Timing skew extraction by time derivative

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August 2, 2023

Abstract

Study note on timing skew extraction/correction of time-interleaving ADC by the use of time derivative of sub-ADC output.

Introduction

We attempt to correct timing skews of sub-ADCs of an time-interleaving ADC under the assumption that mean-square of time-derivative of ADC output sampled at $t_n = t_0 + nT_s$,

$$\left\langle \left(\frac{dx}{dt}\right)^2 \right\rangle = \lim_{M\to\infty} \frac{1}{M} \sum_{n=0}^{M-1} \left(\frac{dx}{dt}\right)^2_{t=t_n},$$

does not depend on initial time t_0, where T_s is the sampling period and n is an integer and x is ADC output.

Skew extraction

Suppose that we have an N-way time-interleaving ADC working at sampling frequency of F_s or sampling period of $T_s = 1/F_s$. Therefore each sub-ADC is working at sampling frequency of F_s/N or sampling period of NT_s.

Let us number each sub-ADC from 0 to $N-1$ and suppose that each sub-ADC works in this order, i.e., the 1st sub-ADC is activated after 0-th ADC, and the 2nd is activated after the 1st and so on, then finally 0th sub-ADC is activated after the $(N\!-\!1)$th ADC. Let δt_j be the timing skew of the j-th sub-ADC, where $j = \{0, ..., N\!-\!1\}$. Time of m-th sampling is, with n which satisfies $m = Nn + j$,

$$t_m = T_s\,(Nn + j) \; + \; \delta t_j . \qquad (m = Nn + j)$$

Time interval ΔT_j from $(m\!-\!1)$-th sample to m-th sample is

$$\begin{aligned} \Delta T_j = t_m - t_{m-1} &= T_s + \delta T_j , \\ &= T_s \left(1 + \frac{\delta T_j}{T_s} \right) , \end{aligned}$$

where

$$\delta T_j \;=\; \begin{cases} \delta t_0 - \delta t_{N-1} & (j = 0) \, , \\ \delta t_j - \delta t_{j-1} & (j \neq 0) \, . \end{cases} \tag{1}$$

One can easily show that

$$\delta T_0 + ... + \delta T_{N-1} = 0. \tag{2}$$

Let Δx be difference between two adjacent time points and Δx_j is subset of it:

$$\begin{aligned} \Delta x \;&=\; x[m] - x[m-1], \\ \Delta x_j \;&=\; \begin{cases} x[Nn] - x[Nn-1], & (j = 0) \, , \\ x[Nn + j] - x[Nn + j - 1], & (j \neq 0) \, . \end{cases} \end{aligned}$$

When $x[m]$ is moving slowly over the time period of T_s, they can also be written as

$$\begin{aligned} \Delta x \;&=\; \left(\frac{\Delta x}{\Delta T} \right) \cdot \Delta T, \\ \Delta x_j \;&=\; \left(\frac{\Delta x}{\Delta T} \right) \cdot \Delta T_j , \end{aligned}$$

where $\left(\frac{\Delta x}{\Delta T}\right)$ is the time derivative of ADC output and $\Delta T = t_m - t_{m-1}$. Let's take mean square of Δx_j, since ΔT_j is deterministic, it can be brought out from the average:

$$\begin{aligned} \left\langle (\Delta x_j)^2 \right\rangle \;&=\; \left\langle \left(\frac{\Delta x}{\Delta T} \right)^2 \right\rangle \cdot (\Delta T_j)^2 \, , \\ &= \left\langle \left(\frac{\Delta x}{\Delta T} \right)^2 \right\rangle \cdot T_s^2 \cdot \left(1 + \frac{\delta T_j}{T_s} \right)^2 , \\ &= \left\langle \left(\frac{\Delta x}{\Delta T} \right)^2 \right\rangle \cdot T_s^2 \cdot \left(1 + 2 \cdot \frac{\delta T_j}{T_s} \right) , \end{aligned} \tag{3}$$

where we used the fact that $\delta T_j/T_s$ is much smaller than unity and dropped its second order term. On the other hand, $\left\langle (\Delta x)^2 \right\rangle$ is average of $\left\langle (\Delta x_j)^2 \right\rangle$ over j.

$$
\begin{aligned}
\left\langle (\Delta x)^2 \right\rangle &= \left\langle \frac{1}{N} \sum_{j=0}^{N-1} (\Delta x_j)^2 \right\rangle, \\
&= \left\langle \frac{1}{N} \sum_{j=0}^{N-1} \left(\frac{\Delta x}{\Delta T}\right)^2 \cdot T_s^2 \cdot \left(1 + 2 \cdot \frac{\delta T_j}{T_s}\right) \right\rangle, \\
&= \left\langle \left(\frac{\Delta x}{\Delta T}\right)^2 \right\rangle \cdot T_s^2 \;+\; \frac{1}{N} \sum_{j=0}^{N-1} 2 \cdot \frac{\delta T_j}{T_s} \cdot \left\langle \left(\frac{\Delta x}{\Delta T}\right)^2 \right\rangle \cdot T_s^2.
\end{aligned}
$$

Because of Eq. (2), the last term vanishes. Therefore

$$
\left\langle (\Delta x)^2 \right\rangle = \left\langle \left(\frac{\Delta x}{\Delta T}\right)^2 \right\rangle \cdot T_s^2.
$$

Here, $\langle (\frac{\Delta x}{\Delta T})^2 \rangle$ in Eq. (3) is mean square of subset of $\frac{\Delta x}{\Delta T}$ associated only to the index j, but we assume it gives the same value for all j. Therefore, above $\langle (\frac{\Delta x}{\Delta T})^2 \rangle$, which is mean square of full set, gives the same value as $\langle (\frac{\Delta x}{\Delta T})^2 \rangle$ in Eq. (3). Combining this and Eq. (3) yields,

$$
\frac{\delta T_j}{T_s} = \frac{1}{2} \left(\frac{\left\langle (\Delta x_j)^2 \right\rangle}{\left\langle (\Delta x)^2 \right\rangle} - 1 \right).
$$

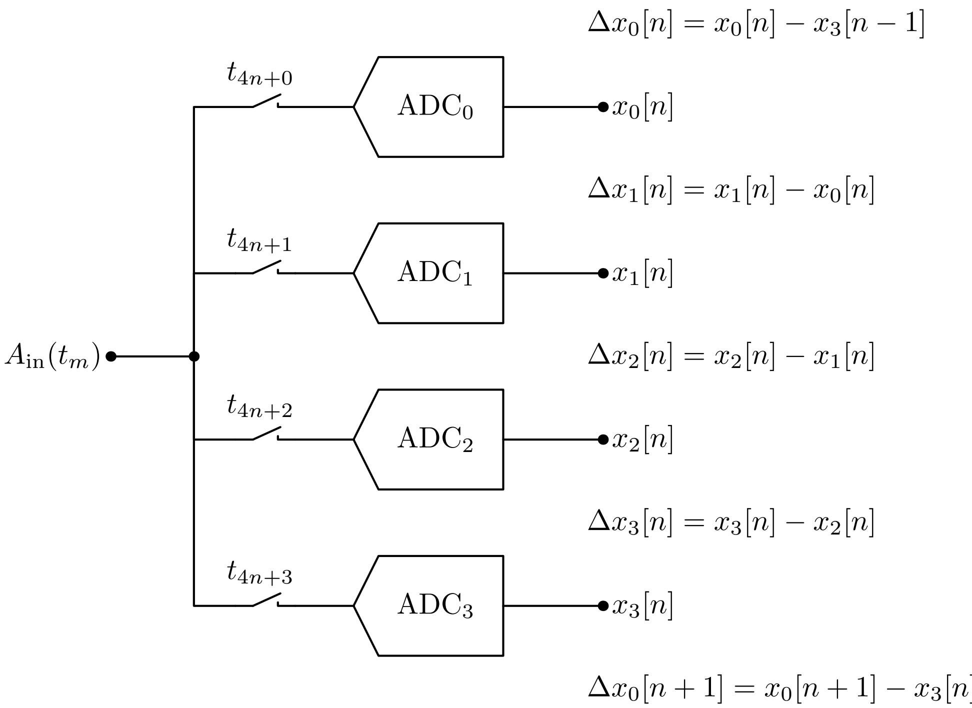
Recalling the definition of δT_j, Eq. (1), δt_j can be written using δT_j as follows:

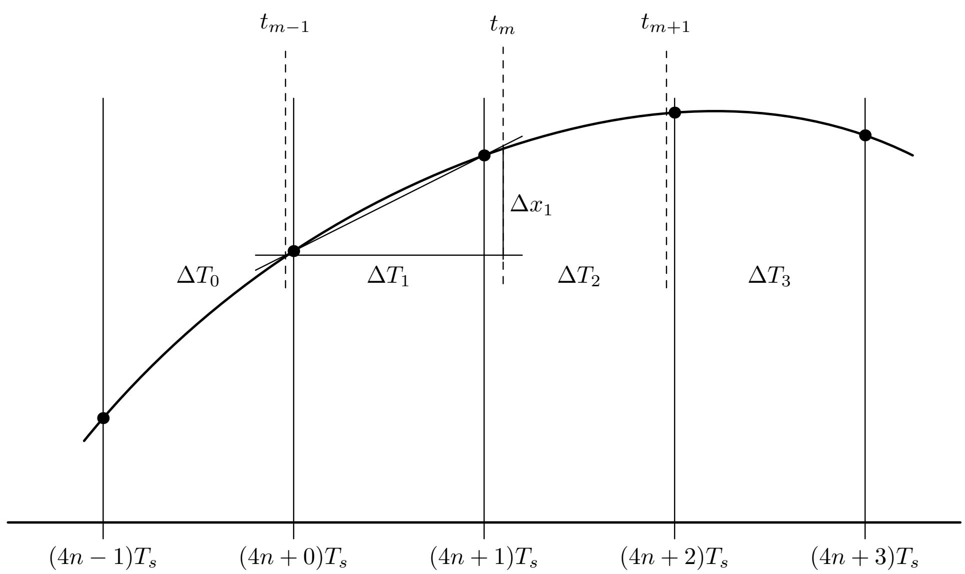
$$
\begin{aligned}
\delta t_0 &= \delta T_0, \\
\delta t_1 &= \delta T_0 + \delta T_1, \\
\delta t_2 &= \delta T_0 + \delta T_1 + \delta T_2, \\
&\ldots \\
\delta t_{N-1} &= \delta T_0 + \delta T_1 + \delta T_2 + \ldots + \delta T_{N-1} = 0,
\end{aligned}
$$

where we set $\delta t_{N-1} = 0$ to make $(N-1)$-th sub-ADC's as the reference of skew. With $\delta_j = \delta t_j/T_s$, t_m can be written as follows,

$$
t_m = T_s \left(Nn + j + \delta_j \right).
$$

Note that δ_j is not a function of T_s and can be calculated solely from ADC's output. Finally, $\delta t_j = T_s\, \delta_j$ is the extracted timing skew.


$\Delta x_0[n] = x_0[n] - x_3[n-1]$
t_{4n+0}
ADC_0
$x_0[n]$
$\Delta x_1[n] = x_1[n] - x_0[n]$
t_{4n+1}
ADC_1
$x_1[n]$
$A_{\mathrm{in}}(t_m)$
$\Delta x_2[n] = x_2[n] - x_1[n]$
t_{4n+2}
ADC_2
$x_2[n]$
$\Delta x_3[n] = x_3[n] - x_2[n]$
t_{4n+3}
ADC_3
$x_3[n]$
$\Delta x_0[n+1] = x_0[n+1] - x_3[n]$


t_{m-1}
t_m
t_{m+1}
Δx_1
ΔT_0
ΔT_1
ΔT_2
ΔT_3
$(4n-1)T_s$
$(4n+0)T_s$
$(4n+1)T_s$
$(4n+2)T_s$
$(4n+3)T_s$