

# Feynman units

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# Feynman units!?

- It is a clickbait, there is no such unit system.
- But I would like to go over constants, units and notations of electrodynamics
- including what Feynman used in this *Lectures on Physics (FLP)*
- To summarize
  - ▶ Among  $\varepsilon_0$ ,  $\mu_0$ ,  $c$ , we only need two of them
  - ▶ Feynman used  $\varepsilon_0$  and  $c$  and I think this makes more sense
  - ▶  $\mathbf{B}$  in Gaussian units is different quantity than  $\mathbf{B}$  in SI

# Microscopic Maxwell's equations

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0$$

$$\nabla \cdot \mathbf{E} = \rho / \epsilon_0$$

$$\nabla \times \mathbf{B} - \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} = \mu_0 \mathbf{j}$$

$$\nabla \cdot \mathbf{B} = 0$$

We have two constants:  $\epsilon_0$   $\mu_0$ .

## Two constants: $\epsilon_0$ $\mu_0$

$\epsilon_0$  and  $\mu_0$  relates charge and current to mechanical force, respectively.

$$F = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q_1 Q_2}{r^2} \qquad F = \frac{\mu_0}{2\pi} \cdot \frac{I_1 I_2}{r^2}$$

Since current is flow of charge,  $\epsilon_0$  and  $\mu_0$  cannot be independent. Indeed, they have following relationship:

$$\epsilon_0 \mu_0 = \frac{1}{c^2} \quad \text{or} \quad \mu_0 = \frac{1}{\epsilon_0 c^2}$$

While  $c$  (the speed of light) does not depend on how we chose the unit of charge

How about using  $\epsilon_0$  and  $c$  instead of  $\epsilon_0$  and  $\mu_0$ ?

Using  $\varepsilon_0$  and  $c$  instead of  $\varepsilon_0$  and  $\mu_0$

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0 \qquad \nabla \cdot \mathbf{E} = \rho / \varepsilon_0$$

$$\nabla \times \mathbf{B} - \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} = \mu_0 \mathbf{j} \qquad \nabla \cdot \mathbf{B} = 0$$

- ▶ This is what Feynman used in his “*Lectures on Physics*”
- ▶ Notice  $\mathbf{E}$  and  $\mathbf{B}$  are in different dimension ( $[\nabla] = \text{L}^{-1}$ ,  $[\frac{\partial}{\partial t}] = \text{T}^{-1}$ )
- ▶ It is  $c\mathbf{B}$  that has the same dimension as  $\mathbf{E}$  ( $[\nabla] = [\frac{1}{c} \frac{\partial}{\partial t}] = \text{L}^{-1}$ )
- ▶ And coefficient of  $\mathbf{j}$  is now  $1/\varepsilon_0 c$

Using  $\varepsilon_0$  and  $c$  instead of  $\varepsilon_0$  and  $\mu_0$

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Coefficient of  $j$ :  $1/\varepsilon_0 c$

Recalling that  $1/c = \sqrt{\varepsilon_0 \mu_0}$

$$\frac{1}{\varepsilon_0 c} = \frac{1}{\varepsilon_0} \cdot \sqrt{\varepsilon_0 \mu_0} = \sqrt{\frac{\mu_0}{\varepsilon_0}} \sim 377 \Omega$$

This is impedance of free space

Now

$$\nabla \times \mathbf{E} + \frac{1}{c} \frac{\partial c\mathbf{B}}{\partial t} = 0$$

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- ▶ We have two constants,  $\varepsilon_0$  and  $c$
- ▶  $\varepsilon_0$  is for charge.  $c$  is for electromagnetic field
- ▶ Coefficient of  $\mathbf{j}$  is impedance of free space,  $377\Omega$
- ▶ We treat  $c\mathbf{B}$  as a single symbol as it has the same dimension as  $\mathbf{E}$

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$$\nabla \times \mathbf{E} + \frac{1}{\textcolor{red}{c}} \frac{\partial \mathbf{cB}}{\partial t} = 0$$

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# Why dimension is so important?

Dimension check is primary method to detect errors in your calculation

You cannot equate, add or subtract quantities in different dimension

Simpler dimension makes error detection easier

$u$  and  $\mathbf{S}$

$$u = \frac{\varepsilon_0}{2} (|\mathbf{E}|^2 + |c\mathbf{B}|^2)$$

$$\mathbf{S} = \varepsilon_0 c (\mathbf{E} \times c\mathbf{B})$$

Note that  $\varepsilon_0$  is capacitance per length,  $\mathbf{E}$  and  $c\mathbf{B}$  is voltage per length,  
and  $1/\varepsilon_0 c$  is resistance.

$u$  and  $\mathbf{S}$

$$u = \frac{\varepsilon_0}{2} (|\mathbf{E}|^2 + |c\mathbf{B}|^2)$$

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$$\begin{aligned}\text{Energy/Volume} &= \text{Capacitance/Length} \times (\text{Voltage/Length})^2 \\ &= \text{Capacitance} \times \text{Voltage}^2/\text{Volume}\end{aligned}$$

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Energy Flux = Conductance  $\times$  Voltage/Length  $\times$  Voltage/Length  
= Power/Area

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$$\begin{aligned}\text{Energy Flux} &= \text{Capacitance/Length} \times \text{Velocity} \times (\text{Voltage/Length})^2 \\ &= \text{Energy density} \times \text{Velocity}\end{aligned}$$



# Gaussian units

$$\nabla \times \mathbf{E} + \frac{1}{c} \frac{\partial \mathbf{B}}{\partial t} = 0$$

$$\nabla \cdot \mathbf{E} = 4\pi\rho$$

$$\nabla \times \mathbf{B} - \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} = \frac{4\pi}{c} \mathbf{j}$$

$$\nabla \cdot \mathbf{B} = 0$$

- ▶ This is Maxwell's equations in Gaussian units
- ▶ Notice that  $\mathbf{E}$  and  $\mathbf{B}$  are in the same unit (dimension) 🍷
- ▶ It has dimensionless number  $4\pi$  instead of  $1/\epsilon_0$ , i.e.,  $F = \frac{Q_1 Q_2}{r^2}$  in Gaussian
- ▶  $4\pi/c$  is still impedance of free space, but in seconds per centimeter!

# Gaussian units is popular among physicists

*“Unfortunately one of the results of the completely disconnected way in which electricity and magnetism have been taught in the past has been the growing acceptance of the mks over the cgs system of units. We have no special preference for centimeters over meters or of grams over kilograms. We do, however, require a system wherein the electric field  $\mathbf{E}$  and the magnetic field  $\mathbf{B}$  are in the same unit.”*

— Melvin Schwarts, *Principles of Electrodynamics*, (1972)

## Gaussian units is popular among physicists, but ...

*"My tardy adoption of the universally accepted SI system is recognition that almost all undergraduate physics texts, as well as engineering book at all levels, employ SI units throughout. For many years Ed Purcell (1912–1997) and I had a pact to support each other in the use of Gaussian units. Now I have betrayed him!"*

— John David Jackson, *Classical Electrodynamics*, (1998)

*"For 50 years, Edward Purcell's classic textbook has introduced students to the world of electricity and magnetism. This third edition has been brought up to date and is now in SI units."*

— Edward M. Purcell and David J. Morin, *Electricity and Magnetism*, (2013)

## Gaussian units with $\epsilon_0$

$$\nabla \times \mathbf{E} + \frac{1}{c} \frac{\partial \mathbf{B}}{\partial t} = 0 \qquad \nabla \cdot \mathbf{E} = \rho / \epsilon_0$$

$$\nabla \times \mathbf{B} - \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} = \frac{1}{\epsilon_0 c} \mathbf{j} \qquad \nabla \cdot \mathbf{B} = 0$$

- ▶ Notice similarity to equations in SI with  $\epsilon_0$  and  $c$
- ▶ Substitute  $1/\epsilon_0$  with  $4\pi$  to go to Gaussian
- ▶ Substitute  $\mathbf{B}$  with  $c\mathbf{B}$  to go to SI
- ▶  $\mathbf{B}$  in SI is not the same quantity as  $\mathbf{B}$  in Gaussian units! ( $c\mathbf{B}$  is)

# Hall coefficient

Now we see why Hall coefficient is different.

In SI,  $R_H = 1/nq$ ,

$$E_y = R_H j_x B = \frac{1}{nq} j_x B = \frac{1}{nqc} j_x cB \quad (\text{SI})$$

Perform  $c\mathbf{B} \rightarrow \mathbf{B}$  to go to Gaussian

$$E_y = \frac{1}{nqc} j_x B \quad (\text{Gaussian})$$

Therefore

$$R_H = \frac{1}{nqc} \quad (\text{Gaussian})$$

# Statics

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- ▶ Remove time derivatives
- ▶  $\mathbf{E}$  and  $c\mathbf{B}$  are independent. Electrostatics and magnetostatics are distinct
- ▶ May make sense to use  $\mu_0$ , because  $\mathbf{E}$  is not related to  $\mathbf{B}$

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- ▶  $\mathbf{B}$  vanishes.  $c$  has to be finite for  $\mathbf{B}$  to exist
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## $\rho$ and $\mathbf{j}$

$\rho$  and  $\mathbf{j}$  includes all the charge, electrons and ions.

$$\rho = \sum_i q_i \delta^3(\mathbf{r} - \mathbf{r}_i) \quad \mathbf{j} = \sum_i \mathbf{v}_i q_i \delta^3(\mathbf{r} - \mathbf{r}_i)$$

We introduce  $\mathbf{P}$  and  $\mathbf{M}$  to bridge microscopic world to macroscopic world.

$$\rho = \rho^{(f)} - \nabla \cdot \mathbf{P} \quad \mathbf{j} = \mathbf{j}^{(f)} + \frac{\partial \mathbf{P}}{\partial t} + \nabla \times \mathbf{M}$$

## $\mathbf{D}$ and $\mathbf{H}$

$$\nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0} \rho$$

$$\nabla \times c\mathbf{B} - \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} = \frac{1}{\epsilon_0 c} \mathbf{j}$$

- ▶ Insert previous page's definition and move  $\mathbf{P}$  and  $\mathbf{M}$  to the other side
- ▶ I wish I could use  $\mathbf{D} = \mathbf{E} + \mathbf{P}/\epsilon_0$  and  $\mathbf{H} = c\mathbf{B} - \mathbf{M}/\epsilon_0 c$  (it's cleaner 🤔)
- ▶ We customary use  $\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$  and  $\mathbf{H} = \epsilon_0 c^2 \mathbf{B} - \mathbf{M}$  (more units 🤔)
- ▶ Feynman used  $\mathbf{H} = \mathbf{B} - \mathbf{M}/\epsilon_0 c^2$  to make  $\mathbf{H}$  to have the same units as  $\mathbf{B}$



## $\mathbf{D}$ and $\mathbf{H}$

$$\nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0} (\rho^{(\text{f})} - \nabla \cdot \mathbf{P})$$

$$\nabla \times c\mathbf{B} - \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} = \frac{1}{\epsilon_0 c} \left( \mathbf{j}^{(\text{f})} + \frac{\partial \mathbf{P}}{\partial t} + \nabla \times \mathbf{M} \right)$$

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- ▶ We customary use  $\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$  and  $\mathbf{H} = \epsilon_0 c^2 \mathbf{B} - \mathbf{M}$  (more units 🤔)
- ▶ Feynman used  $\mathbf{H} = \mathbf{B} - \mathbf{M}/\epsilon_0 c^2$  to make  $\mathbf{H}$  to have the same units as  $\mathbf{B}$

## $\mathbf{D}$ and $\mathbf{H}$

$$\nabla \cdot (\mathbf{E} + \mathbf{P}/\epsilon_0) = \frac{1}{\epsilon_0} \rho^{(f)}$$

$$\nabla \times (c\mathbf{B} - \mathbf{M}/\epsilon_0 c) - \frac{1}{c} \frac{\partial (\mathbf{E} + \mathbf{P}/\epsilon_0)}{\partial t} = \frac{1}{\epsilon_0 c} \mathbf{j}^{(f)}$$

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# $\mathbf{D}$ and $\mathbf{H}$

$$\nabla \cdot (\varepsilon_0 \mathbf{E} + \mathbf{P}) = \rho^{(f)}$$

$$\nabla \times (\varepsilon_0 c^2 \mathbf{B} - \mathbf{M}) - \frac{\partial(\varepsilon_0 \mathbf{E} + \mathbf{P})}{\partial t} = \mathbf{j}^{(f)}$$

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## $\mathbf{D}$ and $\mathbf{H}$

$$\nabla \cdot (\mathbf{E} + \mathbf{P}/\epsilon_0) = \rho^{(f)}/\epsilon_0$$

$$c^2 \nabla \times (\mathbf{B} - \mathbf{M}/\epsilon_0 c^2) - \frac{\partial (\mathbf{E} + \mathbf{P}/\epsilon_0)}{\partial t} = \frac{\mathbf{j}^{(f)}}{\epsilon_0}$$

- ▶ Insert previous page's definition and move  $\mathbf{P}$  and  $\mathbf{M}$  to the other side
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# Macroscopic Maxwell's equations (SI with $\varepsilon_0$ and $c$ )

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0 \qquad \nabla \cdot \mathbf{B} = 0$$

$$\nabla \cdot \mathbf{D} = \rho^{(\text{f})} \qquad \mathbf{D} = (\varepsilon_0 \mathbf{E} + \mathbf{P})$$

$$\nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = \mathbf{j}^{(\text{f})} \qquad \mathbf{H} = (\varepsilon_0 c^2 \mathbf{B} - \mathbf{M})$$

$\mathbf{E}$ ,  $\mathbf{B}$ ,  $\mathbf{D}$ ,  $\mathbf{H}$  are in different units

Some people in the past thought this is cleaner, because constants are hidden

# Macroscopic Maxwell's equations (Feynman)

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0 \qquad \nabla \cdot \mathbf{B} = 0$$

$$\nabla \cdot (\varepsilon_0 \mathbf{E} + \mathbf{P}) = \rho_{\text{other}} \qquad \mathbf{D}/\varepsilon_0 = (\mathbf{E} + \mathbf{P}/\varepsilon_0)$$

$$c^2 \nabla \times \left( \mathbf{B} - \frac{\mathbf{M}}{\varepsilon_0 c^2} \right) = \frac{\mathbf{j}_{\text{cond}}}{\varepsilon_0} + \frac{\partial}{\partial t} \left( \mathbf{E} - \frac{\mathbf{P}}{\varepsilon_0} \right) \qquad \mathbf{H} = \left( \mathbf{B} - \frac{\mathbf{M}}{\varepsilon_0 c^2} \right)$$

Please read *FLP* Vol II Chap 36

# Macroscopic Maxwell's equations (Gaussian units)

$$\nabla \times \mathbf{E} + \frac{1}{c} \frac{\partial \mathbf{B}}{\partial t} = 0 \qquad \nabla \cdot \mathbf{B} = 0$$

$$\nabla \cdot \mathbf{D} = 4\pi \rho^{(\text{f})} \qquad \mathbf{D} = (\mathbf{E} + 4\pi \mathbf{P})$$

$$\nabla \times \mathbf{H} - \frac{1}{c} \frac{\partial \mathbf{D}}{\partial t} = \frac{4\pi}{c} \mathbf{j}^{(\text{f})} \qquad \mathbf{H} = (\mathbf{B} - 4\pi \mathbf{M})$$

$\mathbf{E}$ ,  $\mathbf{B}$ ,  $\mathbf{D}$ ,  $\mathbf{H}$  are in the same unit

$\mathbf{D}$  and  $\mathbf{H}$  in Gaussian are different quantities than those in SI



## Macroscopic Maxwell's equations (Gaussian units with $\varepsilon_0$ )

$$\nabla \times \mathbf{E} + \frac{1}{c} \frac{\partial \mathbf{B}}{\partial t} = 0 \qquad \nabla \cdot \mathbf{B} = 0$$

$$\nabla \cdot \mathbf{D} = \rho^{(\text{f})} / \varepsilon_0 \qquad \mathbf{D} = (\mathbf{E} + \mathbf{P} / \varepsilon_0)$$

$$\nabla \times \mathbf{H} - \frac{1}{c} \frac{\partial \mathbf{D}}{\partial t} = \frac{1}{\varepsilon_0 c} \mathbf{j}^{(\text{f})} \qquad \mathbf{H} = (\mathbf{B} - \mathbf{M} / \varepsilon_0)$$

Substitute  $4\pi$  with  $1/\varepsilon_0$  to give charge a dimension

## Macroscopic Maxwell's equations (Gaussian units with $\varepsilon_0$ )

$$\nabla \times \mathbf{E} + \frac{1}{c} \frac{\partial \mathbf{B}}{\partial t} = 0 \qquad \nabla \cdot \mathbf{B} = 0$$

$$\nabla \cdot \mathbf{E} = \frac{1}{\varepsilon_0} (\rho^{(\text{f})} - \nabla \cdot \mathbf{P})$$

$$\nabla \times \mathbf{B} - \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} = \frac{1}{\varepsilon_0 c} \left( \mathbf{j}^{(\text{f})} + \frac{\partial \mathbf{P}}{\partial t} + \nabla \times c \mathbf{M} \right)$$

EM field in the left, material in the right hand side

You can go to SI with  $\mathbf{B} \rightarrow c\mathbf{B}$  and  $\mathbf{M} \rightarrow \mathbf{M}/c$

$c$     $\epsilon_0$     $q_e$

“ $\sim$ ” means measured, “ $=$ ” means defined    $q_e$  is elementary charge

► Gaussian

$$c \sim 2.998 \times 10^{10} \text{ cm/s} \quad \epsilon_0 = \frac{1}{4\pi} \quad q_e \sim 4.803 \times 10^{-10} \text{ statC} \left( \text{cm}^{3/2} \text{g}^{1/2} \text{s}^{-1} \right)$$

► SI before 2019

$$c = 299792458 \text{ m/s} \quad \epsilon_0 = \frac{10^7}{4\pi (c/(\text{m/s}))^2} \text{ F/m} \quad q_e \sim 1.602 \times 10^{-19} \text{ C}$$

► SI after 2019

$$c = 299792458 \text{ m/s} \quad \epsilon_0 \sim 8.854 \times 10^{-12} \text{ F/m} \quad q_e = 1.602176634 \times 10^{-19} \text{ C}$$

Gaussian units cannot be accurate theory any longer, because you can't modify  $1/4\pi$

# Dimensions for SI quantities ( $[\circ]$ reads dimension of $\circ$ )

$$[\mathbf{E}] = [c\mathbf{B}] = \left[ \frac{\text{Voltage}}{\text{Length}} \right] \quad [\rho] = \left[ \frac{\text{Charge}}{\text{Length}^3} \right] \quad [\mathbf{j}] = \left[ \frac{\text{Current}}{\text{Length}^2} \right] \quad [\varepsilon_0] = \left[ \frac{\text{Cap}}{\text{Length}} \right]$$

$$\left[ \nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0} \right] = \left[ \frac{\text{Voltage}}{\text{Length}^2} = \frac{\text{Charge}}{\text{Cap} \cdot \text{Length}^2} \right] \quad \left( V = \frac{Q}{C} \right) \quad \left[ \varepsilon_0 \frac{S}{d} \right] = [\text{Cap}]$$

$$[\text{Time}] = [\text{Res} \cdot \text{Cap}] \quad (\tau = RC) \quad [1/\varepsilon_0 c] = [\text{Time}/\text{Cap}] = [\text{Res}]$$

$$\left[ \nabla \times c\mathbf{B} - \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} = \frac{1}{\varepsilon_0 c} \mathbf{j} \right] = \left[ \frac{\text{Voltage}}{\text{Length}^2} = \text{Res} \cdot \frac{\text{Current}}{\text{Length}^2} \right] \quad (V = RI)$$

$$[\mathbf{j} = \sigma \mathbf{E}] = \left[ \frac{\text{Current}}{\text{Length}^2} = \frac{1}{\text{Res} \cdot \text{Length}} \cdot \frac{\text{Voltage}}{\text{Length}} \right] \quad \left( I = \frac{V}{R} \right)$$

*“The difficulty of science are to a large extent the difficulties of notation, the units, and all the other artificialities which are invented by man, not by nature.”*

— Richard P. Feynman, *The Feynman Lectures on Physics*

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