LIMITS

1) $\lim_{x \to a} \left[\dfrac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \right]$

2) $\lim_{y \to a} \left[\dfrac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \right]$

3) $\lim_{x \to \pi/6} \left[\dfrac{\cos x - \sqrt{3}\sin x}{\pi - 6x} \right]$

4) $\lim_{x \to \infty} \left[\dfrac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}} \right]$

5] Examine function & find whether the function is continuous or not.

i) $f(x) = \dfrac{\sin 2x}{\sqrt{1-\cos 2x}}, \quad 0 \le x \le \dfrac{\pi}{2}$

$= \dfrac{\cos x}{\pi - 2x}, \quad \dfrac{\pi}{2} < x < \pi$

at $x = \dfrac{\pi}{2}$.

ii) $f(x) = \dfrac{x^2-9}{x-3}, \quad 0 < x < 3.$

$= x+3, \quad 3 \le x < 6.$

$= \dfrac{x^2-9}{x+3}, \quad 6 \le x < 9$

at $x = 3$ & $x = 6$.

Find the value of k so that the function $f(x)$ is continuous at the indicated point.

$$f(x) = \begin{cases} \dfrac{1-\cos 4x}{x^2} & x < 0 \\ k & x = 0 \end{cases} \quad \text{at } x=0$$

$$f(x) = \begin{cases} \cancel{\cos} \cdot (\sec^2 x)^{\cot^2 x} & x \neq 0 \\ k & x = 0 \end{cases} \quad \text{at } x=0$$

$$f(x) = \begin{cases} \dfrac{\sqrt{3}-\tan x}{\pi - 3x} & , x \neq \pi/3 \\ k & , x = \pi/3 \end{cases} \quad \text{at } x=\frac{\pi}{3}$$

Discuss the continuity of the following function which of these function has removable discontinuity? Redefine function so as to remove the discontinuity.

$$f(x) = \begin{cases} \dfrac{1-\cos 3x}{x\tan x} & x \neq 0 \\ 9 & \end{cases} \quad \text{at } x=0$$

$$\frac{1}{3}\lim_{x\to a}\cdot\frac{2\sqrt{a}}{2\sqrt{3}\sqrt{a}} =$$

$$= b \; \frac{2}{3\sqrt{3}}.$$

2) $$\lim_{y\to a}\left[\frac{\sqrt{a+y}-\sqrt{a}}{y\sqrt{a+y}}\right]$$

$$= \lim_{y\to 0}\left[\frac{\sqrt{a+y}-\sqrt{a}}{y\sqrt{a+y}} \times \frac{\sqrt{a+y}+\sqrt{a}}{\sqrt{a+y}+\sqrt{a}}\right]$$

$$= \lim_{y\to 0}\left[\frac{(a+y-a)}{y\sqrt{a+y}\,(\sqrt{a+y}+\sqrt{a})}\right]$$

$$\lim_{y\to 0}\left[\frac{1}{\sqrt{a+y}\,(\sqrt{a+y}+\sqrt{a})}\right].$$

By applying limits.

$$= \frac{1}{\sqrt{a+0}\,(\sqrt{a+0}+\sqrt{a})}$$

$$= \frac{1}{\sqrt{a}\times(\sqrt{a}+\sqrt{a})}$$

$$= \frac{1}{\sqrt{a}\,(2\sqrt{a})} = \frac{1}{2a}.$$

3) $\lim_{x \to \frac{\pi}{6}} \left[\dfrac{\cos x - \sqrt{3} \sin x}{\pi - 6x} \right]$

put $x - \dfrac{\pi}{6} = h$

$x = \dfrac{\pi}{6} + h$

as $x \to \dfrac{\pi}{6} \Rightarrow h \to 0$

$$\lim_{x \to \frac{\pi}{6}} \left[\frac{\cos\left(\frac{\pi}{6} - h\right) - \sqrt{3} \sin\left(\frac{\pi}{6} - h\right)}{\pi - 6\left(\frac{\pi}{6} + h\right)} \right]$$

$$\lim_{x \to \frac{\pi}{6}} \left[\frac{\left(\cos\frac{\pi}{6} \cdot \cos h - \sin\frac{\pi}{6} \cdot \sin h\right) - \sqrt{3}\left(\sin\frac{\pi}{6} \cdot \cos h - \cos\frac{\pi}{6} \cdot \sin h\right)}{\pi - \pi - 6h} \right]$$

$$\lim_{x \to \frac{\pi}{6}} \left[\frac{\left(\frac{\sqrt{3}}{2} \cdot \cos h - \frac{1}{2} \sin h\right) - \left(\frac{\sqrt{3}}{2} \cos h - \frac{\sqrt{3}}{2} \cdot \sin h\right)}{-6h} \right]$$

$$\lim_{x \to \frac{\pi}{6}} \left[\frac{-\frac{1}{2} \sin h - \frac{3}{2} \sin h}{-6h} \right]$$

$$\lim_{x\to\frac{\pi}{6}}\left[\frac{-\frac{4}{2}\sin h}{-6h}\right]$$

$$\lim \frac{2}{6}\lim_{h\to\frac{\pi}{6}}\frac{\sin h}{h}.$$

$$= \frac{1}{3}.$$

$$\lim_{x\to\infty}\left[\frac{\sqrt{x^2+5}-\sqrt{x^2-3}}{\sqrt{x^2+3}-\sqrt{x^2+1}}\right].$$

$$\lim_{x\to\infty}\left(\frac{\sqrt{x^2+5}-\sqrt{x^2-3}}{\sqrt{x^2+3}-\sqrt{x^2+1}}\times\frac{\sqrt{x^2+5}+\sqrt{x^2-3}}{\sqrt{x^2+5}+\sqrt{x^2-3}}\times\frac{\sqrt{x^2+3}+\sqrt{x^2+1}}{\sqrt{x^2+3}+\sqrt{x^2+1}}\right.$$

$$\lim_{x\to\infty}\left[\frac{x^2+5-x^2+3}{x^2+3-x^2-1}\times\frac{\sqrt{x^2+3}+\sqrt{x^2+1}}{\sqrt{x^2+5}+\sqrt{x^2-3}}\right].$$

$$\lim_{x\to 0}\left[\frac{8}{2}\;\frac{\sqrt{x^2(1+3/x^2)}+\sqrt{x^2+(1+1/x^2)}}{\sqrt{x^2(1+5/x^2)}+\sqrt{x^2(1-3/x^2)}}\right]$$

$$= 4\lim_{x\to 0} 0 \,//$$

$$= 4.$$

6.3 $\sqrt{1-\cos 2x}$.

$$\lim_{x \to \frac{\pi}{2}} \frac{2\sin x \cos x}{\sqrt{2\sin^2 x}}$$

$$\lim_{x \to \frac{\pi}{2}} \frac{2\cos x}{\sqrt{2}}$$

$$\frac{2}{\sqrt{2}} \lim_{x \to \frac{\pi}{2}} \cos h$$

L.H.L $\neq$ R.H.L.

$\therefore$ f is not continous at $x = \pi/2$.

2). 1: $f(3) = x + 3 = 3 + 3 = 6$.

$$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} \frac{x^2+9}{x-3} = 6.$$

L.H.L = R.H.L.

f is continous at $x = 3$.

for $x = 6$.

i) $f(6) = \frac{x^2+9}{x+3} = 3.$

$$\lim_{x \to 6^+} = \frac{x^2-9}{x+3}.$$

$$\lim_{x \to 6^+} (x-3) = 3.$$

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$$\lim_{x \to 6^-} x+3 = 3+6 = 9.$$

$= L.H.L \neq R.H.L.$

$\therefore f$ is not continous.

6) $f(x) = \frac{1-\cos 4x}{x^2}.$

$$\lim_{x \to 0} f(x) = f(0).$$

$$\lim_{x \to 0} \frac{1-\cos 4x}{x^2} = k.$$

$$= \lim_{x \to 0} \frac{2\sin^2 2x}{x^2} = k,$$

$$2 \lim_{x \to 0} \left(\frac{\sin^2 2x}{2x^{\cancel{2}}}\right)^2 =$$

$= 2 \times 4 \cancel{\times 2}$

$\cancel{= 8}$

$= k.$

ii) $f(x) = (\sec^2 x)^{\cot^2 x}$.
$= k$,

$$\sqrt{3} - \frac{3\tan h - \sqrt{3} - \tan h}{\frac{1 - \sqrt{3}\tan h}{-3h}}$$

$$= \frac{-4\tan h}{-3h(1-\sqrt{3}\tan h)}$$

$$= \left(\frac{4}{3}\right)\left(\frac{1}{1-0}\right)$$

$$= 4.$$

$$f(x) = (\sec^2 x)^{\cot^2 x}.$$

$$\lim_{x \to 0} (1 + \tan^2 x)^{\frac{1}{\tan^2 x}}.$$

$$= e$$

$$= k.$$

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7). $f(x) = \dfrac{1-\cos 3x}{x\tan x}$.

$f(x) = 9$

$f(x) = \dfrac{1-\cos 3x}{x\tan x}$.

$= \dfrac{2\sin^2 3/2\, x}{x\tan x}$.

$= 2\lim_{x\to 0} \dfrac{\dfrac{\sin^2 3/2\, x}{x^2}}{\dfrac{x\tan x}{x^2}}$.

$= 2\lim_{x\to 0} \left(\dfrac{3}{2}\right)^2 / 1$.

$= 2 \times \dfrac{9}{4}$.

$= \dfrac{9}{2}$.

$\lim_{x\to 0} f(x) = \dfrac{9}{2} \neq f(0)$.

redefine function -

$f(x) = \dfrac{1-\cos 3x}{\tan x}$

$= \dfrac{9}{2}$.

ii) $f(x) = \frac{e^{x^2} - \cos x}{x^2}$

is continuous at $x = 0$.

f is continuous at $x = 0$.

$\lim_{x \to 0} f(x) = f(0)$

$= \frac{e^{x^2} - \cos x}{x^2} = f(0).$

$\lim_{x \to 0} \frac{e^{x^2} - \cos x + 1 - 1}{x^2}.$

$\lim_{x \to 0} \frac{(e^{x^2} - 1) - (\cos x - 1)}{x^2}.$

$= \frac{e^{x^2} - 1}{x^2} - \frac{(\cos x - 1)}{x^2}.$

$= \lim_{x \to 0} \log e - \frac{2\sin^2 x/2}{x^2}$

$= \cancel{\log e} + 2\left(\frac{\sin^2 \frac{x}{2}}{x}\right)^2$

$= \log e + 2 \times \frac{1}{4}$

$= \cancel{\log 1}\; 1 + 2 \times \frac{1}{4}$

$= \frac{3}{2} - f(0).$

8) $f(x) = \frac{(e^{3x}-1)\sin x^{\circ}}{x^2}$

$= \pi/6.$

$\lim_{x \to 0} \frac{(e^{3\pi}-1)\sin\left(\frac{\pi x}{180}\right)}{x^2}$

$= 3\lim_{x\to 0} \log e \frac{\pi}{180} = \frac{\pi}{60} = f(0)$

f is contion at $x=0$.

9) $f(x) = \frac{\sqrt{2}-\sqrt{1+\sin x}}{\cos^2 x}$

$f(0)$ is continous at $x=\frac{\pi}{2}$.

$\lim_{x\to\frac{\pi}{2}} \frac{\sqrt{2}-\sqrt{1+\sin x}}{\cos^2 x} \times \frac{\sqrt{2}+\sqrt{1+\sin x}}{\sqrt{2}+\sqrt{1+\sin x}}$

$= 2+1+\sin x$

$= \lim_{x\to\frac{\pi}{2}} \frac{1+\sin x}{(1-\sin^2 x)(\sqrt{2}+\sqrt{1+\sin x})}$

036

$$\lim_{x \to \frac{\pi}{2}} \frac{1}{(1-\sin x)(\sqrt{2}+\sqrt{1+\sin x})}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{1}{(1-\sin x)(\sqrt{2}+\sqrt{1+\sin x})}$$

$$= \frac{1}{2(\sqrt{2}+\sqrt{2})}$$

$$= f\left(\frac{\pi}{2}\right) = \frac{1}{4\sqrt{2}}$$

02/12/19

PRACTICAL - 02

DERIVATIVES.

Q 1] Show that the following function defined from $\mathbb{R}$ to $\mathbb{R}$ are differentiable.

i) $\cot x$.

$\rightarrow$ $f(x) = \cot x$.

$$Df(x) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to a} \frac{\frac{1}{\tan x} - \frac{1}{\tan a}}{x - a}$$

$$= \lim_{x \to a} \frac{\tan a - \tan x}{(x-a)\tan x \cdot \tan a}$$

put $x - a = h$.

$\therefore x = a + h$

as $x \to a$, $h \to 0$

$$Df(x) = \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{(a + h - a)\tan(a+h)\tan a}$$

$$= \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{h \times \tan(a+h)\tan a}$$

formula: $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \cdot \tan B}$.

$\tan A - \tan B = \tan(A-B)(1 + \tan A \cdot \tan B)$.

$$= \lim_{h \to 0} \frac{\tan(a - a - h) - (1 + \tan a \cdot \tan(a+h))}{h \times \tan(a+h)\tan a.}$$

$$= \lim_{h \to 0} -\frac{\tan h}{h} \times \frac{1 + \tan a \tan(a+h)}{\tan(a+h)\tan a.}$$

$$= -1 \times \frac{1 + \tan^2 a}{\tan^2 a.}$$

$$= \frac{-\sec^2 a}{\tan^2 x}$$

$$= \frac{-1}{\cos^2 a} \times \frac{\cos^2 a}{\sin^2 a}$$

$$= -\operatorname{cosec}^2 a$$

$Df(a) = -\cos^2 a$

$\therefore$ f is differentiable $\forall a \in R$.

) cosec x.

$f(x) = \operatorname{cosec} x$

$$Df(x) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to a} \frac{\operatorname{cosec} x - \operatorname{cosec} a}{x - a}$$

$$= \lim_{x \to a} \frac{\frac{1}{\sin x} - \frac{1}{\sin a}}{x - a}$$

put $x - a = h$

$x = a + h$

as $x \to a$, $h \to 0$.

$$Df(x) = \lim_{h \to 0} \frac{\sin a - \sin(a+h)}{(a+h-a)\sin a \cdot \sin(a+h)}$$

formula:

$$\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right) \cdot \sin\left(\frac{C-D}{2}\right)$$

$$= \lim_{h\to 0} \frac{2\cos\left(\frac{a+a+h}{2}\right)\cdot\sin\left(\frac{a-a-h}{2}\right)}{h \times \sin a \cdot \sin(a+h)}$$

$$= \lim_{h\to 0} \frac{-\sin\frac{h}{2}}{\frac{h}{2}} \times \frac{1}{2} \times \frac{2\cos\left(\frac{2a+h}{2}\right)}{\sin a \cdot \sin(a+h)}$$

$$= -\frac{1}{2} \times \frac{2\cos\left(\frac{2a+0}{2}\right)}{\sin(a+0)}.$$

$$= \frac{-\cos a}{\sin^2 a}$$

$$= -\cot a \cdot \operatorname{cosec} a.$$

Sec x.

$f(x) = \sec x$

$$Df(x) = \lim_{x\to a} \frac{f(x) - f(a)}{x-a}$$

$$= \lim_{x\to a} \frac{\sec x - \sec a}{x-a}$$

$$= \lim_{x\to a} \frac{\frac{1}{\cos x} - \frac{1}{\cos a}}{x-a}$$

$$= \lim_{x\to a} \frac{\cos a - \cos x}{(x-a)\cos x \cdot \cos a}$$

put $x - a = h.$

$x = a + h$

as $x \to a$, $h \to 0$

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$$Df(h) = \lim_{h \to 0} \frac{\cos a - \cos(a+h)}{(x-a)(\cos a \cdot \cos x)}.$$

038

Formula: $-2\sin\left(\frac{C+D}{2}\right) \cdot \sin\left(\frac{C-D}{2}\right)$

$$= \lim_{h \to 0} \frac{-2\sin\left(\frac{a+a+h}{2}\right) \cdot \sin\left(\frac{a-a-h}{2}\right)}{h \times \cos a \cdot \cos(a+h)}.$$

$$= \lim_{h \to 0} \frac{-2\sin\left(\frac{2a+h}{2}\right) \cdot \sin -\frac{h}{2}}{\cos a \cdot \cos(a+h) \times -\frac{h}{2}} \times -\frac{1}{2}$$

$$= -\frac{1}{2} \times \frac{-2\sin\left(\frac{2a+0}{2}\right)}{\cos a \cdot \cos(a+0)}.$$

$$= -\frac{1}{2} \times -2 \frac{\sin a}{\cos a \cdot \cos a}.$$

$$= \tan a . \sec a.$$

Q.2] If $f(x) = 4x+1$, $x \leq 2$.

$= x^2+5$, $x > 0$ at $x=2$, then find function is differentiable or not.

→ L.H.D:

$$Df(2^-) = \lim_{x \to 2^-} \frac{f(x) - f(a)}{x-a}$$

$$= \lim_{x \to 2^-} \frac{4x+1-(4\times2+1)}{x-2}.$$

$$= \lim_{x \to 2^-} \frac{4x+1-9}{x-2}.$$

$$= \lim_{x \to 2^-} \frac{4x-8}{x-2}.$$

$$= \lim_{x \to 2^-} \frac{4(x-2)}{x-2}$$

$= 4$

$Df(2^-) = 4.$

R.H.D : $Df(2^+) = \lim_{x \to 2^+} \frac{x^2 + 5 - 9}{x - 2}$

$= \lim_{x \to 2^+} \frac{x^2 - 4}{x - 2}$

$= \lim_{x \to 2^+} \frac{(x+2)(x-2)}{x-2}$

$= 2 + 2 = 4.$

$\therefore Df(2^+) = 4$

R.H.D = L.H.D.

$\therefore$ f is differentiable at $x = 2$

3). If $f(x) = 4x + 7$, $x < 3$.
$= x^2 + 3x + 1$, $x \geq 3$ at $x = 3$,
Find f is differentiable or not.

$\rightarrow$ R.H.D :

$Df(3^+) = \lim_{x \to 3^+} \frac{f(x) - f(a)}{x - a}$

$= \lim_{x \to 3^+} \frac{x^2 + 3x + 1 - (3^2 + 3 \times 3 + 1)}{x - 3}$

$= \lim_{x \to 3^+} \frac{x^2 + 3x + 1 - 19}{x - 3}$

$= \lim_{x \to 3^+} \frac{x^2 + 3x - 18}{x - 3}$

$= \lim_{x \to 3^+} \frac{x^2 + 6x - 3x - 18}{x - 3}$

$= \lim_{x \to 3^+} \frac{x(x+6) - 3(x+6)}{x - 3}$

$= \lim_{x \to 3^+} x + 6$

$Df(3^+) = 9.$

$$L.H.D = Df(3^-)$$

$$= \lim_{x\to 3^-} \frac{f(x)-f(3)}{x-3}.$$

$$= \lim_{x\to 3^-} \frac{4x+7-19}{x-3}.$$

$$= \lim_{x\to 3^-} \frac{4x-12}{x-3}.$$

$$= \lim_{x\to 3} \frac{4(x-3)}{x-3}.$$

$Df(3^+) = 4.$

$\therefore$ R.H.D $\neq$ L.H.D

$\therefore$ f is not differentiable

4) If $f(x) = 8x-5, \quad x \leq 2.$

$= 3x^2-4x+7 \quad , x > 2$ at $x=2$,

find f is differentiable or not.

$\rightarrow$ $f(2) = 8\times 2-5.$

$= 16-5 = 11.$

R.H.D =

$$Df(2^+) = \lim_{x\to 2^+} \frac{f(x)-f(2)}{x-2}.$$

$$= \lim_{x\to 2^+} \frac{3x^2-4x+7-11}{x-2}.$$

$$= \lim_{x\to 2^+} \frac{3x^2-4x-4}{x-2}.$$

$$= \lim_{x\to 2^+} \frac{3x^2-6x+2x-4}{x-2}.$$

$$= \lim_{x \to 2^+} \frac{3x(x-2) + 2(x-2)}{x-2}$$

$$= \lim_{x \to 2^+} \frac{(3x+2)(x-2)}{x-2}$$

$$= \cancel{3x}\ 3 \times 2 + 2$$

$$= 8$$

$$Df(2^+) = 8$$

L.H.D :

$$Df(2^-) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x-2}$$

$$= \lim_{x \to 2^-} \frac{8x - 5 - 11}{x-2}$$

$$= \lim_{x \to 2^-} \frac{8x - 16}{x-2}$$

$$= \lim_{x \to 2^-} \frac{8(x-2)}{(x-2)}$$

$$= 8$$

$$Df(2^-) = 8$$

L.H.D = R.H.D

$\therefore f$ is differentiable at $x = 3$.

09/12/19

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PRACTICAL - 03.

Application of Derivative.

040

1) Find the intervals in which function is increasing or decreasing.

i) $f(x) = x^3 - 5x - 11$.

$f'(x) = 3x^2 - 5$

$\therefore$ f is increasing iff $f(x) > 0$

$\therefore 3x^2 - 5 > 0$

$3(x^2 - 5/3) > 0$.

$(x - \sqrt{5/3})(x + \sqrt{5/3}) > 0$.

$x \in (-\infty, -\sqrt{5}/3) \cup (\sqrt{5}/3, \infty)$.

2) f is decreasing iff $f'(x) < 0$.

$3x^2 - 5 < 0$.

$3(x^2 - 5/3) < 0$.

$3(x - \sqrt{5}/3)(x + \sqrt{5}/3) < 0$.

ii) $f(x) = x^2 - 4x$.

$f'(x) = 2x - 4$.

f is increasing iff $f(x) > 0$.

$\therefore 2x - 4 > 0$

$2(x - 2) > 0$

$x - 2 > 0$.

$x \in (2, \infty)$

2) f is decreasing iff $f'(x) < 0$.

$\therefore 2x - 4 < 0$

$2(x - 2) < 0$

$x - 2 < 0$

$x \in (-\infty, 2)$.

iii) $f(x) = 2x^3 + x^2 - 20x + 4$

$\therefore f'(x) = 6x^2 + 2x - 20$

$\therefore$ f is increasing if $f'(x) \geq 0$

$6x^2 + 2x - 20 \geq 0$

$2(3x^2 + x - 10) > 0$

$3x^2 + x - 10 > 0$

$3x^2 + 6x - 5x - 10 > 0$

$3x(x+2) - 5(x+2) > 0$

$(x+2)(3x-5) > 0$

$x \in (-\infty, -2) \cup (5/3, \infty)$

& f is decreasing if $f'(x) < 0$

$\therefore 6x^2 + 2x - 20x < 0$

$2(3x^2 + x - 10) < 0$

$3x^2 + x - 10 < 0$

$3x^2 + 6x - 5x - 10 < 0$

$3x(x+2) - 5(x-20) < 0$

$(3x-5)(x+2) < 0$

$+ \quad -$

$x = -2 \qquad 5/3$

$x \in (-2, 5/3)$

iv) $f(x) = 2x^3 - 9x^2 - 24x + 69$

$f'(x) = 6x^2 - 18x - 24$

$\therefore$ f is increasing iff $f'(x) > 0$

$\therefore 6x^2 - 18x - 24 > 0$

$6(x^2 - 3x - 4) > 0$

$x^2 - 4x + x - 4 > 0$

$x(x-4) + 2(x-4) > 0$

$(x-4)(x+2) \geq 0$

$\therefore x \in (-\infty, -1) \cup (4, \infty)$

2) f is decreasing iff $f'(x) < 0$

$\therefore 6x^2 - 18x - 24 < 0$

$6(x^2 - 3x - 4) < 0$

$x^2 - 4x + x - 4 < 0$

$x(x-4) + 1(x-4) < 0$

$(x-4)(x+1) < 0$

$+ \qquad - \qquad +$

$-2 \qquad 4$

$x \in (-1, 4)$.

$f(x) = x^3 - 27x + 5$.

$f'(x) = 3x^2 - 27$.

$\therefore$ f is increasing iff $f'(x) > 0$.

$\therefore 3(x^2 - 9) > 0$

$(x-3)(x+3) > 0$.

$+ \qquad - \qquad +$

-3

$\therefore x \in (-\infty, -3) \cup (3, \infty)$.

c) if

f is decreasing iff $f'(x) < 0$.

$\therefore 3(x^2 - 9) < 0$.

$(x-3)\cdot(x+3) < 0$.

$+ \qquad - \qquad +$

$-3 \qquad 3$

$\therefore x \in (-3, 3)$.

Q.2) i) $Y = 3x^2 - 2x^3$.

$\therefore f(x) = 3x^2 - 2x^3$.

$\therefore f'(x) = 6x - 6x^2$.

$\therefore f''(x) = 6 - 12x$.

f is concave upward iff $f''(x) > 0$

$\therefore (6 - 12x) > 0$.

$\therefore 12(6/12 - x) > 0$.

$x - 1/2 > 0$.

$\therefore x > \frac{1}{2}$.

$\therefore x \in (1/2, \infty)$

ii) $y = (x^4 - 6x^3 + 12x^2 + 5x + 7)$

$f(x) = 4x^4 - 6x^3 + 12x^2 + 5x + 7$

$f'(x) = 4x^3 - 18x^2 + 24x + 5$

$f''(x) = 12x^2 - 36x + 24$

f is concave upward iff $f''(x) > 0$

$\therefore 12(x^2 - 3x + 2) > 0$

$\therefore x^2 - 2x - x + 2 > 0$

$\therefore (x-2)(x-1) > 0$

$x \in (-\infty, 1) \cup (2, \infty)$

iii) $y = x^3 - 27x + 5$

$f(x) = 3x^2 - 27$

$f''(x) = 6x$

f is concave upward iff $f''(x) > 0$

$\therefore 6x > 0$

$\therefore x > 0$ $\quad \therefore x \in (0, \infty)$

iv).

F

v).

042

$y = 2x^3 + x^2 - 20x + 4$

$f(x) = 24x^3 + x^2 - 20x + 4$

$f'(x) = \cancel{6x} \; 6x^2 + 24 - 20$

$f''(x) = 124 + 2$

is concave upward iff $f''(x) > 0$

$\therefore f''(x) > 0$

$\therefore 12x + 2 > 0$

$\therefore 12(x + 2/12) > 0$

$\therefore x + 1/6 > 0$

$\therefore x < -1/6.$

$\therefore f''(x) \not> 0$

$\therefore$ There exist ~~no~~ interval

$x \in (-1/6, \infty)$ //.

$y = 69 - 24x - 9x^2 + 2x^3$

$f(x) = 24^3 - 9x^2 - 24x + 64$

$f'(x) = 6x^2 - 18x - 24$

$f''(x) = 12x - 18$

f. is concave upward iff $f''(x) > 0$

$\therefore 12x - 18 > 0$

$\therefore 12(x - 18/12) > 0$

$\therefore x - 3/2 > 0 \quad \therefore x > 3/2$

$\therefore x \in (3/2, \infty)$ //.

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$\therefore$ f has minimum at $x=2$

$\therefore f(2) = 2^2 + 16/2^2$

$= 4 + 16/4$

$= 4+4$

$= \boxed{8}$.

$\therefore f''(-2) = 2 + 96/(-2)^4$

$= 2 + \frac{96}{16}$

$= 8 > 0$.

$\therefore$ min at $x = -2$.

$\therefore$ function reaches min value at $x=2$. and $x=-2$.

$f(x) = 3 - 5x^3 + 3x^5$

$f'(x) = -15x^2 + 15x^4$

consider, $f'(x) = 0$

$\therefore -15x^2 + 15x^4 = 0$

$15x^4 = 15x^2$

$x^2 = 1$

$x = \pm 1$.

$\therefore f''(x) = -30x + 60x^3$

$f(1) = -30 + 60$

$= 30 > 0$. $\therefore$ f has min value at $x=2$

~~$f(-7) = 3-5$~~

$f(1) = 3 - 5(1)^3 + 3(1)^5$.

$= 6 - 5$

$= \boxed{1}$.

$f(-1) = 3 - 5(-1)^3 + 3(-1)^5$.

$= 3 + 5 - 3$

$= \boxed{5}$.

$f''(-1) = -30(-1) + 60(-1)^3$

$= 30 - 60$

$= -30 < 0$ $\therefore$ f has max value at $x = -1$

$\therefore$ f has max value 5 at $x = -1$ and has the minimum value 1 at $x = 1$.

$f(x) = x^3 - 3x^2 + 7$

$\therefore f'(x) = 3x^2 - 6x$

consider, $f'(x) = 0$

$3x^2 - 6x = 0$

$3x(x - 2) = 0$

$3x = 0$ or $x - 2 = 0$

$x = 0$ or $x = 2$

$\therefore f''(x) = 6x - 6$

$f''(0) = 6(0) - 6$

$= -6 < 0$ $\therefore$ f has max value at $x = 0$.

$\therefore f(0) = (0)^3 - 3(0)^2 + 1 = 1$

$\therefore f''(2) = 6(2) - 6$

$= 12 - 6$

$= 6 > 0$ $\therefore$ f has min value at $x = 2$.

$\therefore f(2) = (2)^3 - 3(2)^2 + 1$

$= 8 - 12 + 1$

$= -4 + 1$

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f has max value 1 at $x=0$ and f has min value -3 at $x=2$.

Q44

$F(x) = 2x^3 - 3x^2 - 12x + 1$

$\therefore f'(x) = 6x^2 - 6x - 12$

consider, $f'(x) = 0$

$6(x^2 - x - 2) = 0$

$x^2 - x - 2x - 2 = 0$

$x(x+1) - 2(x+1) = 0$

$(x-2)(x+1) = 0$

$x = 2$ or $x = -1$

$\therefore f''(x) = 12x - 6$

$f''(2) = 12(2) - 6$

$= \boxed{18 > 0}$.

$\therefore$ f has minimum value at $x=2$

$\therefore f(2) = 2(2)^3 - 3(2)^2 - 12(2) + 1$

$= 2(8) - 3(4) - 24 + 1$

$= 16 - 12 - 24 + 1$

$= \boxed{-19}$.

$\therefore f''(-1) = 12(-1) - 6$

$= \boxed{18 < 0}$.

f has max value at $x = -1$

$\therefore f(-1) = 2(-1) - 3(-1)^2 - 12(1) + 1$

$= -2 - 3 + 12 + 1$

$= \boxed{8}$.

$\therefore$ f has max value 8 at $x = -1$ and min value -19 at $x=2$.

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Q.2) i) $f(x) = x^3 - 3x^2 - 55x + 9.5$

$f'(x) = 3x^2 - 6x - 55$

By Newton's Method

$x_{n+1} = x_n - f(x_n) / f'(x_n)$

$\therefore x_1 = x_0 - f(x_0) / f'(x_0)$

$x_1 = 0 + 9.5/55$

$x_1 = 0.1727$.

$\therefore f(x_1) = (0.1727)^3 - 3(0.1727)^2 - 55(0.1727) + 9.5$

$= 0.0051 - 0.0895 - 9.4985 + 9.5$

$= -0.0829$.

$\therefore f'(x_1) = 3(0.1727)^2 - 6(0.1727) - 55$

$= 0.0895 - 1.0362 - 55$

$= -55.9467$.

$\therefore x_2 = x_1 - f(x_1) / f'(x_1)$.

$= 0.1727 - 0.0829/55.9467$.

$= \cancel{0.0077}.\ 0.1712$.

$f''(x_2) = (0.1712)^3 - 3(0.1712)^3 - 55(0.1712) + 9.5$

$= 0.0050 - 0.0879 - 9.416 + 9.5$

$= 0.0011$.

$f'(x_2) = 3(0.1712)^3 - 6(0.1712) - 55$

$= 0.0879 - 1.0272 - 55$.

$= -55.9393$.

$x_3 = x_2 - f(x_2) / f'(x_2)$.

$= 0.1712 + 0.011 / 55.9393$

$= 0.1712$.

$\therefore$ The root of the equation is 0.1712

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ii) $f(x) = x^3 - 4x - 9$ $[2, 3]$.

$f'(x) = 3x^2 - 4$.

$f(2) = 2^3 - 4(2) - 9$.

$= 8 - 8 - 9$.

$= -9$.

$f(3) = 3^3 - 4(3) - 9$.

$= 27 - 12 - 9$

$= 6$.

let $x_0 = 3$ be the initial approximation.

$\therefore$ By Newton's method.

$x_{n+1} = x_n - f(x_n)/f'(x_n)$.

$x_1 = x_0 - f(x_0)/f'(x_0)$.

$= 3 - 6/23$.

$= 2.7392$.

$f(x_1) = (2.7392)^3 - 4(2.7392) - 9$.

$= 20.5528 - 10.9568 - 9$.

$= 0.596$.

$f'(x_1) = 3(2.7392)^2 - 4$.

$= 22.5096 - 4$.

$= 18.5096$.

$x_2 = x_1 - f(x_1)/f'(x_1)$.

$= 2.7392 - 0.596/18.5096$.

$= 2.7071$.

$f(x_2) = (2.7071)^3 - 4(2.7071)$.

$= 19.8386 - 10.8284$

$= 0.0102$.

$f'(x_3) = 3(2.7041)^2 - 4$

$= 21.9351 - 4$

$= 17.9351$

$= 2.7041 - \frac{0.0907}{17.9351}$

$= 2.7041 - 0.0026$

$= 2.7015$

$f(x_4) = (2.7015)^3 - 4(2.7015) - 9$

$= 19.7152 - 10.806 - 9$

$= -0.0901$

$f'(3) = 3(2.7015)^2 - 4$

$= 21.8948 - 4$

$= 17.8948$

$x_4 = 2.7015 + 0.0901/17.8948$

$= 2.7015 + 0.0050$

$= 2.7065$

$f(x) = x^3 - 1.2x^2 - 10x + 17 \quad [1, 2]$

$f'(x) = 3x^2 - 2.4x - 10$

$f(1) = (1)^3 - 1.2(1)^2 - 10(1) + 17$

$= -1.2 - 10 + 17$

$= 6.2$

$f(2) = (2)^3 - 1.2(2)^2 - 10(2) + 17$

$= 8 - 1.2(4) - 20 + 17$

$= 8 - 7.2 - 3$

$= -2.2$

let $x_0 = 2$ be initial approximation

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By Newton's method.

P46

$x_{n+1} = x_n - f(x_n)/f'(x_n)$

$x_1 = x_0 - f(x_2)/f'(x_2)$

$x_1 = x_0 - f(x_2)/f'(x_2)$.

$= 2 - 2.2/5.2$

$= 2 - 0.4230$

$= 1.577$

$x_1) = (1.577)^3 - 1.8(1.577)^2 - 10(1.577) + 17$

$= 3.9219 - 4.4764 - 15.77 + 17$

$= 0.6755$

$) = 3(1.577)^2 - 3.6(1.577) - 10$

$= 7.4608 - 5.6772 - 10$

$= -8.2164$.

$x_2 = x_1 - f(x_1)/f'(x_1)$.

$= 1.577 + 0.6755/8.2164$.

$= 1.577 + 0.0822$

$= 1.6592$.

$_2) = (1.6592)^3 - 1.8(1.6592)^2 - 10(1.6592) + 17$

$= 4.5677 - 4.9553 - 16.592 + 17$

$= 0.0204$.

$_2) = 3(1.6592)^2 - 3.6(1.6592) - 10$.

$= 8.2588 - 5.97312 - 10$.

$= -7.7143$.

$= x_2 - f(x_2)/f'(x_2)$.

$= 1.6592 + 0.0204/7.7143$.

$= 1.6618$

$) = (1.6618)^3 - 1.8(1.6618)^2 - 10(1.6618) + 17$.

$= 4.5892 - 4.9708 - 16.618 + 17$.

$= 0.0004$.

$f'(x_3) = 3(1.6618)^2 - 3.6(1.6618) - 10.$

$= 8.2847 - 5.9824 - 10.$

$= -7.6977.$

$x_4 = x_3 - f(x_3)/f'(x_4)$

$= 1.6618 + 0.0004/7.6977.$

$= 1.6618.$

PRACTICAL - 05.

INTEGRATION.

Q.1) Solve the following Integration:

1) $\dfrac{dx}{\sqrt{x^2+2x-3}}$.

2) $\int (4e^{3x}+1)\,dx$.

3) $\int (2x^2 - 3\sin x + 5\sqrt{x})\,dx$.

4) $\int \dfrac{x^3+3x+4}{\sqrt{x}}\,dx$.

5) $\int t^7 \sin(2t^4)\,dt$.

6) $\int \sqrt{x}\,(x^2-1)\,dx$.

7) $\int \frac{1}{x^3} \sin\left(\frac{1}{x^2}\right) dx$.

8) $\int \dfrac{\cos x}{3\sqrt{\sin^2 x}}\,dx$.

9) $\int \cancel{co}\, e^{\cos^2 x} \sin 2x\,dx$.

10) $\int \left(\dfrac{x^2-2x}{x^3-3x^2+1}\right) dx$.

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Solutions:

1) $I = \int \frac{dx}{\sqrt{x^2+2x-3}}$

$= \int \frac{dx}{\sqrt{(x+1)^2-(2)^2}}$

comparing with

$\int \frac{dx}{\sqrt{x^2-a^2}} = x^2 = (x+1)^2$

$\therefore I = \log|x+\sqrt{x^2+a^2}|+c$

$= \log|x+1\sqrt{(x+1)^2-(2)^2}|+c.$

2) $I = \int(4e^{3x}+1)dx.$

$= \int 4e^{3x}dx + \int 1\,dx.$

$= \frac{4e^{3x}}{3} + 4+c.$

3) $I = \int(2x^2-3\sin x+5\sqrt{x})dx.$

$= 2\int x^2 - 3\int \sin x + 5\int\sqrt{x}\,dx.$

$= \frac{2}{3}x^3 dx + 3\cos x + \frac{5\times2}{3}x^{3/2}+c.$

$= \frac{2}{3}x^3 + 3\cos x + \frac{10}{3}x^{3/2}+c.$

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048

4) $\int \frac{x^3+3x+4}{\sqrt{x}}\,dx$

$= \int \frac{x^3+3x+4}{\sqrt{x}}\,dx$

$= \int \left(\frac{4x^3}{x^2} + \frac{3x}{x^{1/2}} + \frac{4}{x^{1/2}} \right) dx$

$= \int x^{5/2}\,dx + 3\int x^{1/2}\,dx + 4\int x^{-1/2}\,dx$

$= \frac{2}{7} x^{7/2} + \cancel{x}\, 2^{x^{3/2}} + 8\sqrt{x} + c.$

5) $I = \int t \sin(2t^4)\,dx.$

let $t^4 = x.$

$4t^3 dt = dx.$

$= \frac{1}{4}\int \cancel{t}\, 4t^3 \cdot t^4 \sin 2t^4)\,dt.$

$= \frac{1}{4}(4 \cdot \sin(2x)\,dx.$

$= \frac{1}{4}\left[x\int \sin 2x - \int \left(\int \sin 2x \cdot \frac{d}{dx} x\right]\right.$

$= \frac{1}{4}\left[-\frac{x\cos 2x}{2} + \frac{1}{2}\int \cos 2x^{-1}\right].$

$= \frac{1}{4}\left[-\frac{\cos 2x}{2} + \frac{1}{4}\sin 2x\right] + c$

$= -\frac{1}{8} x \cos 2x + \frac{1}{16}\sin 2x + c.$

$= -\frac{1}{8} t^4 \cos(2t^4) + \frac{1}{16}\sin(2t^4) + c.$

629

PRACTICAL - 8

EULER'S METHOD

1) $\frac{dy}{dx} = y + e^x - 2$ $y(0) = 2$ $h = 0.5$ find $y(2) = ?$

$\rightarrow f(x) = y + e^x - 2$ $x_0 = 0$ $y(0) = 2$ $h = 0.5$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	2	1	2.5
1	0.5	2.5	2.1487	3.5743
2	1	3.5743	4.2925	5.7205
3	1.5	5.7205	8.2021	9.8215
4	2	9.8215		

$\therefore Y(2) = 9.8215.$

2) $\frac{dy}{dx} = 1 + y^2$, $y(0) = 1$ $h = 0.2$ find $y(1) = ?$

$x_0 = 0$, $y_0 = 0$ $h = 0.2.$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	0	1	0.2
1	0.2	0.2	1.04	0.408
2	0.4	0.408	1.664	0.6412
3	0.6	0.6412	1.4111	0.9234
4	0.8	0.9234	1.8526	1.2939
5	1	1.2939		

$Y(1) = 1.2939$

3) $\frac{dy}{dx} = \sqrt{}$

n	x_n
0	0
1	0.
2	0.
3	0.
4	0.
5	1

4) $\frac{dy}{dx}$

n
0
1
2

6) $I = \int \sqrt{x}\,(x^2-1)\,dx$.

$\int \sqrt{x}\,(x^2-1)\,dx$.

$= \int (\sqrt{x}\,.\,x^2 - \sqrt{x})\,dx$.

$= \int (x^{5/2} - \sqrt{x})\,dx$.

$= \frac{2}{7} x^{7/2} - \frac{2}{3} x^{3/2} + c$.

7) $I = \int \frac{1}{x^3} \sin\left(\frac{1}{x^2}\right) dx$.

let $\frac{1}{x^3} = t$.

$x^{-2} = t$.

$= \frac{-2}{x^3}\,dx = dt$.

$I = -\frac{1}{2} \int \frac{-2}{x^3} \sin\left(\frac{1}{x^2}\right) dx$.

$= -\frac{1}{2} \int \sin t$

$= -\frac{1}{2} (-\cos t) + c$

$= \frac{1}{2} \cos t + c$.

Resubstitution $t = \frac{1}{x^2}$.

$\therefore I = \frac{1}{2} \cos\left(\frac{1}{x^2}\right) + c$.

8). $I = \int \frac{\cos x}{3\sqrt{\sin 2x}}$.

let $\sin x = t$.

$\cos x\,dx = dt$.

$I = \int \frac{dt}{3\sqrt{t^2}}$.

$= \int \frac{dt}{t^{2/3}}$.

$= \int t^{-2/3}\,dt$.

$= -3t^{1/3} + c$.

$= 3(\sin x)^{1/3} + c$.

$= 3\sqrt[3]{\sin x} + c$.

9). $I = \int \cos^2 x \cdot \sin 2x \cdot dx$.

let $\cos^2 x = t$.

$= -2\cos x \cdot \sin x\,dx = dt$

$= -2\sin 2x\,dx = dt$.

$I = \int -\sin 2x\, e^{\cos 2x}\,dx$.

$= -\int e^t\,dt$.

$= e^t + c$.

Resubstituting $t = \cos^2 x$.

$I = \int e,$

$I = -e\cos^{2x} + c$.

$$I = \int \left(\frac{x^2 - 2x}{x^2 - 3x^2 + 1} \right) dx.$$

let $x^3 - 3x^2 + 1 = t.$

$$3(x^2 - 2x)\,dx = dt.$$

$$(x^2 - 2x)\,dx = dt/3.$$

$$I = \int 1/t \; dt.$$

$$= \frac{1}{3} \int dt/t$$

$$= \frac{1}{3} \log t + c.$$

Resubstituting $t = x^3 - 3x^2 + 1$

$$I = \frac{1}{3} \log (x^3 - 3x^2 + 1) + c.$$

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PRACTICAL - 6

Application of integration & Numerical integration

P5U

nd the length of the following curve.

$\cdot t \sin t$ $\quad y = 1 - \cos t$

arc length $= \int_0^{?} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^3}$

$dt = \int_0^{2\pi} \sqrt{(1-\cos t)^2 \{\sin t\}^2}\, dt.$

$= \int_0^{2\pi} \sqrt{1 - 2\cos t + \cos^2 t + \sin^2 t} + dt$

$= \sqrt{1 - 2\cos t + \sin^2 t}$

$= \int_0^{2\pi} \sqrt{2 - 2\cos t}\, dt.$

$= \int_0^{2\pi} 2 \sin \frac{1}{2}\, dt.$

$= \left[-9\cos \frac{1}{2}\right]_0^{2\pi}$

$= \left(-4\cos \pi\right) + 4\cos 0.$

$=$ 8 units.

2) $y = \sqrt{4 - x^2} \qquad x \in [-2, 2]$

$\rightarrow \quad l = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

$\frac{dy}{dx} = 2\int_0^2 \sqrt{1 + \left(\frac{-x}{\sqrt{4-x^2}}\right)^2}\, dx$

$= 2\int_0^2 \sqrt{1 + \frac{x^2}{4 - x^2}}\, dx.$

$= 4\int_0^2 \frac{1}{\sqrt{4 - x^2}}\, dx$

$= 4\left(\sin^{-1}\left(\frac{x}{2}\right)\right)_0^2$

$= 2\pi$

3) $y = x^{3/2}$ in $[0, 4]$

$\rightarrow \quad f'(x) = \frac{3}{2} x^{1/2}.$

$[f'(x)]^2 = \frac{9}{4} x$

$l = \int_0^b \sqrt{1 + [f'(x)]^2}\, dx$

$= \int_0^x \sqrt{1 + \frac{3}{4} x}\, dx.$

put $u = 1 + \frac{9}{4} x \quad , \quad du = \frac{9}{4} dx.$

$l = \int_1^{1+9/4 x} \frac{4}{9} \sqrt{u}\, du = \left[\frac{4}{9} \cdot \frac{2}{3} \left(u^{3/2}\right)\right]^{1 + \frac{9}{4} x}$

$= \frac{8}{27}\left[\left(1 + \frac{9x}{4}\right) - 1\right]$

4) $x = 3\sin t$, $y = 3\cos t$.

$$\frac{dx}{dt} = 3\cos t$$

$$\frac{dy}{dt} = -3\sin t$$

$$L = \int_0^{2\pi} \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{(3\cos t)^2 + (-3\sin t)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{9\sin^2 t + 9\cos^2 t}\, dt$$

$$= \int_0^{2\pi} \sqrt{9\sin^2 t + 9\cos^2 t}\, dt$$

$$= \int_0^{2\pi} 3\sqrt{x}\, dt$$

$$= 3\int_0^{2\pi} x\, dt$$

$$= 3[x]_0^{2\pi}$$

$$= 3[2\pi - 0]$$

$$l = 6\pi \text{ units}$$

5) $x = \frac{1}{6}y^3 + \frac{1}{2y}$ on $y = [1, 2]$.

$\rightarrow \frac{dx}{dy} = \frac{y^2}{2} - \frac{1}{2y^2}$

$\frac{dx}{dy} = \frac{y^4-1}{2y^2}$

$L = \int_1^2 \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$

$= \int_1^2 \sqrt{\frac{(y^4+1)^2}{(2y)^2}}\, dy$

$= \int_1^2 \frac{y^4+1}{(2y^2)}\cdot dy$

$= \int_1^2 \cdot \frac{1}{2}\int_1^2 y^2\, dy + \frac{1}{2}\int_1^2 y^{-2}\, dy$

$= \frac{1}{2}\left[\frac{y^3}{3} - \frac{y^{-2}}{1}\right]_1^2$

$= \frac{1}{2}\left[\frac{8}{3} - \frac{1}{2} - \frac{1}{3} + 1\right]$

$= \frac{1}{2}\left[\frac{7}{3} - \frac{1}{2}\right]$

$= 17$ units

q.2] 1) $\int_0^2 e^{x^2} dx$ with $n = 4$.

052

$\rightarrow \int_0^2 e^{x^2} dx = 16.04526$.

In each case the width of the sub interval be $\Delta x = \frac{2-0}{4} = \frac{1}{2}$.

& so the sub intervals will be $[0, 0.5], [0.5, 1], [1, 1.5]\ [1.5, 2]$.

$\therefore$ By Simpson' rule.

$$\int_0^2 e^{x^2} dx = \frac{1/2}{3} \left[y_0 + 4y_1 + 2y_2 + 4y_3 + y_4 \right].$$

$$= \frac{1/2}{3} \left(e^{0^2} + 4e^{(0.5)^2} + 2e^{(1)^2} + 4e^{(1.5)^2} + e^{(2)^{2}} \right)$$

$$= 17.3536.$$

2) $\int_1^4 x^2 dx \quad n = 4$.

$\rightarrow \Delta x = \frac{4-0}{4} = 1$.

$$\int_a^b f(x) dx = \frac{\Delta x}{3} \left[y_0 + 4y_1 + 2y_2 + 4y_3 + y_4 \right].$$

$$= \frac{1}{3} \left[y(0) + 4(1)^2 + 2(2)^2 + 4(3)^2 + 4^2 \right]$$

$$= \frac{1}{3} \left[0^2 + 4(1)^2 + 2(2)^2 + 4(3)^2 + 4^2 \right)$$

$$= \frac{64}{3}.$$

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5) $\int_0^{\pi/3} \sqrt{\sin x}\, dx \quad n = 6.$

$\rightarrow \Delta x = \frac{b-a}{n}$

$$\Delta x = \frac{b-a}{n} = \frac{\pi/3 - 0}{6} = \frac{\pi}{12}$$

x	0	$\pi/8$	$2\pi/12$	$3\pi/12$	$4\pi/12$	$5\pi/12$
y	0	0.4167	0.584	0.707	0.801	0.87
	y_0	y_1	y_2	y_3	y_4	y_5

$$\int_0^{\pi/3} \sqrt{\sin x}\, dx \simeq \frac{\Delta x}{3}\left(y_0 + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)\right)$$

$$+ \frac{\frac{\pi}{12}}{3}\left(0 + 4(0.4167 + 0.707 + 0.875) + 2(0.584 + 0.801)\right) + 0.930$$

$$= 0.681.$$

04.11.2020

PRACTICAL - 7

DIFFERENTIAL EQUATIONS

1) Solve the following differential equations

i) $x\frac{dy}{dx} + \frac{1}{x}y = \frac{e^x}{x}$.

$p(x) = \frac{1}{x}$ $\qquad$ $Q(x) = \frac{e^x}{x}$.

If $e^{\int \frac{1}{x} dx}$ ($\ln x$).

x.

$y(IF) = \int Q(x) \cdot (IF) dx + c$.

$= \int \frac{e^x}{x} \cdot x \, dx + c$

$= \int e^x dx + c$.

$xy = e^x + c$.

2) $e^x \frac{dy}{dx} + 2e^{xy} = 1$.

$\frac{dy}{dx} + \frac{2e^x}{e^x} = \frac{1}{e^x}$

$\frac{dy}{dx} + 2y = \frac{1}{e^x}$.

$\frac{dy}{dx} + 2y = e^{-x}$.

$p(x) = 2$ $\qquad$ $Q(x) = e^{-x}$

$\int p(x) = dx$

$$IF = e^{\int 2 dx}$$
$$= e^{2x}$$
$$y(IF) = \int \varphi(x) \, (IF) dx + c$$
$$= \int e^{x} dx + c$$

$$= y \cdot e^{2x}$$
$$= e^{x} + c.$$

3) $x \frac{dy}{dx} = \frac{\cos x}{x} - 2y$

$\rightarrow$ $x \frac{dy}{dx} = \frac{\cos x}{x} - 2y$

$$\frac{dy}{dx} + \frac{2y}{x} = \frac{\cos x}{x^2}.$$

$p(x) = 2(x)$ $\qquad$ $\varphi(x) = \frac{\cos x}{x^2}$

$$IF = e^{\int P(x) dx}$$
$$= e^{\int 2x \, dx}$$
$$y(IF) = \int \varphi(x) \, (IF) dx + c$$
$$= \int \frac{\cos x}{x^2} - x^2 dx + c$$
$$= \int \cos x + c$$

$\therefore x^2 y = \sin x + c.$

4) $x \cdot \frac{dy}{dx} + 3y = \frac{\sin x}{x^2}$

$\rightarrow \frac{dy}{dx} + \frac{3y}{x} = \frac{\sin x}{x^3}$

$p(x) = 3/x \quad q(x) = \sin x/x^3$

$P(x) = \int 3/x \, dx$

$= x^3$

$IF = e^{\int P(x) dx}$

$= x^3$

$y(IF) = \int q(x)\,(IF)\,dx + c$

$= \int \frac{\sin x}{x^3} \cdot x^3 \, dx + c$

$= \int \sin x + c$

$x^3 y = -\cos x + c$

5) $e^{2x} \frac{dy}{dx} + 2e^{2x} y = 2x$

$\rightarrow \frac{dy}{dx} + 2y = \frac{2x}{e^{2x}}$

$p(x) = 2$

$q(x) = \frac{2x}{e} = 2xe^{-2x}$

$(IF) = e^{\int p(x)\, dx}$

$= e^{\int 2dx}$

$= e^{2x}$

$y(IF) = \int q(x)\,(IF)\,dx + c$

$= \int 2x \; e^{-2x} \; e^{2x} + c$

$ye^{2x} = \int 2x + c = x^2 + c$

6) $\sec^2 x \cdot \tan y \, dx + \sec^2 y \tan x \, dy = 0$

$\rightarrow \sec^2 x \tan y \, dx = -\sec^2 x \tan x \, dy$

$$\frac{\sec^2 x}{\tan x} dx = -\frac{\sec^2 x}{\tan y} dy$$

$$\int \frac{\sec^2 x}{\tan x} dx = -\int \frac{\sec^2 x}{\tan y} dy$$

$$\cancel{\int \frac{\sec^2 x}{\tan x}}$$

$$\log|\tan x| = -\log|\tan y| + c$$

$$\log|\tan x + \tan y| = c$$

$$\tan x + \tan y = e^c$$

7) $\frac{dy}{dx} = \sin^2(x - y + 1)$

put $x - y + 1 = v$

$$x - y + 1 = v$$

$$1 - \frac{dy}{dx} = \frac{dv}{dx}$$

$$1 - \frac{dv}{dx} = \sin^2 v$$

$$\frac{dv}{dx} = 1 - \sin^2 v$$

$$\frac{dv}{dx} = \cos^2 \cancel{x}\, v$$

$$\frac{dv}{\cos^2 \cancel{x}\, v} = dx$$

$$\int \sec^2 v \, dv = \int dx$$

$$\tan v = x + c$$

$$\tan(x - y + 1) = x + c.$$

$$\frac{dy}{dx} = \frac{2x+3y-1}{6x+9y+6}$$

put $2x+3y = v$

$$2 + 3\frac{dy}{dx} = \frac{dv}{dx}$$

$$\frac{dy}{dx} = \frac{1}{3}\left(\frac{dv}{dx} - 2\right)$$

$$= \frac{1}{3}\left(\frac{dv}{dx} - 2\right) = \frac{1}{3}\left(\frac{v-1}{v+2}\right)$$

$$\frac{dv}{dx} = \frac{v-1}{v+2} + 2 = \frac{v-1+2v+4}{v+2}$$

$$= \frac{3v+3}{v+2}$$

$$= \frac{3(v+1)}{(v+2)}$$

$$= \int \frac{v+2}{v+1}\, dv = 3dx$$

$$= \int \frac{v+1}{v}\, dx + \int \frac{1}{v+1}\, dv = \int 3dx$$

$$v \log|v| = 3x + c$$

$$2x + 3y + \log|2x+3y+1| = 3x + c$$

$$3y = x - \log|2x+3y+1| + c$$

$\frac{dy}{dx} = \sqrt{\frac{x}{y}}$ $y(0) = 1$ $h = 0.2$ $y(1) = ?$

P56

$x_0 = 0$, $y(0) = 1$ $h = 0.2$

x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	0	1
0.2	1	0.4472	1.0894
0.4	1.0894	0.6059	1.2105
0.6	1.2105	0.7040	1.3513
0.8	1.3513	0.7696	1.5051
1	1.5051		

$\therefore y(1) = 1.5051$

$\frac{dy}{dx} = 3x^2 + 1$ $y(1) = 2$, find $y(2)$ a. $h = 0.5$

$h = 0.25$

$y_0 = 2$ $x_0 = 1$ $h = 0.5$

x_n	y_n	$f(x_n, y_n)$	y_{n+1}
1	2	4	4
1.5	4	7.75	7.875
2	7.875		

$y(2) = 7.875$

4) $y_0 = 2$ $x_0 = 1$ $h = 0.25$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	3
1	1.25	3	5.6875	4.4218
2	1.5	4.4218	59.6567	19.3360
3	1.75	19.3360	1122.6426	299.9960
4	2	299.9960		

$y(2) = 299.9960$

5) $\frac{dy}{dx} = \sqrt{xy} + 2$ $y(1) = 1$ $h = 0.2$

$x_0 = 1$ $y_0 = 1$ $h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	1	3	3.6
1	1.2	3.6		

$y(1.2) = 3.6$

20/01/2020

PRACTICAL - 9

1) $\lim_{(x,y)\to(-4,-1)} \frac{x^3 - 3y + y^2 - 1}{xy + 5}$

at $(-4,-1)$, Denominator $\neq 0$.

$\therefore$ By applying limit

$= \frac{(-4)^3 - 3(-1) + (-1)^2 - 1}{(-4)(-1) + 5}$

$= \frac{-64 + 3 + 1 - 1}{4+5} = -\frac{61}{9}$

ii) $\lim_{(x,y)\to(2,0)} \frac{(y+1)(x^2+y^2-4x)}{x+3y}$

at $(2,0)$.

$\therefore$ By applying limit

$= \frac{(0+1)((2)^2 + 0 - 4(2))}{2+0}$

$= \frac{1(4+0-8)}{2}$

$= \frac{-4}{2}$

$= -2.$

iii) $\lim_{(x,y,z)\to(1,1,1)} \dfrac{x^2-y^2z^2}{x^3-x^2yz}$

$\therefore \lim_{(x,y,z)\to(1,1,1)} \dfrac{x^2-y^2z^2}{x^3-x^2yz}$

$= \lim_{(x,y,z)\to(1,1,1)} \dfrac{(x-yz)(x+yz)}{x^2(x-yz)}$

$= \dfrac{x+yz}{x^2}$

on applying limit

$= \dfrac{1+1(1)}{(1)^2} = 2.$

Q.2] i) $f(x,y) = xye^{x^2}+y^2$

$f_x = \dfrac{\partial}{\partial x} f((x,y))$

$= \dfrac{\partial}{\partial x}(xye^{x^2}+y^2)$

$= ye^{x^2}+y^2(2x)$

$\therefore f_x = 2xye^{x^2}+y^2$

$f_y = \dfrac{\partial}{\partial y}(f(x,y))$

$= \dfrac{\partial}{\partial y}(xye^{x^2}+y^2)$

$= xe^{x^2}+y^2(2y)$

$\therefore f_y = 2yxe^{x^2}+y^2$

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i) $f(x,y) = e^x \cos y$

$f_x = \frac{\partial}{\partial x}(f(x,y))$

$= \frac{\partial}{\partial x}(e^x \cos y)$

$\therefore f_x = e^x \cos y$

$f_y = \frac{\partial}{\partial y}(f(x,y))$

$= \frac{\partial}{\partial y}(e^x \cos y)$

$\therefore f_y = -e^x \sin y$

ii) $f(x,y) = x^3y^2 - 3x^2y + y^3 + 1$

$f_x = \frac{\partial}{\partial x}(f(x,y))$

$= \frac{\partial}{\partial x}(x^3y^2 - 3x^2y + y^3 + 1)$

$\therefore f_x = 3x^2y^2 - 6xy$

$f_y = \frac{\partial}{\partial y}(f(x,y))$

$= \frac{\partial}{\partial y}(x^3y^2 - 3x^2y + y^3 + 1)$

$\therefore f_y = 2x^3y - 3x^2 + 3y^2$

Q.3 (i) $f(x,y) = \dfrac{2x}{1+y^2}$.

$$f_x = \frac{\partial}{\partial x}\left(\frac{2x}{1+y^2}\right)$$

$$= \frac{1+y^2 \dfrac{\partial}{\partial x}(2x) - 2x \dfrac{\partial}{\partial x}(1+y^2)}{(1+y^2)^2}$$

$$= \frac{2+2y^2-0}{(1+y^2)^2}$$

$$= \frac{2(1+y^2)}{(1+y^2)(1+y)^2} = \frac{2}{1+y^2}.$$

at $(0,0)$ $= \dfrac{2}{1+0} = 2$.

$$f_y = \frac{\partial}{\partial y}\left(\frac{2x}{1+y^2}\right)$$

$$= \frac{1+y^2 \dfrac{\partial}{\partial x}(2x) - 2x \dfrac{\partial}{\partial x}(1+y^2)}{(1+y^2)^2}.$$

$$= \frac{1+y^2(0) - 2x(2y)}{(1+y^2)^2}$$

$$= \frac{-4xy}{(1+y^2)^2}$$

at $(0,0)$

$$= \frac{-4(0)(0)}{(1+0)^2} = 0.$$

$$f(x,y) = \frac{y^2 - xy}{x^2}$$

$$f_x = \frac{x^2 \frac{\partial}{\partial x}(y^2 - xy) - (y^2 - xy)\frac{\partial}{\partial x}(x^2)}{(x^2)^2}$$

$$= \frac{x^2(-y) - (y^2 - xy)(2x)}{x^4}$$

$$= \frac{-x^2y - 2x(y^2 - xy)}{x^4}$$

$$f_y = \frac{2y - x}{x^2}$$

$$f_{xx} = \frac{\partial}{\partial x}\left(\frac{-x^2y - 2x(\cancel{x}y^2 - xy)}{x^4}\right)$$

$$= \frac{x^4\left(\frac{\partial}{\partial x}(-x^2y - 2xy + 2x^2y)\right) - \left(-x^2y - 2xy + 2x^2y\right.}{(x^4)^2}$$

$$= \frac{x^4(-2xy - 2y^2 + 4xy) - 4x^3(-x^2y - 2xy + 2}{x^6}$$

$$f_{yy} = \frac{\partial}{\partial y}\left(\frac{2y - x}{x^2}\right)$$

$$= \frac{2 - 0}{x^2} = \frac{2}{x^2} \quad \text{—} (2)$$

060

$$f_{xy} = \frac{\partial}{\partial y}\left(\frac{-x^2y - 2xy^2 + 2x^2y}{x^4}\right)$$

$$= \frac{-x^2 - 4xy + 2x^2}{x^4} \quad \text{— (3)}$$

$$f_{yx} = \frac{\partial}{\partial x}\left(\frac{2y - x}{x^2}\right)$$

$$= \frac{x^2 \frac{\partial}{\partial x}(2y - x) - (2y - x)\frac{\partial}{\partial x}(x^2)}{(x^2)^2}$$

$$= \frac{-x^2 - 4xy + 2x^2}{x^4} \quad \text{— (4)}$$

$\therefore$ from 3 & 4

$f_{xy} = f_{yx}$

$$f(x, y) = x^3 + 3x^2y^2 - \log(x^2+1)$$

$$f_x = \frac{\partial}{\partial x}(x^3 + 3x^2y^2 - \log(x^2+1))$$

$$= 3x^2 + 6xy^2 - \frac{2x}{x^2+1}.$$

$$f_y = \frac{\partial}{\partial y}(x^3 + 3x^2y^2 - \log(x^2+1))$$

$$= 0 + 6x^2y - 0$$

$$= 6x^2y$$

$$f_{xx} = 6x + 6y^2 - \left(\frac{(x^2+1)\frac{\partial}{\partial x}(2x) - 2x\frac{\partial}{\partial x}(x^2+1)}{(x^2+1)^2}\right)$$

$$f_{yy} = 6x + 6y^2 - \left(\frac{2(x^2+1) - 4x^2}{(x^2+1)^2}\right) \quad \text{—} (1)$$

$$f_{yy} = \frac{\partial}{\partial y}(6x^2 y)$$

$$= 6x^2 \quad \text{—} (2)$$

$$f_{xy} = \frac{\partial}{\partial y}\left(3x^2 + 6xy^2 - \frac{2x}{x^2+1}\right)$$

$$= 0 + 12xy - 0$$

$$= 12xy \quad \text{—} (3)$$

$$f_{yx} = \frac{\partial}{\partial x}(6x^2 y)$$

$$= 12xy \quad \text{—} (4)$$

from (3) & (4)

$$f_{xy} = f_{yx}$$

iii) $f(x,y) = \sin(xy) + e^{x+y}$

$$f_x = y\cos(xy) + e^{x+y}(1)$$

$$= y\cos(xy) + e^{x+y}$$

$$f_y = x\cos(xy) + e^{\cdot}$$

$$= x(\cos(xy) + e$$

$$f_{xx} = \frac{\partial}{\partial x}(y\cos(x,y) + e^{x+y})$$

$$= -y\sin(xy).(y) + e^{x+y}$$

$$= -y^2\sin(xy) + e^{x+y} \quad \text{—} (1)$$

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$f_{yy} = \frac{\partial}{\partial y}(x\cos(xy) + e^{x+y})$.

$= -x\sin(xy)(x) + e^{x+y}(1)$

$= -x^2\sin(xy) + e^{x+y}$ —— (2)

$f_{yx} = \frac{\partial}{\partial x}(x\cos(xy) + e^{x+y})$.

$= -x^2\sin(xy) + \cos(xy) + e^{x+y}$ —— (4)

from 3 & (4)

$f_{xy} \neq f_{yx}$.

i) $f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$

$f(1,1) = \sqrt{(1)^2+(1)^2} = \sqrt{2}$.

$f_x = \frac{1}{2\sqrt{x^2+y^2}}(2x)$ $\quad$ $f_y = \frac{1}{2\sqrt{x^2+y^2}}(2y)$.

$= \frac{x}{\sqrt{x^2+y^2}}$ $\quad$ $= \frac{y}{\sqrt{x^2+y^2}}$

at $(1,1) = \frac{1}{\sqrt{2}}$. $\quad$ f_y at $(1,1) = \frac{1}{\sqrt{2}}$

$l(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1) + \frac{1}{\sqrt{2}}(y-1)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1+y-1)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x+y-2)$.

$$= \sqrt{2} + \frac{1}{\sqrt{2}} x + \frac{1}{\sqrt{2}} y - \frac{2}{\sqrt{2}}$$

$$= \frac{x+y}{\sqrt{2}}$$

$f(x, y) = 1 - x + y \sin x$ at $\left(\frac{\pi}{2}; 0\right)$

$$f\left(\frac{\pi}{2}, 0\right) = 1 - \frac{\pi}{2} + 0 = 1 - \frac{\pi}{2}$$

$f_x = 0 - 1 + y \cos x$ $\qquad$ $f_y = 0 - 0 + \sin x$

f_x at $\left(\frac{\pi}{2}, 0\right) = -1 + 0 = \cancel{0} - 1$ $\qquad$ at $\left(\frac{\pi}{2}, 0\right) = \sin \frac{\pi}{2}$

$$\therefore L(x, y) = f(a, b) + f_x(a, b)(x - a) + f_y(a, b)$$

$$= 1 - \frac{\pi}{2} + (-1)\left(x - \frac{\pi}{2}\right) + 1 \cdot (y - 0)$$

$$= 1 - \frac{\pi}{2} - x + \frac{\pi}{2} + y$$

$$= 1 - x + y$$

$f(x, y) = \log x + \log y$ at $(1, 1)$.

$f(1, 1) = \log(1) + \log(1) = 0$

$f_x = \frac{1}{x} + 0$ $\qquad$ $f_y = 0 + \frac{1}{y}$

f_x at $(1, 1) = 1$ $\qquad$ f_y at $(1, 1) = 1$

$$\therefore L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$$
$$= 0 + 1(x-1) + 1(y-1)$$
$$= x - 1 + y - 1$$
$$= x + y - 2.$$

Q.1) Find the directional derivative at points a in the direction of given u.

i) $f(x,y) = x+2y-3$ $a=(1,-1)$ $u = 3i - j$

Here, $u = 3i - j$ is not a unit vector.

$|\bar{u}| = \sqrt{(3)^2+(-1)^2} = \sqrt{9+1} = \sqrt{10}$.

unit vector along u is $\frac{u}{|u|} = \frac{1}{\sqrt{10}}(3,-1)$.

$\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$.

$f(a) = f(1,-1)$ $\cancel{+h\left(\frac{3}{\sqrt{}}\right.}$

$= (1)+2(-1)-3 = 1-2-3 = -4$

$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$.

$= f\left(1+\frac{3}{\sqrt{10}}\right), \left(-1, \frac{-h}{\sqrt{10}}\right)$

$f(a+hu) = \left(1+\frac{3}{\sqrt{10}}\right) + 2\left(-1, \frac{-h}{\sqrt{10}}\right) - 3$

$= 1 + \frac{3}{\sqrt{10}} - 2 - \frac{2h}{\sqrt{10}} - 3$

$f(a+hu) = -4 + \frac{h}{\sqrt{10}}$.

$D_u f(a) = \lim_{h\to 0} \frac{f(a+hu)-f(a)}{h}$.

$\lim_{h\to 0} \frac{-4 + h/\sqrt{10} + 4}{h}$.

$D_u f(a) = \frac{1}{\sqrt{10}}$

$f(x) = y^2 - 4x + 1$ $\quad a = (3, 4)$ $\cdot$ $4 = i + 5y$

Here $v = i + 5j$ is not unit vector

$|v| = \sqrt{(1)^2 + (5)^2} = \sqrt{26}$

unit vector $\frac{v}{|v|} = \frac{1}{\sqrt{26}}(1, 5)$

$= \left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$.

$f(a) = f(3,4) = (4)^2 - 4(3) + 1 = 5$

$f(a + hv) = f(3,4) + h\left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$.

$= f\left(3 + \frac{h}{\sqrt{26}}, 4 + \frac{5h}{\sqrt{26}}\right)$.

$f(a + hv) = \left(4 + \frac{5h}{\sqrt{26}}\right)^2 - 4\left(3 + \frac{h}{\sqrt{26}}\right) + 1$

$= 16 + \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - 12 - \frac{4h}{\sqrt{26}} + 1$.

$= \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - \frac{4h}{\sqrt{26}} + 5$

$= \frac{25h^2}{26} + \frac{40h - 4h}{\sqrt{26}} + 5$.

$= \frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5$

$D_v f(a) = \lim_{h \to 0} \frac{\frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5 - 5}{h}$.

$$= h\left(\frac{25}{26}h + \frac{36}{\sqrt{26}}\right)$$

$$\therefore D_u f(a) = \cancel{\alpha(1,2)} \frac{25}{26}h + \frac{36}{\sqrt{26}}$$

iii) $2x + 3y$. $\alpha = (1,2)$ $u = (3i + 4j)$.

Here $u = 3i + 4j$ is not a unit vector.

$|\vec{u}| = \sqrt{(3)^2 + (4)^2} = \sqrt{25} = 5$

$$\frac{u}{|u|} = \frac{1}{5}(3,4)$$

$$= \left(\frac{3}{5}, \frac{4}{5}\right).$$

$f(a) = f(1,2) = 2(1) + 3(2) = 8$.

$f(a + hu) = f(1,2) + h\left(\frac{3}{5}, \frac{4}{5}\right)$.

$$= f\left(1 + \frac{3h}{5}, 2 + \frac{4}{5}\right).$$

$$= 2\left(1 + \frac{3h}{5}\right) + 3\left(2 + \frac{4h}{5}\right)$$

$$= 2 + \frac{6h}{5} + 6 + \frac{12h}{5}$$

$D_uf(a) = \lim_{h \to 0} \frac{\frac{18h}{5} + 2 - 2}{h}$

$= \frac{18}{5}$

Q.2)

i) $f(x,1) = x^y + y^x \quad a = (1,1)$

$f_x = y \cdot x^{y-1} + y^x \log y$

$f_y = x^y \log x + \cancel{y^x \log} \; xy^{x-1}$

$\nabla f(x,y) = (f_x, f_y)$

$= (yx^{y-1} + y^x \log y, \; x^y \log x + xy^{x-1})$

$f(1,1) = (1+0, \; 1+0)$

$= (1,1)$

ii) $f(x,y) = (\tan^{-1}x) \cdot y^2 \quad a = (1,-1)$

$f_x = \frac{1}{1+x^2} \cdot y^2$

$f_y = 2y \tan^{-1}x$

$\nabla f(x,y) = (f_x, f_y)$

$= \left(\frac{y^2}{1+x^2}, \; 2y \tan^{-1}x\right)$

$f(1,-1) = \left(\frac{1}{2}, \; \tan^{-1}(1)(-2)\right)$

$= \left(\frac{1}{2}, \; \frac{\pi}{4} \frac{(-2)}{2}\right)$

$= \left(\frac{1}{2}, -\frac{\pi}{2}\right)$

Q. 3) i) $x^2 \cos y + e^{xy} = 2$ at $(1, 0)$.

$f_x = \cos y \, 2x + e^{xy} y$

$f_y = x^2(-\sin y) + e^{xy} \cdot x$.

$(x_0, y_0) = (1, 0)$. $\therefore x_0 = 1$, $y_0 = 0$.

$f_x(x - x_0) + f_y(y - y_0) = 0$

$f_x(x_0, y_0) = \cos 0 \, 2(1) + e^0 0$.

$= 1(2) + 0$

$= 2$.

$f_y(x_0, y_0) = (1)^2(-\sin 0) + e^0 \cdot 1$

$= 0 + 1 \cdot 1$

$= 1$.

$2(x-1) + 1(y-0) = 0$

$2x - 2 + y = 0$

$2x + y - 2 = 0$.

Eq of Normal.

$ax + by + c = 0$

$bx + ay + d = 0$

$1(1) + 2(y) + d = 0$

$1 + 2y + d = 0$

$1 + 2(0) + d = 0$

$d + 1 = 0$

$\therefore d = -1$

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$3xyz - x - y + z = -4$ at $(1, -1, 2)$
$3xyz - x - y + z + 4 = 0$ at $(1, -1, 2)$

$f_x = 3yz - 1 - 0 + 0 + 0$
$= 3yz - 1$

$f_y = 3xz - 0 - 1 + 0 + 0.$
$= 3xz - 1.$

$f_z = 3xy - 0 - 0 + 1 + 0$
$= 3xy + 1$

$(x_0, y_0, z_0) = (1, -1, 2)$ ..
$\therefore (x_0 = 1, y_0 = -1, z_0 = 2.$
$f_x(x_0, y_0, z_0) = 3(-1)(2)(-1) = -7$
$f_y(x_0, y_0, z_0) = 3(+1)(2) - 1 = 5$
$f_z = 3(1)(-1) + 1 = -2.$

Eq of tangent.
$-7(x-1) + 5(y+1) - 2(z-2) = 0$
$-7x + 7 + 5y + 5 - 2z + 4 = 0$
$-7x + 5y - 2z + 16 = 0.$

Eq of Normal
$\frac{x - x_0}{f_x} = \frac{y - y_0}{f_y} = \frac{z - z_0}{f_z}.$
$= \frac{x-1}{-7} = \frac{y+1}{5} = \frac{z-2}{-2}.$

(a)

i) $f(x, y) = 3x^2 + y^2 - 3xy + 6x - 4y$

$f_x = 6x + 0 - 3y + 6 - 0$
$= 6x - 3y + 6.$

$f_y = 0 + 2y - 3x + 0 - 4$
$= 2y - 3x.$

$f_x = 0$

$6x - 3y + 6 = 0.$

$3(2x - y + 2) = 0$

$2x - y + 2 = 0.$

$2x - y = -2.$ — (i)

$f_y = 0.$

$2y - 3x - 4 = 0$

$2y - 3x = 4.$

$4x - 2y = -4$

$2y - 3x = 4$

$x = 0.$

x in eq (i)

$2(0) - y = -2.$

$-y = -2.$

$\therefore y = 2$

$\therefore$ critical points are $(0, 2)$

066

$r = f_{xx} = 6$

$t = f_{yy} = 2.$

$s = f_{xy} = -3$

Here $r > 0$

$rt - s^2$

$6(2) - \cancel{8}(-3)^2$

$= 12 - 9$

$= 3 > 0.$

$\therefore f$ has maximum at $(0, 2)$

$x^2 + y^2 - 3xy + 6x - 4y$ at $(0, 2)$

$3(0)^2 + (2)^2 - 3(0)(2) + 6(0) - 4(2).$

$0 + 4 - 0 + 0 - 8$

$= -4.$

$f(x, y) = x^2 - y^2 + 2x + 8y - 70.$

$f_x = 2x + 2.$

$f_y = -2y + 8$

$f_x = 0 \quad \therefore 2x + 2 = 0$

$x = -1$

$f_y = 0 \quad -2y + 8 = 0.$

$y = \frac{-8}{-2} = 4.$

$\therefore$ critical pts is $(-1, 4).$

$r = f_{xx} = 2.$

$t = f_{yy} = -2$

$s = f_{xy} = 0.$

$r < 0$

$rt - s^2 = 2(-2) - (0)^2$

$= -4 - 0$

$= -4 < 0$

$f(x, y)$ at $(-1, 4)$

$= (-1)^2 - (4)^2 + 2(-1) + 8(4) - 70$

$= 1 + 16 - 2 + 32 - 70$

$= 17 + 30 - 70$

$= 37 - 70$

$= 33$

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