Earnings-based borrowing constraints and pecuniary externalities*

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Abstract

Prices in financial constraints give rise to pecuniary externalities, so policy can improve market outcomes in which agents do not internalize the effects of their decisions on prices. This paper studies the pecuniary externalities that arise from earnings-based borrowing constraints, which are common for US firms. While *asset prices* in traditional collateral constraints typically result in 'over-borrowing' relative to the social optimum, *input prices* in earnings-based constraints can lead to 'under-borrowing.' In particular, we show in a model that borrowing decisions today are suboptimal when firms do not internalize that they impact future wages and thereby change future earnings, which in turn impacts their future borrowing capacity. A numerical application of the model demonstrates that incorrectly rolling out a tax policy derived under the assumption of asset-based constraints in an economy where firms actually borrow based on earnings leads to substantial welfare losses. Optimal macroprudential policy thus critically depends on the specific form of financial constraints that is prevalent in a credit market.

JEL Codes: D62, E32, E44, G28.

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1 Introduction

Should policy-makers intervene in financial markets? If so, why and how? This paper studies pecuniary externalities that arise when prices affect financial constraints but firms and households do not internalize the consequences of their choices on these prices. Our central contribution is to examine the way in which credit limits that link firms' earnings and their debt access, which have shown to be widespread in the US corporate sector, induce sub-optimal borrowing decisions. Our findings highlight that optimal macroprudential policy critically depends on the specific form of financial constraints, and that designing macroprudential policy under imprecise assumptions about the relevant borrowing frictions can lead to drastic welfare losses.

Recent research distinguishes between two types of credit constraints faced by US companies: asset-based and earnings-based constraints (Lian and Ma, 2020, Drechsel, 2022). An asset-based borrowing limit ties credit access to the value of an asset, such as a building or machine. With earnings-based constraints, the ability to obtain funds is linked to the borrowing firm's earnings, usually measured before interest, taxes, depreciation and amortization (EBITDA). The normative implications of asset-based constraints have been studied comprehensively (see e.g. Dávila and Korinek, 2018, Bianchi and Mendoza, 2018).[1] However, although the growing research on earnings-based borrowing constraints has found them to be much more prevalent for US corporations than asset-based constraints, there is still a limited understanding of the pecuniary externalities that arise from this type of credit limit.[2]

The contribution of this paper is to advance this understanding. At the heart of our analysis is a macroeconomic model, inspired by Dávila and Korinek (2018). In a two-agent three-period structure with capital, an intratemporal production input (labor), and financial asset markets, we first characterize the competitive equilibrium and planner solution for a general formulation of credit constraints, in which various prices and quantities may enter. We then specialize this general formulation to study the welfare effects of different specifications of financial constraints linked to earnings, and contrast them with the so far more widely studied asset-based borrowing constraints. Our introduction of labor markets to the model is crucial to study earnings-based constraints, with wages being a key price that affects firms' costs and thereby

[1] Asset-based constraints have also been studied extensively from a positive point of view, going back at least to the seminal work of Kiyotaki and Moore (1997).

[2] There are a few exceptions in which earnings do play a role in credit constraints, such as Bianchi (2016). We analyze the differences between the earnings-based borrowing constraint studied in this paper and existing formulations of financial constraints.

earnings. Modeling labor markets also brings additional challenges in determining the sign of pecuniary externalities, something that the literature has generally pointed out as a difficulty. Our analysis lays out additional theoretical conditions on wage determination that allow signing the relevant externalities.[3]

Our findings are the following. First, an earnings-based borrowing constraint, in which the borrower's debt-to-earnings ratio is restricted by a maximum value, leads to 'under-borrowing' from a welfare point of view. The intuition is that when borrowing increases in the current period, borrower net worth will be lower next period. Under relevant conditions in our model, this reduction in borrower net worth leads real wages to fall next period. A lower real wage means lower costs and higher earnings for firms, which through the earnings-based borrowing constraint allows for more credit. However, when firms borrow today they do not take into account this positive impact of their decisions today on the future borrowing limit through wages. Therefore firms borrow a smaller amount in the current period than what a social planner would implement as a constrained efficient allocation. That is, they under-borrow.

Second, we clarify that this result is the opposite to what holds under an asset-based constraint. With that constraint, the resale value of the borrower's capital serves as collateral. In the model, lower borrower net worth leads to lower capital prices next period, the same way it leads to lower real wages. However with a collateral constraint the price reduction tightens rather than relaxes the future constraint. Borrowers in the current period do not internalize this negative effect of future debt access. They therefore 'over-borrow' relative to the social optimum, in line with previous findings in the literature. In essence, in an earnings-based credit constraint an *input price* (through the wage bill) enters with a negative sign, while in an asset-based constraint an *asset price* (through the value of capital) enters with a positive sign. When real wages and the price of capital respond with the same sign to current borrowing decisions, then the implications for under- vs. over-borrowing are the opposite for the two constraints.

Third, we find that an interest coverage constraint can lead to either over-borrowing or under-borrowing. Interest coverage constraints are also linked to firms' earnings, but impose a minimum on the ratio of earnings to interest expenses, rather limiting the debt-to-earnings ratio. They are frequently observed for US companies, as emphasized by Greenwald (2019). The intuition we provide for their ambiguous normative

[3]Building on Dávila and Korinek (2018), we disentangle different channels through which externalities operate, labeled *distributive effects* and *constraint effects*. Our formal theoretical analysis characterizes whether the constraint effects, those that operate directly through the credit limit, lead to 'over-borrowing' or 'under-borrowing.' In a numerical application of our model, we allow for additional externality channels to be present, including distributive effects through borrowing and investment.

implications is that interest coverage constraints feature the same pecuniary externality through wages as the one described in our first result, but the presence of interest expenses in the constraint gives rise to an opposing force. Interest rates are inversely related to bond prices, so the interest coverage constraint links higher bond prices with looser credit constraints, for a given level of earnings. Therefore, when bond prices move in the same direction as the price of capital in response borrower net worth changes, then the presence of interest payments affects the constraint in the same direction as the price of capital in an asset-based constraint. As a consequence, from welfare point of view an interest coverage constraint can be interpreted as a mixture between an asset-based and earnings-based constraint, with pecuniary externalities operating through both wages and interest rates in opposite directions. This paper is the first to uncover this property of interest coverage constraints.

Fourth, to understand the normative consequences of earnings-based borrowing constraints from quantitative point of view, we study a numerical application. Our above welfare results hold under fairly general conditions, but are tied to pecuniary externalities that directly operate through the credit constraint. By choosing functional forms for preferences and technology, calibrating the model's parameters, and imposing specific macroprudential tax policies, we can also examine their effects through the full array of possible channels. This includes additional distributive externalities, which cannot be generally signed in our framework but which have recently been shown to be important in the context of collateral constraints by Lanteri and Rampini (2021). Specifically, we study the following experiment. A planner calculates an optimal set of taxes, assuming that the economy features asset-based constraints. In an equally calibrated economy where firms actually borrow based on earnings, we then impose this set of 'incorrect' taxes. We find that rolling out a tax policy derived under such imprecise assumptions about firms' borrowing constraints leads to large welfare losses. For example, relative to imposing the optimal policy, the wrongly designed tax policy leads to a loss of up to 2.55% in aggregate consumption.

Finally, we study several extensions. First, we examine a setting with working capital constraints (Bianchi and Mendoza, 2010; Jermann and Quadrini, 2012; Bianchi, 2016). We find that when firms need to pre-finance wages and in addition face earnings-based limits on credit, the pecuniary externality through wages is magnified, so the under-borrowing effect becomes even stronger. Studying this extension also clarifies the differences between our mechanism and those in Bianchi and Mendoza (2010) and Bianchi (2016). Second, while we study a closed economy setting with endogenous interest rates, we connect our findings to the important macroprudential policy considerations in small open economies (Mendoza, 2006, 2010, Bianchi, 2011). Third, in our setting the price of consumption goods is normalized to one and the prices of capital, labor and debt are expressed in relative terms. We discuss implications of relaxing this assumption.

Our results have important implications for the design of an effective regulatory system. Macroprudential policy guided solely by an asset-based collateral mechanism is counterproductive in credit markets where earnings-based borrowing constraints are dominant. Indeed, our numerical application shows that such a policy could lead to drastic welfare losses. The evidence motivating our analysis focuses on nonfinancial companies, so the regulation of corporate credit is where our results are most applicable. Collateral constraints are likely a more central force in household mortgage markets, where real estate serves as collateral, or in trade between financial institutions, where financial assets are pledged in repurchase agreements. This paper makes the case for studying carefully which pecuniary externalities are critical in which types of credit markets, and shows that the distinction between asset and input prices in credit constraint is of first-order importance for determining optimal policy.

Contribution to the literature. Our work contributes to two strands of research. The first strand studies pecuniary externalities with financial constraints.[4] We build on the framework of Dávila and Korinek (2018). The main difference is that our setting features labor markets as well as additional types of financial constraints. The introduction of labor markets provides new challenges in signing the externalities of interest, and a contribution of this paper is to explore relevant model restrictions. Our insight that higher wages tighten financial constraints is complementary to a related mechanism in Bianchi (2016), where firms face working capital and equity constraints, and do not internalize that when they hire workers, wages increase, which in turn tightens other firms' equity constraints.[5] A few other studies consider income-related

[4]Important contributions to this research agenda include Greenwald and Stiglitz (1986), Gromb and Vayanos (2002), Mendoza (2006, 2010), Lorenzoni (2008), Jeanne and Korinek (2010), Korinek (2011), Bianchi (2011), Stein (2012), Benigno et al. (2013), Bianchi (2016), Bianchi and Mendoza (2018). A related line of research studies *aggregate demand externalities* (Schmitt-Grohé and Uribe, 2016; Farhi and Werning, 2016; Korinek and Simsek, 2016). These externalities do not work through financial constraints, but through the combination of nominal wage rigidities and other constraints, such as the ZLB or a fixed nominal exchange rate. Wolf (2020) studies pecuniary externalities that arise from wage rigidities independently of financial constraints and aggregate demand channels.

[5]The pecuniary externality in Bianchi (2016) works through higher *labor demand* having a contemporaneous negative effect on other firms' dividend constraints. In our framework, the pecuniary

rather than asset-based credit constraints in normative analysis, for example Bianchi (2011) where tradable and nontradable income restrict the economy's external position. Benigno et al. (2013) and Schmitt-Grohé and Uribe (2020) also note the possibility of under-borrowing, but through channels that are different from ours. In Benigno et al. (2013), higher wage income relaxes rather than tightens the borrowing constraint faced by a representative household. In Schmitt-Grohé and Uribe (2020) under-borrowing is a result of precautionary savings in the face of self-fulling crises. Fazio (2021) proposes a framework with earnings-based constraints on firms to study the implications of a credit crunch at the zero lower bound (ZLB) on interest rates. What distinguishes our paper from all of the above is that we jointly consider a variety of credit constraints that are observed in microeconomic data on firms and study subtleties in their policy implications. In particular, we are the first to compare the normative consequences of asset-based, debt-to-earnings and interest coverage constraints within the same formal framework. Another aspect that differentiates our paper from the literature is that we study pecuniary externalities in the context of a general labor market structure, with an explicit analysis of both labor demand and labor supply effects.[6] Finally, a related paper is Ottonello, Perez, and Varraso (2022) which focuses on the timing of collateral constraints and shows that conclusions can change depending on whether current or future prices of collateral affect credit access. We instead focus on different variables entering the constraint, going beyond asset-based constraints.[7]

The second strand of research we contribute to provides our empirical background. Recent studies, in particular Lian and Ma (2020) and Drechsel (2022), highlight the distinction between asset-based and earnings-based constraints, but do not consider normative implications. We survey this literature in Section 2.2.

Structure of the paper. Section 2 previews the intuition behind our main insights, and provides the empirical motivation. Section 3 presents the model. Section 4 carries out the efficiency analysis in the model and presents the main results. Section 5 studies the numerical policy experiment. Section 6 explores extensions. Section 7 concludes.

externality and resulting under-borrowing effect arise from firms' current borrowing exerting a positive effect on future credit limits through *labor supply*.

[6] Bianchi and Mendoza (2010), Bianchi (2016), and Fazio (2021) all focus on Greenwood, Hercowitz, and Huffman (1988) (GHH) preferences which eliminate wealth effects on labor supply. Our setting features a more general labor supply specification, which is one way to distinguishes our findings from the ones in these papers.

[7] We also analyze the timing for the earnings-based constraint, and find that the presence of under-borrowing effects are not sensitive to whether current or future earnings enter the constraint.

2 Main intuition and empirical background

This section previews the main insights of our paper, by illustrating some key economic relationships with minimal formality. It also provides our empirical motivation, by drawing on recent studies of corporate borrowing constraints.

2.1 Financial constraints, prices, and pecuniary externalities

Consider a generic financial constraint faced by an economic agent

$$\Phi(x', \boldsymbol{z}, \widetilde{\boldsymbol{z}}) \geq 0 \tag{1}$$

where x' is the net position in a financial asset (negative values of x' indicate borrowing, positive values saving). The $'$-notation indicates that the choice is made in the current period, with repayment in the next period. $\boldsymbol{z}$ is a vector of endogenous variables chosen by the agent, and $\widetilde{\boldsymbol{z}}$ is a vector of endogenous or exogenous variables that the agent takes as given. $\boldsymbol{z}$ and $\widetilde{\boldsymbol{z}}$ may contain past, current and expected future realizations of variables. Φ is some function. When prices are included in $\widetilde{\boldsymbol{z}}$, and these prices are affected by the agent's choices in equilibrium, then pecuniary externalities arise: the agent does not internalize that their choices move prices in (1).

The *direction* of these price movements is critical for the normative implications of financial constraints. Consider the widely studied version of (1) in which the borrowing agent pledges collateral. Let k' be the current choice of capital and q its price. We define $\boldsymbol{z} = k'$ and $\widetilde{\boldsymbol{z}} = q$ and $\Phi(x', k', q) = x' + \phi q k'$, which gives

$$x' \geq -\phi q k' \tag{2}$$

where ϕ is a parameter such that $0 < \phi < 1$. This financial constraint imposes that borrowing cannot exceed a fraction of the market value of capital $\phi q k'$. Importantly, the agent chooses x' and k', but takes q as given. q is a market price and a function of the economy's aggregate state variables $q = q(X, K)$, where capitalized letters denote aggregate states. Aggregate states are taken as given by the agent, that is, the agent does not internalize how their individual choice of say x' influences X' and thereby moves prices in the following period.

Now suppose that the equilibrium response of q to an increase in aggregate net worth is positive and loosens the financial constraint. In this case, the fact that agents do not internalize this equilibrium effect is a source of inefficiency. Borrowing by an individual agent today reduces future aggregate net worth of borrowers in the economy, which in turn, all else equal, decreases future capital prices and thus tightens future borrowing limits. Not internalizing this pecuniary externality, the agent over-borrows today relative to the social optimum.

At the most general level, the insight that will emerge from our analysis is that there are financial constraints in which $\widetilde{\boldsymbol{z}}$ contains prices other than that of collateral, and that the equilibrium movements of these prices may have the opposite effect on credit constraints. The leading example we consider is when a firm's debt access is limited by its earnings. Formally, (1) is written with $\boldsymbol{z} = [y\ \ell]$ and $\widetilde{\boldsymbol{z}} = w$ and $\Phi(x', [y\ \ell], w) = x' + \widetilde{\phi}(y - w\ell)$:

$$x' \geq -\widetilde{\phi}(y - w\ell) \tag{3}$$

y is the firm's output, ℓ is the input choice (labor), and w is the input price (wage). $\widetilde{\phi} > 0$ is a parameter. y is related to input ℓ through a production function. The difference between sales and input costs $y - w\ell$ defines the firms' earnings (EBITDA), which restricts debt access. Wages depend on the aggregate state variables, $w = w(X, K)$. Suppose now that wages also respond positively to an increase in aggregate borrower net worth, the way the price of capital does. Then the pecuniary externality from w in the constraint based on earnings has the opposite effect of that from q in the collateral constraint. While q enters with negative sign in (2) (it positively affects debt space), w enters with positive sign in (3) (it negatively affects debt space).[8] When q and w respond with the same sign to changes in aggregate state variables, then these price changes move the two borrowing limits in the opposite direction.

Unlike the asset-based borrowing constraint, the earnings-based constraint therefore leads to under-borrowing: individual firms do not internalize that borrowing reduces future aggregate borrower net worth, which lowers wages and relaxes credit constraints in the future. They thus borrow less today than what is socially desired. We characterize this effect more generally in our formal theoretical analysis, where we lay out the formal conditions that need to hold for wages to indeed respond in the same direction to changes in aggregate states as the price of capital does, and where we also consider a variation of (3) where interest payments enter. Before moving to the full model, we review the recent microeconomic evidence on firms' borrowing constraints.

[8] In the case of household rather than firm debt, wages could relax rather than tighten constraints, as for example in Benigno et al. (2013). The motivation and focus of our study apply to firm credit.

2.2 Evidence for earnings-based vs. asset-based credit

There is mounting microeconomic evidence in favor of (3) being a relevant constraint for firms. Earnings-based borrowing constraints can arise through debt covenants, which are legal provisions that link debt access to earnings indicators, but also through credit rating methods or through bankruptcy procedures in which recovered debt payments are calculated based on EBITDA.[9]

Lian and Ma (2020) develop a procedure to classify corporate debt contracts into primarily asset-based or earnings-based. Combining a variety of data sources, they find that only 20% of US firm credit is asset-based, while 80% is earnings-based. Motivated by this evidence, their paper investigates the marginal effects of changes in different firm-level variables on borrowing and investment, and find that changes in EBITDA have a strong effects while changes in real estate values have a limited impact. Lian and Ma (2020) also discuss that an earnings-based constraint can insulate firms from fire sale dynamics. While not examined in a normative context, this effect also works through prices in the constraints.[10]

Drechsel (2022) studies how earnings-based borrowing constraints affect the transmission of macroeconomic shocks. In a theoretical model, investment-specific shocks lower the value of collateral but raise earnings, and should therefore allow for more borrowing with an earnings-based constraint but less borrowing with an asset-based constraint. The empirical dynamics in both macro and firm-level data are in line with the predictions that hold with earnings-based constraints. Furthermore, Drechsel (2022) studies the implications of earnings-based constraints for the behavior of price markups in New Keynesian models, and shows that the constraints affects macroeconomic stabilization tradeoffs.

Greenwald (2019) studies constraints in which interest payments are restricted by earnings. Such interest coverage constraints often appear alongside the earnings-to-debt restrictions emphasized by Lian and Ma (2020) and Drechsel (2022). Greenwald (2019) calculates that they are present in over 80% of firms with covenants. Using a combination of model and data, he shows that interest coverage constraints amplify changes in monetary policy. The simultaneity with other constraints makes the transmission of interest rate changes dependent on the level of interest rates.

[9] See Chava and Roberts (2008) for a study on debt covenants. Lian and Ma (2020) explain in detail how creditor claims in the event of bankruptcy are calculated in different types of debt contracts.

[10] The price of capital drives financial amplification, as in the seminal work of Kiyotaki and Moore (1997), while amplification is muted with earnings. As shown by Dávila and Korinek (2018), amplification effects are not necessary or sufficient for inefficiencies to arise.

The literature on earnings-based borrowing constraints is fast growing. While the aforementioned papers use evidence from public companies, a recent study using supervisory data by Caglio, Darst, and Kalemli-Özcan (2021) shows that earnings-based are prevalent for private small and medium-sized companies (SMEs). di Giovanni et al. (2022) investigate the role of earnings-based borrowing constraints in the effects of government procurement auctions in Spain on firm-level outcomes.

Importantly, while all existing studies in the literature examine the origins and the positive consequences of earnings-based constraints at the firm-level and the macroeconomic level, our analysis is the first to focus on their normative implications.

3 Model

Our model is based on Dávila and Korinek (2018) [henceforth 'DK18']. We make two distinct contributions to their framework. First, we generalize it to feature a market for intratemporal production inputs (labor). Second, we allow for a number of additional types of credit constraints. In combination, these two novelties enable us to examine in particular the pecuniary externalities that operate through input prices (wages) in earnings-based credit constraints.

3.1 Economic environment

There are three discrete time periods $t = 0, 1, 2$. The economy is populated by a unit measure of both borrowers and lenders, denoted by index $i \in \{b, l\}$. The state of nature is realized at date $t = 1$ and is denoted by $\theta \in \Theta$.

Preferences. Agent type i derives utility from consumption $c_t^i \geq 0$ and labor $\ell_{st}^i \geq 0$ according to the time separable utility function

$$U^i = \mathbb{E}_0 \left[\sum_{t=0}^{2} \beta^t u^i(c_t^i, \ell_{st}^i) \right] \tag{4}$$

where $u^i(\cdot, \cdot)$ is strictly increasing and weakly concave in consumption, and strictly decreasing and weakly convex in labor. While we think of variable $\ell \geq 0$ as labor, the model is general enough to think of it as any intratemporal production input that can be produced by incurring a utility cost. We set $u^i(c_0^i, \ell_{s0}^i) = u^i(c_0^i)$ as there is no production and input choice at date $t = 0$.

Endowments and production technology. There are consumption goods and capital goods. $e_t^{i,\theta}$ is the endowment of consumption goods agent i receives at date $t = 1, 2$ given state θ. Time-0 endowments are denoted by e_0^i. At date $t = 0$, agents can invest $h^i(k_1^i)$ units of consumption good to produce k_1^i units of date-1 capital goods.[11] The functions $h^i(\cdot)$ are increasing and convex and satisfy $h^i(0) = 0$. k_1^i can be used for the production of consumption goods in period $t = 1$ and be carried over for production in period $t = 2$. $k_2^{i,\theta}$ denotes the amount of capital that agent i carries from date 1 to 2. Capital fully depreciates after date 2. To produce consumption goods in $t \geq 1$, agent i employs both capital and labor to produce $F^i(k_t^{i,\theta}, \ell_{dt}^{i,\theta})$ units of the consumption good. $\ell_{dt}^{i,\theta}$ is labor demanded by agent i at date t. The production functions $F^i(\cdot,\cdot)$ are strictly increasing and weakly concave in each argument and satisfy $F^i(0,0) = 0$. They are allowed to be different across agents $i \in \{b, l\}$.

Market structure. At date $t = 0$, agents trade state-contingent assets that pay 1 unit of the consumption good in period $t = 1$ and state θ. $x_1^{i,\theta}$ denotes the date-0 state-θ purchases by agent i and m_1^θ is the corresponding asset price, taken as given by the agent. Agent i spends $\int_{\theta\in\Theta} m_1^\theta x_1^{i,\theta} d\theta$ in total on these securities. Without further uncertainty between $t = 1$ and $t = 2$, agents trade non-contingent one-period bonds $x_2^{i,\theta}$ at time $t = 1$ at price m_2^θ. There is a competitive labor market. Wages at date $t \geq 1$ and state θ are denoted by w_t^θ.[12] There is also a market to trade capital at a price q^θ at date 1 after production has taken place. There is no trading of capital at date 2 because of the full depreciation. Taken together, the budget constraints of agent $i \in \{b, l\}$ are

$$c_0^i + h^i(k_1^i) + \int_{\theta\in\Theta} m_1^\theta x_1^{i,\theta} d\theta = e_0^i \tag{5}$$

$$c_1^{i,\theta} + q^\theta \Delta k_2^{i,\theta} + m_2^\theta x_2^{i,\theta} = e_1^{i,\theta} + x_1^{i,\theta} + F^i(k_1^i, \ell_{d1}^{i,\theta}) - w_1^\theta \ell_{d1}^{i,\theta} + w_1^\theta \ell_{s1}^{i,\theta}, \quad \forall\theta \tag{6}$$

$$c_2^{i,\theta} = e_2^{i,\theta} + x_2^{i,\theta} + F^i(k_2^{i,\theta}, \ell_{d2}^{i,\theta}) - w_2^\theta \ell_{d2}^{i,\theta} + w_2^\theta \ell_{s2}^{i,\theta}, \quad \forall\theta \tag{7}$$

where $\Delta k_2^{i,\theta} \equiv k_2^{i,\theta} - k_1^i$. Recall that the state of nature θ materializes in $t = 1$ so there is one set of choices made in the initial period (in expectation of the possible states occurring in the future), whereas choices in the subsequent two period are made conditional on the realized state of nature.

[11] Note that $k_1^{i,\theta} = k_1^i$ since it is chosen in $t = 0$, thus not conditional on the state of nature θ.

[12] Both borrower and lender demand and supply labor from and to each other. In the context of earnings-based borrowing constraints, the borrower is typically a firm (see Drechsel, 2022). We also restrict the borrower to demanding labor and the lender to supplying it. As we will discuss below, this nested version of the model features fewer sources of externalities, but generates similar results.

Financial constraints. We assume that there are constraints on the holdings of securities between periods $t = 0$ and $t = 1$, as well as between periods $t = 1$ and $t = 2$. At date $t = 0$, borrowers' holdings of $x_1^b = \{x_1^{b,\theta}\}_{\theta\in\Omega}$ are subject to a constraint

$$\Phi_1^b(x_1^b, k_1^b) \geq 0 \tag{8}$$

At date $t = 1$, borrowers' holdings of $x_2^{b,\theta}$ are subject to a state-dependent constraint

$$\Phi_2^{b,\theta}(x_2^{b,\theta}, k_2^{b,\theta}, \{\ell_{dt}^{b,\theta}, \ell_{st}^{b,\theta}\}_{t=1}^2; q^\theta, w_1^\theta, w_2^\theta, m_2^\theta) \geq 0,\ \forall\theta \tag{9}$$

This is a general formulation of a financial constraint in this economy, in which *any quantities and prices that are not predetermined at the beginning of period* $t = 1$ may restrict access to credit for the borrower. This includes capital and labor, as well as capital prices, wages and asset prices. Section 4 studies the efficiency properties of various types of credit constraints that are special cases of (9). We assume $\Phi_1^l(\cdot) = \Phi_2^{l,\theta}(\cdot) = 0$, that is, lenders are financially unconstrained.

3.2 Decentralized equilibrium

A decentralized equilibrium is defined by the set of real allocations $\{c_0^i, c_1^{i,\theta}, c_2^{i,\theta}, k_1^i, k_2^{i,\theta}, \ell_{d1}^{i,\theta}, \ell_{d2}^{i,\theta}, \ell_{s1}^{i,\theta}, \ell_{s2}^{i,\theta}\}_{i\in\{b,l\},\theta\in\Theta}$, asset allocations $\{x_1^{i,\theta}, x_2^{i,\theta}\}_{i\in\{b,l\},\theta\in\Theta}$, and prices $\{q^\theta, w_1^\theta, w_2^\theta, m_1^\theta, m_2^\theta\}_{\theta\in\Theta}$, such that agents solve their optimization problems and markets clear. The market clearing conditions are given by

$$\sum_i [c_0^i + h^i(k_1^i)] \leq \sum_i e_0^i \tag{10}$$

$$\sum_i c_t^{i,\theta} \leq \sum_i [e_t^i + F^i(k_t^{i,\theta}, \ell_{dt}^{i,\theta})],\ t = 1, 2,\ \forall\theta \tag{11}$$

$$\sum_i k_2^{i,\theta} \leq \sum_i k_1^i,\ \forall\theta \tag{12}$$

$$\sum_i \ell_{dt}^{i,\theta} = \sum_i \ell_{st}^{i,\theta},\ t = 1, 2, \forall\theta \tag{13}$$

$$\sum_i x_t^{i,\theta} = 0,\ t = 1, 2,\ \forall\theta \tag{14}$$

Solution for periods 2 and 1. The solution for the decentralized equilibrium can be obtained via backward induction. Optimal choices at time $t = 2$ are purely intratemporal decisions on consumption and labor supply and demand. Asset positions are settled. In $t = 1$, two sets of variables fully characterize the state of the economy. The first is the holdings of capital by both agents k_1^i. The second one is agents' net worth $n_1^{i,\theta} \equiv e_1^{i,\theta} + x_1^{i,\theta}$.[13] Since agents take aggregate states as given it is helpful to distinguish individual states $\{n_1^{b,\theta}, n_1^{l,\theta}, k_1^b, k_1^l\}$ from aggregate states $\{N_1^{b,\theta}, N_1^{l,\theta}, K_1^b, K_1^l\}$. We further define $N_1^\theta \equiv \{N_1^{b,\theta}, N_1^{l,\theta}\}$ and $K_1 \equiv \{K_1^b, K_1^l\}$, and note that the equilibrium prices are functions of the aggregate state variables: $q^\theta(N_1^\theta, K_1)$, $m_2^\theta(N_1^\theta, K_1)$, $w_1^\theta(N_1^\theta, K_1)$, and $w_2^\theta(N_2^\theta(N_1^\theta, K_1), K_2(N_1^\theta, K_1)) = w_2^\theta(N_1^\theta, K_1)$. The optimization problem of an individual agent i at time $t = 1$ conditional on state θ is a function of both sets of state variables

$$V^{i,\theta}(n_1^{i,\theta}, k_1^i; N_1^\theta, K_1) = \max_{\{c_1^{i,\theta}, c_2^{i,\theta}, k_2^{i,\theta}, x_2^{i,\theta}, \ell_{dt}^{i,\theta}, \ell_{st}^{i,\theta}\}} \left\{ u^i(c_1^{i,\theta}, \ell_{s1}^{i,\theta}) + \beta u^i(c_2^{i,\theta}, \ell_{s2}^{i,\theta}) \right\} \tag{15}$$

subject to

$$c_1^{i,\theta} + q^\theta \Delta k_2^{i,\theta} + m_2^\theta x_2^{i,\theta} = e_1^{i,\theta} + x_1^{i,\theta} + F^i(k_1^i, \ell_{d1}^{i,\theta}) - w_1^\theta \ell_{d1}^{i,\theta} + w_1^\theta \ell_{s1}^{i,\theta} \qquad [\lambda_1^{i,\theta}] \tag{16}$$

$$c_2^{i,\theta} = e_2^{i,\theta} + x_2^{i,\theta} + F^i(k_2^{i,\theta}, \ell_{d2}^{i,\theta}) - w_2^\theta \ell_{d2}^{i,\theta} + w_2^\theta \ell_{s2}^{i,\theta} \qquad [\lambda_2^{i,\theta}] \tag{17}$$

$$\Phi_2^{b,\theta}(x_2^{b,\theta}, k_2^{b,\theta}, \{\ell_{dt}^{b,\theta}, \ell_{st}^{b,\theta}\}_{t=1}^2; q^\theta, w_1^\theta, w_2^\theta, m_2^\theta) \geq 0 \qquad [\kappa_2^{i,\theta}] \tag{18}$$

where $\lambda_1^{i,\theta}$, $\lambda_2^{i,\theta}$, and $\kappa_2^{i,\theta}$ are the Lagrange multipliers for each constraint. The first-order conditions for the period-1 maximization problem with respect to $x_2^{i,\theta}$ and $k_2^{i,\theta}$ are

$$m_2^\theta \lambda_1^{i,\theta} = \beta \lambda_2^{i,\theta} + \kappa_2^{i,\theta} \Phi_{2x^\theta}^{i,\theta}, \tag{19}$$

$$q^\theta \lambda_1^{i,\theta} = \beta \lambda_2^{i,\theta} F_{2k}^{i,\theta}(k_2^{i,\theta}, \ell_{d2}^{i,\theta}) + \kappa_2^{i,\theta} \Phi_{2k}^{i,\theta}, \ \forall i, \theta \tag{20}$$

Equations (19) and (20) are the Euler equations for the financial asset and physical investment. Remember that $\Phi_2^{b,\theta}$ is given by (9) and $\Phi_2^{l,\theta} = 0$.

Distributive effects and constraint effects. Our welfare analysis will rely on studying how changes in aggregate states affect welfare. DK18 show that such changes consist of two components: *distributive effects* and *collateral effects*. We refer to the latter type of effects with a slightly more general terminology as *constraint effects*. This is because we study credit constraints that do not necessarily contain "collateral" in the sense of physical assets. Alternatively, one could re-label for example an earnings-based

[13] DK18 include production output as part of net worth. In our model, the quantity $F^i(k_1^i, \ell_{d1}^{i,\theta})$ is not predetermined because of the labor choice that happens during period $t = 1$. We therefore do not include it as part of the state variable $n_1^{i,\theta}$. We have formally verified that this change would not alter any of the results in the original framework of DK18, see details in Appendix A.1.

borrowing constraint as a "collateral constraint" in which earnings serve as collateral. We choose to instead refer to collateral more narrowly as the presence of physical k in the borrowing constraint. Relative to DK18, both distributive and constraint effects feature additional economic forces in our model.[14] Lemma 1 characterizes relevant properties of the date 1 equilibrium.

Lemma 1 *The effects of changes in the aggregate state variables $N_1^{j,\theta}$ and K_1^j on agent i's indirect utility at date 1 are given by*

$$V_{N_1^j}^{i,\theta} \equiv \frac{dV^{i,\theta}(\cdot)}{dN_1^{j,\theta}} = \lambda_1^{i,\theta}\mathcal{D}_{1N^j}^{i,\theta} + \lambda_2^{i,\theta}\mathcal{D}_{2N^j}^{i,\theta} + \kappa_2^{i,\theta}\mathcal{C}_{N^j}^{i,\theta} \tag{21}$$

$$V_{K_1^j}^{i,\theta} \equiv \frac{dV^{i,\theta}(\cdot)}{dK_1^j} = \lambda_1^{i,\theta}\mathcal{D}_{1K^j}^{i,\theta} + \lambda_2^{i,\theta}\mathcal{D}_{2K^j}^{i,\theta} + \kappa_2^{i,\theta}\mathcal{C}_{K^j}^{i,\theta} \tag{22}$$

where $\mathcal{D}_{1N^j}^{i,\theta}$, $\mathcal{D}_{1K^j}^{i,\theta}$, $\mathcal{D}_{2N^j}^{i,\theta}$ and $\mathcal{D}_{2K^j}^{i,\theta}$ are called the distributive effects

$$\mathcal{D}_{1N^j}^{i,\theta} \equiv -\frac{\partial q^\theta}{\partial N_1^{j,\theta}}\Delta K_2^{i,\theta} - \frac{\partial m_2^\theta}{\partial N_1^{j,\theta}}X_2^{i,\theta} - \frac{\partial w_1^\theta}{\partial N_1^{j,\theta}}\ell_{d1}^{i,\theta} + \frac{\partial w_1^\theta}{\partial N_1^{j,\theta}}\ell_{s1}^{i,\theta} \tag{23}$$

$$\mathcal{D}_{1K_1^j}^{i,\theta} \equiv -\frac{\partial q^\theta}{\partial K_1^j}\Delta K_2^{i,\theta} - \frac{\partial m_2^\theta}{\partial K_1^j}X_2^{i,\theta} - \frac{\partial w_1^\theta}{\partial K_1^j}\ell_{d1}^{i,\theta} + \frac{\partial w_1^\theta}{\partial K_1^j}\ell_{s1}^{i,\theta} \tag{24}$$

$$\mathcal{D}_{2N^j}^{i,\theta} \equiv -\frac{\partial w_2^\theta}{\partial N_1^{j,\theta}}\ell_{d2}^{i,\theta} + \frac{\partial w_2^\theta}{\partial N_1^{j,\theta}}\ell_{s2}^{i,\theta} \tag{25}$$

$$\mathcal{D}_{2K^j}^{i,\theta} \equiv -\frac{\partial w_2^\theta}{\partial K_1^j}\ell_{d2}^{i,\theta} + \frac{\partial w_2^\theta}{\partial K_1^j}\ell_{s2}^{i,\theta} \tag{26}$$

and $\mathcal{C}_{N^j}^{i,\theta}$ and $\mathcal{C}_{K^j}^{i,\theta}$ are called the constraint effects

$$\mathcal{C}_{N^j}^{b,\theta} \equiv \frac{\partial \Phi_2^{b,\theta}}{\partial q^\theta}\frac{\partial q^\theta}{\partial N_1^{j,\theta}} + \frac{\partial \Phi_2^{b,\theta}}{\partial m_2^\theta}\frac{\partial m_2^\theta}{\partial N_1^{j,\theta}} + \frac{\partial \Phi_2^{b,\theta}}{\partial w_1^\theta}\frac{\partial w_1^\theta}{\partial N_1^{j,\theta}} + \frac{\partial \Phi_2^{b,\theta}}{\partial w_2^\theta}\frac{\partial w_2^\theta}{\partial N_1^{j,\theta}} \tag{27}$$

$$\mathcal{C}_{K^j}^{b,\theta} \equiv \frac{\partial \Phi_2^{b,\theta}}{\partial q^\theta}\frac{\partial q^\theta}{\partial K_1^j} + \frac{\partial \Phi_2^{b,\theta}}{\partial m_2^\theta}\frac{\partial m_2^\theta}{\partial K_1^j} + \frac{\partial \Phi_2^{b,\theta}}{\partial w_1^\theta}\frac{\partial w_1^\theta}{\partial K_1^j} + \frac{\partial \Phi_2^{b,\theta}}{\partial w_2^\theta}\frac{\partial w_2^\theta}{\partial K_1^j} \tag{28}$$

$$\mathcal{C}_{N^j}^{l,\theta} = \mathcal{C}_{K^j}^{l,\theta} = 0 \tag{29}$$

for $i \in \{b, l\}$, $j \in \{b, l\}$ and $\theta \in \Theta$.

Proof. The effects of changes in the aggregate state variables (N_1^θ, K_1) on agents'

[14] In their Online Appendix, DK18 also provide a generalization of the constraint, by allowing it to directly depend on net worth, in addition to the price of capital. Our addition of labor markets allows to focus on specific additional cases that are empirically motivated and deliver new results.

indirect utility are derived by taking partial derivatives of $V^{i,\theta}$ as defined by equations (15) to (18). We make use of the envelope theorem, according to which the derivatives of $\left\{u^i(c_1^{i,\theta}, \ell_{s1}^{i,\theta}) + \beta u^i(c_2^{i,\theta}, \ell_{s2}^{i,\theta})\right\}$ with respect to the state variables are 0. We further impose a symmetric equilibrium in which $n^{i,\theta} = N^{i,\theta}$ and $k_1^i = K_1^i$. ■

Remarks on Lemma 1. $\mathcal{D}_{1N^j}^{i,\theta}$, $\mathcal{D}_{1K^j}^{i,\theta}$, $\mathcal{D}_{2N^j}^{i,\theta}$ and $\mathcal{D}_{2K^j}^{i,\theta}$ are called *distributive effects* because

$$\sum_i \mathcal{D}_{1N^j}^{i,\theta} = \sum_i \mathcal{D}_{2N^j}^{i,\theta} = \sum_i \mathcal{D}_{1K^j}^{i,\theta} = \sum_i \mathcal{D}_{2K^j}^{i,\theta} = 0 \tag{30}$$

from the market clearing conditions, that is, they are "zero sum" effects across agents, state by state. Such a relation does not hold for the *constraint effects* $\mathcal{C}_{N^j}^{i,\theta}$ and $\mathcal{C}_{K^j}^{i,\theta}$. These collect any derivatives that multiply the shadow price on the financial constraint $\kappa_2^{i,\theta}$. Comparing Lemma 1 to its analogue in DK18, both our inclusion of labor markets and our more general financial constraint change this characterization. First, wage changes generate both distributive effects and constraint effects. Second, these wage changes occur in both periods 1 and 2, since labor market conditions in $t = 2$ depend on changes in the state of the economy in $t = 1$.[15] These two observations will be important for the earnings-based constraint. Third, we also allow equation (9) to include the asset price m_2^θ so the constraint effects include partial derivatives with respect to this variable.

Solution for period 0. The optimization problem of agent i at time $t = 0$ is

$$\max_{\{c_0^i, k_1^i, x_1^{i,\theta}\}} u^i(c_0^i) + \beta \mathbb{E}_0[V^{i,\theta}(n_1^{i,\theta}, k_1^i; N_1^\theta, K_1)] \tag{31}$$

subject to the time-0 budget constraint (5) and financial constraint (8). Using the envelope conditions $\frac{\partial V^{i,\theta}(\cdot,\cdot)}{\partial n_1^{i,\theta}} = \lambda_1^{i,\theta}$ and $\frac{\partial V^{i,\theta}(\cdot,\cdot)}{\partial k_1^i} = \lambda_1^{i,\theta}(q^\theta + F_{1k}^{i,\theta}(k_1^i, l_{d1}^{i,\theta}))$, the first-order conditions with respect to the asset holding and capital are derived as

$$m_1^\theta \lambda_0^i = \beta \lambda_1^{i,\theta} + \kappa_1^i \Phi_{1x^\theta}^i, \tag{32}$$

$$h^{i\prime}(k_1^i)\lambda_0^i = \mathbb{E}_0[\beta \lambda_1^{i,\theta}(F_{1k}^{i,\theta}(k_1^i, \ell_{d1}^{i,\theta}) + q^\theta)] + \kappa_1^i \Phi_{1k}^i, \ \forall i, \theta \tag{33}$$

where λ_0^i is Lagrange multiplier for (5) and κ_1^i is Lagrange multiplier for (8).

[15] More precisely, wages in $t = 2$ depend on states in $t = 2$ and states in $t = 2$ in turn depend on states in $t = 1$. As we have emphasized notationally, $w_2^\theta(N_2^\theta(N_1^\theta, K_1), K_2(N_1^\theta, K_1)) = w_2^\theta(N_1^\theta, K_1)$.

4 Efficiency analysis with different credit constraints

This section studies the pecuniary externalities arising from different credit constraints. We first determine the constrained efficient allocation by solving a (constrained) planner problem. This allocation can be implemented using a set of tax rates, which in turn are shown to depend on a set of sufficient statistics related to the distributive and constraint effects derived in Lemma 1. After introducing additional model restrictions required to sign the externalities, we characterize the sources and direction of the externalities for various special cases of financial constraints. Finally, we put the results in the context of macroprudential regulation in practice.

4.1 Constrained efficient allocation and sufficient statistics

Social planner problem. The social planner chooses allocations in $t = 0$ subject to the same period-0 constraints as the private agents, and subject to optimal behavior of the agents in periods $t = 1, 2$. This corresponds to a constrained Ramsey planner who can levy taxes in $t = 0$. Relative to DK18, the planner in our settings also takes into account the decentralized labor market in period $t = 1, 2$. All externalities have their root in the agents' period-0 decisions.[16] Formally, the social planner problem is

$$\max_{\{C_0^i \geq 0, K_1^i, X_1^{i,\theta}\}} \sum_i \alpha^i \{u^i(C_0^i) + \beta \mathbb{E}_0[V^{i,\theta}(N_1^{i,\theta}, K_1^i; N^\theta, K_1)]\} \tag{34}$$

$$\text{s.t.} \sum_i [C_0^i + h^i(K_1^i) - e_0^i] \leq 0 \quad (v_0) \tag{35}$$

$$\sum_i X_1^{i,\theta} = 0, \quad \forall \theta \quad (v_1^\theta) \tag{36}$$

$$\Phi_1^i(X_1^i, K_1^i) \geq 0, \ \forall i \quad (\alpha_i \kappa_1^i) \tag{37}$$

Note that α^b and α^l are Pareto weights that the social planner applies to borrowers and lenders, respectively. The variables in brackets denote Lagrange multipliers. The presence of $V^{i,\theta}(N_1^{i,\theta}, K_1^i; N^\theta, K_1)$, which is described by equation (15) to (18) above, makes clear that the planner takes the private equilibrium of periods $t = 1$ and $t = 2$ as given. In particular, the social planner internalizes the impact of changing N^θ and K_1 on prices in equilibrium.

[16] This also makes the nature of our externalities through wages distinct from those in the related model of Bianchi (2016). In his framework pecuniary externalities arise from contemporaneous decisions, in particular about labor demand. In ours, current borrowing decisions affect future borrowing constraints through prices, arising from labor supply changes.

Constrained efficient allocation and implementation. The economy's constrained efficient allocation is described by quantities $(C_0^i, K_1^i, X_1^{i,\theta})$, Pareto weights $\alpha^b/\alpha^l = \lambda_0^l/\lambda_0^b$ and shadow prices v_0,v_1^θ, and κ_1^i satisfying the optimality conditions and constraints of the social planner's problem. This allocation can be implemented with a set of tax rate on financial asset and capital purchases. Since the solution of the planner's problem is similar to DK18, we relegate the details to Appendix A.2. The final set of tax rates is

$$\tau_x^{i,\theta} = -\Delta MRS_{01}^{ij,\theta}\mathcal{D}_{1N^i}^{i,\theta} - \Delta MRS_{02}^{ij,\theta}\mathcal{D}_{2N^i}^{i,\theta} - \tilde{\kappa}_2^{b,\theta}\mathcal{C}_{N^i}^{b,\theta},\ \forall i,\theta \tag{38}$$

$$\tau_k^i = -\mathbb{E}_0[\Delta MRS_{01}^{ij,\theta}\mathcal{D}_{1K^i}^{i,\theta}] - \mathbb{E}_0[\Delta MRS_{02}^{ij,\theta}\mathcal{D}_{2K^i}^{i,\theta}] - \mathbb{E}_0[\tilde{\kappa}_2^{b,\theta}\mathcal{C}_{K^i}^{b,\theta}],\ \forall i \tag{39}$$

$\Delta MRS_{0t}^{ij,\theta} \equiv MRS_{0t}^{i,\theta} - MRS_{0t}^{j,\theta}$ for $t = 1,2$ denotes the difference between agents in the marginal rate of substitution (MRS) across time, $MRS_{01}^{j,\theta} \equiv \beta\lambda_1^{j,\theta}/\lambda_0^j$, $MRS_{02}^{j,\theta} \equiv \beta\lambda_2^{j,\theta}/\lambda_0^j$. We define $\tilde{\kappa}_2^{b,\theta} \equiv \beta\kappa_2^{b,\theta}/\lambda_0^b$ as the relative shadow price. A positive $\tau_x^{i,\theta}$ implies that agent i saves too much (borrows too little) in the market outcome. The planner thus wants to impose a tax on savings (remember that $x_1^i > 0$ implies saving, $x_1^i < 0$ borrowing). A positive τ_k^i means that agent i invests too much in capital relative to the constrained efficient allocation, so the planner imposes a tax on investment. In our formal welfare analysis, we focus on over-/under-borrowing since over-/under-investment effects cannot be signed in the DK18 framework. In the numerical application of the model, we do allow for both forces.

Nature of externalities and sufficient statistics. The optimal tax wedges, in combination with the distributive effects $\mathcal{D}$ and the constraint effects $\mathcal{C}$ derived in Lemma 1, allow us to characterize the externalities in this economy. In essence, by analyzing and interpreting the different terms in (38) and (39), we can understand how outcomes in the market economy deviate from the constrained efficient allocation and how such distortions could be corrected. Building on the earlier terminology we distinguish *distributive externalities* and *constraint externalities*.

The sign and magnitude of *distributive externalities* are determined by the product of:

(i) The difference in MRS of agents in periods 1 and 2, $\Delta MRS_{01}^{ij,\theta}$ and $\Delta MRS_{02}^{ij,\theta}$

(ii) The net trading positions on capital $\Delta K_2^{i,\theta}$, financial assets $X_2^{i,\theta}$, labor supply in periods 1 and 2 $\ell_{s1}^{i,\theta}, \ell_{s2}^{i,\theta}$, and labor demand in periods 1 and 2 $\ell_{d1}^{i,\theta}, \ell_{d2}^{i,\theta}$

(iii) The sensitivity of equilibrium prices to changes in aggregate state variables $\frac{\partial q^\theta}{\partial N_1^{j,\theta}}$,

$\frac{\partial m_2^\theta}{\partial N_1^{j,\theta}}, \frac{\partial w_1^\theta}{\partial N_1^{j,\theta}}, \frac{\partial q^\theta}{\partial K_1^j}, \frac{\partial m_2^\theta}{\partial K_1^j}, \frac{\partial w_1^\theta}{\partial K_1^j}$

The sign and magnitude of *constraint externalities* are determined by the product of:

(i) The relative shadow price of the financial constraint $\tilde{\kappa}_2^{i,\theta}$

(ii) The sensitivity of the financial constraint to the price of capital, asset price and wages for period 1 and 2 $\partial\Phi_2^{i,\theta}/\partial q^\theta$, $\partial\Phi_2^{i,\theta}/\partial m_2^\theta$, $\partial\Phi_2^{i,\theta}/\partial w_1^\theta$, $\partial\Phi_2^{i,\theta}/\partial w_2^\theta$

(iii) The sensitivity of the equilibrium capital price, asset price and wages in periods 1 and 2 to changes in aggregate states $\frac{\partial q^\theta}{\partial N_1^{j,\theta}}, \frac{\partial m_2^\theta}{\partial N_1^{j,\theta}}, \frac{\partial w_1^\theta}{\partial N_1^{j,\theta}}, \frac{\partial w_2^\theta}{\partial N_1^{j,\theta}}, \frac{\partial q^\theta}{\partial K_1^j}, \frac{\partial m_2^\theta}{\partial K_1^j}, \frac{\partial w_1^\theta}{\partial K_1^j}, \frac{\partial w_2^\theta}{\partial K_1^j}$

Remarks on the externalities. The lists above reveal how distortions in the model can be parsed into a compact list of sufficient statistics. Distributive externalities, those driven by effects which are "zero sum," depend on the difference in marginal rates of substitution in combination with the positions that agents take in quantities of capital, labor and financial assets in equilibrium. If these externalities were fully corrected, these quantities would be such that marginal rates of substitution equalize across agents. Logically, constraint externalities depend on the shadow price on the financial constraint, in combination with how the constraint moves with prices changes. Finally, both types of externalities depend on how prices react to changes in the aggregate states, making clear any externalities ultimately operate through price changes.

Determining the sign of externalities. Establishing the direction of pecuniary externalities is inherently difficult, even in relatively simple neoclassical settings. Distributive externalities as well as the constraint externalities that operate through changes in aggregate capital cannot be signed, a finding that DK18 refer to as "anything goes." Fortunately, the sign of constraint externalities that operate through changes net worth can be pinned down based on plausible additional assumptions, and this can provide useful insights into the normative consequences of financial constraints. It is a contribution of our paper to show that a model with labor markets brings about new subtleties in the determination of the sign of pecuniary externalities, and to lay out relevant additional assumptions for such a model. The next section introduces and discusses these additional assumptions, before we examine the normative implications of different types of financial constraints in the following section. In our numerical application of the model further below, we allow distributive externalities to be operational in addition to constraint externalities.

4.2 Additional model restrictions

This section lays out conditions that specialize the economic setting enough to determine the sign of the constraint externalities for each of the financial constraints of interest. Specifically, we introduce restrictions on different price responses to changes in sector-wide net worth in the borrowing and lending sector.

Condition required to analyze collateral constraints. Condition (40) is imposed to characterize the normative implications of asset-based collateral constraints:

$$\frac{\partial q^{\theta}}{\partial N_1^{i,\theta}} \geq 0, \ \forall i \tag{40}$$

This restriction is discussed in DK18, who show that it holds under standard preferences. DK18 also show that the failure of (40) leads to multiplicity and unstable equilibria. The economic assumption is that the price of capital increases in sector-wide net worth. In period $t = 1$, aggregate capital supply does not move with all-else-equal changes in net worth. Therefore the response of q^{θ} to changes in $N_1^{i,\theta}$ is driven by changes in the demand for new capital (K_2^{θ}). It is plausible that an increase in resources, holding the amount of available capital in the economy fixed, will increase the capital demand and thus put upward pressure on its price. Our graphical analysis below illustrates the role that the capital market and condition (40) play for the implications of collateral constraints.[17]

Condition required to analyze earnings-based borrowing constraints. To study the normative implications of earnings-based constraints, we introduce labor markets into the DK18 framework. The motivation is that wages crucially affect firms' costs and thereby their earnings. An important insight of this paper is that a general model with labor markets and earnings-based credit constraints requires further restrictions to be able to determine the sign of the relevant pecuniary externalities. In particular, we impose condition (41), restricting the model to an environment in which additional sector-wide net worth puts upward pressure on current wages in equilibrium:

[17]There is also an externality that operates through the choice of capital itself. Recall from our previous discussion that this over-investing vs. under-investing effect can not generally be signed. In our formal theoretical analysis, we therefore focus on over-borrowing vs. under-borrowing decisions in $t = 0$ which operate through the effect of net worth on prices in $t = 1$. In our numerical model experiments, we do allow for welfare changes that occur through over-investing vs. under-investing as well.

$$\frac{\partial w_1^\theta}{\partial N_1^{i,\theta}} \geq 0, \ \forall i \tag{41}$$

Relative to condition (40), restricting wage responses requires a different economic reasoning. In the case of capital, supply at the beginning of period $t = 1$ is fixed so price responses to changes in net worth are given by shifts in capital demand. For labor, an *intratemporal* production input, we need to examine whether and how labor demand and supply depend on net worth. First, we focus on *labor demand*. The optimal choice of labor by the firm is pinned down by its optimality conditions given the installed capital available for production. Installed capital is predetermined at the beginning of period $t = 1$. As a result, labor demand does not move when sector-wide net worth changes. Second, we examine *labor supply* movements in response to net worth changes. We do so formally in Appendix B.1. In this appendix we show that, holding the labor demand curve fixed, as long as the labor demand curve is downward sloping, the labor supply curve is upward sloping, and there is a sufficiently strong direct positive equilibrium effect from changes in net worth on the demand for leisure, then labor supply decreases in changes in sector-wide net worth. Taken together, with labor demand constant and labor supply decreasing in net worth, wages unambiguously rise with higher sector-wide net worth, and condition (41) holds. The graphical analysis of the model that will follow further below illustrates the role that the labor market and condition (41) play for the normative implications of earnings-based credit constraints.

Providing the reasoning for signing pecuniary externalities with labor demand and labor supply is a central insight of our paper. If there were no effects of changes in net worth on labor supply, then wage changes would purely be driven by changes in labor demand. As our subsequent analysis will show, labor demand is pinned down by the pre-determined capital stock available for production, so the derivative in (41) would be 0, and the allocation under an earnings-based borrowing limit would be constrained efficient, as far as constraint externalities are considered. Of course, there would still be distributive externalities due to the constraint. We show this in our numerical application in Section 5.

One assumption that shuts off the effect of changes in net worth on labor supply are GHH preferences. This assumption is made in related work of Bianchi and Mendoza (2010), Bianchi (2016) and Fazio (2021). Interestingly, in these papers labor demand forces do impact constrained efficiency, which distinguishes their setting from ours. We elaborate on this difference further in Section 6.1.

Conditions required to analyze interest coverage constraints. In order to study the normative consequences of interest coverage constraints, two restrictions are needed:

$$\frac{\partial m_2^\theta}{\partial N_1^{i,\theta}} \geq 0, \ \forall i \tag{42}$$

$$\frac{\partial w_2^\theta}{\partial N_1^{i,\theta}} \geq 0, \ \forall i \tag{43}$$

We introduce (42) because interest payments (the price of the financial asset) enter the interest coverage constraint. We justify it based on a logic similar to (40), stating that the price of savings increases in net worth. Intuitively, with higher sector-wide net worth, all else equal, agents desire to save more to smooth consumption, so the price of savings m_2^θ rises. Indeed, given the unconstrained agents' Euler equations, the price of capital and the financial asset are linked through a no-arbitrage restriction, so the bond price should tend to move in the same way after an all-else-equal changes in net worth as the price of capital, which increases in sector-wide net worth because of (40).

Condition (43) is an extension of (41) to future rather than current wages. Since in the model interest payments are made in $t = 2$ and the interest coverage constraint is written with relation to the ratio of earnings to interest payments in the same period, this constraint requires a restriction on w_2 rather than w_1. In direct analogy to (41), we impose that future wages respond positively to a rise in sector wide net worth, the same way current wages do. It turns out that it is more difficult to prove conditions for this derivative in the general model. We apply the informal argument that we want to characterize earnings-to-interest constraints under similar assumptions as debt-to-earnings constraints, and therefore assume the same wage response in both cases.

Validating the restrictions for specific model case. In the version of our model that we specify and calibrate for the numerical application, we can verify whether the above conditions hold for that particular specification of the general framework. Under the functional forms for preferences and technology chosen there, the relevant conditions are indeed satisfied. More details follow in Section 5.

4.3 Main results: welfare with different credit constraints

We now turn to the heart of our analysis, the efficiency properties of different forms of financial constraints. Based on our empirical motivation, we examine different functional forms of $\Phi_2^{b,\theta}$ in (18): collateral constraints, earnings-based constraints and interest coverage constraints. In each case, we study the constraint externalities that operate through borrowing decisions in the initial period.

4.3.1 Pecuniary externalities with a collateral constraint

We begin with the familiar case of a collateral-based financial constraint, in which physical capital limits the access to debt. Formally, when making decisions in period $t = 1$, the borrower's financial constraint (18) takes the following form:

$$\Phi_2^{b,\theta}(\cdot) = x_2^{b,\theta} + \phi q^\theta k_2^{b,\theta} \geq 0 \tag{44}$$

where $0 < \phi < 1$. The borrower maximizes her objective with respect to this constraint as well as the budget constraints (16) and (17). The constraint corresponds to equation (2) in the preview we provided in Section 2.

Proposition 1. *A collateral constraint as defined by (44), as long as it binds, gives rise to non-negative constraint externalities. This implies that there is an over-borrowing effect that operates through the constraint externalities.*

Proof. From (44), $\phi > 0$ and $k_2^{b,\theta} \geq 0$ it follows that $\frac{\partial \Phi_2^{b,\theta}}{\partial q^\theta} \geq 0$. According to condition (40), $\frac{\partial q^\theta}{\partial N_1^{i,\theta}} \geq 0$. Therefore $C_{N^i}^{b,\theta} = \frac{\partial \Phi_2^{b,\theta}}{\partial q^\theta} \frac{\partial q^\theta}{\partial N_1^{i,\theta}} \geq 0$. If the constraint binds, $\tilde{\kappa}_2^{b,\theta}$ is non-negative. It follows that the constraint externality resulting from the constraint is non-negative, that is, $\tilde{\kappa}_2^{b,\theta} C_{N^i}^{b,\theta} \geq 0$. This implies that there is over-borrowing operating through the constraint externalities: as is visible in equation (38), the social planner imposes subsidies on savings $\tau_x^{i,\theta}$ in order to induce less borrowing. ■

Interpretation. Proposition 1 confirms one of the main insights of DK18 and the existing literature more generally, and it formalizes the intuition on collateral constraints we previewed in Section 2. The borrower's decisions exert an externality through the market price of capital. As borrowers increase their debt position in period $t = 0$, they reduce aggregate net worth in the borrowing sector in period $t = 1$. Since the price of capital positively depends on sector-wide net worth by condition (40), it falls in $t = 1$.[18] Through the collateral constraint, the lower price of capital limits the

[18] While borrowing more reduces future aggregate net worth in the borrowing sector, it also increases future net worth in the lending sector. By condition (40), the latter effect actually puts upward pressure

ability to borrow between $t = 1$ and $t = 2$. As borrowers in $t = 0$ do not internalize this negative effect on future borrowing capability, the amount of debt taken on in $t = 0$ is suboptimally high, that is, there is over-borrowing. The social planner internalizes this relation, and thus discourages borrowing in $t = 0$ through subsidies on saving (for any given level of distributive externalities).

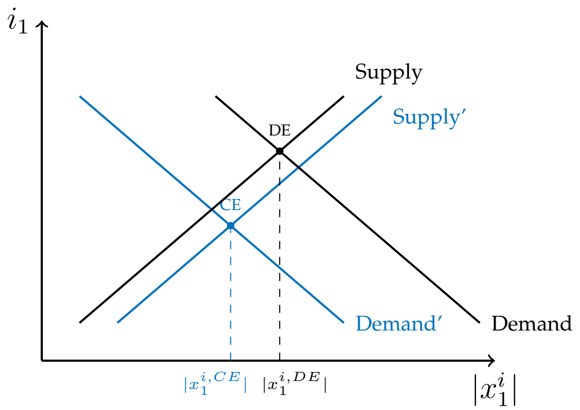
Graphical representation. Figure 1 provides the intuition behind Proposition 1 graphically. This graphical analysis will be especially helpful as a benchmark for the results with the earnings-based constraint below. It shows the period-0 credit market, period-1 capital market, and period-1 credit market. In each panel, points CE and DE represent the constrained efficient allocation and the decentralized equilibrium, respectively. The figure conveys how externalities emerge from borrowing decisions in $t = 0$, which through changes in the price of capital affect credit constraints in $t = 1$.

To explain Figure 1, we focus first on the decentralized equilibrium, point DE across Panels (a)-(d). The difference between Panels (a) and (b) only becomes relevant for implementing constrained efficiency, so for now consider Panel (a) to understand the period-0 credit market. The horizontal axis depicts the financial asset position of each agent in absolute value, that is, borrowing or credit demand $-x_1^{b,\theta}$, and saving or credit supply $x_1^{l,\theta}$. The vertical axis captures the interest rate between periods 0 and 1, $i_1^\theta = 1/m_1^\theta - 1$. Due to market clearing, borrowing and saving positions net out to 0, so $x_1^{b,\theta,DE} + x_1^{l,\theta,DE} = 0 \Rightarrow |x_1^{b,\theta,DE}| = |x_1^{l,\theta,DE}|$. Decisions on the credit market in $t = 0$ impact future net worth and thereby affect investment decisions in period $t = 1$. This is visible in Panel (c), which plots the capital supply curve (given by the vertical line indicating K_1) and the capital demand curve (given by the downward sloping relation between K_2^θ and q_1^θ). Capital supply is in general governed by an upward sloping relationship between K_1 and $q_1^\theta, \forall\theta$. However, since the analysis in the figure traces out the effects of period-0 borrowing externalities, and how these operate through changes in period-1 net worth, capital supply is effectively predetermined at the beginning of period $t = 1$.[19] The location of the demand curve does depend on the realization of aggregate net worth. Finally, the capital market equilibrium is linked to the period-1 credit market through the collateral constraint. Panel (d) shows credit supply and credit demand in period 1, by plotting $-x_2^{b,\theta}$ and $x_2^{l,\theta}$ in absolute value against the interest rate i_2^θ. The

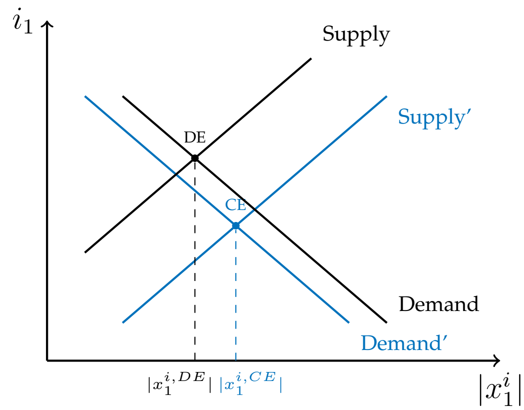
on the price of capital. However, the net effect of changes in borrower and lender net worth leads to a fall in the price of capital. We highlight this in the graphical illustration we provide further below.

[19]This would be different in a graphical analysis of pecuniary externalities that operate through over- and under-investment between $t = 0$ and $t = 1$.

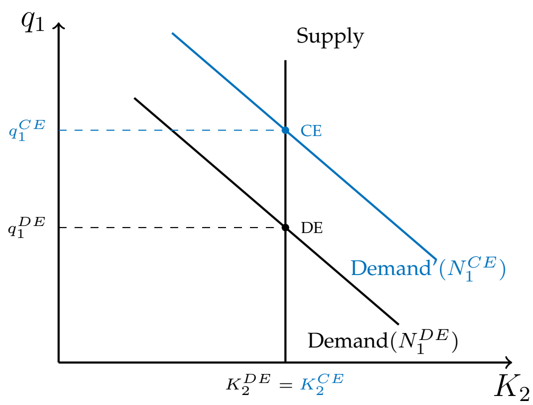
Figure 1: MARKET VS. PLANNER ALLOCATIONS: COLLATERAL CONSTRAINT



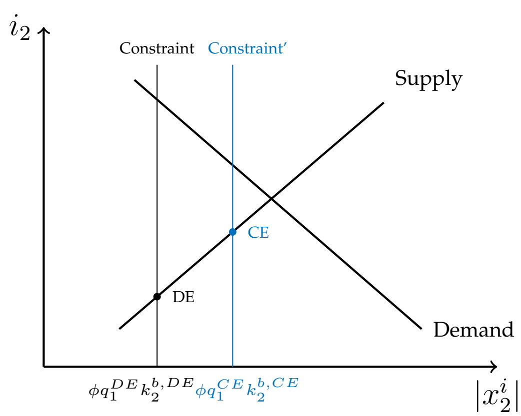
(a) Period-0 credit market (case 1: $|\tau_x^b| > |\tau_x^l|$)



(b) Period-0 credit market (case 2: $|\tau_x^b| < |\tau_x^l|$)



(c) Period-1 capital market (both cases)



(d) Period-1 credit market (both cases)

Notes. Decentralized equilibrium (DE) and constrained efficient equilibrium (CE) in the period-0 credit market, period-1 capital market and period-1 credit market of the model. State θ is omitted from the notation in the labeling. The figure distinguishes case 1 ($\partial q_1^\theta / \partial N_1^{b,\theta} > \partial q_1^\theta / \partial N_1^{l,\theta} \Leftrightarrow |\tau_x^{b,\theta}| > |\tau_x^{l,\theta}|$) and case 2 ($\partial q_1^\theta / \partial N_1^{b,\theta} < \partial q_1^\theta / \partial N_1^{l,\theta} \Leftrightarrow |\tau_x^{b,\theta}| < |\tau_x^{l,\theta}|$) as described in the text. In both cases, the social planner internalizes that period-0 borrowing decisions reduce equilibrium prices in the market for physical capital in period 1, which tightens the collateral constraint. The constrained efficient allocation features higher capital prices and more credit in period 1, as more saving (less borrowing) is incentivized through taxes/subsidies in period 0.

collateral constraint (44) puts a cap $\phi q_1^{\theta,DE} k_2^{\theta,DE}$ on the amount of credit, represented by a vertical line. Importantly, its location is determined by the market clearing price of capital $q_1^{\theta,DE}$. The decentralized equilibrium in the period-1 credit market is given by the intersection of the constraint and the credit supply curve.

By Proposition 1, the decentralized equilibrium is not efficient: the social planner distorts borrowing decisions in period 0 to drive up capital prices and thereby relax borrowing constraints in period 1. Under condition (40), sector-wide net worth of both borrowers and lenders positively impacts the price of capital. For the graphical analysis of the constrained efficient allocation, point CE across Panels (a)-(d), two finer cases can be distinguished: in case 1 the impact of the borrower sector net worth on wages is stronger than that of net worth in the lender sector ($\partial q_1^\theta / \partial N_1^{b,\theta} > \partial q_1^\theta / \partial N_1^{l,\theta}$) and in case 2, the opposite is true ($\partial q_1^\theta / \partial N_1^{b,\theta} < \partial q_1^\theta / \partial N_1^{l,\theta}$). In both cases, the social planner alters borrower and lender equilibrium net worth such that capital prices increase in $t = 1$. However, depending on the relative impact of net worth in the different sectors on the price of capital, the planner will tax borrowing (subsidize saving) more heavily for either the borrower or the lender to achieve the desired increase in the price of capital: in case 1, $|\tau_x^{b,\theta}| > |\tau_x^{l,\theta}|$, while in case 2, $|\tau_x^{b,\theta}| < |\tau_x^{l,\theta}|$. In other words, the planner reverts the over-borrowing of that agent more heavily whose decisions have a stronger impact on capital prices, making capital prices in period 1 rise in either case.[20] This is visible in Panels (a) and (b) which show the constrained efficient equilibrium for cases 1 and 2. In both cases, the planner incentivizes lenders to save more and borrowers to borrow less, to counteract the *over-borrowing* motive of both agents.[21] As a result, the credit supply curve is located to the right, and the credit demand curve to the left relative to their counterparts in the decentralized case. However, in Panel (a) (case 1), $|\tau_x^{b,\theta}| > |\tau_x^{l,\theta}|$, so the decrease in demand from the borrower is larger than the increase in supply from the lender, and the equilibrium quantity of credit is below that of the decentralized equilibrium. With a smaller amount of equilibrium borrowing, borrower net worth in period 1 will be higher while lender net worth will be lower relative to the decentralized equilibrium. Since $\partial q_1^\theta / \partial N_1^{b,\theta} > \partial q_1^\theta / \partial N_1^{l,\theta}$, capital prices are higher. In

[20]This can be seen as follows. According to Proposition 1, the constraint externality from the collateral constraint is non-negative, meaning that through equation (38) the planner desires a negative $\tau_x^{i,\theta}$ for $i \in \{b, l\}$. By equation (38), the size of the tax rate the planner chooses to implement the constrained efficient equilibrium is proportional to the size of the derivative of capital prices to sector wide net worth, that is, $\tilde{\kappa}_2^{b,\theta} \mathcal{C}_{N^i}^{b,\theta} \propto \partial q_1^\theta / \partial N_1^{i,\theta}$. As a result, when constraint externalities are corrected by the planner, the relative magnitude of $\partial q_1^\theta / \partial N_1^{b,\theta}$ and $\partial q_1^\theta / \partial N_1^{l,\theta}$ determines the relative magnitude of $\tau_x^{b,\theta}$ and $\tau_x^{l,\theta}$.

[21]This explanation highlights that in principle, in the case of the lender one could alternatively call the over-borrowing force an 'under-saving' effect.

Panel (b) (case 2), $|\tau_x^{b,\theta}| < |\tau_x^{l,\theta}|$ so there is a greater amount of equilibrium borrowing, and borrower net worth in period 1 will be lower while lender net worth will be higher. Since $\partial q_1^\theta / \partial N_1^{b,\theta} < \partial q_1^\theta / \partial N_1^{l,\theta}$, capital prices are higher, as in case 1. This makes clear that while the collateral constraint induces over-borrowing motives (borrowers want to borrow too much, savers want to save too little), a corrective policy may actually increase or decrease equilibrium credit.

In both cases 1 and 2, the corrective wedges introduced by the planner lead capital demand to shift upward, while changes the net worth induced by the planner do not move the capital supply curve, all else equal. These effects, shown in Panel (c), are the graphical counterpart to our discussion of condition (40) above.[22] As a result, capital prices in the constrained efficient equilibrium in period $t = 1$ are higher relative to the decentralized equilibrium. As in the decentralized case, the period-1 credit market, shown in Panel (d), is connected to the capital market through the price of capital. An increase in the price of capital loosens the collateral constraint, moving the intersection of the vertical line with the credit supply curve in Panel (d) to the right relative to the decentralized equilibrium. The planner internalizes the effect of period-0 borrowing decisions on future prices, and in turn on future borrowing space. The over-borrowing force in $t = 0$ is corrected through a tax wedge so that borrowers can obtain more credit between period 1 and 2 in the constrained efficient economy.

4.3.2 Pecuniary externalities with an earnings-based borrowing constraint

Consider now the case of an earnings-based borrowing constraint in the spirit of Drechsel (2022). As shown in Section 2.2, there is ample empirical evidence that this is a relevant constraint for US nonfinancial companies. (18) is specified as

$$\Phi_2^{b,\theta}(\cdot) = x_2^{b,\theta} + \widetilde{\phi}(F^b(k_1^b, \ell_{d1}^{b,\theta}) - w_1^\theta \ell_{d1}^{b,\theta}) \geq 0 \tag{45}$$

where $\widetilde{\phi} > 0$. This constraint implies that access to debt is restricted by the agent's period earnings, calculated as sales minus labor input costs. The constraint corresponds to equation (3) in our illustrative preview in Section 2.

[22]Recall that in the formal welfare analysis we focus on pecuniary externalities that operate through changes in net worth, and do not characterize over- or under-investment effects. In the graphical depiction, we therefore abstract from any difference in investment in $t = 0$ that may occur between the decentralized equilibrium and the constrained efficient allocation that the planner implements. In the numerical application of the model in Section 5, we also allow for over- und under-investment.

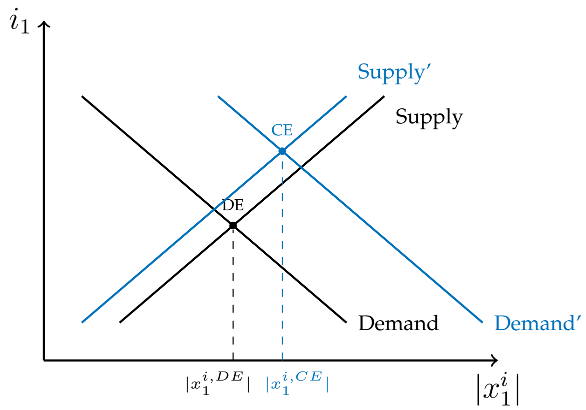
Proposition 2. *An earnings-based borrowing constraint as defined by (45), as long as it binds, gives rise to non-positive constraint externalities. This implies that there is an under-borrowing effect that operates through the constraint externalities.*

Proof. From (45), $\widetilde{\phi} > 0$ and $\ell_{d1}^{b,\theta} \geq 0$ it follows that $\frac{\partial \Phi_2^{b,\theta}}{\partial w_1^\theta} \leq 0$. According to (41), $\frac{\partial w_1^\theta}{\partial N_1^{i,\theta}} \geq 0$. Therefore, $C_{N^i}^{b,\theta} = \frac{\partial \Phi_2^{b,\theta}}{\partial w_1^\theta} \frac{\partial w_1^\theta}{\partial N_1^{i,\theta}} \leq 0$. If the constraint binds, $\tilde{\kappa}_2^{b,\theta}$ is non-negative. It follows that the constraint externality resulting from the constraint is non-positive, $\tilde{\kappa}_2^{b,\theta} C_{N^i}^{b,\theta} \leq 0$. This implies that there is under-borrowing operating through the constraint externalities: as is visible in equation (38) the planner imposes taxes on savings $\tau_x^{i,\theta}$ in order to induce more borrowing. ■

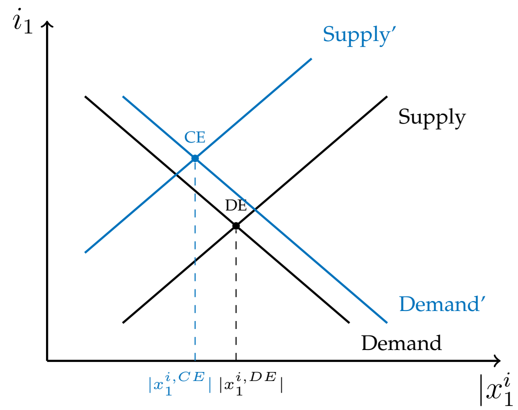
Interpretation. Proposition 2 delivers one of our main theoretical insights, previewed less formally in Section 2. An earnings-based borrowing constraint implies that the borrower takes a debt position that is too small relative to the social optimum. The mechanics of the model are similar to our explanation of Proposition 1, but operate through the real wage rate rather than the price of capital. A larger debt position in $t = 0$ reduces net worth in the borrowing sector in $t = 1$, which in turn reduces wages due to condition (41) (recall the discussion around labor demand and labor supply). Borrowers in $t = 0$ do not internalize that lower wages increase earnings and provide slack in the borrowing limit in $t = 1$. Therefore, in the market economy, agents under-borrow. The social planner internalizes the positive effect of borrowing in $t = 0$ on debt capacity in $t = 1$ through wages, and subsidizes (lowers the tax on) borrowing in period $t = 0$ (for a given level of distributive externalities).

Graphical representation. Figure 2 presents a graphical analysis for the case of the earnings-based borrowing constraint. As in Figure 1, points CE and DE represent the constrained efficient allocation and the decentralized equilibrium. The figure conveys how externalities emerge from borrowing decisions in $t = 0$, which through wage determination in the labor market affect credit constraints in $t = 1$. Relative to the case of the collateral constraint, Panel (c) now depicts the labor market in $t = 1$ rather than the market for physical capital. The earnings-based constraint (45) is represented by a vertical line in Panel (d), putting a cap $\widetilde{\phi}\pi(w_1^\theta) = \widetilde{\phi}(F^b(k_1^b, \ell_{d1}^{b,\theta}) - w_1^\theta \ell_{d1}^{b,\theta})$ on the amount of credit. Its location is affected by the market clearing wage. Similar to the collateral constraint and Figure 1, there is a refinement of condition (41) on the response of wages to changes in net worth. In both cases, according to Proposition 2, the decentralized

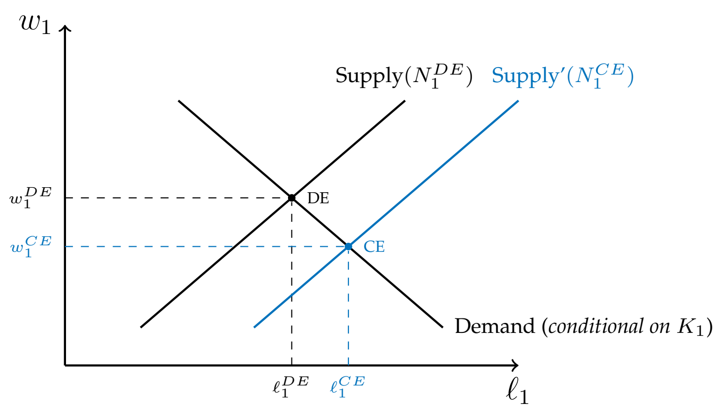
Figure 2: MARKET VS. PLANNER ALLOCATIONS: EARNINGS-BASED BORROWING CONSTRAINT



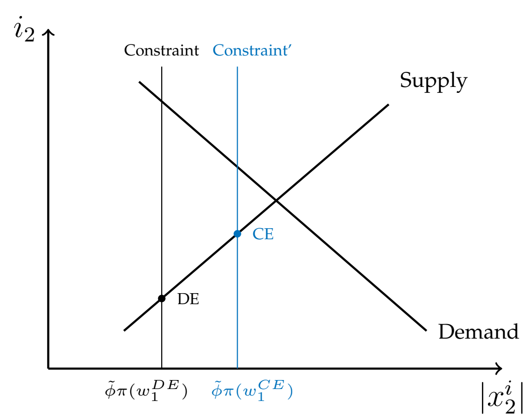
(a) Period-0 credit market (case 1: $|\tau_x^b| > |\tau_x^l|$)



(b) Period-0 credit market (case 2: $|\tau_x^b| < |\tau_x^l|$)



(c) Period-1 labor market (both cases)



(d) Period-1 credit market (both cases)

Notes. Decentralized equilibrium (DE) and constrained efficient equilibrium (CE) in the period-0 credit market, period-1 labor market and period-1 credit market of the model. State θ is omitted from the notation in the labeling. The figure distinguishes case 1 ($\partial w_1^\theta / \partial N_1^{b,\theta} > \partial w_1^\theta / \partial N_1^{l,\theta} \Leftrightarrow |\tau_x^{b,\theta}| > |\tau_x^{l,\theta}|$) and case 2 ($\partial w_1^\theta / \partial N_1^{b,\theta} < \partial w_1^\theta / \partial N_1^{l,\theta} \Leftrightarrow |\tau_x^{b,\theta}| < |\tau_x^{l,\theta}|$) as described in the text. In both cases, the social planner internalizes that period-0 borrowing decisions reduce equilibrium wages in period 1, which relaxes the earnings-based borrowing constraint. The constrained efficient allocation features lower wages and more credit in period 1, as less saving (more borrowing) is incentivized through taxes/subsidies in period 0.

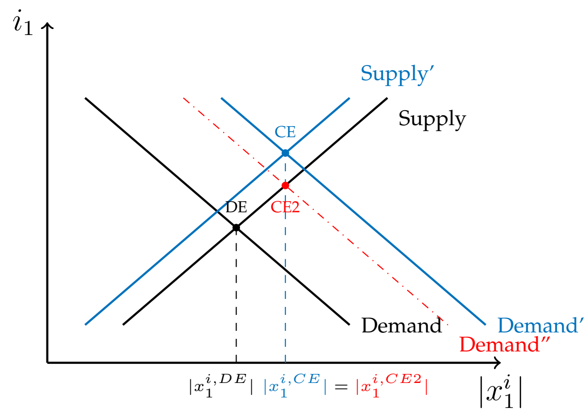
equilibrium features under-borrowing and the social planner subsidizes borrowing (taxes saving) in $t = 0$. In period $t = 0$ agents do not internalize that by reducing net worth in period 1 wages are reduced and this relaxes future borrowing constraints. To lower wages and thus create space for the constrained optimal amount of period-1 credit, the planner induces more debt in period 0 through corrective tax wedges.

The graphical representation of the economy with earnings-based borrowing constraint highlights the new insights that come with signing pecuniary externalities in our model with labor markets. The condition that wages increase with sector wide net worth in $t = 1$ requires understanding the response of labor demand as well as labor supply. Given that the capital available for production (K_1) is predetermined at the beginning of the period, labor demand is already pinned down, while labor supply responds to changes in sector-wide net worth (see Panel (c) of Figure 2). This is different in the market of capital relevant for the collateral constraint case, where the supply of capital is fixed, but the demand for new capital (K_2) increases with net worth (compare Panel (c) of Figure 1). In the presence of earnings-based constraints the planner can therefore induce more borrowing in the initial period, and thereby reduce borrower net worth in $t = 1$ to increase labor supply. This leads wages to fall.

Take-aways from graphical analysis of both constraints. In conclusion to the graphical analysis, the differences between Figures 1 and 2 reveal the sharp contrast between the normative consequences of the earnings-based and the collateral constraint. In the earnings-based constraint an *input price* (through the wage bill) enters with the opposite sign to how an *asset price* (the value of capital) enters the collateral constraint. Since wages and the price of capital respond with the same sign to changes in borrower net worth, all else equal, the implications in terms of whether agents borrow to much or too little in period $t = 0$ from a normative standpoint are the opposite for the two constraint types.

Alternative implementations of constrained efficiency. The set of tax rates τ_x^i, $i \in \{b, l\}$ that implements the constrained efficient equilibrium is not unique. There is an infinite number of combination of τ_x^b and τ_x^l that will alter $N_1^{b,\theta}$ and $N_1^{l,\theta}$ such that the same changes in period-1 prices and credit access are achieved. For the case of the earnings-based borrowing constraint we illustrate this in Figure 3, which is constructed as Panel (a) of Figure 2 but also plots an alternative implementation of the constrained efficient equilibrium (denoted $CE2$). This equilibrium represents the polar case in which only the borrower's financial asset position is taxed (borrowing is subsidized), while the lender is not taxed, $\tau_x^l = 0$. As the graph conveys, there is a choice for τ_x^b that achieves the identical equilibrium credit amount as point CE. As a result, the labor and credit market outcomes in period 1 would be the same as in Figure 2. A similar argument can be made for case 2 in Figure 2 and for both cases of the collateral constraint analyzed in Figure 1.

Figure 3: NON-UNIQUENESS OF IMPLEMENTATION



Period 0 Credit Market ($|\tau_x^b| > |\tau_x^l|$)

Notes. This figure repeats Panel (a) of Figure 2 but also plots an alternative implementation of the constrained efficient equilibrium (denoted $CE2$). Constrained efficiency can be achieved with different sets of tax rates $\tau_x^{i,\theta}$, $i \in \{b, l\}$, which give rise to the same change in aggregate net worth (and resulting wage reduction) in the constrained efficient relative to the decentralized equilibrium. In this case, only the borrowers' savings decisions are taxes (borrowing is subsidized), while $\tau_x^{l,\theta} = 0$. State θ is omitted from the notation in the labeling of the graph.

Timing of earnings. A result analogous to Proposition 2 can be obtained when the earnings-based constraint is written in terms of *future* earnings. See a related discussion on the timing of earning-based credit restrictions in Drechsel (2022). Formally, we modify the constraint to include earnings in period $t = 2$:

$$\Phi_2^{b,\theta}(\cdot) = x_2^{b,\theta} + \widetilde{\phi}(F^b(k_2^{b,\theta}, \ell_{d2}^{b,\theta}) - w_2^\theta \ell_{d2}^{b,\theta}) \geq 0 \tag{46}$$

It is easy to see that in this case, $C_{N^i}^{b,\theta} = \frac{\partial \Phi_2^{b,\theta}}{\partial w_2^\theta} \frac{\partial w_2^\theta}{\partial N_1^{i,\theta}} \leq 0$ because $\frac{\partial \Phi_2^{b,\theta}}{\partial w_2^\theta} \leq 0$ and $\frac{\partial w_2^\theta}{\partial N_1^{i,\theta}} \geq 0$ from (43). Therefore, $\tilde{\kappa}_2^{b,\theta} C_{N^i}^{b,\theta} \leq 0$, so under the restrictions we make on the model this modified version of the constraint also implies under-borrowing. This is interesting in light of the findings of Ottonello, Perez, and Varraso (2022) who emphasize the importance of timing assumptions in the context of collateral constraints.

Possibility of simplified labor market structure. In the model, both borrowers and lenders supply and demand labor. The empirical relevance of earnings-based constraints largely pertains to situations in which we think of the borrower as a firm. We have therefore studied an alternative version of our model in which we restrict the borrower to demanding labor and the lender to supplying it. This version of the model amounts to a special case in which the borrowing sector net worth does not affect equilibrium wages through the borrower's labor supply decisions. Nevertheless, due to the the effect of the lending sector's labor supply, the market equilibrium is still not efficient. The planner would levy a tax that corrects the effect of wages on future borrowing constraints. The details are provided in Appendix B.2.

4.3.3 Pecuniary externalities with an interest coverage constraint

Finally, we consider an interest coverage constraint, which restricts the amount of earnings relative to interest rate payments on the financial asset. This interest coverage ratio is a popular indicator used in debt covenants and its consequences for the transmission of monetary policy shocks have recently been studied by Greenwald (2019). We denote the interest rate in relation to the price of debt as $i_2^\theta = \frac{1}{m_2^\theta} - 1$. The constraint is written as

$$\Phi_2^{b,\theta}(\cdot) = x_2^{b,\theta} + \hat{\phi}\frac{F^b(k_2^{b,\theta}, \ell_{d2}^{b,\theta}) - w_2^\theta \ell_{d2}^{b,\theta}}{i_2^\theta} \geq 0 \tag{47}$$

where $\hat{\phi} > 0$. Equation (47) makes clear that the interest coverage constraints can be interpreted as variant of the earnings-based constraints, where interest payments on debt $i_2^\theta x_2^{b,\theta}$ rather than the level of debt $x_2^{b,\theta}$ are restricted by earnings. Note that we define this debt limit in terms of earnings one period ahead: as interest payments need to be defined *between two periods*, we compute the coverage ratio as earnings in $t = 2$ divided by interest payments between $t = 1$ and $t = 2$.

Proposition 3. *An interest coverage constraint as defined by (47), as long as it binds, gives rise to a product of constraint externalities that cannot be unambiguously signed. It results in either under-borrowing or over-borrowing depending on the relative absolute magnitude of two distinct externalities. As long as both earnings and the interest rate are positive, the first one operates through earnings and is non-positive, the second one operates through the interest rate and is non-negative.*

Proof. The relevant constraint externality for equation (47) is $C^{b,\theta}_{N^i} = \frac{\partial \Phi^{b,\theta}_2}{\partial w^\theta_2}\frac{\partial w^\theta_2}{\partial N^{i,\theta}_1} + \frac{\partial \Phi^{b,\theta}_2}{\partial i^\theta_2}\frac{\partial i^\theta_2}{\partial N^{i,\theta}_1}$. For the first term, as long as $i^\theta_2 > 0$, the same logic as in the proof of Proposition 2 applies, so $\frac{\partial \Phi^{b,\theta}_2}{\partial w^\theta_2}\frac{\partial w^\theta_2}{\partial N^{i,\theta}_1} \leq 0$. For the second term, if $F^b(k^{b,\theta}_2, \ell^{b,\theta}_{d2}) - w^\theta_2 \ell^{b,\theta}_{d2} > 0$, then $\frac{\partial \Phi^{b,\theta}_2}{\partial i^\theta_2} \leq 0$. According to (42), $\frac{\partial i^\theta_2}{\partial N^{i,\theta}_1} \leq 0$. Therefore $\frac{\partial \Phi^{b,\theta}_2}{\partial i^\theta_2}\frac{\partial i^\theta_2}{\partial N^{i,\theta}_1} \geq 0$. If the constraint binds, $\tilde{\kappa}^{b,\theta}_2$ is non-negative. It follows that the constraint externality resulting from the constraint is non-positive if $\left|\frac{\partial \Phi^{b,\theta}_2}{\partial w^\theta_2}\frac{\partial w^\theta_2}{\partial N^{i,\theta}_1}\right| > \left|\frac{\partial \Phi^{b,\theta}_2}{\partial i^\theta_2}\frac{\partial i^\theta_2}{\partial N^{i,\theta}_1}\right|$, which would imply $\tilde{\kappa}^{b,\theta}_2 C^{b,\theta}_{N^i} \leq 0$. This would imply under-borrowing: as is visible in equation (38) the planner imposes taxes on savings $\tau^{i,\theta}_x$ in order to induce more borrowing. The constraint externality is non-negative if $\left|\frac{\partial \Phi^{b,\theta}_2}{\partial w^\theta_2}\frac{\partial w^\theta_2}{\partial N^{i,\theta}_1}\right| < \left|\frac{\partial \Phi^{b,\theta}_2}{\partial i^\theta_2}\frac{\partial i^\theta_2}{\partial N^{i,\theta}_1}\right|$, which would imply $\tilde{\kappa}^{b,\theta}_2 C^{b,\theta}_{N^i} \geq 0$. This would imply over-borrowing: as is visible in equation (38) the planner imposes a subsidy on savings $\tau^{i,\theta}_x$ in order to induce less borrowing. ■

Interpretation. Proposition 3 delivers the novel insight that, on the one hand, interest coverage constraints contain an element of under-borrowing, where similar to an earnings-based constraint a pecuniary externality operates through the wage bill. On the other hand, however, rising prices of financial assets (falling interest rates) induce over-borrowing. The social planner needs to assess quantitatively whether the pecuniary externality operating through wages or the one embodied in interest rate changes is stronger. In one case, borrowing in period $t = 0$ should be supported, in the other case, incentives should be provided to reduce period-0 credit.

Interestingly, the price of the financial asset and the price of capital are linked through no-arbitrage restrictions imposed by the unconstrained agents' Euler equations. This is a restriction by which q^θ and $\frac{1}{i^\theta_2}$ should tend to move in the same way in response to changes in sector-wide net worth. From a welfare point of view, one can therefore interpret the interest coverage constraint as a "mixture" of an earnings-based and an asset-based borrowing constraint. To the best of our knowledge, this paper is the first to uncover this property of interest coverage constraints, which so far have only been studied from a positive angle (see e.g. Greenwald, 2019).

4.4 Discussion of the practical relevance of our results

Our findings highlight that the optimal design of macroprudential interventions depends critically on the specific nature of financial constraints. Proposition 1 guides policy to monitor asset prices whenever they are the limiting factor for borrowers' ability to obtain debt. What comes to mind are mortgage markets, where real estate serves as collateral, or repo markets, where financial assets are pledged. Proposition 2 is motivated by microeconomic evidence on earnings-based constraints faced by nonfinancial companies, and the results points to a role for considering the relation between labor markets and corporate credit markets in supervision.[23] Proposition 3 illustrates the possibility of constraints, again in connection to nonfinancial firms, in which several forces operate simultaneously. To gauge the relative strengths of competing channels, it is imperative to base supervision on sufficient microeconomic detail, and to understand further economic forces that may interact with the pecuniary externalities we characterize. The next section provides a quantitative angle on our results, and allows for additional channels through which externalities operate.

5 Numerical application

This section conducts policy experiments in a parameterized version of the model. Our application quantifies the welfare loss that arises from imposing an 'incorrect' macroprudential policy. We assume that the true model is an economy with earnings-based borrowing constraints, but impose tax rates that are computed as optimal under the assumption that agents instead face asset-based borrowing constraints. We then calculate the resulting welfare change in consumption equivalents. Importantly, in this experiment, both distributive and constraint externalities, as well as both under- and over-borrowing and under- and over-investing, are at play. Hence the analysis not only provides a quantification of the normative implications of earning-based borrowing constraints, but also does so in the presence of multiple externality channels, addressing the "anything goes" limitation of the general DK18 framework. This is important in light of recent work by Lanteri and Rampini (2021) who show distributive externalities to be meaningful in magnitude, even exceeding the impact of constraint externalities.

5.1 Model specification and parameterization

Throughout this application of the model, we assume that there is no uncertainty and no period-0 financial constraint, to fully focus on the constraint and distributive externalities that arise from the financial constraints in period 1.

[23] In fact, it is also the case that mortgages have income-flow related (*payment-to-income*) constraints in addition to asset-based (*loan-to-value*) constraints (Greenwald, 2018), so Proposition 2 may have some relevance for mortgage markets as well.

Preferences. We consider the case where labor supply is inelastic and the case where it is optimally chosen by the agents. In the case of inelastic labor supply, the period utility function follows the log-utility specification

$$u^i(c_t^i, \ell_{st}^i) = \log(c_t^i) \tag{48}$$

In the case of endogenously determined labor supply, the period utility function follows a standard separable utility specification

$$u^i(c_t^i, \ell_{st}^i) = \begin{cases} \log(c_t^i) & \text{if } \ t = 0 \\ \log(c_t^i) - \frac{1}{1+\psi}(\ell_{st}^i)^{1+\psi} & \text{if } \ t \geq 1 \end{cases} \tag{49}$$

where there is no labor choice in $t = 0$ because no production takes place in the initial period. Note here that we choose preferences that generate a wealth effect on labor supply, contrary to related work, for example Bianchi (2016).

Production technology. We assume a constant to returns to scale (CRS) and a decreasing returns to scale (DRS) production function for the borrower and the lender, respectively, following Kiyotaki and Moore (1997). Formally,

$$F^b(k_t^b, \ell_{dt}^b) = z_b(k_t^b)^\alpha(\ell_{dt}^b)^{1-\alpha}, \quad F^l(k_t^l, \ell_{dt}^l) = z_l((k_t^l)^\alpha(\ell_{dt}^l)^{1-\alpha})^\nu \tag{50}$$

where we assume $z_b > z_l$ and $\nu < 1$.

Investment technology. Following Dávila and Korinek (2018), $h^i(k)$ is given by

$$h^i(k) = \frac{\eta}{2}k^2, \quad i \in \{b, l\} \tag{51}$$

Parameter values. Table 1 summarizes our parameterization. We impose a standard value 0.33 for the capital share in production α. We set the discount factor β as 0.9752 following Drechsel (2022) who targets average US corporate loan rates. The Frisch labor supply elasticity ψ and returns to scale ν are set to 2 and 0.75 as in the recent firm financial frictions model of Jungherr and Schott (2021). We set the tightness parameter of the asset-based constraint ϕ as 0.46 following Bianchi (2016), who uses the average leverage ratio of US non-financial corporations of 46% as a calibration target. Based on this value of ϕ, we then calibrate the borrowing limit in earnings-based constraint $\widetilde{\phi}$ to ensure that the debt-to-output ratio is the same across the economies in which we calculate the optimal tax rates and the one in which we impose them. We do this separately for the case with inelastic labor supply and the case with endogenous labor supply. For the remaining parameters, it is difficult to find an exact counterpart in the literature or the data. We set them to ensure that the borrower has a superior production technology ($z_b > z_l$), but lacks the endowments to make capital investment, while the lender can provide the necessary resources to fund the capital investment.

Table 1: Calibration of the model

Parameter	Description	Value	Source / Target
α	Capital share	0.33	Standard for US case
β	Discount factor	0.9752	Drechsel (2022)
ψ	Labor supply elasticity	2	Jungherr and Schott (2021)
ν	Returns to scale - lender	0.75	Jungherr and Schott (2021)
ϕ	Borrowing limit - asset	0.46	Bianchi (2016)
$\widetilde{\phi}$	Borrowing limit - earnings (inelastic labor)	0.534	Match debt-to-output, $\frac{-x_2^b}{y_1^b+y_1^l}$
	Borrowing limit - earnings (endogenous labor)	0.617	Match debt-to-output, $\frac{-x_2^b}{y_1^b+y_1^l}$
η	Investment technology	1	Normalization
(z_b, z_l)	Productivity	(2,1)	
(e_0^b, e_1^b, e_2^b)	Endowments - borrower	(0,0,0)	
(e_0^l, e_1^l, e_2^l)	Endowments - lender	(1,1,0)	

Validity of model restrictions. In the parameterization of the model shown in Table 1, we verify that the model restrictions introduced in Section 4.2, and required to derive our formal theoretical analysis above, indeed hold. This is the case for equations (40) and (41), which are relevant for our policy application. That is, the calibration of the model implies $\frac{\partial q}{\partial N_1^i} \geq 0, \frac{\partial w_1}{\partial N_1^i} \geq 0, \ \forall i$.

5.2 Definition of policy experiment and corrective tax rates

Determining the tax schedule in asset-based economy. We proceed by first solving the planner problem to derive the constrained efficient allocation in an economy with asset-based borrowing constraints. In this social planner problem, we need to pick the welfare weights (α_b, α_l). We set them to achieve the same ratio of period-0 consumption as in the corresponding decentralized equilibrium. This leads to $(\alpha_b, \alpha_l) = (0.05, 0.95)$ for the case with inelastic labor supply and $(\alpha_b, \alpha_l) = (0.20, 0.80)$ for the case with endogenous labor supply. We then compute the optimal corrective taxes $(\tau_x^b, \tau_x^l, \tau_k^b, \tau_k^l)$ at the constrained efficient allocation according to (38) and (39). To be able to separate distributive and constraint externalities in our analysis, we also compute that component of optimal taxes on borrowing/saving decisions that arise from the constraint externalities at the constrained efficient allocation, $\tau_x^{i,c.e.} = -\tilde{\kappa}_2^b \mathcal{C}_{N^i}^b$. $\forall i$.

Imposing the 'wrong' tax schedule in earnings-based economy. Next we consider the 'true' economy with earnings-based borrowing constraints. First, we compute the welfare gain from moving from the decentralized equilibrium to the constrained efficient allocation in this economy. This is done with the same welfare weights as in the asset-based economy. We call this the 'right' policy. Second, we compute the welfare change from imposing the corrective taxes that we optimally derived in the economy with asset-based constraints above. We call this the 'wrong' policy.

To calculate consumption equivalent welfare measures, we follow Jones and Klenow (2016). We compute how much of permanent consumption should be inflated or deflated when we change from allocation B to allocation A, by finding λ such that

$$\begin{aligned} SW^{B,\lambda} &\equiv \alpha_b \sum_{t=0}^{2} \beta^t u((1+\lambda)c_{bt}^B, \ell_{bt}^B) + \alpha_l \sum_{t=0}^{2} \beta^t u((1+\lambda)c_{lt}^B, \ell_{lt}^B) \\ &= \alpha_b \sum_{t=0}^{2} \beta^t u(c_{bt}^A, \ell_{bt}^A) + \alpha_l \sum_{t=0}^{2} \beta^t u(c_{lt}^A, \ell_{lt}^A) \equiv SW^A. \end{aligned}$$

Under log-utility assumption, λ is derived as

$$\lambda = \exp\left((SW^A - SW^B)\frac{1-\beta}{1-\beta^3}\right) \times 100\ (\%), \tag{52}$$

where $SW^B \equiv \alpha_b \sum_{t=0}^{2} \beta^t u(c_{bt}^B, \ell_{bt}^B) + \alpha_l \sum_{t=0}^{2} \beta^t u(c_{lt}^B, \ell_{lt}^B)$.

Finally, we need to determine how taxes are rebated to the agents. Similar to Lanteri and Rampini (2021), we assume that agents are reimbursed a lump-sum amount that corresponds to the amount they paid or received as part of the distortionary taxes.

Optimal corrective taxes in different economies. We first examine the resulting corrective taxes. Table 2 shows the tax rates that implement constrained efficient allocation for each economy. Recall from equations (38) and (39) that the subscripts x and k indicate taxes on saving in the financial asset and saving in capital, respectively. The table shows these two tax rates separately for the lender and the borrower, and additionally reports the component of the corrective taxes on saving due to constraint externalities only, $\tau_x^{b,c.e.}$ and $\tau_x^{l,c.e.}$. The negative sign of these tax rates in the asset-based economy, and the positive sign in the earnings-based economy with endogenous labor supply confirm our findings from Section 4. There is over-borrowing with a collateral constraint, so the social planner levies a negative tax on saving (a debt tax), $\tau_x^{i,c.e.} < 0$. There is under-borrowing with the earnings-based constraint, so the social planner taxes saving to induce borrowing, $\tau_x^{i,c.e.} > 0$. If labor is inelastic, however, the allocation with the earnings-based constraint is already constrained efficient, so $\tau_x^{i,c.e.} = 0$.

Table 2: Optimal corrective taxes in different economies (in $\%$)

Economy	τ_x^b	τ_x^l	τ_k^b	τ_k^l	$\tau_x^{b,c.e.}$	$\tau_x^{l,c.e.}$
Collateral constraints, inelastic labor	-21.1	4.0	-29.1	-29.4	-0.3	-0.1
Earnings-based constraints, inelastic labor	-8.2	-1.3	-26.7	-12.4	0.0	0.0
Collateral constraints, endogenous labor	-1.6	-3.4	-1.0	0.6	-1.9	-3.2
Earnings-based constraints, endogenous labor	0.3	0.4	-2.6	-7.1	0.9	0.3

Table 2 also shows that the fully optimal taxes $(\tau_x^b, \tau_x^l, \tau_k^b, \tau_k^l)$, which address both constraint and distributive externalities, and which are levied on both savings and investment decisions, are large compared to the components that address the constraint externalities only. This finding indicates that distributive externalities and over- and under-investment forces, which cannot be signed in the general DK18 framework, are quantitatively large in our parameterized specification of the model. This is in line with the findings of Lanteri and Rampini (2021) on the importance of distributive externalities for capital reallocation in an infinite horizon version of DK18. The strong impact of distributive externalities is also loosely connected to the work of Itskhoki and Moll (2019), which emphasizes the importance distributive externalities through wages in the context of growth and development policies in the presence of financial frictions.

5.3 Results of numerical policy experiment

We now turn to calculating how much a macroprudential policy designed under imprecise assumptions about financial constraints deteriorates social welfare, the ultimate goal of our numerical application. Table 3, Panel (a) shows the welfare results when both distributive and constraint externalities are operational. In the true economy with earnings-based borrowing constraints, the constrained efficient allocation leads to a 0.60% higher permanent consumption than the decentralized equilibrium in the same economy. Importantly, when the wrong policy is rolled out, consumption equivalent welfare decreases by 1.95% and 0.52% for the economy with inelastic and endogenous labor supply, respectively. The table also reports the difference in consumption equivalents between imposing the right and the wrong policy, which amounts to as much as 2.55% in the economy where labor supply is inelastic. To put the magnitudes of these welfare effects into some context, in Bianchi (2011) the welfare gains from correcting the externality are 0.135% of permanent consumption. In Bianchi and Mendoza (2018) the average welfare gain from implementing the optimal policy is 0.3% in permanent consumption. In light of these numbers, the wrong policy in our application worsens social welfare significantly, relative to the market allocation and even more so relative to imposing the optimal policy. Designing macroprudential policy under imprecise assumptions about the relevant borrowing frictions can evidently lead to drastic welfare losses.

Table 3: Consumption equivalent welfare change in different counterfactuals

Panel (a): all types of externalities

Economy	**Right policy, $\lambda(\%)$**	**Wrong policy, $\lambda(\%)$**	**$\Delta(\%)$**
Earnings-based constraints, inelastic labor	0.60	-1.95	-2.55
Earnings-based constraints, endogenous labor	0.60	-0.52	-1.12

Panel (b): constraint externalities only

Economy	**Right policy, $\lambda(\%)$**	**Wrong policy, $\lambda(\%)$**	**$\Delta(\%)$**
Earnings-based constraints, inelastic labor	0.00	-0.01	-0.01
Earnings-based constraints, endogenous labor	0.06	-0.47	-0.53

Notes. The table shows the welfare impact of policies carried out in the 'true' economy, which features earnings-based constraints. The right policy is the solution to the social planner's problem in that economy. The wrong policy is calculated under the incorrect assumption that agents face asset-based borrowing constraints.

Panel (b) separately breaks out results for the effects of constraint externalities only. As there is no inefficiency through constraint externalities in the earning-based economy with inelastic labor supply, social welfare is not altered through the right policy. With endogenous labor supply, the right policy increases permanent consumption only marginally, by 0.06%. However, the wrong policy decreases permanent consumption by 0.01% and 0.47% for the economy with inelastic and endogenous labor supply. Compared to the optimal policy, a consumption loss of as much as 0.53% is incurred by the agents. These effects are still meaningful, and larger than some results in the literature, though smaller than those operating through all types of externalities discussed in Panel (a).

Taken together, our numerical applications suggest that considering the specific microeconomic details of borrowing constraints is quantitatively important for the design of macroprudential policy. The empirical evidence for the corporate sector points towards earnings-based constraints being the prevalent type of credit limit, while the literature on pecuniary externalities has largely focused on asset-based constraints. Our application makes clear that it is important to narrow this gap, and we think our analysis provides an important first step to do so.

6 Extensions and additional considerations

This section expands our analysis by exploring several modifications of the financial constraints and the economic environment. These modifications connect our findings to relevant insights in the related literature.

6.1 Working capital and labor demand inefficiencies

The constraints in our model limit an intertemporal financial position. In practice, it is common that firms also hold shorter-run or intratemporal debt positions. For example, firms pre-finance production inputs before revenues are collected. An insight that comes naturally out of our framework is that if the access to such *working capital*, in addition to other debt, is limited by an earnings-based constraint, this enhances the strength of the externality that operates through wages. To see this, suppose a firm takes the intertemporal position x' as above, and in addition pre-finances a fraction ψ of its wage bill with an intraperiod working capital loan $x_{wc} = -\psi w\ell$. An earnings-based constraint on *total* borrowing takes the form

$$x' - \psi w\ell \geq -\widetilde{\phi}(F(k,\ell) - w\ell) \tag{53}$$

which can be rearranged to

$$x' \geq -\widetilde{\phi}F(k,\ell) + (\widetilde{\phi} + \psi)w\ell \tag{54}$$

This constraint corresponds to (45), with the only difference that the parameter multiplying the wage bill is $(\widetilde{\phi} + \psi) > \widetilde{\phi}$. The presence of working capital thus strengthens the externality in the earnings-based constraint, leading to a more pronounced under-borrowing effect and an even more important role for macroprudential policy. To see this formally, in the proof of Proposition 2 a larger parameter multiplying the wage increases $\frac{\partial \Phi_2^{b,\theta}}{\partial w_1^{\theta}}$ and thus drives $C_{N^i}^{b,\theta}$ more negative.

The same logic applies to working capital in combination with an interest coverage constraint. Recall that this constraint entails two competing forces, with one operating through earnings (wages) and the other one through interest rates. In combination with working capital, the wage externality becomes stronger and the constraint thus more likely to result in under-borrowing.

In their seminal work, Bianchi and Mendoza (2010) and Bianchi (2016) propose models with working capital and collateral constraints. In Bianchi and Mendoza (2010), working capital payments relate to wages, as in (54), but the sum of intertemporal debt and working capital are restricted by collateral rather than by earnings. In our notation,

$$x' \geq -\phi q k' + \psi w \ell \tag{55}$$

Bianchi and Mendoza (2010) find that in their framework the collateral price effect (through the debt limit) is stronger than the wage effect (through working capital). Our setting with an earnings-based borrowing constraint is quite different because both the debt limit itself and the working capital component depend on wages. While the setting of Bianchi and Mendoza (2010) features two offsetting effects through ϕq and ψw, the earnings-based constraint in combination with working capital gives rise to two effects through $\widetilde{\phi} w$ and ψw which go in the same rather than in opposing directions. This gives a strong under-borrowing force in our framework.

In Bianchi (2016) firms need to pre-finance payments not only to workers but also shareholders and bondholders, which amounts to their full revenue stream appearing on the left hand side of the constraint. Such a formulation is close to that in Jermann and Quadrini (2012), and in our notation reads

$$x' \geq -\phi q k' + \tilde{\psi} F(k, \ell) \tag{56}$$

In addition, firms in Bianchi (2016) face equity constraints. In his setting, firms do not internalize that hiring puts upward pressure on wages, which tightens other firms' constraints contemporaneously. This force is complementary to our mechanism, where the externality unfolds intertemporally: firms' current borrowing exerts a positive effect on future borrowing constraints through net worth affecting equilibrium wages.

Note that households in the models of Bianchi and Mendoza (2010) and Bianchi (2016) have GHH preferences. The more general preferences in our framework allow us study the role of labor supply for pecuniary externalities with financial constraints. In fact, in our framework there are no constrained inefficiencies that operate through labor demand. The reason is that in our setting optimal labor demand with or without an earnings-based constraint maximizes earnings ($F(k, \ell) - w\ell$). Thus, the agents can always choose labor demand that maximizes their unconstrained objective as well as their borrowing capacity. This is not the case for a working capital constraint, where an agent has the incentive to reduce expenditures on labor to ease the working capital constraint. This incentive might go in the opposite direction from the need to incur wage costs to maximize profits.

In brief, inefficiencies that arise from the earnings-based constrained as formulated in this paper operate through labor supply effects. In contrast, the inefficiencies that arise in the work of Bianchi and Mendoza (2010) and Bianchi (2016) operate through labor demand effects. This makes clear that the insights derived from our framework are new and complementary to the existing findings of the literature. Interestingly, the mechanics illustrated by equation (54) suggest that these complementary effects might amplify each other.

6.2 Small open economy vs. endogenous interest rates

Our model features a fully endogenous interest rate $i = \frac{1}{m} - 1$, which is determined by financial market clearing. A body of work that studies financial frictions and pecuniary externalities in small open economies (SOE), where the interest rate is fixed and assumed to be determined in international markets. A prominent example is Bianchi (2011), who studies the welfare consequences of borrowing constraints in an SOE environment and highlights the pecuniary externalities that operate through external financial positions of emerging economies. Other important studies include Mendoza (2006, 2010) and Jeanne and Korinek (2010). Bianchi and Lorenzoni (2022) study capital controls and foreign currency reserves as macroprudential policy tools for open economies in the context of pecuniary and aggregate demand externalities.

We focus on an endogenous interest rate for two reasons. First, the evidence on financial constraints reviewed in Section 2.2 is primarily for the United States, an economy for which the assumption of a fixed interest rate is less suitable. Second, a setting with fixed interest rates would trivially render the interest coverage constraint similar to the debt-to-earnings limit when it comes to welfare consequences. Formally, this is easy to see when replacing i_2^{θ} with a constant $\bar{i}$ in equation (47). The externalities coming from this constraint would have the same sign as those in (45). An endogenous interest rate thus allows us to study a wider range of constraints.

Interestingly, macroeconomic models of emerging markets also feature varying forms of borrowing constraints, in which capital, endowments, (tradable) production output, or combinations of these variables may restrict access to debt. Examples of such constraints in the context of emerging markets can be found in Mendoza (2006), Korinek (2011), and in related work. Since the microeconomic evidence on the specific forms of constraints is thinner for emerging economies, we believe that it would be promising to conduct an analysis similar to what Lian and Ma (2020) do for US and Japanese companies, but with a focus on emerging markets.[24]

6.3 Output prices vs. input prices

In the earnings-based borrowing constraint, w denotes the price of labor, while the price of output is normalized to 1. As a consequence, it is the *relative* price of production inputs through which the externality operates. The fact that we emphasize the role of prices in credit constraints begs the question whether it is relevant to also study output (sales) price variation for firms as a source of pecuniary externalities. Relevant variation in output prices could be introduced by extending our model to a multi-good environment. We provide thoughts on two possibilities.

The first possibility would be to make firms sell in monopolistically competitive markets, which gives them pricing power. In this environment, prices are choice variables of the firm, so firms would actually internalize how their own price setting affects the constraints. However, firms would not internalize how their individual choices affect aggregate inflation, which in turn could affect (nominal) debt limits. This suggests an interesting avenue for further research.

The second possibility would be a competitive multi-good environment, where choices of agents affect relative prices between different goods, and these effects on relative prices are not internalized. This possibility is explored by Fazio (2021), who considers an extension of her model with a manufacturing and a service sector. Manufacturing producers face a credit constraint that depends on their earnings, but take the relative price of manufacturing goods as given, which gives rise to the possibility that manufacturing prices are inefficiently high. Such relative output price externalities can feature simultaneously to the labor price externalities that we have characterized, and studying their combination could be interesting. Relative output price changes could also occur between the internationally tradable and the nontradable sector, relevant for the discussion on the open economy literature above. We leave these ideas for future research.

[24] In emerging economy corporate debt markets, currency mismatches have been emphasizes (see for example Cespedes, Chang, and Velasco, 2004). Less is known about the specific anatomy of the financial constraints, e.g. whether their debt contracts in emerging markets are asset-based or earnings-based. This dimension may be important on top of currency mismatches and it may even interact with them.

7 Conclusion

This paper examines the implications of different credit constraints for optimal macroprudential policy. Our analysis is guided by recent empirical research on credit used by US firms, and shows that whether debt is backed by assets or linked to firms' earnings has sharply different normative consequences. The contrast between the different optimal policies that our analysis uncovers connects to a broader notion of 'good' vs. 'bad' credit booms (Gorton and Ordonez, 2020; Müller and Verner, 2021). In our framework, whether a regulator should encourage or tame an expansion in credit flows depends on the specific prices that relax or tighten agents' financial constraints in different credit markets. While a variety of economic forces must be considered in the design of an effective regulatory system, our theoretical results on how borrowing constraints shape externalities in firm decisions, as well as our finding that incorrect regulation leads to large welfare losses, have important implications for the macroprudential supervision of US corporate debt markets.

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ONLINE APPENDIX TO

Earnings-based borrowing constraints and pecuniary externalities

by Thomas Drechsel and Seho Kim

A Detailed derivations

A.1 Insensitivity to re-definition of net worth

In our model, we do not include production output as part of the definition of net worth. This is because output is not predetermined at the beginning of the period due to labor markets clearing during the period. It therefore cannot be a state variable of the model. To ensure that this definitional change does not affect the results, we show in this Appendix that a re-definition of net worth along the same lines gives identical results in the original Dávila and Korinek (2018) (DK18) framework. This is also useful to interpret our Lemma 1 in relation to its analogue in DK18: in our model, we obtain extra terms that contain additional economically meaningful effects.

We proceed by re-defining net worth in DK18 by excluding production output and prove that the *distributive effects* and *collateral effects* in DK18's version of Lemma 1 are identical. We denote net worth as defined by DK18 as $N_{DK}^{i,\theta} \equiv e_1^{i,\theta} + X_1^{i,\theta} + F_1^{i,\theta}(K_1^i)$. The resulting equilibrium capital and debt price are denoted by $q_{DK}^{\theta}(N_{DK}^{\theta}, K_1)$ and $m_{2,DK}^{\theta}(N_{DK}^{\theta}, K_1)$. We define net worth without production output as $N_{WP}^{i,\theta} \equiv e_1^{i,\theta} + X_1^{i,\theta}$ and the resulting equilibrium capital and debt price are denoted by $q_{WP}^{\theta}(N_{WP}^{\theta}, K_1)$ and $m_{2,WP}^{\theta}(N_{WP}^{\theta}, K_1)$. A simple re-definition of the model's state variables cannot change the prices in equilibrium, so that we can set

$$q_{WP}^{\theta}(N_{WP}^{\theta}, K_1) = q_{DK}^{\theta}(N_{DK}^{\theta}, K_1) \tag{57}$$

$$m_{2,WP}^{\theta}(N_{WP}^{\theta}, K_1) = m_{2,DK}^{\theta}(N_{DK}^{\theta}, K_1) \tag{58}$$

Noting that $N_{DK}^{i,\theta} = N_{WP}^{i,\theta} + F_1^{i,\theta}(K_1^i)$, we differentiate both sides of (57) and (58) with respect to $N_{(\cdot)}^{i,\theta}$ and K_1^i, in order to determine how the derivatives of prices with respect to net worth and capital are related across models. This gives us

$$\frac{\partial q^{\theta}_{WP}}{\partial N^{i,\theta}_{WP}} = \frac{\partial q^{\theta}_{DK}}{\partial N^{i,\theta}_{DK}} \tag{59}$$

$$\frac{\partial m^{\theta}_{2,WP}}{\partial N^{i,\theta}_{WP}} = \frac{\partial m^{\theta}_{2,DK}}{\partial N^{i,\theta}_{DK}} \tag{60}$$

$$\frac{\partial q^{\theta}_{WP}}{\partial K^{i}_{1}} = \frac{\partial q^{\theta}_{DK}}{\partial N^{i,\theta}_{DK}}\frac{\partial N^{i,\theta}_{DK}}{\partial K^{i}_{1}} + \frac{\partial q^{\theta}_{DK}}{\partial K^{i}_{1}} = \frac{\partial q^{\theta}_{DK}}{\partial N^{i,\theta}_{DK}} F'(K^{i}_{1}) + \frac{\partial q^{\theta}_{DK}}{\partial K^{i}_{1}} \tag{61}$$

$$\frac{\partial m^{\theta}_{2,WP}}{\partial K^{i}_{1}} = \frac{\partial m^{\theta}_{2,DK}}{\partial N^{i,\theta}_{DK}}\frac{\partial N^{i,\theta}_{DK}}{\partial K^{i}_{1}} + \frac{\partial m^{\theta}_{2,DK}}{\partial K^{i}_{1}} = \frac{\partial m^{\theta}_{2,DK}}{\partial N^{i,\theta}_{DK}} F'(K^{i}_{1}) + \frac{\partial m^{\theta}_{2,DK}}{\partial K^{i}_{1}} \tag{62}$$

where we used the chain rule for the differentiation with respect to capital. (61) and (62) make clear that the derivatives of prices with respect to capital after the re-definition of net worth "contain" the partial derivatives of $F(\cdot)$ that appear in DK18's Lemma 1. The *distributive effects* in DK18 are the following:

$$\mathcal{D}^{DK,i,\theta}_{N^{j,\theta}_{DK}} = -\left[\frac{\partial q^{\theta}_{DK}}{\partial N^{j,\theta}_{DK}}\Delta K^{i,\theta}_{2} + \frac{\partial m^{\theta}_{2,DK}}{\partial N^{j,\theta}_{DK}} X^{i,\theta}_{2}\right] \tag{63}$$

$$\mathcal{D}^{DK,i,\theta}_{K^{j}_{1}} = F'(K^{i}_{1})\mathcal{D}^{DK,i,\theta}_{N^{j,\theta}_{DK}} - \left[\frac{\partial q^{\theta}_{DK}}{\partial K^{j}_{1}}\Delta K^{i,\theta}_{2} + \frac{\partial m^{\theta}_{2,DK}}{\partial K^{j}_{1}} X^{i,\theta}_{2}\right] \tag{64}$$

The *distributive effects* with the re-definition of net-worth can be derived as

$$\mathcal{D}^{WP,i,\theta}_{N^{j,\theta}_{WP}} = -\left[\frac{\partial q^{\theta}_{WP}}{\partial N^{j,\theta}_{WP}}\Delta K^{i,\theta}_{2} + \frac{\partial m^{\theta}_{2,WP}}{\partial N^{j,\theta}_{WP}} X^{i,\theta}_{2}\right] \tag{65}$$

$$\mathcal{D}^{WP,i,\theta}_{K^{j}_{1}} = -\left[\frac{\partial q^{\theta}_{WP}}{\partial K^{j}_{1}}\Delta K^{i,\theta}_{2} + \frac{\partial m^{\theta}_{2,WP}}{\partial K^{j}_{1}} X^{i,\theta}_{2}\right] \tag{66}$$

Using (59) - (62), we obtain

$$\mathcal{D}^{DK,i,\theta}_{N^{j,\theta}_{DK}} = \mathcal{D}^{WP,i,\theta}_{N^{j,\theta}_{WP}} \tag{67}$$

$$\mathcal{D}^{DK,i,\theta}_{K^{j}_{1}} = \mathcal{D}^{WP,i,\theta}_{K^{j}_{1}} \tag{68}$$

Similarly, it can be shown that

$$\mathcal{C}^{DK,i,\theta}_{N^{j,\theta}_{DK}} = \mathcal{C}^{WP,i,\theta}_{N^{j,\theta}_{WP}} \tag{69}$$

$$\mathcal{C}^{DK,i,\theta}_{K^{j}_{1}} = \mathcal{C}^{WP,i,\theta}_{K^{j}_{1}} \tag{70}$$

This shows that a re-definition of net worth in the original DK18 model gives identical results. Furthermore, these derivations show that Lemma 1 in our model would be identical to Lemma 1 to its counterpart in DK18 if we did not include labor markets and did not have a more general definition of the financial constraint.

A.2 Details on constrained efficient allocation and implementation

Derivation of constrained efficient allocation. These derivations correspond to Proposition 1 (a) and the associated proof in DK18. The Lagrangian of the social planner's problem can be written as

$$\mathcal{L} = \sum_i \alpha^i \{u^i(C_0^i) + \beta \mathbb{E}_0[V^{i,\theta}(N_1^{i,\theta}, K_1^i; N^\theta, K_1)] + \kappa_1^i \Phi_1^i(X_1^i, K_1^i)\}$$
$$+ v_0 \sum_i [e_0^i - (C_0^i + h^i(K_1^i))] - \int_{\theta \in \Theta} v_1^\theta \sum_i X_1^{i,\theta} d\theta.$$

The first-order conditions of the social planner are

$$\frac{d\mathcal{L}}{dC_0^i} = \alpha^i u'^i(C_0^i) - v_0 = 0, \ \forall i \tag{71}$$
$$\frac{d\mathcal{L}}{dX_1^{i,\theta}} = -v_1^\theta + \alpha^i \beta V_n^{i,\theta} + \alpha^i \kappa_1^i \Phi_{1x}^i + \beta \sum_j \alpha_j V_{N^i}^{j,\theta}, \ \forall i, \theta \tag{72}$$
$$\frac{d\mathcal{L}}{dK_1^i} = -v_0 h'^i(K_1^i) + \alpha^i \beta \mathbb{E}_0[V_k^{i,\theta}] + \alpha^i \kappa_1^i \Phi_{1k}^i + \beta \sum_j \alpha^j \mathbb{E}_0[V_{K^i}^{j,\theta}] = 0, \ \forall i \tag{73}$$

Note that there are no expectation terms in the second first-order condition since $X_1^{i,\theta}$ is chosen for each θ.

The first first-order condition in the decentralized equilibrium implies $v_0 = \alpha^i \lambda_0^i$, so $\alpha^b/\alpha^l = \lambda_0^l/\lambda_0^b$. We divide the second FOC by α^i, and use $\alpha^i = v_0/\lambda_0^i$ as well as the envelope condition in the decentralized equilibrium $V_n^{i,\theta} = \lambda_1^{i,\theta}$. This gives us

$$\frac{v_1^\theta}{v_0} \lambda_0^i = \beta_i \lambda_1^{i,\theta} + \kappa_1^i \Phi_{1x^\theta}^i + \beta \sum_j \frac{\alpha^j}{\alpha^i} V_{N^i}^{j,\theta}, \ \forall i, \theta \tag{74}$$

We then use the third first-order condition and the envelope condition $V_k^{i,\theta} = \mathbb{E}_0[\lambda_1^{i,\theta}(F_{1k}^{i,\theta}(K_1^i, l_{1d}^{i,\theta}) + q^\theta)]$ to get

$$h^{i\prime}(K_1^i)\lambda_0^i = \beta \mathbb{E}_0[\lambda_1^{i,\theta}(F_{1k}^{i,\theta}(K_1^i, l_{1d}^{i,\theta}) + q^\theta)] + \kappa_1^i \Phi_{1k}^i + \beta \sum_j \frac{\alpha^j}{\alpha^i} \mathbb{E}_0[V_{K^i}^{j,\theta}], \ \forall i, \tag{75}$$

Equations (74) and (75), together with the constraints of the social planner's problem describe the constrained efficient allocation. Note that variables in $t \geq 1$ are optimal choices by the agents. Lemma 1 gives more detailed expressions being $V^{j,\theta}_{N^i}$ and $V^{j,\theta}_{K^i}$.

Implementation of constrained efficiency. These derivations correspond to Proposition 1 (b) and the associated proof in DK18. The constrained efficient allocation can be implemented by setting taxes on Arrow-Debreu security purchases and capital investment that satisfy

$$\tau_x^{i,\theta} = -\sum_j MRS_{01}^{j,\theta}\mathcal{D}^{j,\theta}_{1N^i} - \sum_j MRS_{02}^{j,\theta}\mathcal{D}^{j,\theta}_{2N^i} - \sum_j \tilde{\kappa}_2^{j,\theta}\mathcal{C}^{j,\theta}_{N^i},\ \forall i,\theta \tag{76}$$

$$\tau_k^{i} = -\sum_j \mathbb{E}_0[MRS_{01}^{j,\theta}\mathcal{D}^{j,\theta}_{1K^i}] - \sum_j \mathbb{E}_0[MRS_{02}^{j,\theta}\mathcal{D}^{j,\theta}_{2K^i}] - \sum_j \mathbb{E}_0[\tilde{\kappa}_2^{j,\theta}\mathcal{C}^{j,\theta}_{K^i}],\ \forall i \tag{77}$$

where $MRS_{01}^{j,\theta} \equiv \beta\lambda_1^{j,\theta}/\lambda_0^j$, $MRS_{02}^{j,\theta} \equiv \beta\lambda_2^{j,\theta}/\lambda_0^j$ and $\tilde{\kappa}_2^{j,\theta} \equiv \beta\kappa_2^{j,\theta}/\lambda_0^j$. This can be shown as follows. Re-write the period-0 first-order conditions (32) and (33) by including tax wedges for security purchases ($\tau_x^{i,\theta}$) and capital investment (τ_k^i). This gives

$$(m_1^\theta + \tau_x^{i,\theta})\lambda_0^i = \beta\lambda_1^{i,\theta} + \kappa_1^i\Phi^i_{1x^\theta} \tag{78}$$

$$(h'^i(k_1^i) + \tau_k^i)\lambda_0^i = \beta\mathbb{E}_0[\lambda_1^{i,\theta}(F^{i,\theta}_{1k}(k_1^i, l^{i,\theta}_{d1}) + q^\theta)] + \kappa_1^i\Phi^i_{1k}\ \forall i \tag{79}$$

Substituting the above tax rates into these optimality conditions replicates the planner's optimality conditions (74) and (75). Note that $m_1^\theta = \frac{v_1^\theta}{v_0}$ in the replicated allocations, i.e., Arrow-Debreu price in the decentralized equilibrium should equal the value of state contingent commodity in the social planner's problem measured by the shadow prices. Importantly, note also that the expressions for the tax rates contain additional terms relative to DK18 due to the presence of labor markets and the more general financial constraint formulation.

Combining equations (76) and (77) with equation (29) and (30) gives equations (38) and (39) in the main text.

B Further derivations on wages and labor markets

B.1 The equilibrium wage response to net worth changes

Pinning down the sign of the constraint externalities requires imposing further restrictions on the economic environment. In this Appendix we examine condition (41) introduced in the main text more closely. This condition restricts the model to an economy in which increases (decreases) in sector-wide net worth move equilibrium wages up (down), all else equal. In any general labor market setting, wage changes in response to variation in net worth could be driven both by changes in labor demand and changes in labor supply. For labor demand, our reasoning is already laid out in the main text, so here we study the responses of labor supply to changes in sector wide net worth.

Formally, consider the labor market clearing condition (13) for period $t = 1$. Labor demand and supply of both borrower and lender are defined according to the following first-order conditions (dropping the notation for θ for simplicity) :

$$w_1 = F^i_\ell(K^i_1, \ell^i_{d1}) \tag{80}$$

$$w_1 u^i_{c1} + u^i_{\ell 1} = 0 \tag{81}$$

where F^i_ℓ is the marginal product of labor, u^i_{c1} is the marginal utility of consumption, and $u^i_{\ell 1}$ is the marginal utility of labor for agent $i \in \{b, l\}$. Equation (80) equates wages to the marginal product of labor (in which capital is predetermined). Equation (81) relates labor supply and consumption choices through their relative price, the wage rate. Note that the consumption choice for agent i in period 1 is a function of state variables and prices w_1,w_2,m_2, and q. Note that in (80) and (81) we ignore the presence of borrowing constraints. In the labor supply equation, we do so because labor supply does not enter the earnings-based borrowing constraint, for which we require condition (41).[1]

Using (80) and (81), we write labor demand and supply as explicit functions of the variables they depend on, that is,

$$\ell^i_{d1} = \ell^i_{d1}(w_1, K^i_1), \tag{82}$$

$$\ell^i_{s1} = \ell^i_{s1}(w_1, c^i_1(w_1, w_2, q, m_2, K^i_1, N^i_1)). \tag{83}$$

Given these relations, we state the labor market clearing condition (13) in period $t = 1$

[1] In the most general case, $\Phi_2^{b,\theta}$ in (18) can constraint also labor supply, but this is not the case in any of the cases we analyze in this paper.

more explicitly as

$$\sum_i \ell^i_{d1}(w_1, K^i_1) = \sum_i \ell^i_{s1}(w_1, c^i_1(w_1, w_2, q, m_2, K^i_1, N^i_1)) \tag{84}$$

The labor market clearing condition, expressed in this way, will be useful to characterize how net worth changes affect wages in period $t = 1$. Formally, we can totally differentiate equation (84) with respect to N^j_1 to determine the sign of $\frac{\partial w_1}{\partial N^j_1}$. (In this differentiation, effects of N^j_1 on ℓ_{d1} are 0). We obtain

$$\left\{\sum_i \left[\underbrace{\frac{\partial \ell^i_{d1}}{\partial w_1}}_{A1} - \underbrace{\left(\frac{\partial \ell^i_{s1}}{\partial w_1} + \frac{\partial \ell^i_{s1}}{\partial c^i_1}\frac{\partial c^i_1}{\partial w_1}\right)}_{B1}\right]\right\}\frac{\partial w_1}{\partial N^j_1} = \underbrace{\sum_i \frac{\partial \ell^i_{s1}}{\partial c^i_1}\left(\frac{\partial c^i_1}{\partial N^j_1} + \frac{\partial c^i_1}{\partial w_2}\frac{\partial w_2}{\partial N^j_1} + \frac{\partial c^i_1}{\partial q}\frac{\partial q}{\partial N^j_1} + \frac{\partial c^i_1}{\partial m_2}\frac{\partial m_2}{\partial N^j_1}\right)}_{C1} \tag{85}$$

Expression (85) makes clear what is required for condition (41) to hold. $\frac{\partial w^\theta_1}{\partial N^j_1}$ is positive as long as the ratio of term $C1$ to terms $A1 - B1$ (summed over both agents) is positive. For each agent, term $A1$ represents the slope of the labor demand curve, and term $B1$ is the slope of an labor supply curve (which is composed of a substitution and income effect). Term $C1$ captures how change in net worth shift the labor supply curves through various equilibrium forces that do operate through wages in $t = 1$. More specifically, this term is composed of two types of effects. First, a direct equilibrium effect of net worth on the consumption-leisure tradeoff, $\sum_i \frac{\partial \ell^i_{s1}}{\partial c^i_1}\left(\frac{\partial c^i_1}{\partial N^j_1}\right)$. This term is negative for both agents under standard preferences such as a constant relative risk aversion (CRRA) utility function. Second, a collection of indirect effects from price changes on the consumption-leisure tradeoff, $\sum_i \frac{\partial \ell^i_{s1}}{\partial c^i_1}\left(\frac{\partial c^i_1}{\partial w_2}\frac{\partial w_2}{\partial N^j_1} + \frac{\partial c^i_1}{\partial q}\frac{\partial q}{\partial N^j_1} + \frac{\partial c^i_1}{\partial m_2}\frac{\partial m_2}{\partial N^j_1}\right)$. It is not possible to unambiguously determine the sign of the combination of these effects, especially since they may have different sign across the two types of agents.

So under what conditions do these labor supply effects give support condition (41)? It is reasonable to assume that labor demand curves are downward-sloping and labor supply curves are upward sloping. In this case the sum across agents of the term $A1 - B1$ is negative. Thus, for condition (41) to hold, term $C1$ needs to be negative. This means that the direct effect of changes in net worth in $C1$ needs be stronger than the net effect of changes in net worth from the combination of indirect effects through other prices across agents. This is the restriction we impose on the model through requiring condition (41) to hold in the main text. In Section 5 of the main text, we verify this reasoning for a specific choice of functional forms for preferences and technology.

B.2 Version with simplified labor market structure

In the model, both borrowers and lenders supply and demand labor. The empirical relevance of earnings-based constraints relates to the borrower being a firm, that is, an agent that typically demands but not supplies labor. In this Appendix, we therefore analyze how the model would change if the labor market structure is simplified such that the borrower only demands labor and the lender only supplies labor. We show that the economy will still be constrained inefficient.

Recall the labor market clearing condition (13) in period $t = 1$,

$$\sum_i \ell_{d1}^i(w_1, K_1^i) = \sum_i \ell_{s1}^i(w_1, c_1^i(w_1, w_2, q, m_2, K_1^i, N_1^i)) \tag{86}$$

where we drop θ for simplicity. Note that we have written labor demand as a function of w_1, K^i coming from the optimal labor demand decision which equates the marginal product of labor with the wages rate. Labor supply is a function of wages and consumption because the household's labor-leisure decision depends on the wage and marginal utilities (see Appendix B.1). For the proof of Proposition 2 we use condition (41). In Appendix B.1, we have shown that this condition can be determined from differentiating (86) with respect to N_1^j. Now if we assume that the lender only supplies labor, then the labor market clearing condition in $t = 1$ instead becomes

$$\ell_{d1}^b(w_1, K^b) = \ell_{s1}^l(w_1, c_1^l(w_1, w_2, q, m_2, K^l, N^l)) \tag{87}$$

that is, there is no summation over $i \in \{b, l\}$ but demand instead only comes from borrowers and supply only from lenders. It is easy to see that (41) can still apply: as in the main text, we reason based on labor demand and labor supply effects separately.

First, labor demand is still pinned down based firm optimality conditional on the predetermined capital stock in period $t = 1$, following the logic in the main text. Second, to study the role of labor supply effects in the simplified labor market, we can derive an equation that corresponds to (85) for the derivative with respect to N_1^l only. By applying similar arguments as in Appendix B.1, we can conclude that $\frac{\partial w_1^\theta}{\partial N_1^{l,\theta}} > 0$. As period-0 borrowing decisions change wages through changes in lender net worth, the market equilibrium is still not efficient, and the planner would levy taxes that correct the externality. This is true even if there were no effects coming through the borrower's labor demand, that is, $\frac{\partial w_1^\theta}{\partial N_1^{b,\theta}} = 0$. In this case, the social planner would levy taxes only on lenders. In this case, however, it might be more appropriate to label the mechanism over-saving, rather than under-borrowing.