An Interactive Approach for Fuzzy Multi-level Programming

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Abstract

This paper is to consider an interactive approach applying to fuzzy multi-level programming (MLP). Simulating the actual decision-making process of the hierarchical structure of an organization, MLP is a practical and useful approach to decentralized planning problems. Because of the complexity of the problems, there are no traditional techniques efficient enough to obtain the numerical solution of a reasonable size problem. Hence, Shih et al. [1] propose a fuzzy approach for MLP, to simplify the complex structure, which is proven to be feasible and efficient. When the coefficients of MLP could not be estimated exactly, then the fuzzy MLP problem will be involved. On account of such a complicated decision, we will take advantage of an interactive technique for obtaining sufficient decision information from the decision maker (DM). Generally speaking, there are two interactive procedures for fuzzy MLP: inside and outside. The former is for the preference of the DM, represented by fuzzy membership functions; latter for the imprecision of coefficients, described by possibility distributions. Special consideration will be given to compensatory operation, positive and negative ideal solutions, and fuzzy membership functions. In the final section, linear-programming type and network-flow type of fuzzy MLP problems will be solved separately as an integrated multi-level system.

1. Introduction

The interactive technique is a promising direction dealing with complex systems, in which decision information can be determined in its process to ensure a rational decision. The concept of this technique provides a learning process about the system, whereby decision maker (DM) can learn to recognize good solutions, the relative importance of factors, and finally design a high-productivity system [2]. According to Aksoy [3], five major advantages of the interactive technique can be summarized as follows: (a) interactive technique does not require preference information which is quite so difficult for the DM to provide; (b) the DM has greater confidence in the solution obtained; (c) the algorithm allows an effective division of labor between the DM and the machine; (d) the DM can clearly learn about his preferences; and (e) the interactive approach would mitigate problems associated with mismatches between the DM's perception and the formalization of the problem via a computerized algorithm.

Multi-level programming (MLP) is developed to solve decentralized planning problems with multiple DMs in a hierarchical organization, where each unit or department seeks its own interests. The MLP problem can be encountered in almost any hierarchical organization such as government agencies, profit or non-profit organizations, manufacturing plants, and network flow problems [4]. Traditional approaches, including vertex enumeration and transformation approaches, cannot provide an efficient algorithm to solving reasonably practical size problems. The simplest bi-level problem in linear form is NP-hard and non-convex. Therefore, Shih et al. [1] suggest a fuzzy approach for MLP to simplify the complex structure, by utilizing the concept of multi-objective decision making (MODM) and the degree of satisfaction, then to transfer it from top-down for simulating the hierarchical decision making process.

Furthermore, the coefficients of MLP could not be estimated precisely in the real world, and the fuzzy MLP problem will be introduced. Typical vagueness of a large hierarchical organization are explored by the use of possibility theory [5]. To process the imprecision of coefficients, we deal with a series of transferred crisp problems based on possibility measure and necessity measure, which need much decision information for processing, such as the confidence of the imprecision, different positive (PISs) and negative (NISs) ideal solutions, and cut-off values. Consequently, an interactive process will be naturally jointed to provide the basic information required. However, there is no existing interactive technique dealing with the fuzziness or the vagueness for MLP [6]. In addition, compensatory operation will be discussed to fit the managerial decision making [7].

All of the above information and other relative important factors will be dialoged with the DMs so that the problems could be solved step by step in our approach.

2. Basic Concepts

2.1 Decision Information

The central part for solving the fuzzy MODM is to handle the conflict of objectives and imprecision of their coefficients. The conflict of objectives can be combined through the concept of global criterion, and the imprecision can be described by possibility distribution. The necessary information for decision making as well as the interactive progress will be discussed.

2.1.1 Compromised Solution

One major technique for MODM is the method of global criterion [8]. The essence of this approach includes the following: (a) reference points, i.e. the concept of an ideal or worse system; (b) distance, i.e. location of alternatives away from reference points; and (c) normalization, i.e. the process to eliminate non-measurability among objectives. Reference points of the technique are PIS $f_k^+(\boldsymbol{x})$ and NIS $f_k^-(\boldsymbol{x})$. PIS is defined as a maximum (or the optimal) solution corresponding to a single-objective maximization problem, or a minimum solution corresponding to a single-objective minimization problem, which is the point DMs like to be adjacent to; on the contrary, NIS works vice versa. In addition, the L_P metric is to define the distance between these two points, i.e. one objective f_k and its ideal solution $f_k^+(\boldsymbol{x})$, in k-dimension space: $d_p = [\sum_k (f_k^+(\boldsymbol{x}) - f_k)^p]^{1/p}$, for $p \geq 1$, where p is the parameter for distance setting. This concept is popular for managing MODM and MLP problems.

2.1.2 Confidence of Imprecision

Possibility is one major concept in fuzzy sets, and it originally tries to describe fuzziness or imprecision of natural languages. For comparison of two fuzzy intervals P and Q, Dubois & Prade [5] propose four fundamental indices, $Pos(\overline{X} \geq \underline{Y})$, $Pos(\overline{X} > \overline{Y})$, $Nec(\underline{X} \geq \underline{Y})$ and $Nec(\underline{X} > \overline{Y})$, where X and Y are variables whose domains are constrained by μ_P and μ_Q, respectively. The relation among these four: $Pos(\overline{X} \geq \underline{Y}) \geq \max\{Pos(\overline{X} > \overline{Y}), Nec(\underline{X} \geq \underline{Y})\} \geq \min\{Pos(\overline{X} > \overline{Y}),$ $Nec(\underline{X} \geq \underline{Y})\} \geq Nec(\underline{X} > \overline{Y})$, represents the confidence of the imprecision in DM's mind and used as the basis for adjusting the imprecision.

For mathematical programming, Buckley [9] proposes an evaluation through the operation expressed as triangular fuzzy membership functions which seeks the minimum of all possibilities in its objective and decision spaces. Negi [10] further extends the problem to *k*-dimension objective space with minimization and maximization based on (strict) exceedance possibility whose distributions are trapezoidal shapes. Cut-off values could be also considered as another factor affecting the imprecision range. For example, if the DM has a lower cut-off value between 0 and 1, the expression will be linear.

2.1.3 Compensatory Operation

In fuzzy mathematical programming, Zimmermann [11] has followed the decision as the intersection of goals and constraints, i.e. μ_D*= $\max\{\min(\mu_G , \mu_C)\}$. This operation is accepted as a general tool for manipulating the linear programming and MODM problems. However, the interpretation of such decision as an intersection or union will result in no compensation (under-achievement) or full compensation (over-achievement). Managerial decisions always have some kind of compensation between either different degrees of goal achievement or decision restrictions [7]. Afterward, Werners [12] proposes two operators which both lead to formulations in linear form with respect to the empirical data. However, "fuzzy and" will be easier for use and is good for MLP problems [13].

2.2 Fuzzy Approach for MLP

The simplest MLP formulation, a bi-level case, is illustrated as follows.

$$\max_{x1} \quad f_1(\boldsymbol{x}_1, \boldsymbol{x}_2) = \boldsymbol{c}_{11}^{\mathrm{T}}\boldsymbol{x}_1 + \boldsymbol{c}_{12}^{\mathrm{T}}\boldsymbol{x}_2 \qquad \text{(upper level)}$$

where $\boldsymbol{x}_2$ solves,

$$\max_{x2} \quad f_2(\boldsymbol{x}_1, \boldsymbol{x}_2) = \boldsymbol{c}_{21}^{\mathrm{T}}\boldsymbol{x}_1 + \boldsymbol{c}_{22}^{\mathrm{T}}\boldsymbol{x}_2 \quad \text{(lower level)}$$

$$\text{s.t.} \quad (\boldsymbol{x}_1, \boldsymbol{x}_2) \in \boldsymbol{X} = \{(\boldsymbol{x}_1, \boldsymbol{x}_2) \mid \boldsymbol{A}_1\boldsymbol{x}_1 + \boldsymbol{A}_2\boldsymbol{x}_2 \leq \boldsymbol{b}, \text{ and } \boldsymbol{x}_1, \boldsymbol{x}_2 \geq \boldsymbol{0}\},$$

where $\boldsymbol{c}_{11}$, $\boldsymbol{c}_{12}$, $\boldsymbol{c}_{21}$, $\boldsymbol{c}_{22}$, and $\boldsymbol{b}$ are linear vectors, $\boldsymbol{A}_1$ and $\boldsymbol{A}_2$ are linear matrices, as well as $\boldsymbol{X}$ denotes the constraint set.

Then, the upper-level DM defines one's objective and decisions with possible tolerances which are described by membership functions of fuzzy set theory. This information then constraints the lower-level DM's feasible space, and we will solve the lower level auxiliary problem as follows.

$$
\begin{aligned}
\text{Max} \quad & \lambda \\
\text{s.t.} \quad & \boldsymbol{x} \in \boldsymbol{X}, \\
& \mu_{f1}(f_1(\boldsymbol{x})) = [f_1(\boldsymbol{x}) - f_1^-] / [f_1^+ - f_1^-] \geq \lambda, \\
& \mu_{x1}(x_1) = [\boldsymbol{x}_1 - (\boldsymbol{x}_1^U - p_1)] / p_1 \geq \lambda, \\
& \mu_{x1}(x_1) = [(\boldsymbol{x}_1^U + p_2) - \boldsymbol{x}_1] / p_2 \geq \lambda, \\
& \mu_{f1}(f_2(\boldsymbol{x})) = [f_2(\boldsymbol{x}) - f_2^-] / [f_2^+ - f_2^-] \geq \lambda,
\end{aligned}
$$

$\lambda \in [0, 1]$, where p_1 and p_2 are the two-side tolerance for decision vector $\boldsymbol{x}_1$ on LHS and RHS, respectively.

A satisfactory solution is reached when the upper and lower level DMs fulfill with the above solution. Otherwise, one needs new membership functions to the lower level DM until a satisfactory solution is reached. Combined with a set of control decisions and goals with tolerance, this solution becomes a satisfactory solution for the original MLP problem. Furthermore, the above expression can be re-formulated by the Werners's "fuzzy and" compensatory operator [12] as follows.

$$
\begin{aligned}
\text{Max} \quad & \mu_{\text{and}} = \lambda + (1-\gamma)(\lambda_1+\lambda_2+\lambda_3) / 3 \\
\text{s.t.} \quad & \\
& \boldsymbol{x} \in \boldsymbol{X}, \\
& \mu_{f1}(f_1(\boldsymbol{x})) \geq (\lambda+\lambda_1), \\
& \mu_{x1}(x_1) \geq (\lambda+\lambda_2), \\
& \mu_{f2}(f_2(\boldsymbol{x})) \geq (\lambda+\lambda_3), \\
& \lambda + \lambda_i \leq 1, \ i = 1, 2, 3
\end{aligned}
$$

γ, λ, λ_1, λ_2, and $\lambda_3 \in [0, 1]$, where γ is the grade of compensation which is between 0 and 1. This auxiliary problem could be easily solved by any linear programming code, e.g. LINDO. In addition, its fuzzy coefficients can be handled by possibility distribution.

2.3 Fuzzy Approach for Fuzzy Multi-level Minimum-Cost Flow Problems

Minimum-cost flow (MCF) problem is used to determine a least shipment cost of a commodity through a capacitated network in order to satisfy demands at certain nodes from available supplies at other nodes. It is a general form of network flow problem which deals with broader types of problems, such as transportation, maximum flow, assignment, shortest path, and transshipment problems [14]. Since cost and capacity parameters at each arc cannot be exactly estimated, a fuzzy MCF problem can be formulated as follows.

$$\text{Min} \quad \tilde{f}(x) = \sum_{(i,j)\in \boldsymbol{A}} \tilde{c}_{ij}\, x_{ij}$$

$$\text{s.t.} \quad \sum_{\{j:(i,j)\in \boldsymbol{A}\}} x_{ij} - \sum_{\{j:(j,i)\in \boldsymbol{A}\}} x_{ji} = b(i),\ \forall\, i \in \boldsymbol{N},$$

$$\tilde{l}_{ij} \le x_{ij} \le \tilde{u}_{ij},\ \forall\,(i,j)\in \boldsymbol{A},$$

$$x_{ij} \ge 0, \text{ and integer}, \ \forall\,(i,j)\in \boldsymbol{A},$$

where $\tilde{f}(x)$, $\tilde{c}_{ij}$, $\tilde{l}_{ij}$, and $\tilde{u}_{ij}$ represent the fuzzy objective, fuzzy cost, fuzzy capacity with lower bound, and fuzzy capacity with upper bound of each arc, respectively. These fuzzy parameters are described by four numbers which represent the four corners of the trapezoidal shape.

Possibilistic linear programming with trapezoidal fuzzy numbers (TFNs) seems well-fitted to any fuzzy programming problem. However, it will has two shortcomings in dealing with network flows. Thus, we re-define the fuzzy arc capacity as: $\alpha(l_{ij2} - l_{ij1}) + l_{ij1} \le x_{ij} \le u_{ij4} - \alpha(u_{ij2} - u_{ij1})\ \forall\, i$ and j [15]. The auxiliary problem is as the following form.

$$\text{Max} \quad \lambda$$

s.t.

$$(f^{1+} - f^{1}(x_{ij})) / (f^{1+} - f^{1-}) \ge \lambda,$$

$$[x^{1}_{ij} - (x^{1\,\mathrm{U}}_{ij} - p^{1}_{ij})] / p^{1}_{ij} \ge \lambda,\ \forall\, i \text{ and } j$$

$$[(x^{1\,\mathrm{U}}_{ij} + p^{2}_{ij}) - x^{1}_{ij}] / p^{2}_{ij} \ge \lambda,\ \forall\, i \text{ and } j$$

$$(f^{2+} - f^{2}(x_{ij})) / (f^{2+} - f^{2-}) \ge \lambda,$$

$$\alpha(l_{ij2} - l_{ij1}) + l_{ij1} \le x_{ij} \le u_{ij4} - \alpha(u_{ij2} - u_{ij1}),\ \forall\, i \text{ and } j,$$

$$\textstyle\sum_i \sum_j c_{ij1}\, x_{ij} \le f^{1} \le \sum_i \sum_j c_{ij1}\, x_{ij}, \text{ and } \theta^{1}_{1} \ge \alpha,$$

$$\textstyle\sum_i \sum_j c_{ij1}\, x_{ij} \le f^{2} \le \sum_i \sum_j c_{ij1}\, x_{ij}, \text{ and } \theta^{2}_{1} \ge \alpha,$$

$$\alpha,\ \theta^{1}_{1} \text{ and } \theta^{2}_{1} \in [0, 1],$$

$$x_{ij} \ge 0 \text{ and integer},$$

where $x_{ij} = (x^{1}_{ij}, x^{2}_{ij})$, and $i = 1, ..., m$ and $j = 1, ..., n$. In addition, the objectives are constituted by two decision variable sets, i.e. $f^{1} = \sum_i\sum_j c^{11}_{ij}\, x^{1}_{ij} + c^{12}_{ij}\, x^{2}_{ij}$ and $f^{2} = \sum_i\sum_j c^{21}_{ij}\, x^{1}_{ij} + c^{22}_{ij}\, x^{2}_{ij}$. In addition, $\theta^{1}_{1} = [f^{1} - \sum_j c_{j1}\, x_j] / [\sum_j c_{j2}\, x_j - \sum_j c_{j1}\, x_j]$ and $\theta^{2}_{1} = [f^{2} - \sum_j c_{j1}\, x_j] / [\sum_j c_{j2}\, x_j - \sum_j c_{j1}\, x_j]$.

Furthermore, the compensatory operator is to replace the max-min operator as the previous expression. Since there are some integer variables involved, this auxiliary problem could be easily solved by any mixed-integer programming code, e.g. LINGO.

3. An Interactive Process for Fuzzy MLP

Figure 1 illustrates the flowchart of the proposed approach. The procedure of the proposed approach is condensed as the following four steps:

Step 1. Starts with decision information pre-process to obtain some basic values, the grade of compensation γ and a cut-off value α for the confidence of imprecision and possibilistic indices. Then go to Step 2.

Step 2. The upper level and lower level DMs solve their problems independently. The imprecise range will be considered by possibility theory, including the following four indices: exceedance possibility, strict exceedance possibility, exceedance necessity and strict exceedance necessity. If DMs satisfy any one of above solutions, a satisfactory solution of the system is obtained. Go to stop; otherwise, go to Step 3. And the above information, including four groups of PISs and NISs with the fixed cut-off value, will be the reference points for the later steps. In addition, the network-type problem will be simplified which depends only on capacity modification.

Step 3. The upper level DM sets up his/her goal and decisions in terms of membership functions. Meanwhile, the lower level DM also sets up his or her goal in terms of another membership function. These membership functions will be extra constraints of their auxiliary crisp problem with compensation.

Step 4. Solve the auxiliary problem for satisfying the compensatory requirement. If the DM at each level satisfies this solution, a compromise is reached. Go to stop. Otherwise, go to Step 3 to adjust their goals and decisions or go to Step 2 to adjust their imprecise range.

To illustrate this approach, let us consider the following examples.

4. Illustrated Examples

Two types of the problems are illustrated here, i.e. a network-flow type and a linear-programming type of problems, as an integrated fuzzy MLP system.

Example 1. A bi-level fuzzy MCF problem with 8 nodes and 11 arcs with trapezoidal fuzzy numbers.

We present a two-level decision-making problem with the figure and data as in Table 1. Our task is to minimize the total cost f^1 for the upper-level DM and to minimize the passing time f^2 for the lower-level DM. Based on the information of PISs and NISs, the fuzzy range for each objective can be established as $f^1 \in [236.5, 409.75]$ and $f^2 \in [176.6, 293.25]$. Assume that the upper-level DM has two control variables, the first control decision x_{14} is around 10 within the interval [4, 15]. The second control decision x_{35} is around 4 within the interval [0, 11]. Thus, this decision information can be substituted into the auxiliary expression with $\lambda = \min\{\lambda^1, \lambda^2_{14}, \lambda^2_{35}, \lambda^3\}$ with no compensation. A reasonable result will be obtained if we utilize Werners's "fuzzy and" operator. The compromised solution is $\boldsymbol{f}^* = (f^{1*}, f^{2*}) = (255.75, 192.05)$ and the flows are $x_{14} = 10$, $x_{35} = 7$, $x_{26} = 10$, $x_{34} = 3$, $x_{47} = 8$, $x_{57} = 7$, $x_{68} = 10$, $x_{23} = 10$, $x_{21} = x_{56} = x_{78} = 0$ with the total degree of satisfaction $\mu_{and} = 0.560$ for compensatory parameter $\gamma = 0.5$ and the cut-off value $\alpha = 0.5$.

Example 2. A trade-off MLP problem between exports and imports with trapezoidal fuzzy numbers. The fuzzy parameters are represented by four numbers in the following problem:

$$\max_{x1} \quad f_1 = (1.5,2,2,2.5)x_1 + (-1.5,-1,-1,-0.5)x_2 \ \text{(upper level)}$$

where x_2 solves,

$$\max_{x2} \quad f_2 = (0.8,1,1,1.4)x_1 + (1.5,2,1.5,2.5)x_2 \ \text{(lower level)}$$

s.t.

$$\begin{aligned}
&(2.5,3,3,3.5)x_1 + (-5.5,-5,-5,-4.5)x_2 \le (10, 25, 25, 26),\\
&(2.8,3,3.2,3.5)x_1 + (-1.5,-1,-1,-0.5)x_2 \le (15,20,25,35),\\
&(2.75,3,3.2,3.2)x_1 + (0.8,1,1,1.3)x_2 \le (20,25,25,42),\\
&(2.5,3,3,3.5)x_1 + (3.6,4,4,4.4)x_2 \le (30,32,35,50),\\
&(0.8,1,1,1.2)x_1 + (2.6,3,3,3.4)x_2 \le (14,15,16,25),\\
&x_1, x_2 \ge 0.
\end{aligned}$$

The fuzzy range for each objective can be established as: $f_1 \in [0, 23.47]$ and $f_2 \in [0, 21.12]$. The first level control decision x_1 is around 11.6 with negative and positive side tolerances 2.06 and 1.94, respectively. The compromised objective is $\boldsymbol{f}^* = (f_1^*, f_2^*) = (22.62, 19.24)$ and the decision is $\boldsymbol{x}^*=(x_1^*, x_2^*)= (10.88, 2.48)$ with the total degree of satisfaction $\mu_{and} = 0.920$ for the grade of compensation $\gamma = 0.5$ and the fixed cut-off value $\alpha = 0.5$.

5. Conclusions

Due to the involvement of DMs, the compromised solution of fuzzy MLP problem is reached with a few iterations. Compared to simulation, our proposed approach is rather efficient and DMs would be more confident about the results. Although all illustrated examples are bi-level cases, the multi-level case is expected to be solved by the same algorithm.

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