta}, N_1^{l,\theta}, K_1^b, K_1^l\}$. We further define $N_1^\theta \equiv \{N_1^{b,\theta}, N_1^{l,\theta}\}$ and $K_1 \equiv \{K_1^b, K_1^l\}$, and note that the equilibrium prices are functions of the aggregate state variables: $q^\theta(N_1^\theta, K_1)$, $m_2^\theta(N_1^\theta, K_1)$, $w_1^\theta(N_1^\theta, K_1)$, and $w_2^\theta(N_2^\theta(N_1^\theta, K_1), K_2(N_1^\theta, K_1)) = w_2^\theta(N_1^\theta, K_1)$. The optimization problem of an individual agent i at time $t = 1$ conditional on state θ is a function of both sets of state variables

$$V^{i,\theta}(n_1^{i,\theta}, k_1^i; N_1^\theta, K_1) = \max_{\{c_1^{i,\theta}, c_2^{i,\theta}, k_2^{i,\theta}, x_2^{i,\theta}, \ell_{dt}^{i,\theta}, \ell_{st}^{i,\theta}\}} \left\{ u^i(c_1^{i,\theta}, \ell_{s1}^{i,\theta}) + \beta u^i(c_2^{i,\theta}, \ell_{s2}^{i,\theta}) \right\} \quad (15)$$

subject to

$$c_1^{i,\theta} + q^\theta \Delta k_2^{i,\theta} + m_2^\theta x_2^{i,\theta} = e_1^{i,\theta} + x_1^{i,\theta} + F^i(k_1^i, \ell_{d1}^{i,\theta}) - w_1^\theta \ell_{d1}^{i,\theta} + w_1^\theta \ell_{s1}^{i,\theta} \quad [\lambda_1^{i,\theta}] \quad (16)$$

$$c_2^{i,\theta} = e_2^{i,\theta} + x_2^{i,\theta} + F^i(k_2^{i,\theta}, \ell_{d2}^{i,\theta}) - w_2^\theta \ell_{d2}^{i,\theta} + w_2^\theta \ell_{s2}^{i,\theta} \quad [\lambda_2^{i,\theta}] \quad (17)$$

$$\Phi_2^{b,\theta}(x_2^{b,\theta}, k_2^{b,\theta}, \{\ell_{dt}^{b,\theta}, \ell_{st}^{b,\theta}\}_{t=1}^2; q^\theta, w_1^\theta, w_2^\theta, m_2^\theta) \geq 0 \quad [\kappa_2^{i,\theta}] \quad (18)$$

where $\lambda_1^{i,\theta}$, $\lambda_2^{i,\theta}$, and $\kappa_2^{i,\theta}$ are the Lagrange multipliers for each constraint. The first-order conditions for the period-1 maximization problem with respect to $x_2^{i,\theta}$ and $k_2^{i,\theta}$ are

$$m_2^\theta \lambda_1^{i,\theta} = \beta \lambda_2^{i,\theta} + \kappa_2^{i,\theta} \Phi_{2x}^{i,\theta}, \quad (19)$$

$$q^\theta \lambda_1^{i,\theta} = \beta \lambda_2^{i,\theta} F_{2k}^{i,\theta}(k_2^{i,\theta}, \ell_{d2}^{i,\theta}) + \kappa_2^{i,\theta} \Phi_{2k}^{i,\theta}, \quad \forall i, \theta \quad (20)$$

Equations (19) and (20) are the Euler equations for the financial asset and physical investment. Remember that $\Phi_2^{b,\theta}$ is given by (9) and $\Phi_2^{l,\theta} = 0$.

Distributive effects and constraint effects. Our welfare analysis will rely on studying how changes in aggregate states affect welfare. DK18 show that such changes consist of two components: *distributive effects* and *collateral effects*. We refer to the latter type of effects with a slightly more general terminology as *constraint effects*. This is because we study credit constraints that do not necessarily contain “collateral” in the sense of physical assets.¹⁶ Relative to DK18, both distributive and constraint effects feature additional economic forces in our model.¹⁷ Lemma 1 characterizes relevant properties of the date 1 equilibrium.

results in the original framework of DK18, see details in Appendix A.1.

¹⁶Alternatively, one may re-label for example an earnings-based borrowing constraint as a “collateral constraint” in which earnings serve as collateral. We choose to refer to collateral more narrowly as the presence of physical k in the borrowing constraint.

¹⁷In their Online Appendix, DK18 also provide a generalization of the constraint, by allowing it to directly depend on net worth, in addition to the price of capital. Our addition of labor markets allows to focus on specific additional cases that are empirically motivated and deliver new results.

Lemma 1 *The effects of changes in the aggregate state variables $N_1^{j,\theta}$ and K_1^j on agent i 's indirect utility at date 1 are given by*

$$V_{N_1^j}^{i,\theta} \equiv \frac{dV^{i,\theta}(\cdot)}{dN_1^{j,\theta}} = \lambda_1^{i,\theta} \mathcal{D}_{1N^j}^{i,\theta} + \lambda_2^{i,\theta} \mathcal{D}_{2N^j}^{i,\theta} + \kappa_2^{i,\theta} \mathcal{C}_{N^j}^{i,\theta} \quad (21)$$

$$V_{K_1^j}^{i,\theta} \equiv \frac{dV^{i,\theta}(\cdot)}{dK_1^j} = \lambda_1^{i,\theta} \mathcal{D}_{1K^j}^{i,\theta} + \lambda_2^{i,\theta} \mathcal{D}_{2K^j}^{i,\theta} + \kappa_2^{i,\theta} \mathcal{C}_{K^j}^{i,\theta} \quad (22)$$

where $\mathcal{D}_{1N^j}^{i,\theta}$, $\mathcal{D}_{1K^j}^{i,\theta}$, $\mathcal{D}_{2N^j}^{i,\theta}$ and $\mathcal{D}_{2K^j}^{i,\theta}$ are called the distributive effects

$$\mathcal{D}_{1N^j}^{i,\theta} \equiv -\frac{\partial q^\theta}{\partial N_1^{j,\theta}} \Delta K_2^{i,\theta} - \frac{\partial m_2^\theta}{\partial N_1^{j,\theta}} X_2^{i,\theta} - \frac{\partial w_1^\theta}{\partial N_1^{j,\theta}} \ell_{d1}^{i,\theta} + \frac{\partial w_1^\theta}{\partial N_1^{j,\theta}} \ell_{s1}^{i,\theta} \quad (23)$$

$$\mathcal{D}_{1K^j}^{i,\theta} \equiv -\frac{\partial q^\theta}{\partial K_1^j} \Delta K_2^{i,\theta} - \frac{\partial m_2^\theta}{\partial K_1^j} X_2^{i,\theta} - \frac{\partial w_1^\theta}{\partial K_1^j} \ell_{d1}^{i,\theta} + \frac{\partial w_1^\theta}{\partial K_1^j} \ell_{s1}^{i,\theta} \quad (24)$$

$$\mathcal{D}_{2N^j}^{i,\theta} \equiv -\frac{\partial w_2^\theta}{\partial N_1^{j,\theta}} \ell_{d2}^{i,\theta} + \frac{\partial w_2^\theta}{\partial N_1^{j,\theta}} \ell_{s2}^{i,\theta} \quad (25)$$

$$\mathcal{D}_{2K^j}^{i,\theta} \equiv -\frac{\partial w_2^\theta}{\partial K_1^j} \ell_{d2}^{i,\theta} + \frac{\partial w_2^\theta}{\partial K_1^j} \ell_{s2}^{i,\theta} \quad (26)$$

and $\mathcal{C}_{N^j}^{i,\theta}$ and $\mathcal{C}_{K^j}^{i,\theta}$ are called the constraint effects

$$\mathcal{C}_{N^j}^{b,\theta} \equiv \frac{\partial \Phi_2^{b,\theta}}{\partial q^\theta} \frac{\partial q^\theta}{\partial N_1^{j,\theta}} + \frac{\partial \Phi_2^{b,\theta}}{\partial m_2^\theta} \frac{\partial m_2^\theta}{\partial N_1^{j,\theta}} + \frac{\partial \Phi_2^{b,\theta}}{\partial w_1^\theta} \frac{\partial w_1^\theta}{\partial N_1^{j,\theta}} + \frac{\partial \Phi_2^{b,\theta}}{\partial w_2^\theta} \frac{\partial w_2^\theta}{\partial N_1^{j,\theta}} \quad (27)$$

$$\mathcal{C}_{K^j}^{b,\theta} \equiv \frac{\partial \Phi_2^{b,\theta}}{\partial q^\theta} \frac{\partial q^\theta}{\partial K_1^j} + \frac{\partial \Phi_2^{b,\theta}}{\partial m_2^\theta} \frac{\partial m_2^\theta}{\partial K_1^j} + \frac{\partial \Phi_2^{b,\theta}}{\partial w_1^\theta} \frac{\partial w_1^\theta}{\partial K_1^j} + \frac{\partial \Phi_2^{b,\theta}}{\partial w_2^\theta} \frac{\partial w_2^\theta}{\partial K_1^j} \quad (28)$$

$$\mathcal{C}_{N^j}^{l,\theta} = \mathcal{C}_{K^j}^{l,\theta} = 0 \quad (29)$$

for $i \in \{b, l\}$, $j \in \{b, l\}$ and $\theta \in \Theta$.

Proof. The effects of changes in the aggregate state variables (N_1^θ, K_1) on agents' indirect utility are derived by taking partial derivatives of $V^{i,\theta}$ as defined by equations (15) to (18). We make use of the envelope theorem, according to which the derivatives of $\{u^i(c_1^{i,\theta}, \ell_{s1}^{i,\theta}) + \beta u^i(c_2^{i,\theta}, \ell_{s2}^{i,\theta})\}$ with respect to the state variables are 0. We further impose a symmetric equilibrium in which $n^{i,\theta} = N^{i,\theta}$ and $k_1^i = K_1^i$. ■

Remarks on Lemma 1. Note that $\mathcal{D}_{1Nj}^{i,\theta}$, $\mathcal{D}_{1Kj}^{i,\theta}$, $\mathcal{D}_{2Nj}^{i,\theta}$ and $\mathcal{D}_{2Kj}^{i,\theta}$ are called *distributive effects* because

$$\sum_i \mathcal{D}_{1Nj}^{i,\theta} = \sum_i \mathcal{D}_{2Nj}^{i,\theta} = \sum_i \mathcal{D}_{1Kj}^{i,\theta} = \sum_i \mathcal{D}_{2Kj}^{i,\theta} = 0 \quad (30)$$

from the market clearing conditions, that is, they are “zero sum” effects across agents, state by state. Such a relation does not hold for the *constraint effects* $\mathcal{C}_{Nj}^{i,\theta}$ and $\mathcal{C}_{Kj}^{i,\theta}$. These collect any derivatives that multiply the shadow price on the financial constraint $\kappa_2^{i,\theta}$. Comparing Lemma 1 to its analogue in DK18, both our inclusion of labor markets and our more general financial constraint change this characterization.¹⁸ First, wage changes generate both distributive effects and constraint effects. Second, these wage changes occur in both periods 1 and 2, since labor market conditions in $t = 2$ depend on changes in the state of the economy in $t = 1$.¹⁹ These two observations will be important for the earnings-based constraint. Third, we also allow equation (9) to include the asset price m_2^θ so the constraint effects include partial derivatives with respect to this variable.

Solution for period 0. The optimization problem of agent i at time $t = 0$ is

$$\max_{\{c_0^i, k_1^i, x_1^{i,\theta}\}} u^i(c_0^i) + \beta \mathbb{E}_0[V^{i,\theta}(n_1^{i,\theta}, k_1^i; N_1^\theta, K_1)] \quad (31)$$

subject to the time-0 budget constraint (5) and financial constraint (8). Using the envelope conditions $\frac{\partial V^{i,\theta}(\cdot, \cdot)}{\partial n_1^{i,\theta}} = \lambda_1^{i,\theta}$ and $\frac{\partial V^{i,\theta}(\cdot, \cdot)}{\partial k_1^i} = \lambda_1^{i,\theta}(q^\theta + F_{1k}^{i,\theta}(k_1^i, l_{d1}^{i,\theta}))$, the first-order conditions with respect to the asset holding and capital are derived as

$$m_1^\theta \lambda_0^i = \beta \lambda_1^{i,\theta} + \kappa_1^i \Phi_{1x^\theta}^i, \quad (32)$$

$$h^{i'}(k_1^i) \lambda_0^i = \mathbb{E}_0[\beta \lambda_1^{i,\theta} (F_{1k}^{i,\theta}(k_1^i, l_{d1}^{i,\theta}) + q^\theta)] + \kappa_1^i \Phi_{1k}^i, \quad \forall i, \theta \quad (33)$$

where λ_0^i is Lagrange multiplier for (5) and κ_1^i is Lagrange multiplier for (8).

¹⁸In addition to economically meaningful changes relative to DK18, in Appendix A.1, we show that the minor definitional change of net worth, does not change Lemma 1.

¹⁹More precisely, wages in $t = 2$ depend on states in $t = 2$ and states in $t = 2$ depend on states in $t = 1$. As we have emphasized notationally, $w_2^\theta(N_2^\theta(N_1^\theta, K_1), K_2(N_1^\theta, K_1)) = w_2^\theta(N_1^\theta, K_1)$.

4 Efficiency analysis with different credit constraints

This section studies the pecuniary externalities arising from different types of credit constraints. We first determine the constrained efficient allocation in the model economy by solving a (constrained) planner problem. This allocation can be implemented using a set of tax rates, which in turn can be shown to depend on a set of sufficient statistics related to the distributive and constraint effects derived in Lemma 1. After introducing some additional model restrictions required to sign the externalities, we formally characterize the sources and direction of the externalities for various special cases of financial constraints. Finally, we put the results in the broader context of macroprudential regulation.

4.1 Constrained efficient allocation and sufficient statistics

Social planner problem. The social planner chooses allocations in $t = 0$ subject to the same period-0 constraints as the private agents, and subject to optimal behavior of the agents in periods $t = 1, 2$. As discussed in DK18, this corresponds to the problem of a constrained Ramsey planner who can levy taxes in $t = 0$. In our setting, this also has the consequence that the planner lets labor allocations be generated by the decentralized labor market in period $t = 1, 2$. All externalities and the resulting over-borrowing or under-borrowing effects in our framework have their root in the agents' period-0 (borrowing) decisions.²⁰ Formally, the social planner problem is

$$\max_{\{C_0^i \geq 0, K_1^i, X_1^{i,\theta}\}} \sum_i \alpha^i \{u^i(C_0^i) + \beta \mathbb{E}_0[V^{i,\theta}(N_1^{i,\theta}, K_1^i; N^\theta, K_1)]\} \quad (34)$$

$$\text{s.t.} \sum_i [C_0^i + h^i(K_1^i) - e_0^i] \leq 0 \quad (v_0) \quad (35)$$

$$\sum_i X_1^{i,\theta} = 0, \quad \forall \theta \quad (v_1^\theta) \quad (36)$$

$$\Phi_1^i(X_1^i, K_1^i) \geq 0, \quad \forall i \quad (\alpha_i \kappa_1^i) \quad (37)$$

Note that α^b and α^l are Pareto weights that the social planner applies to borrowers and lenders, respectively. The presence of $V^{i,\theta}(N_1^{i,\theta}, K_1^i; N^\theta, K_1)$, which is described by equation (15) to (18), makes clear that the planner takes the private equilibrium of

²⁰This also makes the nature of our externalities through wages distinct from those in the related model of Bianchi (2016). In his framework pecuniary externalities arise from contemporaneous decisions (in particular from labor demand). In ours, current borrowing decisions affect future borrowing constraints through prices, both through labor demand and labor supply changes.

periods 1 and 2 as given. The variables in brackets denote Lagrange multipliers.

Constrained efficient allocation and implementation. The economy's constrained efficient allocation is described by quantities $(C_0^i, K_1^i, X_1^{i,\theta})$, Pareto weights $\alpha^b/\alpha^l = \lambda_0^l/\lambda_0^b$ and shadow prices v_0, v_1^θ , and κ_1^i satisfying the optimality conditions and constraints of the social planner's problem. This allocation can be implemented with a set of tax rate on financial asset and capital purchases. Since the solution of the planner's problem is similar to DK18, we relegate the details to Appendix A.2. The final set of tax rates is

$$\tau_x^{i,\theta} = -\Delta MRS_{01}^{ij,\theta} \mathcal{D}_{1N^i}^{i,\theta} - \Delta MRS_{02}^{ij,\theta} \mathcal{D}_{2N^i}^{i,\theta} - \tilde{\kappa}_2^{b,\theta} C_{N^i}^{b,\theta}, \quad \forall i, \theta \quad (38)$$

$$\tau_k^i = -\mathbb{E}_0[\Delta MRS_{01}^{ij,\theta} \mathcal{D}_{1K^i}^{i,\theta}] - \mathbb{E}_0[\Delta MRS_{02}^{ij,\theta} \mathcal{D}_{2K^i}^{i,\theta}] - \mathbb{E}_0[\tilde{\kappa}_2^{b,\theta} C_{K^i}^{b,\theta}], \quad \forall i \quad (39)$$

$\Delta MRS_{0t}^{ij,\theta} \equiv MRS_{0t}^{i,\theta} - MRS_{0t}^{j,\theta}$ for $t = 1, 2$ denotes the difference between agents in the marginal rate of substitution (MRS) across time, $MRS_{01}^{j,\theta} \equiv \beta \lambda_1^{j,\theta} / \lambda_0^j$, $MRS_{02}^{j,\theta} \equiv \beta \lambda_2^{j,\theta} / \lambda_0^j$. We define $\tilde{\kappa}_2^{b,\theta} \equiv \beta \kappa_2^{b,\theta} / \lambda_0^b$ as the relative shadow price. A positive $\tau_x^{i,\theta}$ implies that agent i saves too much (borrows too little) in the market outcome. The planner thus wants to impose a tax on savings (remember that $x_1^i > 0$ implies saving, $x_1^i < 0$ borrowing). A positive τ_k^i means that agent i invests too much in capital relative to the constrained efficient allocation, so the planner imposes a tax on investment. In our welfare analysis, we generally focus on over-/under-borrowing and do not determine if there is over-/under-investment.²¹

Nature of externalities and sufficient statistics. The optimal tax wedges, in combination with the distributive effects \mathcal{D} and the constraint effects \mathcal{C} derived in Lemma 1, allow us to characterize the externalities in this economy. In essence, by analyzing and interpreting the different terms in (38) and (39), we can understand how outcomes in the market economy deviate from the constrained efficient allocation and how such distortions could be corrected. Building on the earlier terminology we distinguish *distributive externalities* and *constraint externalities*.

The sign and magnitude of *distributive externalities* are determined by the product of:

- (i) The difference in MRS of agents in periods 1 and 2, $\Delta MRS_{01}^{ij,\theta}$ and $\Delta MRS_{02}^{ij,\theta}$

²¹DK18 show that in the case of the collateral constraint where they find over-borrowing, either over- or under-investing may occur. Given this indeterminacy, we only focus on over-borrowing vs. under-borrowing effects across the different borrowing constraints that we analyze.

- (ii) The net trading positions on capital $\Delta K_2^{i,\theta}$, financial assets $X_2^{i,\theta}$, labor supply in periods 1 and 2 $\ell_{s1}^{i,\theta}, \ell_{s2}^{i,\theta}$, and labor demand in periods 1 and 2 $\ell_{d1}^{i,\theta}, \ell_{d2}^{i,\theta}$
- (iii) The sensitivity of equilibrium prices to changes in aggregate state variables $\frac{\partial q^\theta}{\partial N_1^{j,\theta}}, \frac{\partial m_2^\theta}{\partial N_1^{j,\theta}}, \frac{\partial w_1^\theta}{\partial N_1^{j,\theta}}, \frac{\partial q^\theta}{\partial K_1^j}, \frac{\partial m_2^\theta}{\partial K_1^j}, \frac{\partial w_1^\theta}{\partial K_1^j}$

The sign and magnitude of *constraint externalities* are determined by the product of:

- (i) The relative shadow price of the financial constraint $\tilde{\kappa}_2^{i,\theta}$
- (ii) The sensitivity of the financial constraint to the price of capital, asset price and wages for period 1 and 2 $\partial \Phi_2^{i,\theta} / \partial q^\theta, \partial \Phi_2^{i,\theta} / \partial m_2^\theta, \partial \Phi_2^{i,\theta} / \partial w_1^\theta, \partial \Phi_2^{i,\theta} / \partial w_2^\theta$
- (iii) The sensitivity of the equilibrium capital price, asset price and wages in periods 1 and 2 to changes in aggregate states $\frac{\partial q^\theta}{\partial N_1^{j,\theta}}, \frac{\partial m_2^\theta}{\partial N_1^{j,\theta}}, \frac{\partial w_1^\theta}{\partial N_1^{j,\theta}}, \frac{\partial w_2^\theta}{\partial N_1^{j,\theta}}, \frac{\partial q^\theta}{\partial K_1^j}, \frac{\partial m_2^\theta}{\partial K_1^j}, \frac{\partial w_1^\theta}{\partial K_1^j}, \frac{\partial w_2^\theta}{\partial K_1^j}$

Remarks on the externalities. The lists above reveals how distortions in the model can be parsed into a compact list of sufficient statistics. Distributive externalities, those driven by effects which are “zero sum,” depend on the difference in marginal rates of substitution in combination with the positions that agents take in quantities of capital, labor and financial assets in equilibrium. If these externalities were fully corrected, these quantities would be such that marginal rates of substitution equalize across agents. Logically, constraint externalities depend on the shadow price on the financial constraint, in combination with how the constraint moves with prices changes. Finally, both types of externalities depend on how prices react to changes in the aggregate states, making clear any externalities ultimately operate through price changes.

Determining the sign of externalities. Establishing the direction of pecuniary externalities is inherently difficult, even in relatively simple neoclassical settings. Distributive externalities as well as the constraint externalities that operate through changes in aggregate capital cannot be signed, a finding that DK18 refer to as “anything goes.” Fortunately, the sign of constraint externalities that operate through changes net worth can be pinned down based on plausible additional assumptions, and this can provide useful insights into the normative consequences of financial constraints. It is a contribution of our paper to show that a model with labor markets brings about new subtleties in the determination of the sign of pecuniary externalities, and to lay out relevant additional assumptions for such a model. The next section introduces and discusses these additional assumptions, before we examine the normative implications of different types of financial constraints in the following section.

4.2 Additional model restrictions

Before specifying the different borrowing constraints, this section lays out conditions that specialize the economic setting enough to determine the sign of the constraint externalities for each of the financial constraints of interest. This requires introducing restrictions on different price responses to changes in sector-wide net worth in the borrowing and lending sector.

Condition required to analyze collateral constraints. Condition (40) is imposed to characterize the normative implications of asset-based collateral constraints:

$$\frac{\partial q^\theta}{\partial N_1^{i,\theta}} \geq 0, \forall i \quad (40)$$

This restriction is already discussed in DK18, who show that it holds under standard preferences.²² The assumption is that the price of capital increases in sector-wide net worth. We emphasize that in $t = 1$, aggregate capital supply (K_1) is upward sloping in the price of capital, but does not move with all-else-equal changes in net worth. Therefore the response of q^θ to changes in $N_1^{i,\theta}$ is driven by changes in capital demand (K_2^θ). It is plausible that an increase in resources, holding the amount of available capital in the economy fixed, will increase the demand for capital and thus put upward pressure on its price. Our graphical analysis below illustrates the role that the capital market and condition (40) play for the normative implications of collateral constraints.²³

Condition required to analyze earnings-based borrowing constraints. To study the normative implications of earnings-based constraints, we introduce labor markets into the DK18 framework. The motivation is that wages are a key price that affects firms' costs and thereby their earnings. An important insight of this paper is that a general model with labor markets and earnings-based credit constraints requires further restrictions to allow us to determine the sign of the relevant pecuniary externalities. In particular, we impose condition (41), restricting the model to an economic environment in which additional sector-wide net worth puts upward pressure on current wages in

²²DK18 also show that the failure of (40) leads to multiplicity and locally unstable equilibria.

²³There is also an externality that operates through K_1 itself, but recall from DK18 and from our previous discussion that this over-investing vs. under-investing effect can generally not be signed. We therefore focus on over-borrowing vs. under-borrowing forces, which arise from borrowing decisions in $t = 0$ and operate through the effect of net worth on prices in $t = 1$.

equilibrium:

$$\frac{\partial w_1^\theta}{\partial N_1^{i,\theta}} \geq 0, \forall i \quad (41)$$

Relative to condition (40), restricting wage responses in the labor market requires a more involved argument. The reason is that in the case of capital, supply at the beginning of period $t = 1$ is fixed so price responses to changes in net worth are given by shifts in capital demand. In general, this is different for labor, an *intratemporal* production input, for which both supply and demand can shift in response to variation in net worth. Our justification for the specific condition (41) follows a two-step argument, separately for labor demand and labor supply.

First, for *labor demand* we apply the argument that an all-else-equal increase in resources increases the demand for labor in the production process, the same way it does for the other production factor, capital, according to (40). This argument does not rely on the fact that capital demand increases with net worth and that capital and labor are complementary production factors. A capital demand increase will affect capital in the production process in the following period, while labor is demanded for production in the current period. Instead, the argument relies on the presence of financial constraints: for any type of binding borrowing constraint, more net worth should imply a loosening in the constraint, all else equal. This reduces the effective cost of hiring labor, so labor demand increases.²⁴ Second, we study the case of *labor supply* movements in response to net worth changes formally in Appendix B.1. In this Appendix we show that, holding the labor demand curve fixed, as long as the labor demand curve is downward sloping, the labor supply curve is upward sloping, and there is a sufficiently strong direct positive equilibrium effect from changes in net worth on the demand for leisure, then labor supply decreases in changes in sector-wide net worth. Taken together, with demand increasing and supply decreasing in net worth, wages unambiguously rise with higher sector-wide net worth, and (41) holds.

Providing the reasoning for (41) based on understanding both labor demand and labor supply is a central insight of our paper. Note that if agents in the model had GHH preferences, wage changes would purely be driven by changes in labor demand. The GHH assumption is made in related work of Bianchi and Mendoza (2010), Bianchi (2016) and Fazio (2021). The graphical analysis of the model that will follow further below illustrates the role that the labor market and condition (41) play for the normative implications of earnings-based credit constraints.

²⁴A similar relation is present in the model of Bianchi and Mendoza (2010) where more savings carried into a time period raise labor demand and wages in the presence of financial constraints.

Conditions required to analyze interest coverage constraints. In order to study the normative consequences of interest coverage constraints, two restrictions are needed:

$$\frac{\partial m_2^\theta}{\partial N_1^{i,\theta}} \geq 0, \forall i \quad (42)$$

$$\frac{\partial w_2^\theta}{\partial N_1^{i,\theta}} \geq 0, \forall i \quad (43)$$

We introduce (42) because interest payments (the price of the financial asset) enter the interest coverage constraint. We justify it based on a logic similar to (40), stating that the price of savings increases in net worth. Intuitively, with higher sector-wide net worth, all else equal, agents desire to save more to smooth consumption, so the price of savings m_2^θ rises. Indeed, given the unconstrained agents' Euler equations, the price of capital and the financial asset are linked through a no-arbitrage restriction, so the bond price should tend to move in the same way after an all-else-equal changes in net worth as the price of capital, which increases in sector-wide net worth because of (40).

Condition (43) is an extension of (41) to future rather than current wages. Since in the model interest payments are made in $t = 2$ and the interest coverage constraint is written with relation to the ratio of earnings to interest payments in the same period, this constraint requires a restriction on w_2 rather than w_1 . In direct analogy to (41), we impose that future wages respond positively to a rise in sector wide net worth, the same way current wages do.²⁵

4.3 Main results: welfare with different credit constraints

We now turn to the heart of our analysis, the efficiency properties of different forms of financial constraints. Based on our empirical motivation, we examine different functional forms of $\Phi_2^{b,\theta}$ in (18): collateral constraints, earnings-based constraints and interest coverage constraints. In each case, we study the constraint externalities that operate through borrowing decisions in the initial period.

²⁵It turns out that it is more difficult to pin-point this condition in the general model. For example, the reasoning through labor demand behind condition (41) cannot be applied in this case, because there are no further borrowing constraints that restrict decisions in $t = 2$. We apply the more informal argument that we want to characterize earnings-to-interest constraints under similar assumption as we do for debt-to-earnings constraints, and therefore assume the same wage response in both cases.

4.3.1 Pecuniary externalises with a collateral constraint

We begin with the familiar case of a collateral-based financial constraint, in which physical capital limits the access to debt. Formally, when making decisions in period $t = 1$, the borrower's financial constraint (18) takes the following form:

$$\Phi_2^{b,\theta}(\cdot) = x_2^{b,\theta} + \phi q^\theta k_2^{b,\theta} \geq 0 \quad (44)$$

where $0 < \phi < 1$. The borrower maximizes her objective with respect to this constraint as well as the budget constraints (16) and (17).²⁶ The constraint corresponds to equation (2) in the preview we provided in Section 2.

Proposition 1. *A collateral constraint as defined by (44), as long as it binds, gives rise to non-negative constraint externalities. This implies that there is an over-borrowing effect that operates through the constraint externalities.*

Proof. From (44), $\phi > 0$ and $k_2^{b,\theta} \geq 0$ it follows that $\frac{\partial \Phi_2^{b,\theta}}{\partial q^\theta} \geq 0$. According to condition (40), $\frac{\partial q^\theta}{\partial N_1^{i,\theta}} \geq 0$. Therefore $C_{N^i}^{b,\theta} = \frac{\partial \Phi_2^{b,\theta}}{\partial q^\theta} \frac{\partial q^\theta}{\partial N_1^{i,\theta}} \geq 0$. If the constraint binds, $\tilde{\kappa}_2^{b,\theta}$ is non-negative. It follows that the constraint externality resulting from the constraint is non-negative, that is, $\tilde{\kappa}_2^{b,\theta} C_{N^i}^{b,\theta} \geq 0$. This implies that there is over-borrowing operating through the constraint externalities: as is visible in equation (38), the social planner imposes subsidies on savings $\tau_x^{i,\theta}$ in order to induce less borrowing. ■

Interpretation. Proposition 1 confirms one of the main insights of DK18 and the existing literature more generally, and it formalizes the intuition on collateral constraints we previewed in Section 2. The borrower's decisions exert an externality through the market price of capital. As borrowers increase their debt position in period $t = 0$, they reduce aggregate net worth in the borrowing sector in period $t = 1$. Since the price of capital positively depends on sector-wide net worth by condition (40), it falls in $t = 1$.²⁷ Through the collateral constraint, the lower price of capital limits the ability to borrow between $t = 1$ and $t = 2$. As borrowers in $t = 0$ do not internalize this negative effect on future borrowing capability, the amount of debt taken on in $t = 0$ is

²⁶The state θ materializes at time $t = 1$, so decisions are made conditional on the realization of θ and there is no further uncertainty between periods 1 and 2.

²⁷While borrowing more reduces future aggregate net worth in the borrowing sector, it also increases future net worth in the lending sector. By condition (40), the latter effect actually puts upward pressure on the price of capital. However, the net effect of changes in borrower and lender net worth leads to a fall in the price of capital. We highlight this in the graphical illustration we provide further below.

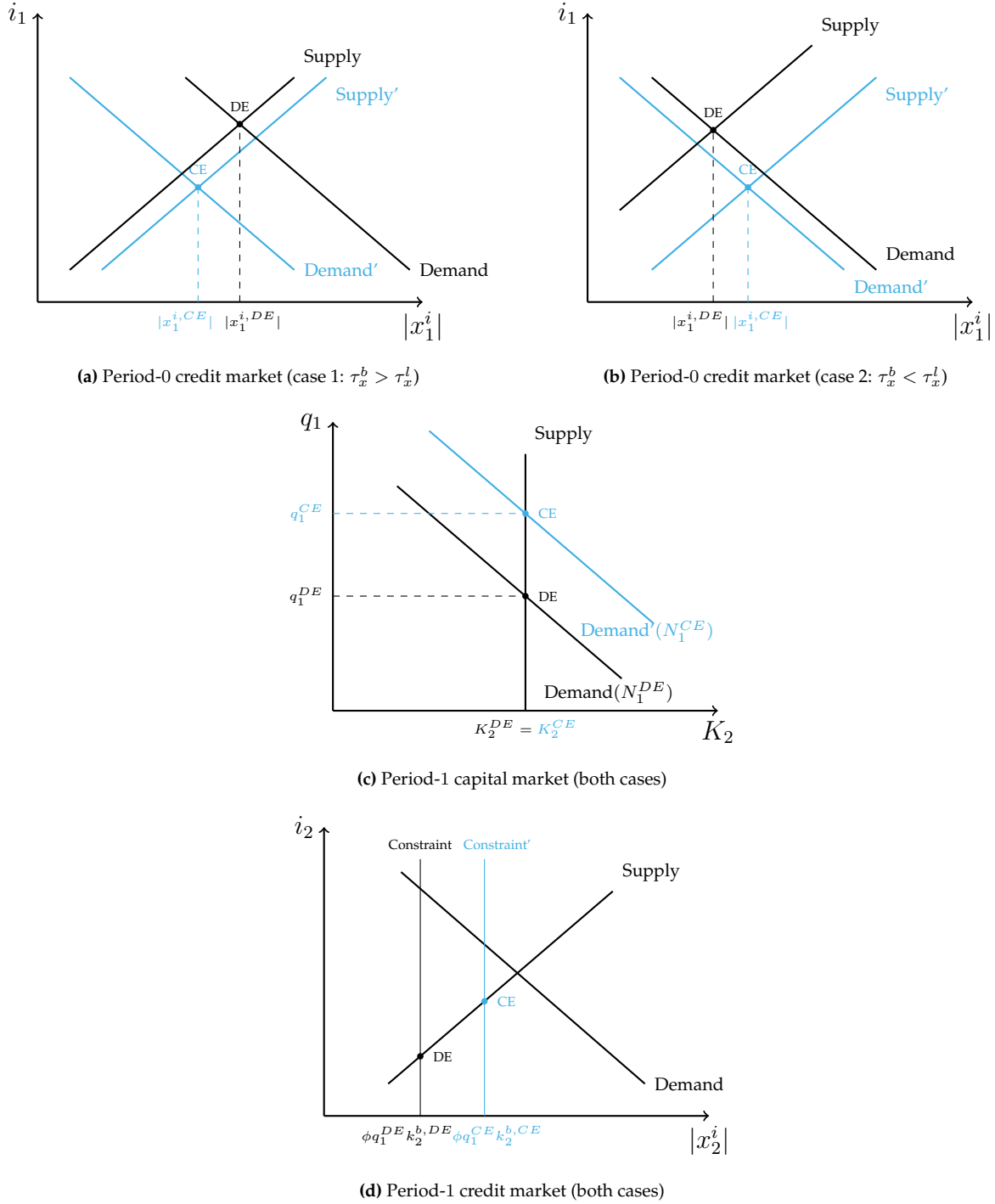
suboptimally high, that is, there is over-borrowing. The social planner internalizes this relation, and thus discourages borrowing in $t = 0$ through subsidies on saving (for any given level of distributive externalities).

Graphical representation. Figure 1 provides the intuition behind Proposition 1 graphically. This graphical analysis will be especially helpful as a benchmark for the results with the earnings-based constraint below. It shows the period-0 credit market, period-1 capital market, and period-1 credit market. In each panel, points CE and DE represent the constrained efficient allocation and the decentralized equilibrium, respectively. The figure conveys how externalities emerge from borrowing decisions in $t = 0$, which through changes in the price of capital affect credit constraints in $t = 1$.

To explain Figure 1, we focus first on the decentralized equilibrium, point DE across Panels (a)-(d). The difference between Panels (a) and (b) only becomes relevant for implementing constrained efficiency, so for now consider Panel (a) to understand the period-0 credit market. The horizontal axis depicts the financial asset position of each agent in absolute value, that is, borrowing or credit demand $-x_1^{b,\theta}$, and saving or credit supply $x_1^{l,\theta}$. The vertical axis captures the interest rate between periods 0 and 1, $i_1^\theta = 1/m_1^\theta - 1$. In the decentralized equilibrium, borrowing and saving positions net out to 0, so $x_1^{b,\theta,DE} + x_1^{l,\theta,DE} = 0 \Rightarrow |x_1^{b,\theta,DE}| = |x_1^{l,\theta,DE}|$. Decisions on the credit market in $t = 0$ impact future net worth and thereby affect investment decisions in period $t = 1$. This is visible in Panel (c), which plots the capital supply curve (given by the vertical line indicating K_1) and the capital demand curve (given by the downward sloping relation between K_2^θ and q_1^θ). Capital supply is in general governed by an upward sloping relationship between K_1 and $q_1^\theta, \forall \theta$. However, since the analysis in the figure traces out the effects of period-0 borrowing externalities, and how these operate through changes in period-1 net worth, capital supply is effectively predetermined at the beginning of period $t = 1$.²⁸ The location of the demand curve does depend on the realization of aggregate net worth. Finally, the capital market equilibrium is linked to the period-1 credit market through the collateral constraint. Panel (d) shows credit supply and credit demand in period 1, by plotting $-x_2^{b,\theta}$ and $x_2^{l,\theta}$ in absolute value against the interest rate i_2^θ . The collateral constraint (44) puts a cap $\phi q_1^{\theta,DE} k_2^{\theta,DE}$ on the amount of credit, represented by a vertical line. Importantly, its location is determined by the market clearing price of capital $q_1^{\theta,DE}$. The decentralized equilibrium in the period-1 credit market is given by the intersection of the constraint and the credit supply curve.

²⁸This would be different in a graphical analysis of pecuniary externalities that operate through over- and under-investment between $t = 0$ and $t = 1$.

Figure 1: MARKET VS. PLANNER ALLOCATIONS: COLLATERAL CONSTRAINT



Notes. Decentralized equilibrium (DE) and constrained efficient equilibrium (CE) in the period-0 credit market, period-1 capital market and period-1 credit market of the model. State θ is omitted from the notation in the labeling. The figure distinguishes case 1 ($\partial q_1^\theta / \partial N_1^{b,\theta} > \partial q_1^\theta / \partial N_1^{l,\theta} \Leftrightarrow \tau_x^{b,\theta} > \tau_x^{l,\theta}$) and case 2 ($\partial q_1^\theta / \partial N_1^{b,\theta} < \partial q_1^\theta / \partial N_1^{l,\theta} \Leftrightarrow \tau_x^{b,\theta} < \tau_x^{l,\theta}$) as described in the text. In both cases, the social planner internalizes that period-0 borrowing decisions reduce equilibrium prices in the market for physical capital in period 1, which tightens the collateral constraint. The constrained efficient allocation features higher capital prices and more credit in period 1, as more saving (less borrowing) is incentivized through taxes/subsidies in period 0.

By Proposition 1, the decentralized equilibrium is not efficient: the social planner distorts borrowing decisions in period 0 to drive up capital prices and thereby relax borrowing constraints in period 1. Under condition (40), sector-wide net worth of both borrowers and lenders positively impacts the price of capital. For the graphical analysis of the constrained efficient allocation, point CE across Panels (a)-(d), two finer cases can be distinguished: in case 1 the impact of the borrower sector net worth on wages is stronger than that of net worth in the lender sector ($\partial q_1^\theta / \partial N_1^{b,\theta} > \partial q_1^\theta / \partial N_1^{l,\theta}$) and in case 2, the opposite is true ($\partial q_1^\theta / \partial N_1^{b,\theta} < \partial q_1^\theta / \partial N_1^{l,\theta}$). In both cases, the social planner alters borrower and lender equilibrium net worth such that capital prices increase in $t = 1$. However, depending on the relative impact of net worth in the different sectors on the price of capital, the planner will tax borrowing (subsidize saving) more heavily for either the borrower or the lender to achieve the desired increase in the price of capital: in case 1, $\tau_x^{b,\theta} > \tau_x^{l,\theta}$, while in case 2, $\tau_x^{b,\theta} < \tau_x^{l,\theta}$. In other words, the planner reverts the over-borrowing of that agent more heavily whose decisions have a stronger impact on capital prices, making capital prices in period 1 rise in either case.²⁹ This is visible in Panels (a) and (b) which show the constrained efficient equilibrium for cases 1 and 2. In both cases, the planner incentivizes lenders to save more and borrowers to borrow less, to counteract the *over-borrowing* motive of both agents.³⁰ As a result, the credit supply curve is located to the right, and the credit demand curve to the left relative to their counterparts in the decentralized case. However, in Panel (a) (case 1), $\tau_x^{b,\theta} > \tau_x^{l,\theta}$, so the decrease in demand from the borrower is larger than the increase in supply from the lender, and the equilibrium quantity of credit is below that of the decentralized equilibrium. With a smaller amount of equilibrium borrowing, borrower net worth in period 1 will be higher while lender net worth will be lower relative to the decentralized equilibrium. Since $\partial q_1^\theta / \partial N_1^{b,\theta} > \partial q_1^\theta / \partial N_1^{l,\theta}$, capital prices are higher. In Panel (b) (case 2), $\tau_x^{b,\theta} < \tau_x^{l,\theta}$ so there is a greater amount of equilibrium borrowing, and borrower net worth in period 1 will be lower while lender net worth will be higher. Since $\partial q_1^\theta / \partial N_1^{b,\theta} < \partial q_1^\theta / \partial N_1^{l,\theta}$, capital prices are higher, as in case 1. This makes clear that while the collateral constraint induces over-borrowing motives (borrowers want

²⁹This can be seen as follows. According to Proposition 1, the constraint externality from the collateral constraint is non-negative, meaning that through equation (38) the planner desires decreasing $\tau_x^{i,\theta}$ for $i \in \{b, l\}$. By equation (38), the size of the tax rate the planner chooses to implement the constrained efficient equilibrium is proportional to the size of the derivative of capital prices to sector wide net worth, that is, $\tilde{\kappa}_2^{b,\theta} C_{N^i}^{b,\theta} \propto \partial q_1^\theta / \partial N_1^{i,\theta}$. As a result, when constraint externalities are corrected by the planner, the relative magnitude of $\partial q_1^\theta / \partial N_1^{b,\theta}$ and $\partial q_1^\theta / \partial N_1^{l,\theta}$ determines the relative magnitude of $\tau_x^{b,\theta}$ and $\tau_x^{l,\theta}$.

³⁰This explanation highlights that in principle, in the case of the lender one could alternatively call the over-borrowing force an ‘under-saving’ effect.

to borrow too much, savers want to save too little), a corrective policy may actually increase or decrease equilibrium credit.

In both cases 1 and 2, the corrective wedges introduced by the planner lead capital demand to shift upward, while changes the net worth induced by the planner do not move the capital supply curve, all else equal. These effects, shown in Panel (c), are the graphical counterpart to our discussion of condition (40) above.³¹ As a result, capital prices in the constrained efficient equilibrium in period $t = 1$ are higher relative to the decentralized equilibrium. As in the decentralized case, the period-1 credit market, shown in Panel (d), is connected to the capital market through the price of capital. An increase in the price of capital loosens the collateral constraint, moving the intersection of the vertical line with the credit supply curve in Panel (d) to the right relative to the decentralized equilibrium. The planner internalizes the effect of period-0 borrowing decisions on future prices, and in turn on future borrowing space. The over-borrowing force in $t = 0$ is corrected through a tax wedge so that borrowers can obtain more credit between period 1 and 2 in the constrained efficient economy.

4.3.2 Pecuniary externalises with an earnings-based borrowing constraint

Consider now the case of an earnings-based borrowing constraint in the spirit of Drechsel (2020). As shown in Section 2.2, there is ample empirical evidence that this is a relevant constraint for US nonfinancial companies. (18) is specified as

$$\Phi_2^{b,\theta}(\cdot) = x_2^{b,\theta} + \tilde{\phi}(F^b(k_1^b, \ell_{d1}^{b,\theta}) - w_1^\theta \ell_{d1}^{b,\theta}) \geq 0 \quad (45)$$

where $\tilde{\phi} > 0$. This constraint implies that access to debt is restricted by the agent's period earnings, calculated as sales minus labor input costs. The constraint corresponds to equation (3) in our illustrative preview in Section 2.

Proposition 2. *An earnings-based borrowing constraint as defined by (45), as long as it binds, gives rise to non-positive constraint externalities. This implies that there is an under-borrowing effect that operates through the constraint externalities.*

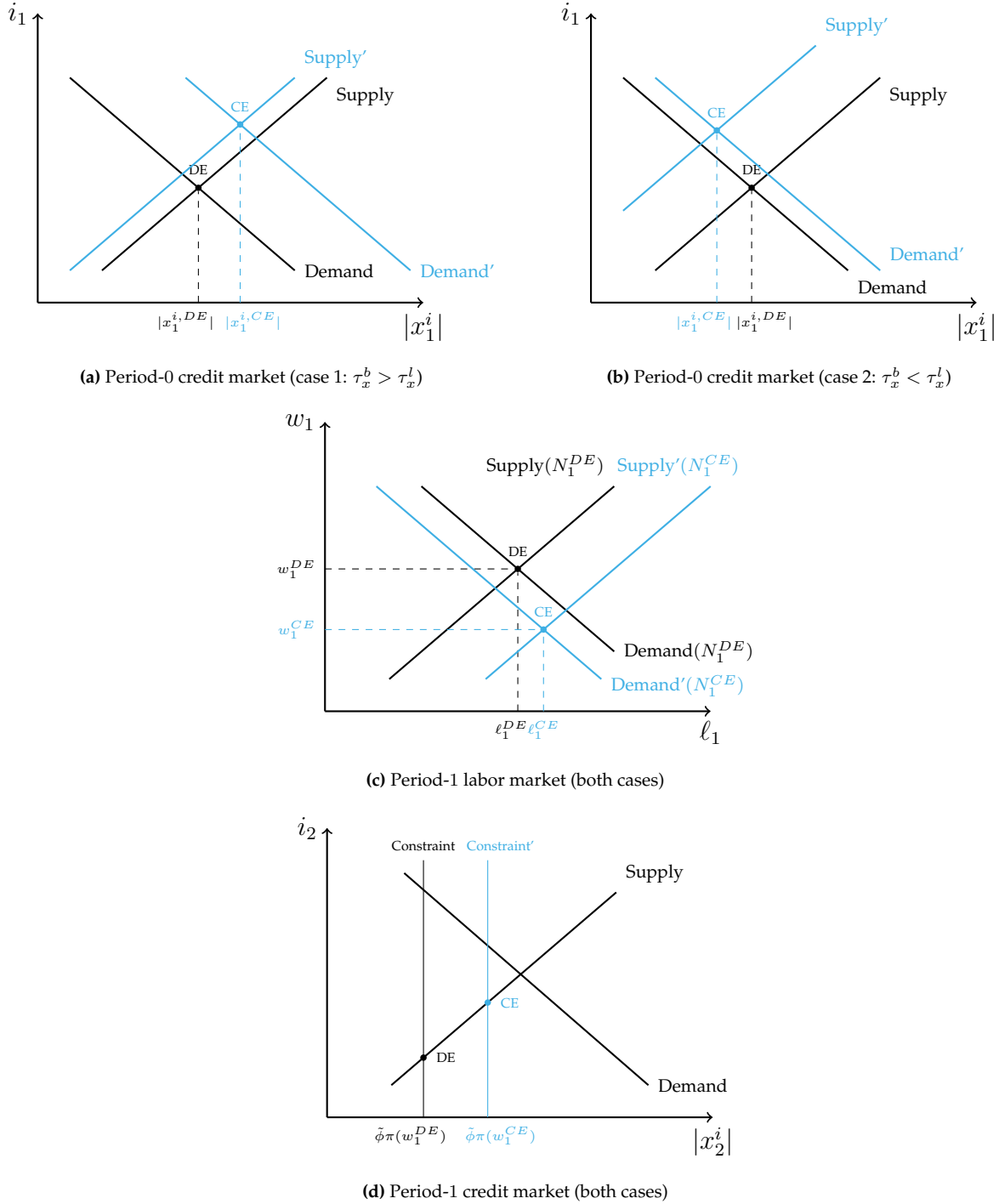
³¹Recall that we focus on pecuniary externalities that operate through changes in net worth, and do not characterize over- or under-investment effects with any of the constraint that we study. In the graphical analysis, we thus abstract from any difference in investment in $t = 0$ that may occur between the decentralized equilibrium and the constrained efficient allocation that the planner implements.

Proof. From (45), $\tilde{\phi} > 0$ and $\ell_{d1}^{b,\theta} \geq 0$ it follows that $\frac{\partial \Phi_2^{b,\theta}}{\partial w_1^\theta} \leq 0$. According to (41), $\frac{\partial w_1^\theta}{\partial N_1^{i,\theta}} \geq 0$. Therefore, $C_{Ni}^{b,\theta} = \frac{\partial \Phi_2^{b,\theta}}{\partial w_1^\theta} \frac{\partial w_1^\theta}{\partial N_1^{i,\theta}} \leq 0$. If the constraint binds, $\tilde{\kappa}_2^{b,\theta}$ is non-negative. It follows that the constraint externality resulting from the constraint is non-positive, $\tilde{\kappa}_2^{b,\theta} C_{Ni}^{b,\theta} \leq 0$. This implies that there is under-borrowing operating through the constraint externalities: as is visible in equation (38) the planner imposes taxes on savings $\tau_x^{i,\theta}$ in order to induce more borrowing. ■

Interpretation. Proposition 2 delivers one of our main theoretical insights, previewed less formally in Section 2. An earnings-based borrowing constraint implies that the borrower takes a debt position that is too small relative to the social optimum. The mechanics of the model are similar our explanation of Proposition 1, but operate through the real wage rate rather than the price of capital. A larger debt position in $t = 0$ reduces net worth in the borrowing sector in $t = 1$, which in turn reduces wages due to condition (41). Borrowers in $t = 0$ do not internalize that lower wages increase earnings and provide slack in the borrowing limit in $t = 1$. Therefore, in the market economy, agents under-borrow. The social planner internalizes the positive effect of borrowing in $t = 0$ on debt capacity in $t = 1$ through wages, and subsidizes (lowers the tax on) borrowing in period $t = 0$ (for a given level of distributive externalities).

Graphical representation. Figure 2 presents a graphical analysis for the case of the earnings-based borrowing constraint. As in Figure 1, points CE and DE represent the constrained efficient allocation and the decentralized equilibrium. The figure conveys how externalities emerge from borrowing decisions in $t = 0$, which through wage determination in the labor market affect credit constraints in $t = 1$. Relative to the case of the collateral constraint, Panel (c) now depicts the labor market in $t = 1$ rather than the market for physical capital. The earnings-based constraint (45) is represented by a vertical line in Panel (d), putting a cap $\tilde{\phi}\pi(w_1^\theta) = \tilde{\phi}(F^b(k_1^b, \ell_{d1}^{b,\theta}) - w_1^\theta \ell_{d1}^{b,\theta})$ on the amount of credit. Its location is affected by the market clearing wage. Similar to the collateral constraint and Figure 1, there is a refinement of condition (41) on the response of wages to changes in net worth (see notes below the figure for details). In both cases, according Proposition 2, the decentralized equilibrium features under-borrowing and the social planner subsidizes borrowing (taxes saving) in $t = 0$. The reason is that in period-0 agents do not internalize that by reducing net worth in period 1 wages are reduced and this relaxes future borrowing constraints. To lower wages and thus create space for the constrained optimal amount of period-1 credit, the planner induces more debt in

Figure 2: MARKET VS. PLANNER ALLOCATIONS: EARNINGS-BASED BORROWING CONSTRAINT



Notes. Decentralized equilibrium (DE) and constrained efficient equilibrium (CE) in the period-0 credit market, period-1 labor market and period-1 credit market of the model. State θ is omitted from the notation in the labeling. The figure distinguishes case 1 ($\partial w_1^\theta / \partial N_1^{b, \theta} > \partial w_1^\theta / \partial N_1^{l, \theta} \Leftrightarrow \tau_x^{b, \theta} > \tau_x^{l, \theta}$) and case 2 ($\partial w_1^\theta / \partial N_1^{b, \theta} < \partial w_1^\theta / \partial N_1^{l, \theta} \Leftrightarrow \tau_x^{b, \theta} < \tau_x^{l, \theta}$) as described in the text. In both cases, the social planner internalizes that period-0 borrowing decisions reduce equilibrium wages in period 1, which relaxes the earnings-based borrowing constraint. The constrained efficient allocation features lower wages and more credit in period 1, as less saving (more borrowing) is incentivized through taxes/subsidies in period 0.

period 0 through corrective tax wedges.

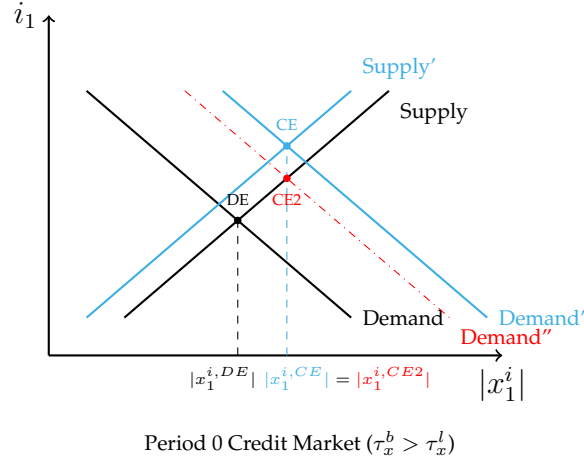
The graphical analysis of the earnings-based borrowing constraint highlights the difficulty that comes with signing pecuniary externalities in our model with labor markets. The condition that wages increase with sector wide net worth in $t = 1$ requires understanding the response of labor demand as well as labor supply, which both move in response to changes in sector wide net worth (see Panel (c) of Figure 2). This is different in the market of capital relevant for the collateral constraint case, where the supply of capital is predetermined at the beginning of the period (compare Panel (c) of Figure 1). When the planner induces more borrowing in the initial period, and thereby reduces borrower net worth in $t = 1$, labor demand falls, while labor supply increases. This unambiguously leads wages to fall, as stated by condition (41), and discussed in detail in Section 4.2.

In conclusion to the graphical analysis, the differences between Figures 2 and 1 reveal the sharp contrast between the normative consequences of the earnings-based and the collateral constraint. In the earnings-based constraint the wage bill enters with the opposite sign to how the value of capital enters the latter constraint. Since wages and the price of capital react with the same sign to changes in borrower net worth, the implications in terms of whether agents borrow too much or too little in period 0 are the opposite for the two constraint types.

Alternative implementations of constrained efficiency. The set of tax rates τ_x^i , $i \in \{b, l\}$ that implements the constrained efficient equilibrium is not unique. There is an infinite number of combination of τ_x^b and τ_x^l that will alter $N_1^{b,\theta}$ and $N_1^{l,\theta}$ such that the same changes in period-1 prices and credit access are achieved. For the case of the earnings-based borrowing constraint we illustrate this in Figure 3, which is constructed as Panel (a) of Figure 2 but also plots an alternative implementation of the constrained efficient equilibrium (denoted *CE2*). This equilibrium represents the polar case in which only the borrower's financial asset position is taxed (borrowing is subsidized), while the lender is not taxed, $\tau_x^l = 0$. As the graph conveys, there is a choice for τ_x^b that achieves the identical equilibrium credit amount as point *CE*. As a result, the labor and credit market outcomes in period 1 would be the same as in Figure 2. A similar argument can be made for case 2 in Figure 2 and for both cases of the collateral constraint analyzed in Figure 1.

Timing of earnings. A result analogous to Proposition 2 can be obtained when the earnings-based constraint is written in terms of *future* earnings. See a related discussion

Figure 3: NON-UNIQUENESS OF IMPLEMENTATION



Notes. This figure repeats Panel (a) of Figure 2 but also plots an alternative implementation of the constrained efficient equilibrium (denoted $CE2$). Constrained efficiency can be achieved with different sets of tax rates $\tau_x^{i,\theta}$, $i \in \{b, l\}$, which give rise to the same change in aggregate net worth (and resulting wage reduction) in the constrained efficient relative to the decentralized equilibrium. In this case, only the borrowers' savings decisions are taxes (borrowing is subsidized), while $\tau_x^{l,\theta} = 0$. State θ is omitted from the notation in the labeling of the graph.

on the timing of earning-based credit restrictions in Drechsel (2020). Formally, we modify the constraint to include earnings in period $t = 2$:

$$\Phi_2^{b,\theta}(\cdot) = x_2^{b,\theta} + \tilde{\phi}(F^b(k_2^{b,\theta}, \ell_{d2}^{b,\theta}) - w_2^\theta \ell_{d2}^{b,\theta}) \geq 0, \quad (46)$$

It is easy to see that in this case, $C_{N^i}^{b,\theta} = \frac{\partial \Phi_2^{b,\theta}}{\partial w_2^\theta} \frac{\partial w_2^\theta}{\partial N_1^{i,\theta}} \leq 0$ because $\frac{\partial \Phi_2^{b,\theta}}{\partial w_2^\theta} \leq 0$ and $\frac{\partial w_2^\theta}{\partial N_1^{i,\theta}} \geq 0$ from (43). Therefore, $\tilde{\kappa}_2^{b,\theta} C_{N^i}^{b,\theta} \leq 0$, so under the restrictions we make on the model this modified version of the constraint also implies under-borrowing. This is interesting in light of the findings of Ottonello, Perez, and Varraso (2019) who emphasize the importance of timing assumptions in the context of collateral constraints.

Possibility of simplified labor market structure. In the model, both borrowers and lenders supply and demand labor. The empirical relevance of earnings-based constraints largely pertains to situations in which we think of the borrower as a firm. We have therefore studied an alternative version of our model in which we restrict the borrower to demanding labor and the lender to supplying it. This version of the model amounts to a special case in which the borrowing sector net worth does not affect equilibrium wages through the borrower's labor supply decisions. Nevertheless, due to the presence of labor demand effects, and the effect of the lending sector's labor

supply, the market equilibrium is still not efficient. The planner would levy a tax that corrects the effect of wages on future borrowing constraints. The details are provided in Appendix B.2.

4.3.3 Pecuniary externalities with an interest coverage constraint

Finally, we consider an interest coverage constraint, which restricts the amount of (future) earnings relative to interest rate payments on the financial asset.³² This interest coverage ratio is a popular indicator used in debt covenants and its consequences for the transmission of monetary policy shocks have recently been studied by Greenwald (2019). We denote the interest rate in relation to the price of debt as $i_2^\theta = \frac{1}{m_2^\theta} - 1$. The constraint is written as

$$\Phi_2^{b,\theta}(\cdot) = x_2^{b,\theta} + \hat{\phi} \frac{F^b(k_2^{b,\theta}, \ell_{d2}^{b,\theta}) - w_2^\theta \ell_{d2}^{b,\theta}}{i_2^\theta} \geq 0 \quad (47)$$

where $\hat{\phi} > 0$. Equation (47) makes clear that the interest coverage constraints can be interpreted as variant of the earnings-based constraints, where interest payments on debt $i_2^\theta x_2^{b,\theta}$ rather than the level of debt $x_2^{b,\theta}$ are restricted by earnings.

Proposition 3. *An interest coverage constraint as defined by (47), as long as it binds, gives rise to a product of constraint externalities that cannot be unambiguously signed. It results in either under-borrowing or over-borrowing depending on the relative absolute magnitude of two distinct externalities. As long as both earnings and the interest rate are positive, the first one operates through earnings and is non-positive, the second operates one through the interest rate and is non-negative.*

Proof. The relevant constraint externality for equation (47) is $C_{Ni}^{b,\theta} = \frac{\partial \Phi_2^{b,\theta}}{\partial w_2^\theta} \frac{\partial w_2^\theta}{\partial N_1^{i,\theta}} + \frac{\partial \Phi_2^{b,\theta}}{\partial i_2^\theta} \frac{\partial i_2^\theta}{\partial N_1^{i,\theta}}$. For the first term, as long as $i_2^\theta > 0$, the same logic as in the proof of Proposition 2 applies, so $\frac{\partial \Phi_2^{b,\theta}}{\partial w_2^\theta} \frac{\partial w_2^\theta}{\partial N_1^{i,\theta}} \leq 0$. For the second term, if $F^b(k_2^{b,\theta}, \ell_{d2}^{b,\theta}) - w_2^\theta \ell_{d2}^{b,\theta} > 0$, then $\frac{\partial \Phi_2^{b,\theta}}{\partial i_2^\theta} \leq 0$. According to (42), $\frac{\partial i_2^\theta}{\partial N_1^{i,\theta}} \leq 0$. Therefore $\frac{\partial \Phi_2^{b,\theta}}{\partial i_2^\theta} \frac{\partial i_2^\theta}{\partial N_1^{i,\theta}} \geq 0$. If the constraint binds, $\tilde{\kappa}_2^{b,\theta}$ is non-negative. It follows that the constraint externality resulting from the constraint is non-positive if $\left| \frac{\partial \Phi_2^{b,\theta}}{\partial w_2^\theta} \frac{\partial w_2^\theta}{\partial N_1^{i,\theta}} \right| > \left| \frac{\partial \Phi_2^{b,\theta}}{\partial i_2^\theta} \frac{\partial i_2^\theta}{\partial N_1^{i,\theta}} \right|$, which would imply $\tilde{\kappa}_2^{b,\theta} C_{Ni}^{b,\theta} \leq 0$.

³²Similar to (45), this debt limit links to earnings (EBITDA), but we define it in terms of earnings one period ahead: as interest payments need to be defined *between two periods*, we compute the coverage ratio as earnings in $t = 2$ divided by interest payments between $t = 1$ and $t = 2$.

This would imply under-borrowing: as is visible in equation (38) the planner imposes taxes on savings $\tau_x^{i,\theta}$ in order to induce more borrowing. The constraint externality is non-negative if $\left| \frac{\partial \Phi_2^{b,\theta}}{\partial w_2^\theta} \frac{\partial w_2^\theta}{\partial N_1^{i,\theta}} \right| < \left| \frac{\partial \Phi_2^{b,\theta}}{\partial i_2^\theta} \frac{\partial i_2^\theta}{\partial N_1^{i,\theta}} \right|$, which would imply $\tilde{\kappa}_2^{b,\theta} C_{N^i}^{b,\theta} \geq 0$. This would imply over-borrowing: as is visible in equation (38) the planner imposes a subsidy on savings $\tau_x^{i,\theta}$ in order to induce less borrowing. ■

Interpretation. Proposition 3 delivers the novel insight that, on the one hand, interest coverage constraints contain an element of under-borrowing, where similar to an earnings-based constraint a pecuniary externality operates through the wage bill. On the other hand, however, rising prices of financial assets (falling interest rates) induce over-borrowing. The social planner needs to assess quantitatively whether the pecuniary externality operating through wages or the one embodied in interest rate changes is stronger. In one case, borrowing in period $t = 0$ should be supported, in the other case, incentives should be provided to reduce period-0 credit.

Interestingly, the price of the financial asset and the price of capital are linked through no-arbitrage restrictions imposed by the unconstrained agents' Euler equations. This is a restriction by which q^θ and $\frac{1}{i_2^\theta}$ should tend to move in the same way in response to changes in sector-wide net worth. From a welfare point of view, one can therefore interpret the interest coverage constraint as a "mixture" of an earnings-based and an asset-based borrowing constraint. To the best of our knowledge, this paper is the first to uncover this property of interest coverage constraints, which so far have only been studied from a positive angle (see e.g. [Greenwald, 2019](#)).

4.4 The importance of the results

Our findings highlight that the optimal design of macroprudential interventions depends critically on the specific nature of financial constraints. Bearing in mind that our analysis remains limited to constraint externalities that operate through borrowing decision, the above findings reveal mechanisms through which meaningful inefficiencies operate. Proposition 1 provides guidance for policy to monitor asset prices whenever these asset prices are an important limiting factor on borrowers' ability to obtain debt. What comes to mind is the regulation of mortgage markets. Proposition 2 is motivated by microeconomic evidence on earnings-based constraints faced by nonfinancial companies, and the results points to a role for considering the relation

between labor markets and corporate credit markets in supervision.³³ Proposition 3 illustrates the possibility of constraints, again in connection to nonfinancial firms, in which several forces operate simultaneously. To gauge the relative strengths of competing channels, it is imperative to base supervision on sufficient microeconomic detail, and to understand further economic forces that may interact with the pecuniary externalities we characterize.

5 Extensions and additional considerations

This section broadens our analysis by considering several modification of the financial constraints and the economic environment. We also connect our findings to relevant insights in the related literature.

5.1 Working capital

The constraints in our model limit an intertemporal financial position. In practice, it is common that firms also hold shorter-run (or intratemporal) debt positions, for example to pre-finance production inputs before revenues are collected. An insight that comes naturally out of our framework is that if the access to such *working capital* (in addition to other debt) is limited by an earnings-based constraint, this enhances the strength of the externality that operates through wages. To see this, suppose a firm takes the intertemporal position x' as above, and in addition pre-finances a fraction ψ of its wage bill with an intraperiod working capital loan $x_{wc} = -\psi w\ell$. Noting that the total financial position of the firm is $x' + x_{wc}$, an earnings-based constraint on total debt and working capital takes the form

$$x' - \psi w\ell \geq -\tilde{\phi}(F(k, \ell) - w\ell) \quad (48)$$

which can be rearranged to

$$x' \geq -\tilde{\phi}F(k, \ell) + (\tilde{\phi} + \psi)w\ell \quad (49)$$

It is easy to see that this constraint corresponds to equation (45), with the only difference that the parameter multiplying the wage bill is $(\tilde{\phi} + \psi) > \tilde{\phi}$. The presence of working

³³In fact, it is also the case that mortgages have income-flow related (*payment-to-income*) constraints in addition to asset-based (*loan-to-value*) constraints (Greenwald, 2018), so Proposition 2 may have some relevance for mortgage markets as well.

capital thus enhances the strength of the externality coming from an earnings-based constraint, and may therefore lead to a more pronounced under-borrowing effect and an even more important role for macroprudential policy.³⁴

The same logic applies to working capital in combination an interest coverage constraint. Recall that this constraint entails two competing forces, with one operating through earnings (wages) and the other one through interest rates. In combination with working capital, the wage externality becomes stronger and the constraint thus more likely to result in under-borrowing.

In their seminal work, [Bianchi and Mendoza \(2010\)](#) and [Bianchi \(2016\)](#) propose models with working capital and collateral constraints. In [Bianchi and Mendoza \(2010\)](#), working capital payments relate to wages, as in (49), but the sum of intertemporal debt and working capital are restricted by collateral rather than by earnings. In our notation, this constraint is of the form

$$x' \geq -\phi q k' + \psi w \ell \quad (50)$$

[Bianchi and Mendoza \(2010\)](#) find that in their framework the collateral price effect (through the debt limit) is stronger than the wage effect (through working capital). Our setting with an earnings-based borrowing constraint is quite different because both the debt limit itself and the working capital component depend on wages. While the setting of [Bianchi and Mendoza \(2010\)](#) features two offsetting effects through ϕq and ψw , the earnings-based constraint in combination with working capital gives rise to two effects through $\tilde{\phi} w$ and ψw which go in the same rather than in opposing directions. This gives to a potentially strong under-borrowing force.

In [Bianchi \(2016\)](#) firms need to pre-finance payments not only to workers but also shareholders and bondholders, which amounts to their full revenue stream appearing on the left hands side of the constraint. Such a formulation is close to that in [Jermann and Quadrini \(2012\)](#), and in our notation reads

$$x' \geq -\phi q k' + \tilde{\psi} F(k, \ell) \quad (51)$$

In addition, firms in [Bianchi \(2016\)](#) face equity constraints. In his setting, firms do not internalize that hiring puts upward pressure on wages, which tightens other firms' constraints contemporaneously. This is different and complementary to our mechanism, where the externality unfolds intertemporally: firms' current borrowing

³⁴To see this formally, in the proof of Proposition 2 a larger parameter multiplying the wage increases $\frac{\partial \Phi_2^{b,\theta}}{\partial w_1^\theta}$ and thus makes $C_{N^i}^{b,\theta}$ more negative.

exerts a positive effect on borrower constraints through borrower net worth affecting equilibrium wages. Note also that households in the models of [Bianchi and Mendoza \(2010\)](#) and [Bianchi \(2016\)](#) have GHH preferences.³⁵ The more general preferences in our framework allow study the role of both labor supply and labor demand for pecuniary externalities with financial constraints in which wages play a central role.

5.2 Sticky wages

As our formal analysis has highlighted, the strength of pecuniary externalities is influenced by the response of prices to aggregate state variables. If wages display rigidities, they would respond less to changes in aggregate conditions and the externalities might be less pronounced. This can be illustrated by considering an ad-hoc wage rule in the spirit of [Schmitt-Grohé and Uribe \(2016\)](#):

$$w = \chi w^* + (1 - \chi)w_{-1}, \quad 0 < \chi < 1 \quad (52)$$

where w^* is a flexible wage component and w_{-1} is the previous period's wage. The earnings-based constraint would be

$$x' \geq -\tilde{\phi}F(k, \ell) + \tilde{\phi}\chi w^* \ell + \tilde{\phi}(1 - \chi)w_{-1} \ell \quad (53)$$

Since $0 < \chi < 1$, we have that $\tilde{\phi}\chi < \tilde{\phi}$, so the derivative of the constraint with respect to the flexible wage component (the one reacting contemporaneously to changes in state variables) is smaller, and the under-borrowing force is less pronounced than in the case where wages are fully flexible. To quantify the overall impact of the under-borrowing effect from earnings-based constraint, policy-makers therefore need a good sense of the wage determination process and the degree of wages stickiness.

Importantly, however, rigid wages themselves would give rise to additional externalities independent of financial constraints. For example, [Wolf \(2020\)](#) builds a model with downward wage rigidities in which firms do not internalize that their hiring decisions in a boom drive up wages, which makes the downward nominal wage rigidity more likely to bind in a future downturn. In addition, sticky wages will give rise to aggregate demand externalities of the type highlighted by [Schmitt-Grohé and Uribe \(2016\)](#) and [Farhi and Werning \(2016\)](#).³⁶ The planner would need to take these

³⁵[Fazio \(2021\)](#) studies an earnings-based constraint in a setting with GHH preferences, but without working capital constraints.

³⁶[Korinek and Simsek \(2016\)](#) focus on price rigidities and aggregate demand externalities.

additional externalities into account for a complete welfare characterization. Studying the joint effect of different sources of pecuniary externalities through wages and wage stickiness would be required for sharper insights for macroprudential policy.

5.3 Small open economy vs. endogenous interest rates

Our model features a fully endogenous interest rate $i = \frac{1}{m} - 1$, which is determined by financial market clearing. This stands in contrast with a body of work that has studied financial frictions and pecuniary externalities in small open economies (SOE), where the interest rate is fixed and assumed to be determined in international markets. A prominent example is [Bianchi \(2011\)](#), who studies the welfare consequences of borrowing constraints in an SOE environment and highlights the pecuniary externalities that operate through external financial positions of emerging economies (see also [Mendoza, 2006, 2010](#), [Jeanne and Korinek, 2010](#)).

There are two reasons why we focus on an endogenous interest rate. First, the empirical evidence on financial constraints reviewed in Section 2.2 is largely provided for the United States, an economy for which the assumption of a fixed interest rate is less suitable. Second, a setting with fixed interest rates would trivially render the interest coverage constraint similar to the earnings-based constraint when it comes to welfare consequences. Formally, this is easy to see when replacing i_2^θ with a constant \bar{i} in equation (47). An endogenous interest rates thus allows us to study a wider range of credit constraints.

Interestingly, in the context of emerging markets, macroeconomic models in the literature also feature varying forms of borrowing constraints, in which capital, endowments, (tradable) production output, or combinations of these variables may restrict access to debt.³⁷ Since the microeconomic evidence on the specific forms of constraints is thinner for emerging economies, we believe that it would be promising to conduct an analysis similar to what [Lian and Ma \(2020\)](#) do for US and Japanese companies, but with a focus on emerging markets.³⁸

³⁷Examples of such constraints in the context of emerging markets can be found in [Mendoza \(2006\)](#), [Korinek \(2011\)](#), and in related work.

³⁸When it comes to emerging market debt of non-financial firms, there has been emphasis on understanding currency mismatches (see for example [Céspedes, Chang, and Velasco, 2004](#)). Less is known about the specific anatomy of the financial constraints that these firms face, e.g. whether their contracts are asset-based or earnings-based. This dimension may be important on top of currency mismatches and it may even interact with them.

5.4 Output prices vs. input prices

In the earnings-based borrowing constraint, w denotes the price of labor, while the price of output is normalized to 1. As a consequence, in the context of our real model, it is the *relative* price of production inputs through which the externality operates. The fact that we emphasize the role of prices in credit constraints begs the question whether it is relevant to also study output (sales) price variation for firms as a source of pecuniary externalities.

Meaningful variation in output prices could be introduced by extending our model to a multi-good environment. We provide thoughts on two possibilities. The first would be to make firms producers in monopolistically competitive markets, which gives them pricing power. In this environment, prices are choice variables of the firm, so firms would actually internalize how their own price setting affects the constraints. However, firms would not internalize how their individual choices affect aggregate inflation, which in turn could affect (nominal) debt limits. This suggests an interesting avenue for further research.³⁹

The second possibility would be a competitive multi-good environment, where choices of agents affect relative prices between different goods, and these effects on relative prices are not internalized. This possibility is explored by [Fazio \(2021\)](#), who considers an extension of her model with a manufacturing and a service sector. Manufacturing producers face a credit constraint that depends on their earnings, but take the relative price of manufacturing goods as given, which gives rise to the possibility that manufacturing prices are inefficiently high. Relative output price externalities can feature alongside the input (labor) price externalities resulting from earning-based constraints that we have characterized. Relative output price changes could also occur between the internationally tradable and the nontradable sector, relevant for the discussion on the open economy literature above.

6 Conclusion

This paper examines the implications of different types of credit constraints for macroprudential policy. Our analysis is guided by recent empirical research, and shows that whether debt is backed by collateral or linked to firms' earnings flows has sharply different normative consequences. The pecuniary externality that operates through

³⁹[Drechsel \(2020\)](#) studies earnings-based and collateral constraints in a model with monopolistically competitive firms and nominal rigidities, but does not derive normative implications.

wages in earnings-based borrowing constraints generally prescribes that a regulatory authority should encourage nonfinancial firms to borrow, as they do not internalize the positive effect of their current borrowing on future borrowing capacity. While a variety of competing economic forces must be considered in the design of an effective regulatory system, this paper provides an internally consistent theoretical treatment of how the specific nature of borrowing constraints shapes firm decisions, which can help to inform macroprudential policy.

References

- BENIGNO, G., H. CHEN, C. OTROK, A. REBUCCI, AND E. R. YOUNG (2013): “Financial crises and macro-prudential policies,” *Journal of International Economics*, 89, 453–470.
- BIANCHI, J. (2011): “Overborrowing and Systemic Externalities in the Business Cycle,” *American Economic Review*, 101, 3400–3426.
- (2016): “Efficient Bailouts?” *American Economic Review*, 106, 3607–59.
- BIANCHI, J. AND E. G. MENDOZA (2010): “Overborrowing, Financial Crises and ‘Macro-prudential’ Taxes,” Working Paper 16091, NBER.
- (2018): “Optimal Time-Consistent Macroprudential Policy,” *Journal of Political Economy*, 126, 588–634.
- CAGLIO, C. R., R. M. DARST, AND S. KALEMLI-ÖZCAN (2021): “Risk-Taking and Monetary Policy Transmission: Evidence from Loans to SMEs and Large Firms,” Tech. rep., National Bureau of Economic Research.
- CESPEDES, L. F., R. CHANG, AND A. VELASCO (2004): “Balance Sheets and Exchange Rate Policy,” *American Economic Review*, 94, 1183–1193.
- CHAVA, S. AND M. R. ROBERTS (2008): “How Does Financing Impact Investment? The Role of Debt Covenants,” *The Journal of Finance*, 63, 2085–2121.
- DÁVILA, E. AND A. KORINEK (2018): “Pecuniary Externalities in Economies with Financial Frictions,” *The Review of Economic Studies*, 85, 352–395.
- DRECHSEL, T. (2020): “Earnings-based borrowing constraints and macroeconomic fluctuations,” *Working paper*.
- FARHI, E. AND I. WERNING (2016): “A Theory of Macroprudential Policies in the Presence of Nominal Rigidities,” *Econometrica*, 84, 1645–1704.
- FAZIO, M. (2021): “Financial Stabilisation Policies in a Credit Crunch: Zombie Firms and the Effective Lower Bound,” *Working paper, London School of Economics*.
- GREENWALD, B. C. AND J. E. STIGLITZ (1986): “Externalities in Economies with Imperfect Information and Incomplete Markets,” *The Quarterly Journal of Economics*, 101, 229–264.
- GREENWALD, D. (2018): “The mortgage credit channel of macroeconomic transmission,” *Working Paper*.
- (2019): “Firm debt covenants and the macroeconomy: The interest coverage channel,” *Working paper*.

- GREENWOOD, J., Z. HERCOWITZ, AND G. W. HUFFMAN (1988): "Investment, capacity utilization, and the real business cycle," *The American Economic Review*, 402–417.
- GROMB, D. AND D. VAYANOS (2002): "Equilibrium and welfare in markets with financially constrained arbitrageurs," *Journal of Financial Economics*, 66, 361–407.
- JEANNE, O. AND A. KORINEK (2010): "Excessive Volatility in Capital Flows: A Pigouvian Taxation Approach," *American Economic Review*, 100, 403–07.
- JERMANN, U. AND V. QUADRINI (2012): "Macroeconomic Effects of Financial Shocks," *American Economic Review*, 102, 238–71.
- KIYOTAKI, N. AND J. MOORE (1997): "Credit Cycles," *Journal of Political Economy*, 105, 211–248.
- KORINEK, A. (2011): "Excessive dollar borrowing in emerging markets: Balance sheet effects and macroeconomic externalities," *Working paper*.
- KORINEK, A. AND A. SIMSEK (2016): "Liquidity Trap and Excessive Leverage," *American Economic Review*, 106, 699–738.
- LIAN, C. AND Y. MA (2020): "Anatomy of Corporate Borrowing Constraints*," *The Quarterly Journal of Economics*, 136, 229–291.
- LORENZONI, G. (2008): "Inefficient Credit Booms," *The Review of Economic Studies*, 75, 809–833.
- MENDOZA, E. G. (2006): "Lessons from the Debt-Deflation Theory of Sudden Stops," *The American Economic Review*, 96, 411–416.
- (2010): "Sudden Stops, Financial Crises, and Leverage," *American Economic Review*, 100, 1941–66.
- OTTONELLO, P., D. J. PEREZ, AND P. VARRASO (2019): "Are Collateral-Constraint Models Ready for Macroprudential Policy Design?" *Unpublished Manuscript*.
- SCHMITT-GROHÉ, S. AND M. URIBE (2016): "Downward Nominal Wage Rigidity, Currency Pegs, and Involuntary Unemployment," *Journal of Political Economy*, 124, 1466–1514.
- (2020): "Multiple Equilibria in Open Economies with Collateral Constraints," *The Review of Economic Studies*, 88, 969–1001.
- STEIN, J. C. (2012): "Monetary Policy as Financial Stability Regulation," *The Quarterly Journal of Economics*, 127, 57–95.
- WOLF, M. (2020): "Pecuniary externalities in economies with downward wage rigidity," *Journal of Monetary Economics*, 116, 219 – 235.

APPENDIX FOR

Earnings-based borrowing constraints and pecuniary externalities

by Thomas Drechsel and Seho Kim

A Detailed derivations

A.1 Insensitivity to re-definition of net worth

In our model, we do not include production output as part of the definition of net worth. This is because output is not predetermined at the beginning of the period due to labor markets clearing during the period. It therefore cannot be a state variable of the model. To ensure that this definitional change does not affect the results, we show in this Appendix that a re-definition of net worth along the same lines gives identical results in the original [Dávila and Korinek \(2018\)](#) (DK18) framework. This is also useful to interpret our Lemma 1 in relation to its analogue in DK18: in our model, we obtain extra terms that contain additional economically meaningful effects, but there are also some terms.

We proceed by re-defining net worth in DK18 by excluding production output and prove that the *distributive effects* and *collateral effects* in DK18's version of Lemma 1 are identical. We denote net worth as defined by DK18 as $N_{DK}^{i,\theta} \equiv e_1^{i,\theta} + X_1^{i,\theta} + F_1^{i,\theta}(K_1^i)$. The resulting equilibrium capital and debt price are denoted by $q_{DK}^\theta(N_{DK}^\theta, K_1)$ and $m_{2,DK}^\theta(N_{DK}^\theta, K_1)$. We define net worth without production output as $N_{WP}^{i,\theta} \equiv e_1^{i,\theta} + X_1^{i,\theta}$ and the resulting equilibrium capital and debt price are denoted by $q_{WP}^\theta(N_{WP}^\theta, K_1)$ and $m_{2,WP}^\theta(N_{WP}^\theta, K_1)$. A simple re-definition of the model's state variables cannot change the prices in equilibrium, so that we can set

$$q_{WP}^\theta(N_{WP}^\theta, K_1) = q_{DK}^\theta(N_{DK}^\theta, K_1) \tag{54}$$

$$m_{2,WP}^\theta(N_{WP}^\theta, K_1) = m_{2,DK}^\theta(N_{DK}^\theta, K_1) \tag{55}$$

Noting that $N_{DK}^{i,\theta} = N_{WP}^{i,\theta} + F_1^{i,\theta}(K_1^i)$, we differentiate both sides of (54) and (55) with respect to $N_{(\cdot)}^{i,\theta}$ and K_1^i , in order to determine how the derivatives of prices with respect to net worth and capital are related across models. This gives us

$$\frac{\partial q_{WP}^\theta}{\partial N_{WP}^{i,\theta}} = \frac{\partial q_{DK}^\theta}{\partial N_{DK}^{i,\theta}} \quad (56)$$

$$\frac{\partial m_{2,WP}^\theta}{\partial N_{WP}^{i,\theta}} = \frac{\partial m_{2,DK}^\theta}{\partial N_{DK}^{i,\theta}} \quad (57)$$

$$\frac{\partial q_{WP}^\theta}{\partial K_1^i} = \frac{\partial q_{DK}^\theta}{\partial N_{DK}^{i,\theta}} \frac{\partial N_{DK}^{i,\theta}}{\partial K_1^i} + \frac{\partial q_{DK}^\theta}{\partial K_1^i} = \frac{\partial q_{DK}^\theta}{\partial N_{DK}^{i,\theta}} F'(K_1^i) + \frac{\partial q_{DK}^\theta}{\partial K_1^i} \quad (58)$$

$$\frac{\partial m_{2,WP}^\theta}{\partial K_1^i} = \frac{\partial m_{2,DK}^\theta}{\partial N_{DK}^{i,\theta}} \frac{\partial N_{DK}^{i,\theta}}{\partial K_1^i} + \frac{\partial m_{2,DK}^\theta}{\partial K_1^i} = \frac{\partial m_{2,DK}^\theta}{\partial N_{DK}^{i,\theta}} F'(K_1^i) + \frac{\partial m_{2,DK}^\theta}{\partial K_1^i} \quad (59)$$

where we used the chain rule for the differentiation with respect to capital. (58) and (59) make clear that the derivatives of prices with respect to capital after the re-definition of net worth “contain” the partial derivatives of $F(\cdot)$ that appear in DK18’s Lemma 1. The *distributive effects* in DK18 are the following:

$$\mathcal{D}_{N_{DK}^{j,\theta}}^{DK,i,\theta} = - \left[\frac{\partial q_{DK}^\theta}{\partial N_{DK}^{j,\theta}} \Delta K_2^{i,\theta} + \frac{\partial m_{2,DK}^\theta}{\partial N_{DK}^{j,\theta}} X_2^{i,\theta} \right] \quad (60)$$

$$\mathcal{D}_{K_1^j}^{DK,i,\theta} = F'(K_1^i) \mathcal{D}_{N_{DK}^{j,\theta}}^{DK,i,\theta} - \left[\frac{\partial q_{DK}^\theta}{\partial K_1^j} \Delta K_2^{i,\theta} + \frac{\partial m_{2,DK}^\theta}{\partial K_1^j} X_2^{i,\theta} \right] \quad (61)$$

The *distributive effects* with the re-definition of net-worth can be derived as

$$\mathcal{D}_{N_{WP}^{j,\theta}}^{WP,i,\theta} = - \left[\frac{\partial q_{WP}^\theta}{\partial N_{WP}^{j,\theta}} \Delta K_2^{i,\theta} + \frac{\partial m_{2,WP}^\theta}{\partial N_{WP}^{j,\theta}} X_2^{i,\theta} \right] \quad (62)$$

$$\mathcal{D}_{K_1^j}^{WP,i,\theta} = - \left[\frac{\partial q_{WP}^\theta}{\partial K_1^j} \Delta K_2^{i,\theta} + \frac{\partial m_{2,WP}^\theta}{\partial K_1^j} X_2^{i,\theta} \right] \quad (63)$$

Using (56) - (59), we obtain

$$\mathcal{D}_{N_{DK}^{j,\theta}}^{DK,i,\theta} = \mathcal{D}_{N_{WP}^{j,\theta}}^{WP,i,\theta} \quad (64)$$

$$\mathcal{D}_{K_1^j}^{DK,i,\theta} = \mathcal{D}_{K_1^j}^{WP,i,\theta} \quad (65)$$

Similarly, it can be shown that

$$\mathcal{C}_{N_{DK}^{j,\theta}}^{DK,i,\theta} = \mathcal{C}_{N_{WP}^{j,\theta}}^{WP,i,\theta} \quad (66)$$

$$\mathcal{C}_{K_1^j}^{DK,i,\theta} = \mathcal{C}_{K_1^j}^{WP,i,\theta} \quad (67)$$

This shows that a re-definition of net worth in the original DK18 model gives identical results. Furthermore, these derivations show that Lemma 1 in our model would be identical to Lemma 1 to its counterpart in DK18 if we did not include labor markets and did not have a more general definition of the financial constraint.

A.2 Details on constrained efficient allocation and implementation

Derivation of constrained efficient allocation. These derivations correspond to Proposition 1 (a) and the associated proof in DK18. The Lagrangian of the social planner's problem can be written as

$$\begin{aligned}\mathcal{L} = & \sum_i \alpha^i \{u^i(C_0^i) + \beta \mathbb{E}_0[V^{i,\theta}(N_1^{i,\theta}, K_1^i; N^\theta, K_1)] + \kappa_1^i \Phi_1^i(X_1^i, K_1^i)\} \\ & + v_0 \sum_i [e_0^i - (C_0^i + h^i(K_1^i))] - \int_{\theta \in \Theta} v_1^\theta \sum_i X_1^{i,\theta} d\theta.\end{aligned}$$

The first-order conditions of the social planner are

$$\frac{d\mathcal{L}}{dC_0^i} = \alpha^i u'^i(C_0^i) - v_0 = 0, \quad \forall i \quad (68)$$

$$\frac{d\mathcal{L}}{dX_1^{i,\theta}} = -v_1^\theta + \alpha^i \beta V_n^{i,\theta} + \alpha^i \kappa_1^i \Phi_{1x}^i + \beta \sum_j \alpha_j V_{N^i}^{j,\theta}, \quad \forall i, \theta \quad (69)$$

$$\frac{d\mathcal{L}}{dK_1^i} = -v_0 h'^i(K_1^i) + \alpha^i \beta \mathbb{E}_0[V_k^{i,\theta}] + \alpha^i \kappa_1^i \Phi_{1k}^i + \beta \sum_j \alpha_j \mathbb{E}_0[V_{K^i}^{j,\theta}] = 0, \quad \forall i \quad (70)$$

Note that there are no expectation terms in the second first-order condition since $X_1^{i,\theta}$ is chosen for each θ .

The first first-order condition in the decentralized equilibrium implies $v_0 = \alpha^i \lambda_0^i$, so $\alpha^b/\alpha^l = \lambda_0^l/\lambda_0^b$. We divide the second FOC by α^i , and use $\alpha^i = v_0/\lambda_0^i$ as well as the envelope condition in the decentralized equilibrium $V_n^{i,\theta} = \lambda_1^{i,\theta}$. This gives us

$$\frac{v_1^\theta}{v_0} \lambda_0^i = \beta \lambda_1^{i,\theta} + \kappa_1^i \Phi_{1x}^i + \beta \sum_j \frac{\alpha_j}{\alpha^i} V_{N^i}^{j,\theta}, \quad \forall i, \theta \quad (71)$$

We then use the third first-order condition and the envelope condition $V_k^{i,\theta} = \mathbb{E}_0[\lambda_1^{i,\theta}(F_{1k}^{i,\theta}(K_1^i, l_{1d}^{i,\theta}) + q^\theta)]$ to get

$$h''(K_1^i) \lambda_0^i = \beta \mathbb{E}_0[\lambda_1^{i,\theta}(F_{1k}^{i,\theta}(K_1^i, l_{1d}^{i,\theta}) + q^\theta)] + \kappa_1^i \Phi_{1k}^i + \beta \sum_j \frac{\alpha_j}{\alpha^i} \mathbb{E}_0[V_{K^i}^{j,\theta}], \quad \forall i, \quad (72)$$

Equations (71) and (72), together with the constraints of the social planner's problem describe the constrained efficient allocation. Note that variables in $t \geq 1$ are optimal choices by the agents. Lemma 1 gives more detailed expressions being $V_{N^i}^{j,\theta}$ and $V_{K^i}^{j,\theta}$.

Implementation of constrained efficiency. These derivations correspond to Proposition 1 (b) and the associated proof in DK18. The constrained efficient allocation can be implemented by setting taxes on Arrow-Debreu security purchases and capital investment that satisfy

$$\tau_x^{i,\theta} = - \sum_j MRS_{01}^{j,\theta} \mathcal{D}_{1N^i}^{j,\theta} - \sum_j MRS_{02}^{j,\theta} \mathcal{D}_{2N^i}^{j,\theta} - \sum_j \tilde{\kappa}_2^{j,\theta} \mathcal{C}_{N^i}^{j,\theta}, \quad \forall i, \theta \quad (73)$$

$$\tau_k^i = - \sum_j \mathbb{E}_0[MRS_{01}^{j,\theta} \mathcal{D}_{1K^i}^{j,\theta}] - \sum_j \mathbb{E}_0[MRS_{02}^{j,\theta} \mathcal{D}_{2K^i}^{j,\theta}] - \sum_j \mathbb{E}_0[\tilde{\kappa}_2^{j,\theta} \mathcal{C}_{K^i}^{j,\theta}], \quad \forall i \quad (74)$$

where $MRS_{01}^{j,\theta} \equiv \beta \lambda_1^{j,\theta} / \lambda_0^j$, $MRS_{02}^{j,\theta} \equiv \beta \lambda_2^{j,\theta} / \lambda_0^j$ and $\tilde{\kappa}_2^{j,\theta} \equiv \beta \kappa_2^{j,\theta} / \lambda_0^j$. This can be shown as follows. Re-write the period-0 first-order conditions (32) and (33) by including tax wedges for security purchases ($\tau_x^{i,\theta}$) and capital investment (τ_k^i). This gives

$$(m_1^\theta + \tau_x^{i,\theta}) \lambda_0^i = \beta \lambda_1^{i,\theta} + \kappa_1^i \Phi_{1x^\theta}^i \quad (75)$$

$$(h^i(k_1^i) + \tau_k^i) \lambda_0^i = \beta \mathbb{E}_0[\lambda_1^{i,\theta} (F_{1k}^{i,\theta}(k_1^i, l_{d1}^{i,\theta}) + q^\theta)] + \kappa_1^i \Phi_{1k}^i \quad \forall i \quad (76)$$

Substituting the above tax rates into these optimality conditions replicates the planner's optimality conditions (71) and (72). Note that $m_1^\theta = \frac{v_1^\theta}{v_0}$ in the replicated allocations, i.e., Arrow-Debreu price in the decentralized equilibrium should equal the value of state contingent commodity in the social planner's problem measured by the shadow prices. Importantly, note also that the expressions for the tax rates contain additional terms relative to DK18 due to the presence of labor markets and the more general financial constraint formulation.

Combining equations (73) and (74) with equation (29) and (30) gives equations (38) and (39) in the main text.

B Further derivations on wages and labor markets

B.1 The equilibrium wage response to net worth changes

Pinning down the sign of the constraint externalities requires imposing further restrictions on the economic environment. In this Appendix we examine condition (41) introduced in the main text more closely. This condition restricts the model to an economy in which increases (decreases) in sector-wide net worth move equilibrium wages up (down), all else equal. As explained in Section 4.2 of the main text, in our general labor market setting wage changes in response to variation in net worth can be driven both by changes in labor demand and changes in labor supply. For labor demand, the reasoning is explained in the main text, so here we hold the relation between labor demand and net worth fixed, and study the responses of labor supply to changes in sector wide net worth.

Formally, consider the labor market clearing condition (13) for period $t = 1$. Labor demand and supply of both borrower and lender are defined according to the following first-order conditions (dropping the notation for θ for simplicity) :

$$w_1 = F_\ell^i(K_1^i, \ell_{d1}^i) \quad (77)$$

$$w_1 u_{c1}^i + u_{\ell 1}^i = 0 \quad (78)$$

where F_ℓ^i is the marginal product of labor, u_{c1}^i is the marginal utility of consumption, and $u_{\ell 1}^i$ is the marginal utility of labor for agent $i \in \{b, l\}$. Equation (77) equates wages to the marginal product of labor (in which capital is predetermined). Equation (78) relates labor supply and consumption choices through their relative price, the wage rate. Note that the consumption choice for agent i in period 1 is a function of state variables and prices w_1, w_2, m_2 , and q . Note that in (77) and (78) we ignore the presence of borrowing constraints. In the labor supply equation, we do so because labor supply does not enter the earnings-based borrowing constraint, for which we require condition (41).¹ In the labor demand equation, we do so because we already establish arguments in the main text about how financial constraints give rise to a positive impact of sector wide net worth on wages, and in what follows we want to study the pure labor supply mechanism.

Using (77) and (78), we write labor demand and supply as explicit functions of the variables they depend on, that is,

¹In the most general case, $\Phi_2^{b,\theta}$ in (18) can constraint also labor supply, but this is not the case in any of the cases we analyze in this paper.

$$\ell_{d1}^i = \ell_{d1}^i(w_1, K_1^i), \quad (79)$$

$$\ell_{s1}^i = \ell_{s1}^i(w_1, c_1^i(w_1, w_2, q, m_2, K_1^i, N_1^i)). \quad (80)$$

Given these relations, we state the labor market clearing condition (13) in period $t = 1$ more explicitly as

$$\sum_i \ell_{d1}^i(w_1, K_1^i) = \sum_i \ell_{s1}^i(w_1, c_1^i(w_1, w_2, q, m_2, K_1^i, N_1^i)) \quad (81)$$

The labor market clearing condition, expressed in this way, will be useful to characterize how net worth changes affect wages in period $t = 1$. Formally, we can totally differentiate equation (81) with respect to N_1^j to determine the sign of $\frac{\partial w_1}{\partial N_1^j}$. As explained above, we carry out this differentiation by ignoring any terms through which ∂N_1^j may affect ℓ_{d1} (e.g. through financial constraints). We obtain

$$\left\{ \sum_i \left[\underbrace{\frac{\partial \ell_{d1}^i}{\partial w_1}}_{A1} - \underbrace{\left(\frac{\partial \ell_{s1}^i}{\partial w_1} + \frac{\partial \ell_{s1}^i}{\partial c_1^i} \frac{\partial c_1^i}{\partial w_1} \right)}_{B1} \right] \right\} \frac{\partial w_1}{\partial N_1^j} = \underbrace{\sum_i \frac{\partial \ell_{s1}^i}{\partial c_1^i} \left(\frac{\partial c_1^i}{\partial N_1^j} + \frac{\partial c_1^i}{\partial w_2} \frac{\partial w_2}{\partial N_1^j} + \frac{\partial c_1^i}{\partial q} \frac{\partial q}{\partial N_1^j} + \frac{\partial c_1^i}{\partial m_2} \frac{\partial m_2}{\partial N_1^j} \right)}_{C1} \quad (82)$$

Expression (82) makes clear what is required for condition (41) to hold. $\frac{\partial w_1}{\partial N_1^j}$ is positive as long as the ratio of term $C1$ to terms $A1 - B1$ (summed over both agents) is positive. For each agent, term $A1$ represents the slope of the labor demand curve, and term $B1$ is the slope of an labor supply curve (which is composed of a substitution and income effect). Term $C1$ captures how change in net worth shift the labor supply curves through various equilibrium forces that do operate through wages in $t = 1$. More specifically, this term is composed of two types of effects. First, a direct equilibrium effect of net worth on the consumption-leisure tradeoff, $\sum_i \frac{\partial \ell_{s1}^i}{\partial c_1^i} \left(\frac{\partial c_1^i}{\partial N_1^j} \right)$. This term is negative for both agents under standard preferences such as a constant relative risk aversion (CRRA) utility function. Second, a collection of indirect effects from price changes on the consumption-leisure tradeoff, $\sum_i \frac{\partial \ell_{s1}^i}{\partial c_1^i} \left(\frac{\partial c_1^i}{\partial w_2} \frac{\partial w_2}{\partial N_1^j} + \frac{\partial c_1^i}{\partial q} \frac{\partial q}{\partial N_1^j} + \frac{\partial c_1^i}{\partial m_2} \frac{\partial m_2}{\partial N_1^j} \right)$. It is not possible to unambiguously determine the of sign the combination of these effects, especially since they may have different sign across the two types of agents.

So under what conditions do these labor supply effects give support condition (41)? It is reasonable to assume that labor demand curve and downward-sloping and labor

supply curves are upward sloping. In this case the sum across agents of the term $A1 - B1$ is negative. Thus, for condition (41) to hold, term $C1$ needs to be negative. This means that the direct effect of changes in net worth in $C1$ needs to be stronger than the net effect of changes in net worth from the combination of indirect effects through other prices across agents. This is the restriction we impose on the model through requiring condition (41) to hold in the main text.

B.2 Version with simplified labor market structure

In the model, both borrowers and lenders supply and demand labor. The empirical relevance of earnings-based constraints relates to the borrower being a firm, that is, an agent that typically demands but not supplies labor. In this Appendix, we therefore analyze how the model would change if the labor market structure is simplified such that the borrower only demands labor and the lender only supplies labor. We show that the economy will still be constrained inefficient.

Recall the labor market clearing condition (13) in period $t = 1$,

$$\sum_i \ell_{d1}^i(w_1, K_1^i) = \sum_i \ell_{s1}^i(w_1, c_1^i(w_1, w_2, q, m_2, K_1^i, N_1^i)) \quad (83)$$

where we drop θ for simplicity. Note that we have written labor demand as a function of w_1, K^i coming from the optimal labor demand decision which equates the marginal product of labor with the wages rate. Labor supply is a function of wages and consumption because the household's labor-leisure decision depends on the wage and marginal utilities (see Appendix B.1). For the proof of Proposition 2 we use condition (41). In Appendix B.1, we have shown that this condition can be determined from differentiating (83) with respect to N_1^j . Now if we assume that the lender only supplies labor, then the labor market clearing condition in $t = 1$ instead becomes

$$\ell_{d1}^b(w_1, K^b) = \ell_{s1}^l(w_1, c_1^l(w_1, w_2, q, m_2, K^l, N^l)) \quad (84)$$

that is, there is no summation over $i \in \{b, l\}$ but demand instead only comes from borrowers and supply only from lenders. It is easy to see that (41) can still apply: as in the main text, we can reason based labor demand and labor supply effects separately.

First, in the simplified labor market setting based on equation (84) there can still be positive effects of changes in net worth in the (financially constrained) borrowing sector on wages through labor demand, by the same reasoning applied to the general labor

market structure in the main text. Second, to study the role of labor supply effects in the simplified labor market, we can derive an equation that corresponds to (82) for the derivative with respect to N_1^l only. By applying similar arguments as in Appendix B.1, we can conclude that $\frac{\partial w_1^\theta}{\partial N_1^{l,\theta}} > 0$. As period-0 borrowing decisions change wages through changes in lender net worth, the market equilibrium is still not efficient, and the planner would levy taxes that correct the externality. This is true even if there were no effects coming through the borrower's labor demand, that is, $\frac{\partial w_1^\theta}{\partial N_b^{l,\theta}} = 0$. In this case, the social planner would levy taxes only on lenders. In this case, however, it might be more appropriate to label the mechanism over-saving, rather than under-borrowing.