European Journal of Operational Research 199 (2009) 867–872

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Structures in decision making: On the subjective geometry of hierarchies and networks

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ARTICLE INFO

Article history:
Received 30 November 2007
Accepted 31 January 2009
Available online 1 April 2009

Keywords:
Hierarchy
Network
Structure
Decision making
Problem solving
Subjective geometry

ABSTRACT

In the field of decision making, creating a structure is the first step in organizing, representing and solving a problem. A structure is a model, an abstraction of a problem. It helps us visualize and understand the relevant elements within it that we know from the real world and then use our understanding to solve the problem represented in the structure with greater confidence. In general, there are two kinds of structures used to represent problems: hierarchies and networks. Both rely to a varying degree on the interactions. Some examples are given followed by a discussion about how to structure the problem. At a minimum, a structure must satisfy two requirements: that it be logical in identifying and grouping similar things together, and that it relates them accurately according to the flow of influence among them. It must be complete with nothing left out that has an important influence. The structure is then tested as to whether it helps solve the problem to one's *satisfaction*.

1. Introduction – concept construction

Geometry is a strange dimension that constantly haunts our thinking processes. The word geometry conjures up images of triangles and circles, of similarities and congruences, of lengths, areas and higher dimensions and of linearity and curvature out there in the real world. But that is not the only kind of geometry. There is also a geometry that is associated with our subjective thought processes, a geometry of vertices and edges that connect them, of paths and cycles.

When thinking, our brain identifies objects and ideas and deals with them in terms of their properties. We group elements that have the same property together, and they are further related according to their influence on our goals. We attempt to assign importance to the influences and focus on the most important ones. It might be said that we all associate something like vertices to the things we think about and lines and paths to the perceived influence connections among them.

The outcome of identifying our purposes and relating them to goals in light of the influences is a structure of nodes that depict the elements and lines with arrows that depict the connections. We always have some underlying understanding about the flow of influence among elements with respect to other elements that are our objectives and goals. The question is: What kinds of generic structures are there to represent this understanding and how do we characterize them?

We need geometric structures to represent our abstract understanding of influences among elements in the real world and their connections and interactions (Garuti and Spencer, 2007). The human mind, a decision maker, is a collection of cognitive processes involving perception, interpretation, imagination, memory, reasoning, and language, which are unique when compared with other animals. A fundamental process of the human brain is thinking; we use it to understand and solve problems, and deal with them to further our goals and purposes. Thinking involves mental manipulation of information to form concepts, reason, solve problems, and make decisions.

It is worth our while to clarify the relation between thinking and decision making. Not all decision making involves thinking because many of our decisions are automatic reflexes or are done intuitively (Kenning and Plassmann, 2005). In fact most of us rely more on our intuition to make decisions than on explicit and detailed reasoning. But when we think through a decision, we have to structure it, make judgments, choose, and then act accordingly. We need to systematize and make a science out of the thinking involved in making considered decisions.

Structuring a decision is the first step a thinking mind takes to organize and to represent any problem and in particular a decision problem that needs action. Problem solving usually begins with a feeling of dissatisfaction with an existing situation that may border on unhappiness. Our minds are conditioned and guided by feelings

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doi:10.1016/j.ejor.2009.01.064

of satisfaction or dissatisfaction that are fulfilled to different degrees as Maslow showed in his hierarchy of needs (Maslow, 1943). To deal with a problem, one attempts to identify the elements that relate to it, their connections and interactions, the cause(s) that give rise to the problem, and possible ways of solution. To do this we have to structure the problem. Although judgment is needed to guide us from one step to another later during the analysis, in this paper we will only focus on the structuring process and the kinds of structures that are helpful. Thus, we identify all the elements of a problem that we think of by brainstorming the goal which they affect (Osborn, 1963), the criteria that serve the fulfillment of that goal, the influences, the actors involved and the actions to be taken. Then we group those elements systematically into components of similar elements and have the choice to arrange them in levels of a hierarchy or as clusters in a network.

In passing we note that there is at least one other general way for representing influences that are better represented not as being tracked from one point to another point but along a manifold. For example the influence of the sun spreads out in all directions but cannot be always thought of as being transmitted from a source point to a sink point. Similarly winds and hurricanes have a widespread influence and so does the flow of water when not confined in a pipe, heat and cold, electromagnetic fields in which radio and TV signals are transmitted and other similar things like the dissemination of ideas in communication. To analyze such influences requires that we deal with a continuum number of points for their representation. Human thinking is better able to relate elements represented by points to other elements also represented by points. When the number is very large we have difficulty capturing what it is that we should have in mind. The mathematics of this subject has already been explored in the literature in decision making by the first author (Saaty, 2004).

This paper is organized as follows: The next section deals with hierarchies in greater detail and provides examples illustrating many different forms of hierarchies applied in planning and decision making. It is then followed by a section on networks with examples. Section 4 is dedicated to the details of the process of structuring problems as a hierarchy or a network and how to validate that a structure is meaningful. Conclusions are made in Section 5.

2. Hierarchic structures

A hierarchy is a powerful way of classification used by the mind to order information gained from experience or from our own thinking to understand the complexity of the world around us according to the order and distribution of influences that make certain outcomes happen. Decisions do not occur in isolation – there are all sorts of influence that affect the potential outcome of a decision. It can all be regarded as a network of influences of which hierarchies are a special case. In this section, we focus on hierarchies. One needs to consider who or what is most influential when important decisions are made in an organization and who gets to make these decisions and who should be involved because of the special knowledge and understanding they have: internal people, external experts, managers and their subordinates, or should they be only superiors, and so on. One also needs to include those people and things affected by the decision and how taking that decision serves their interests. Any influence or interaction between elements would be connected by a link. Only after all the elements and interactions are represented well in a hierarchy, can an effective structure be considered to be adequately formulated. The structure of a hierarchy further depends on who interprets it and how they interpret it. Structures can vary for different types of problems, from one person to another having the same problem, even at different times by the same person with the same problem because that person's perspective may be different. To diminish the effect of current perspective, the problem can be embedded in a very broad setting that includes social, political, economic, environmental and other "control" perspectives.

A hierarchy can be defined formally in mathematical terms (Saaty, 1980). It is a stratified system for organizing people, ideas or things, whereby each element of the system, except for the top element which is the goal of the hierarchy, falls in a level and is subordinate to other elements in the level above. We can identify the world from galaxies to small living organisms in different ways within hierarchical structures. The influences in a hierarchy can be expanded to include metaphysical intangibles, such as ideas and beliefs and is generalized so we can understand the world more fully.

As the examples and graphical representations of hierarchies that we will give, suggest, we may consider a hierarchy a special type of ordered set, or a particular case of a graph. We have chosen the first interpretation as the basis of our formal definition, and the second as an illustration. No doubt, the roles could be reversed.

Definition 1. An ordered set is any set S with a binary relation $\leqslant$ which satisfies the reflexive, antisymmetric, and transitive laws:
Reflexive
For all $x, x \leqslant x$.
Antisymmetric If $x \leqslant y$ and $y \leqslant x$, then $x = y$.
Transitive
If $x \leqslant y$ and $y \leqslant z$, then $x \leqslant z$.

For any relation $x \leqslant y$ (read, y includes x) of this type, we may define $x < y$ to mean that $x \leqslant y$ and $x \neq y$. y is said to cover (dominate) x if $x < y$ and if $x < t < y$ is possible for no t.

Ordered sets with a finite number of elements can be conveniently represented by a directed graph. Each element of the system is represented by a vertex so that an arc is directed from a to b if $b < a$.

Definition 2. A simply or totally ordered set (also called a chain) is an ordered set with the additional property that if $x, y \in S$ then either $x \leqslant y$ or $y \leqslant x$.

Definition 3. A subset E of an ordered set S is said to be bounded from above if there is an element $s \in S$ such that $x \leqslant s$ for every $x \in E$. The element s is called an *upper bound*of E. We say E has a supremum or least upper bound in S if E has upper bounds and if the set of upper bounds U has an element u_1 such that $u_1 \leqslant u$ for all $u \in U$. The element u_1 is unique and is called the supremum of E in S. The symbol sup is used to represent a supremum. (For finite sets largest elements and upper bounds are the same.)

Similar definitions may be given for sets bounded from below, a *lower bound* and *infimum*. The symbol inf is used.

There are many ways of defining a hierarchy. The one which suits our needs best here is the following:

We use the notation $x^- = \{y|x \text{ covers } y\}$ and $x^+ = \{y|y \text{ covers } x\}$, for any element x in an ordered set.

Definition 4. Let H be a finite partially ordered set with largest element b.

H is a *hierarchy* if it satisfies the conditions

(1) There is a partition of H into sets L_k, $k = 1, \ldots, h$ where $L_1 = \{b\}$.
(2) $x \in L_k$ implies $x^- \subset L_{k+1} k = 1, \ldots, h-1$
(3) $x \in L_k$ implies $x^+ \subset L_{k+1} k = 2, \ldots, h$.

For each $x \in H$, there is a suitable weighting function (whose nature depends on the phenomenon being hierarchically structured):

$$w_x : x^- \to [0,1] \quad \text{such that} \sum_{y \in x^-} w_x(y) = 1.$$

The sets L_i are the *levels* of the hierarchy, and the function w_x is the *priority function*of the element in one level with respect to the objective x. We observe that even if $x^- \neq L_{k+1}$ (for some level L_k), w_x may be defined for all of L_k by setting it equal to zero for all elements in L_{k+1} not in x^-.

The weighting function, we feel, is a significant contribution towards application of the Analytic Hierarchy Process.

Definition 5. A hierarchy is *complete* if, for all $x \in L_k x^+ = L_{k-1}$ for k=2,...,h.

It is the very definition of a hierarchy in the abstract that entails the idea of levels in that hierarchy. The nature or kind of elements of a particular decision determines the number of levels. Depending on their potential interactions, the homogeneous elements in levels are connected by links to some or all of the elements in the level immediately above and immediately below. There would be different types of elements involved in the levels such as scenarios, environmental factors, actors their objectives and their policies, people and other things influenced by the policies and actions of the actors, the objectives of these people and how they are served by certain actions to be taken by someone. Scenarios are descriptions of syntheses of possible conditions that can prevail in the environment, so certain actions judged under that set of conditions are more plausible than under other conditions. The likelihoods of the scenarios then enable one to focus on the best mixed strategy to follow. The environment consists of the external conditions: the resources and stimuli with which the system in which the decision is being made interacts. There are two kinds of factors in the environment, controllable and uncontrollable. Uncontrollable factors are restricted by the physical limitations of the people and other factors involved in the system. Controllable factors are amenable to actions and can be influenced by the people involved. Actors are the active players who control the influences that restrict the actions to be taken which influence people who are affected by the influences according to their importance. Actions are needed to satisfy needs. If the actions work as expected, the result would be a feeling of satisfaction. To understand the effectiveness of the actions taken, one examines the effect to change the environment of the problem. One can study the influences from a top-down or from a bottom-up point of view. The first attempts to emphasize what one thinks is more important to drive the outcome, the second emphasizes the virtue of the actions to be taken as they are influenced by the factors above them. The bottom-up approach has a greater flavor of practicality than the idealism or legalism of the top-down approach. It provides the support to what is thought to be necessary above. In the top-down approach it is thought that the lower levels are parts or decompositions of what is above and are therefore only smaller parts of a larger whole.

An example for choosing the level of water in a dam for example full or half-full, is considered as a single hierarchy (Saaty, 1990). Seven levels, from focus, decision criteria, decision makers, factors, group affected, objectives, and to the alternatives, are used to represent all the elements involved and their interactions. One specific feature of this conceptually very useful hierarchy is the different actors included. One group of actors is the decision makers located in the third level of the hierarchy because of their different concerns that affect the outcome of a decision. The other actors are groups that are affected by the decision located in fifth level of the structure. The details are shown in Fig. 1.

Although all the elements of a hierarchy are arranged in a suitable position in a graph with links in the structure, the actual performance of the elements cannot be illustrated graphically. We need to think about their performance and influence and their importance in our own minds to make the judgments we use to develop performance measures or priorities. Sometimes control hierarchies are used that depict specific kinds of homogeneous influence like economic or political or environmental influences and only that influence is to be considered when making judgments. Our job would be to combine these influences into a single overall appropriately mixed influence according to priority. Every element of a hierarchy must ultimately be related to some or all the alternative actions or outcomes either directly or through a path. No element should be isolated from the alternatives.

Hundreds of examples of decision hierarchies, many of which are commonly encountered in decision situations are illustrated in The Hierarchon (Saaty and Forman, 1993). There are some common structures consisting of hierarchies: a single hierarchy; two hierarchies, one that deals only with benefits and another that deals only with costs and pains; four hierarchies, one each for benefits (B), opportunities (O), costs (C), and risks (R) collectively referred to as BOCR. In fact a decision may involve many hierarchies depending on number and structure of the control criteria used to track influences. That is, economic influence may have its own hierarchy, social influence another hierarchy, and political influences yet another and so on. This entire approach is repeated for each of the four BOCR merits. Because of space limitation, we need brevity in our presentation of the examples.

3. Network structures

A network involves a grouping of elements (scenarios, environmental factors, actors, objectives, actions) into clusters that are not organized in any particular way, unlike the levels of a hierarchy. Its clusters are not grouped in levels. The concept of dependence is central in defining a network. To develop it at length here would take us far afield from the scope of this paper. Suffice it to say that when elements are dependent they can influence one another in the appropriate direction of dependence. For most purposes in practice, influence can be thought of as a primitive notion: a cause that can change the state of a system.

Definition 6. Let $\mathfrak{S}$ be a family of nonempty sets (clusters) $\mathfrak{S}_1, \mathfrak{S}_2, \ldots, \mathfrak{S}_n$ where $\mathfrak{S}_i$ consists of the elements $\{e_{ij}, j = 1, \ldots, m_i\}$, $i = 1, 2, \ldots, n$. $\mathfrak{S}$ is a network if it is a graph with directed arcs whose vertices are $\mathfrak{S}_i$ and whose arcs are defined through the concept of dependence; thus, given two components $\mathfrak{S}_i$ and $\mathfrak{S}_j \varepsilon \mathfrak{S}$, there is an arc from $\mathfrak{S}_i$ to $\mathfrak{S}_j$ if some or elements in $\mathfrak{S}_j$ are dependent on some or all the elements in $\mathfrak{S}_i$.

Let $\mathfrak{S}$ be a network consisting of the clusters $C_1, C_2, \ldots, C_n$. For each C_i there is some C_j so that either C_i depends on C_j or C_j depends on C_i, or both.

It is easy to see that a hierarchy is a special case of a network in which the subsets are arranged linearly in an ascending or a descending order.

A network can refer to any interconnected group, cluster, or system, which consists of nodes or elements that are connected by links. Nodes may be joined by more than one link, but no node is isolated.

The concept of a network is useful in helping us to portray the complex relations of real-world problems. There are many examples of networks in transportation, computer science, neurology, operations research, flow problems, business, marketing and in human society. When a network is used to show connections from points to points without regard to direction, it involves the use of

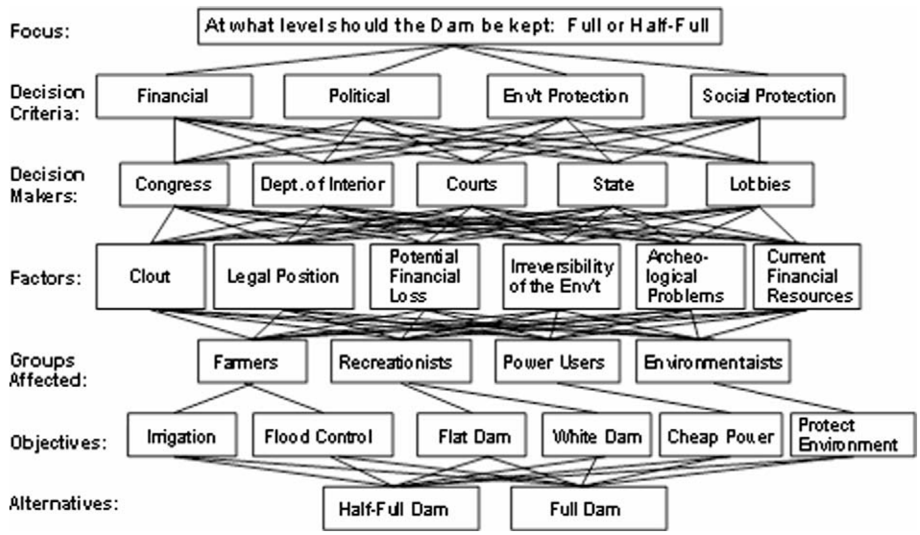

Fig. 1. Hierarchy for level of a dam: full or half-full.

chains for linear connections along a sequence of points, starting from a first point and ending with a last point. It uses circuits to represent closed chains that return to their starting point. When direction is important, arrows are used to represent links and the chains are referred as paths and circuits as cycles. Detailed examples and applications of networks in decision making are abundantly illustrated in The Encyclicon (Saaty and Ozdemir, 2005) and The Encyclicon, Volume 2 (Saaty and Cillo, 2008).

Networks also deal with control structures. What is a control network? We said before about hierarchies that it is a structure used to study a particular kind of influence such as economic influence, social influence and political influence. The analysis is made by such a decomposition of influences in separate structures whose results are then appropriately combined into an overall result.

As with hierarchies, although there are many types of networks, generally a real-world problem cannot be represented by a single network, only several. There are two typical structures for networks used to represent decisions: a single network with both inner and outer dependence or multiple networks of benefits, opportunities, costs and risks (BOCR).

4. How to structure a problem

Hierarchic and network structures are forms that help us to think about what a problem is, and offer conceptual guides to solve that problem. A hierarchy or a network is our logical conceptualization of a problem that pares it down to its essentials. In general, if the elements and their connections are easily located in levels of dominance with connections that transmit influence downwards, a hierarchic structure fits the decision best. On the other hand, if the elements and their connections are complicated and can only be grouped in clusters that do not fit well in defined levels, a network structure is more appropriate. There is no better way to determine which to use than the ability to abbreviate and summarize now influences work and reach the level of alternatives without undue need to represent interdependence among the elements in the levels or feedback from lower to upper levels. However, other than an urgent need to obtain a quick answer, it is risky to use a hierarchy in a complex decision because many influences can be lost by not representing them with the needed connections. It is safe to say that the more important and complex a decision is, the more likely that it needs a network for its structure in the BOCR form.

How to structure a problem is an essential concern of problem solving. If the problem can be structured in a systematic way, what one has to do with it would be much like a computational process. No matter how complex the structure may be, the computational process tends to be algorithmic and so that it can be executed on a computer and does not need much human effort. In this situation the problem has become a well-defined structured problem.

But often it is more difficult to deal with how to structure the problem since it involves a considerably greater degree of abstraction than does solving that problem. A substantial amount of human thinking and imagination are needed to create a structure for a complex problem. These two processes human capabilities are the most valuable human qualities that cannot be replaced by computers. They are related to our goals of enhancing our survival.

In the real world, we always face many problems to be solved. Most existing techniques show us what to solve but not always how to solve. However, we need a way of inventive ideation (Ross, 2006) to structure of a problem. An approach that relies not on psychology but on technology, the inventive problem solving method known widely by the name of TRIZ (Theory of Solving Inventive Problems) is widely taught and widely used to help us explore for solutions in fields other than our own. It provides some guidelines for overcoming our "psychological inertia".

In general, a problem begins with some dissatisfaction with an ongoing situation, or as Altshuller (1974), the creator of TRIZ put it, a problem is a contradiction in the mind. We think of taking action to rid ourselves of the situation. To do that, we identify the relevant elements in the environment and organize them by using our understanding and our memory of past situations of similar problems; if we are successful a pattern emerges that describes our problem. This type of thinking is a complex mental process involving cognition, pattern matching, associative memory and knowledge, judgment, comparisons, and imagination. To find a good structure and to identify a new alternative which is a possible way to act, imagination, one of our most treasured attributes, can be extremely helpful. We really do not know if a good structure will emerge and what it might be like. It could pop out with a rough structure that can be revised to better fit our problem. We generally use a combination of our habitual ways of perceiving, thinking, responding, and acting, together with their formation, dynamics, and basis in experience and knowledge, or what is called our habitual domain (Yu, 1991). If our habitual domain sufficiently rich, a creative structure for a problem might be generated.

There are two kinds of problem people face: those with generally known solutions and those with unknown solutions. Those with known solutions can usually be solved using information found in books, technical journals, or with the help of experts in the field. Such solutions follow the general pattern of problem solving, and the steps followed are analogous to those we use to solve other problems.

On the other hand, if a problem is one with no known solution, it can be considered to be an inventive problem and such problems often contain contradictory requirements. Solving it may be helped by notions from the field of psychology where the links between the brain and insight and innovation are studied. Methods such as brainstorming and trial-and-error are commonly used.

Altshuller was led to seek ways to standardize the methods of problem solving. At a minimum, he felt that such a method should satisfy the following conditions:

1. be a systematic, step-by-step procedure
2. be a guide through a broad solution space directed toward an ideal solution
3. be repeatable and reliable and not dependent on psychological tools
4. be able to access the body of inventive knowledge
5. be able to add to the body of inventive knowledge
6. be familiar enough to inventors that they could follow its general approach to problem solving.

Another important concept of TRIZ is that a successful system is captured through idealization. Idealization means that the system has maximum benefits and minimum costs and negative effects, and the outcome should be an ideal final result (IFR). The characteristics of an IFR are four: (i) it reduces the disadvantages of the original system; (ii) it preserves the advantages of the original system; (iii) it does not make the new system more complicated; and (iv) it does not introduce any drawbacks in the new system. When we plan for an IFR, we check for the above four characteristics and think of problem solving as a process of innovation.

Although TRIZ was originally used for product innovation, it can also be helpful in structuring a problem. The essential part of TRIZ is to overcome our psychological inertia and expand the solution space so that some creative ideas can be generated. Then we can follow the step-by-step procedure proposed by Altshuller to develop the structure of the problem.

Step by step structuring process

Step 1: Identify the Problem.It begins with a contradiction or dissatisfaction in our mind. It might need a group meeting or questionnaires to capture the problem in a quantitative and qualitative way. This is the step where the customer's needs and wants are recorded and the elements are identified.

Step 2: Formulate the Problem: the Prism of TRIZRestate the problem in terms of its contradictions and dissonances. Identify the problems that could occur, their consequences, and the ways in which they can occur. What, when and how the actions are selected? What are the advantages and disadvantages of the actions? This step attempts to clarify the problem and the relations among all the elements.

Step 3: Search for Previous Successfully Solved ProblemHere we search for some successful cases to study. Compare their structures and consider the similarities and dissimilarities with our problem.

Step 4: Look for Analogous Solutions and Adapt them to Generate Our SolutionTry to extend previous analogous structures being aware of their advantages and disadvantages to remove any contradictions. Some trade-offs might need to be made so that an inventive structure can be established. There are 40 inventive principles that can help us think better when we structure a problem as: segmentation, extraction, asymmetry, combining, universality, nesting, counterweight, inversion, spheroidality, dynamicity, partial or overdone action, moving to a new dimension, periodic action, continuity of a useful action, rushing through, converting harm into benefit, feedback, mediator, self-service, copying, homogeneity, phase transformation and so on (Altshuller, 1984). For instance, the principle of the segmentation means dividing an object into independent parts, making an object sectional, and increasing the degree of an object's segmentation so that a problem can be broken down.

In the following paragraphs, a procedure is proposed first for hierarchic structures and then for network structures and the latter is then illustrated with an example.

4.1. Structuring hierarchies

The process of structuring a system hierarchically is as follows.

(1) Define the goal or focus of the decision problem at the top level

The purpose or focus will be the desired state or goal of the problem to be solved in the future. For instance, it could be a vision or a mission statement of an organization. It could also be the target value of a performance measure when the problem is solved.

(2) Break down the purpose into some supportive elements in the first level below the goal

There are various ways to make the breakdown. The elements on the first level should be comparable and homogeneous or close in their possession of a common attribute.

The breakdown can be made from the elements in the first level into their sub-elements. For instance, a system can be broken down physically into sub-systems, units, sub-units, components, etc. On the other hand, there might be many combinations or choices in the physical integration of a system. This is the most straightforward way. When we attempt to manage the functions of a system, we characterize its performance according to its purpose. Then we break the purpose down into criteria or figures of merits, sub-criteria, etc.

It is also possible to break down purpose according to long-term planning and short-term planning time horizons. Another way is to break purpose down in terms of strategic planning, tactical planning, and operational planning in an organization.

Sometimes decomposition runs from actions to consequences. There are some policies to support the purpose, and each policy has a set of possible actions and each action has its consequences. Another way is to think of the entire hierarchic decomposition in terms of causes and effects. The relations of cause-effect can form a hierarchy, similar to a fish bone diagram in quality management.

(3) Insert actors into a suitable level

The function of the actors is similar to a filter that screens out some influences at the upper levels. It might be more than one level of actors depending on the requirements.

(4) Establish the bottom level for choice

The bottom level of the hierarchy could be alternatives, actions, consequences, scenarios, or policies to be chosen. These elements or actions are assumed to solve the problem if they are implemented. Hence, a more formal hierarchy can be established.

(5) Examine the hierarchic levels forward and backward

Conceptually, the elements at the high levels can be decomposed into many elements at the lower levels, and the lower levels should support the upper levels. One usually needs to check and revise the elements, and even the levels, backward and forward iteratively to ensure the consistency of the structure. A formal hierarchy is thus determined. It is worth noting that Saaty and Kearns

(1985) provide 13 points to aid in structuring a hierarchy. They are more practical suggestion for constructing a hierarchy.

4.2. Structuring networks

Because of the complex interrelations involved, it may not be easy to classify the elements by levels as in a hierarchy because of the need for feedback and for inner dependence loops and thus a network representation is more appropriate, where there is dependence of what would be upper level elements on lower level ones if structured as a hierarchy. The process of structuring a system as a network is as follows.

(1) Categorize the elements into suitable clusters

The elements related to a decision include goal, criteria, influence, actors, actions, and so on. Besides influence, all the rest can be grouped into a cluster based on the similar characteristics, i.e. homogenous elements are sorted. Some guidelines for breaking down the levels for a hierarchy, in *Step 2* of previous section, could be helpful to categorizing the clusters in a network.

(2) Determine the influences

The influences are different kinds of interactions among elements or clusters of elements. It can happen within a cluster, inner dependence, or between two clusters, outer dependence. One needs to check the influences carefully to ensure that the connections of that represent interactions are correctly made along with their direction.

Note that there can several networks to represent the different kinds of influence, social influence, political influence, economic influence and these are repeated in appropriate fashion for the four BOCR. The different influence criteria are called control criteria because they individually determine the way we provide judgments for that type of influence and follow it by synthesizing all the control type of influences. See Saaty (2005) for details.

(3) Examine the network by clusters forward and backward

There is no clear top-down or bottom-up relations in a network. Examination of the nonlinear structure would depend on the scope of understanding of the problem by the analysts. Usually they need to sketch and revise the elements within each cluster and the relations among the clusters to make sure of the completeness and consistency of the structure.

We believe that the diversity of examples collected in books are gradually turning the process of construction of a hierarchy or a network from art to science (Saaty, 2001).

4.3. Validation of the structures

The structure of a hierarchy or network is a way of representing a real-world problem by the observers. A decision is similar to a model in operations research. But establishing the structure is more of an art than a science, and certain characteristic make it difficult to validate the structure.

There are some possible ways to alleviate the difficulty of validation. Daellenbach (1994) addresses how to perform ongoing evaluations during the modeling process and also proposes three rules for testing validity. These rules deal with field studies so his suggestions indirectly handle validation. In addition, a valid result or outcome is not necessarily equal to a good decision. Validation might not be possible in a decision-making process, but it is needed to avoid the stigma of garbage-in garbage-out. Forecasting outcomes whose occurrence can be documented statistically is one way to increase confidence decision models. Here two guidelines to check the structure that are possibly related to validation.

(1) Is the structure logical?

This consideration is focused on the systematic representation of the influences involved so that the flow of influence among the elements systematically illustrated. The concepts of forward and backward planning might help us sharpen our logical thinking. In addition, reviewing: who, why, what, when, where, and how, might provide some clues toward planning a logical structure.

(2) Is the structure complete?

Because a structure is an abstraction of the problem, it is essential to include the most important elements and their relations in that structure to ensure completeness. It is common to investigate structures from economic, social, political, ethical and environmental points of view to ensure their completeness, but there may be other factors to consider in different situations. The point is to keep an open inquiring mind that is able to include these factors as necessary.

No matter how a structure is validated, group participation with knowledgeable people is a good way to ensure its logicality and completeness.

5. Conclusions

We structure a problem in order to understand it, solve it. It is an aid to systematize our thought processes. A geometry is necessary for us to represent our structure, a new kind of subjective geometry: a graph of a hierarchy or a network. Such a representation makes it easier for us to visualize and understand the relevant issues and their interactions and enables us to solve the problem with greater efficiency, relevance and confidence.

We are grateful to Rozann Whitaker and Mariya Sodenkamp for their careful reading and editing of this paper.

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