# Capacitor mismatch

$$C = \epsilon LW \quad \rightarrow \quad \delta C = \epsilon W \delta L + \epsilon L \delta W \quad \rightarrow \quad \delta C^2 = \epsilon^2 W^2 \delta L^2 + \epsilon^2 L^2 \delta W^2 + 2\epsilon^2 LW \, \delta L \, \delta W$$

$$\begin{aligned}
\langle \delta C^2 \rangle &= \epsilon^2 W^2 \langle \delta L^2 \rangle + \epsilon^2 L^2 \langle \delta W^2 \rangle + 2\epsilon^2 LW \ \langle \delta L \, \delta W \rangle \\
&= (\epsilon LW)^2 \left( \frac{\langle \delta L^2 \rangle}{L^2} + \frac{\langle \delta W^2 \rangle}{W^2} + 2 \frac{\langle \delta L \, \delta W \rangle}{LW} \right) \\
&= \frac{C^2}{LW} \left( \frac{W}{L} \langle \delta L^2 \rangle + \frac{L}{W} \langle \delta W^2 \rangle + 2 \langle \delta L \, \delta W \rangle \right)
\end{aligned}$$

Let's say $\langle \delta L^2 \rangle \sim \langle W^2 \rangle \sim \delta^2$ and $\langle \delta L \, \delta W \rangle \sim c \, \delta^2$

$$\frac{\langle \delta C^2 \rangle}{C^2} = \frac{\delta^2}{LW} \left( \frac{W}{L} + \frac{L}{W} + 2c \right)$$

Therefore (we use $\delta C$ for two different things: standard deviation and error, confusing!)

$$\frac{\sqrt{\langle \delta C^2 \rangle}}{C} = \frac{\delta C}{C} \ \propto \ \frac{1}{\sqrt{LW}}$$

# Capacitor mismatch (square)

$$\frac{\langle \delta C^2 \rangle}{C^2} = \frac{\delta^2}{LW}\left(\frac{W}{L} + \frac{L}{W} + 2c\right)$$

Recalling that

$$\min\left(x + \frac{1}{x}\right) = 2, \quad x_{\min} = 1$$

Square ($L = W$) minimizes mismatch, when $c = 0$.

If capacitor is square ($L = W$)

$$\frac{\langle \delta C^2 \rangle}{C^2} = \frac{\delta^2}{LW}(2 + 2c) = \begin{cases} \dfrac{2\delta^2}{LW} & (c = 0) \\ \dfrac{4\delta^2}{LW} & (c = 1,\ \delta L \sim \delta W) \\ 0 & (c = -1,\ \delta L \sim -\delta W) \end{cases}$$

# Resistor mismatch

Repeat the same procedure as we did for capacitor for resistor,

$$R = \rho \cdot \frac{L}{W}$$

We find

$$\frac{\langle \delta R^2 \rangle}{R^2} = \frac{\delta^2}{LW} \left( \frac{W}{L} + \frac{L}{W} - 2c \right)$$

For resistor $R$ determines $L/W$ and $W/L$.