New Examples

Shlok Mishra

February 3, 2024

Example 1 *Let* $(X_1, \ldots, X_d) \sim \mathcal{N}_d(\mathbf{0}, \mathbf{I})$, $(Y_1, \ldots, Y_d) \sim \mathcal{N}_d(\boldsymbol{\mu}_1, \mathbf{I})$, *and* $(Z_1, \ldots, Z_d) \sim \mathcal{N}_d(\boldsymbol{\mu}_2, \mathbf{I})$, *where* $\mathbf{0}$, $\boldsymbol{\mu}_1$, *and* $\boldsymbol{\mu}_2$ *are distinct mean vectors, and* $\mathbf{I}$ *denotes the identity matrix. Specifically, let* $\boldsymbol{\mu}_1 = (1, 1, 1, 1, 0, \ldots, 0)^T$ *and* $\boldsymbol{\mu}_2 = (2, 2, 2, 2, 0, \ldots, 0)^T$. *Here* $\mathcal{N}_d(\boldsymbol{\mu}, \mathbf{\Sigma})$ *denotes the d-dimensional normal distribution with mean vector* $\boldsymbol{\mu}$ *and covariance matrix* $\mathbf{\Sigma}$.

Example 2 *Suppose that* $(X_1, X_2), (X_3, X_4) \sim \mathcal{N}_2(\mathbf{0}, \mathbf{\Sigma_1})$ *with* $\mathbf{\Sigma_1} = \begin{pmatrix} 1 & 0.9 \\ 0.9 & 1 \end{pmatrix}$, $(Y_1, Y_2), (Y_3, Y_4) \sim \mathcal{N}_2(\mathbf{0}, \mathbf{\Sigma_2})$ *with* $\mathbf{\Sigma_2} = \begin{pmatrix} 1 & -0.9 \\ -0.9 & 1 \end{pmatrix}$, *and* $(Z_1, Z_2), (Z_3, Z_4) \sim \mathcal{N}_2(\mathbf{0}, \mathbf{\Sigma_3})$ *with* $\mathbf{\Sigma_3} = \begin{pmatrix} 1 & 0.5 \\ 0.5 & 1 \end{pmatrix}$. *For each class,* $X_5, \ldots, X_d$, $Y_5, \ldots, Y_d$, *and* $Z_5, \ldots, Z_d$ *are iid* $\mathcal{N}(0, 1)$. *Here,* $(X_1, \ldots, X_4)$, $(X_5, \ldots, X_d)$, $(Y_1, \ldots, Y_4)$, $(Y_5, \ldots, Y_d)$, $(Z_1, \ldots, Z_4)$, *and* $(Z_5, \ldots, Z_d)$ *are mutually independent and 'iid' stands for independent and identically distributed.*