# Interleaving Spurious

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**Abstract**

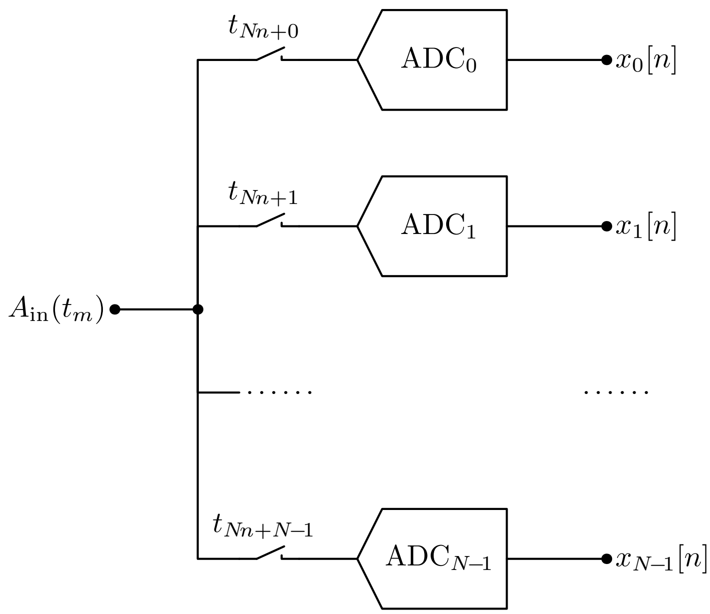
Study note on interleaving spurious. We would like to find a way to extract offset, gain and skew mismatches of interleaving ADC. We first show such mismatches induce spurious tones from a single tone input and then find formulae to extract mismatches from Fourier coefficients of such tones. Offset mismatch gives spurious tones at $kF_s/N$, while gain and skew mismatches gives tones at $f_{in} + kF_s/N$, where $F_s$, $N$, $k$, is sampling frequency, number of interleaving channels and an integer ranging $\{0, ..., N-1\}$, respectively.

## 1 Introduction

Suppose that we have an $N$-way time-interleaving ADC working at sampling frequency of $F_s$ or sampling period of $T_s = 1/F_s$. Therefore total $N$ of each sub-ADC is working at sampling frequency of $F_s/N$ or sampling period of $NT_s$.

Let us number each sub-ADC from 0 to $N-1$ and suppose that each sub-ADC works in this order, i.e., the 1st sub-ADC is activated after 0th ADC, and the 2nd is activated after the 1st and so on, then finally 0th sub-ADC is activated after the $(N-1)$th ADC. Now we introduce sampled output $x[m]$ and $x_j[n]$ and time index $m$, $n$ and $j$ such that

$$x[m] = x[Nn + j] = x_j[n]. \qquad (m = Nn + j)$$




Let $\delta t_j$ be the timing skew of the $j$-th sub-ADC, where $j = \{0, ..., N-1\}$. Time of $m$-th sampling is, with $n$ which satisfies $m = Nn + j$,

$$t_m = (Nn + j)\, T_s + \delta t_j. \qquad (m = Nn + j)$$

Now we would like to feed sine wave of amplitude of $2|A|$, frequency of $f$ to this ADC. Let $o_j$ and $\epsilon_j$ be offset and gain error of $j$-th sub-ADC, respectively, the output can be written as follows:

$$x[m] \;=\; o_j \;+\; A\,(1 + \epsilon_j)\, e^{i2\pi f(mT_s + \delta t_j)} \;+\; c.c., \qquad (m = Nn + j)$$

where *c.c.* is the complex conjugate of the second term which corresponds to the negative frequency and this term makes $x[m]$ real.

# 2 Extracting mismatches from spurious tones

## 2.1 Offset

Since Fourier transform is a linear transform, where principle of superposition applies, we can treat the offset term and the rest separately. Let's take a look at the offset first,

$$x[m] \;=\; o_j. \qquad (m = Nn + j)$$

This sequence is periodic with the period of $N$. Therefore it is convenient to write it in Fourier series,

$$x[m] \;=\; o_j \;=\; \sum_{k=0}^{N-1} O[k]\, e^{i\frac{2\pi jk}{N}}, \qquad O[k] = \frac{1}{N}\sum_{j=0}^{N-1} o_j\, e^{-i\frac{2\pi kj}{N}}$$

Here, notice that we can replace $j$ with $m = Nn + j$. That is

$$x[m] \;=\; \sum_{k=0}^{N-1} O[k]\, e^{i\frac{2\pi km}{N}}.$$

Using $F_s T_s = 1$, we get

$$x[m] \;=\; \sum_{k=0}^{N-1} O[k]\, e^{i2\pi\frac{kF_s}{N}\cdot mT_s}.$$

Now, we see that $N$ spurious tone at $kF_s/N$. In other words, we can calculate each sub-ADC's offset ($o_j$) from the measured Fourier coefficient at $kF_s/N$.

$$o_j \;=\; \sum_{k=0}^{N-1} O[k]\, e^{i\frac{2\pi jk}{N}}.$$

## 2.2 Gain and skew

For gain and skew, we start from this:

$$x[m] \;=\; A\,(1+\epsilon_j)\, e^{i2\pi f(mT_s+\delta t_j)} \;+\; c.c. \qquad (m = Nn + j)$$

Assuming $\delta t_j$ is small,

$$e^{i2\pi f\delta t_j} \;\sim\; 1 + i2\pi f\delta t_j.$$

Therefore

$$x[m] \;=\; A\,(1+\epsilon_j + i2\pi f\delta t_j)\, e^{i2\pi f\cdot mT_s} \;+\; c.c.,$$

where we have dropped second order small quantity. Let's decompose this into linear term ($\bar{x}[m]$) and distortion term ($\tilde{x}[m]$),

$$x[m] \;=\; \bar{x}[m] \;+\; \tilde{x}[m],$$

where

$$\bar{x}[m] = A\, e^{i2\pi f\cdot mT_s} + c.c., \qquad \tilde{x}[m] = d_j\, e^{i2\pi f\cdot mT_s} + c.c.,$$

and

$$d_j = A\,(\epsilon_j \;+\; i2\pi f\delta t_j)\,.$$

Since $d_j$ repeats every $N$ samples, it can be written in Fourier series,

$$d_j = \sum_{k=0}^{N-1} D[k]\, e^{i\frac{2\pi jk}{N}}, \qquad D[k] = \frac{1}{N}\sum_{j=0}^{N-1} d_j\, e^{-i\frac{2\pi kj}{N}}.$$

Therefore,

$$\begin{aligned}\tilde{x}[m] \;&=\; d_j\, e^{i2\pi f\cdot mT_s} \;+\; c.c.,\\ &=\; \sum_{k=0}^{N-1} D[k]\, e^{i\frac{2\pi jk}{N}}\; e^{i2\pi f\cdot mT_s} \;+\; c.c.\end{aligned}$$

Replacing $j$ with $Nn + j = m$ and using $T_sF_s = 1$ yields,

$$\tilde{x}[m] \;=\; \sum_{k=0}^{N-1} D[k]\, e^{i2\pi(f+kF_s/N)\cdot mT_s} \;+\; c.c.$$

Now we see spurious tones of $D[k]$ at $f + kF_s/N$ from the first term and $D^*[k]$ at $-f - kF_s/N$ from the complex conjugate. Therefore we can calculate $d_j$ from those measured Fourier coefficients.

$$d_j = \sum_{k=0}^{N-1} D[k]\, e^{i\frac{2\pi jk}{N}}$$

Note that $D[0]$, which is at the fundamental, is average of mismatches:

$$D[0] \;=\; \frac{1}{N}\sum_{j=0}^{N-1} d_j \;=\; \frac{1}{N}\sum_{j=0}^{N-1} A\left(\epsilon_j + i2\pi f\delta t_j\right).$$

We can always set this to zero by adjusting reference for $\epsilon_j$ and $\delta t_j$. In that case, we can find $A$ at the fundamental tone in measured Fourier coefficients. Let's say $\epsilon'_j$ and $\delta t'_j$ is such re-referenced mismatches, $\epsilon'_j$ and $\delta t'_j$ is real and imaginary part of $d_j/A$, respectively.

$$\epsilon'_j = \mathrm{Re}\left(d_j/A\right), \qquad 2\pi f\delta t'_j = \mathrm{Im}\left(d_j/A\right).$$

# 3 Concluding remarks

So far, we have seen that interleaving mismatches induce tones and found formulae to extract mismatches from Fourier coefficients of tones. However, we can also extract those mismatches by comparing Fourier coefficients of each sub-ADC output, $x_j[n]$. Offset mismatch can be obtained from the DC component. Gain and skew can be obtained amplitude and phase of the fundamental tone. And these two methods are equivalent.