Q1] $$\lim_{x \to a} \left[\frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \right]$$

$$\lim_{x \to a} \left[\frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \times \frac{\sqrt{a+2x} + \sqrt{3x}}{\sqrt{a+2x} + \sqrt{3x}} \times \frac{\sqrt{3a+x} + 2\sqrt{x}}{\sqrt{3a+x} + 2\sqrt{x}} \right]$$

$$\lim_{x \to a} \frac{(a+2x-3x)}{(3a+x-4x)} \frac{(\sqrt{3a+x} + 2\sqrt{x})}{\sqrt{a+2x} + \sqrt{3x})}$$

$$\lim_{x \to a} \frac{(a-x)(\sqrt{3a+x} + 2\sqrt{x})}{(3a-3x)(\sqrt{a+2x} + \sqrt{3x})}$$

$$\frac{1}{3} \lim_{x \to a} \frac{(a-x)(\sqrt{3a+x} + 2\sqrt{x})}{(a-x)(\sqrt{a+2x} + \sqrt{3x})}$$

$$\frac{1}{3} \times \frac{\sqrt{4a} + 2\sqrt{a}}{\sqrt{3a} + \sqrt{3a}}$$

$$\frac{1}{3} \times \frac{2\sqrt{a} + 2\sqrt{a}}{\sqrt{3a} + \sqrt{3a}}$$

$$\frac{1}{3} \times \frac{\overset{2}{\cancel{4}}\sqrt{a}}{\cancel{2}\sqrt{3a}}$$

$$= \frac{2}{3\sqrt{3}}$$

Q2) $\lim_{y\to 0}\left[\frac{\sqrt{a+y}-\sqrt{a}}{y\sqrt{a+y}}\right]$

$\lim_{y\to 0}\left[\frac{\sqrt{a+y}-\sqrt{a}}{y\sqrt{a+y}} \times \frac{\sqrt{a+y}+\sqrt{a}}{\sqrt{a+y}+\sqrt{a}}\right]$

$\lim_{y\to 0} \frac{a+y-a}{y\sqrt{a+y}\,(\sqrt{a+y}+\sqrt{a})}$

$\lim_{y\to 0} \frac{y}{y\sqrt{a+y}\,(\sqrt{a+y}+\sqrt{a})}$

$\frac{1}{\sqrt{a+0}\,\sqrt{a+0}+\sqrt{a}}$

$\frac{1}{\sqrt{a}\,(\sqrt{a}+\sqrt{a})}$

$\frac{1}{\sqrt{a}\,(2\sqrt{a})} = \frac{1}{2a}$

Q3) $\lim_{x\to \pi/6} \frac{\cos x-\sqrt{3}\sin x}{\pi-6x}$

By substituting $x-\frac{\pi}{6}=h$

$x = h+\frac{\pi}{6}$

where $h\to 0$

$\lim_{h\to 0} \frac{\cos\left(h+\frac{\pi}{6}\right)-\sqrt{3}\sin\left(h+\frac{\pi}{6}\right)}{\pi-6\left(h+\frac{\pi}{6}\right)}$

using
$\cos(A+B) = \cos A\cdot\cos B$
$\sin(A+B) = \sin A\cos B+\cos A\cdot\sin B$

$\lim_{h\to 0} \frac{\cos h\cdot\cos\frac{\pi}{6}-\sin h\sin\frac{\pi}{6} - \sqrt{3}\sin h\cos\frac{\pi}{6}+\cosh\sin\frac{\pi}{6}}{\pi-6\left(6h+\frac{\pi}{6}\right)}$

$\lim_{h\to 0} \frac{\cos h\cdot\frac{\sqrt{3}}{2}-\sin h\frac{1}{\sqrt{2}} - \sqrt{3}\left(\sin h\frac{\sqrt{3}}{2}+\cos h\frac{1}{\sqrt{2}}\right)}{\pi-6h+\pi}$

$\lim_{h\to 0} \frac{\cos\frac{\sqrt{3}}{2}h-\sin\frac{h}{2}-\sin\frac{3h}{2}-\cos\frac{\sqrt{3}h}{2}}{-2h}$

$\lim_{h\to 0} \frac{-\frac{\sin 4h}{2}}{-6h}$

$\lim_{h\to 0} \frac{\sin 4h}{12h}$

$\frac{1}{3}\lim_{h\to 0}\frac{\sin h}{h} = \frac{1}{3}\times 1 = \frac{1}{3}$

Qu] $\lim_{x\to\infty} \left[\dfrac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}} \right]$

By rationalizing numerator & Denominator both

$$\lim_{x\to\infty} \left[\frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}} \times \frac{\sqrt{x^2+5} + \sqrt{x^2-3}}{\sqrt{x^2+5} + \sqrt{x^2-3}} \times \frac{\sqrt{x^2+3} + \sqrt{x^2+1}}{\sqrt{x^2+3} + \sqrt{x^2+1}} \right]$$

$$\lim_{x\to\infty} \left[\frac{(x^2+5-x^2+3)(\sqrt{x^2+3} + \sqrt{x^2+1})}{(x^2+3-x^2-1)(\sqrt{x^2+5} + \sqrt{x^2-3})} \right]$$

$$\lim_{x\to\infty} \frac{8(\sqrt{x^2+3} + \sqrt{x^2+1})}{2(\sqrt{x^2+5} + \sqrt{x^2-3})}$$

$$4 \lim_{x\to\infty} \frac{\sqrt{x^2\left(1+\frac{3}{x^2}\right)} + \sqrt{x^2\left(1+\frac{1}{x^2}\right)}}{\sqrt{x^2\left(1+\frac{5}{x^2}\right)} + \sqrt{x^2\left(1-\frac{3}{x^2}\right)}}$$

After applying limit

we get, $= 4$ //

Q5) $f(x) = \dfrac{\sin 2x}{\sqrt{1-\cos 2x}}$, for $0 < x \le \pi/2$

$= \dfrac{\cos x}{\pi - 2x}$, for $\pi/2 < x < \pi$

$\Big\}$ at $x = \pi/2$

$f(\pi/2) = \dfrac{\sin 2\left(\frac{\pi}{2}\right)}{\sqrt{1-\cos 2(\pi/2)}}$ $\therefore f(\pi/2) = 0$

f at $x = \pi/2$ define

ii) $\lim_{x\to\pi/2} f(x) = \lim_{x\to\pi/2^+} \dfrac{\cos x}{\pi - 2x}$

By substituting method

$x - \frac{\pi}{2} = h$

$x = h + \pi/2$

where $h \to 0$

$$\lim_{h\to 0} \frac{\cos\left(h+\frac{\pi}{2}\right)}{\pi - 2\left(h+\frac{\pi}{2}\right)}$$

$$\lim_{h\to 0} \frac{\cos\left(h+\frac{\pi}{2}\right)}{\pi - 2\left(\frac{2h+\pi}{2}\right)}$$

$$\lim_{h\to 0} \frac{\cos\left(h+\frac{\pi}{2}\right)}{-2h}$$ using $\cos(A+B) = \cos A \cos B - \sin A \sin B$

$$\lim_{h\to 0} \frac{\cos h \cdot \cos\frac{\pi}{2} - \sin h \cdot \sin\frac{\pi}{2}}{-2h}$$

$$\lim_{h\to 0} \frac{\cos h \cdot 0 - \sin h}{-2h}$$

$$\frac{1}{2} \lim_{h\to 0} \frac{\sin h}{h}$$

$= \frac{1}{2}$ //

b) $\lim_{x \to \pi/2} f(x) = \lim_{x \to \pi/2} - \frac{\sin 2x}{\sqrt{1-\cos 2x}}$ using $\sin 2x = 2\sin x . \cos x$

$\lim_{x \to \pi/2} - \frac{2\sin x . \cos x}{\sqrt{2\sin^2 x}}$

$\lim_{x \to \pi/2} - \frac{2 \sin x \cos x}{\sqrt{2} \sin x}$

$\frac{2}{\sqrt{2}} \lim_{x \to \frac{\pi}{2}} - \cos x$

$\therefore LHL \neq RHL$

$\therefore f$ is not continuous at $x = \pi/2$

Q5) ii) $f(x) = \frac{x^2-9}{x-3}$ $0 < x < 3$

$= x+3$ $3 \leq x \leq 6$ } at $x = 3$ & $x = 6$

$= \frac{x^2-9}{x+3}$ $6 \leq x < 9$

at $x = 3$

i) $f(3) = \frac{x^2-9}{x-3} = 0$

f at $x = 3$ define

$f(x) = \frac{1-\cos 4x}{x^2}$ $\quad \left.\begin{array}{l} x<0 \\ x=0 \end{array}\right\}$ at $x=0$

f is continuous at $x=0$

$\lim_{x\to 0} f(x) = f(0)$

$\lim_{x\to 0} \frac{1-\cos 4x}{x^2} = k$

$\lim_{x\to 0} \frac{2-\sin^2 2x}{x^2} = k$

$2\lim_{x\to 0} \frac{\sin^2 x}{x^2} = k$

$2\lim_{x\to 0} \left(\frac{\sin \cdot 2x}{x}\right)^2 = k$

$2(2)^2 = k$

$\therefore k = 8$

ii) $f(x) = (\sec^2 x)^{\cot^2 x}$ $\quad \left.\begin{array}{l} x\neq 0 \\ x=0 \end{array}\right\}$ at $x=0$

or $f(x) = (\sec^2 x)^{\cot^2 x}$

Usin

$\sec^2 - \tan^2 x \quad -\sec^2 x = 1$

$\therefore \sec^2 x = 1 + \tan^2 x$

&

$\cot^2 x = \frac{1}{\tan^2 x}$

$\lim_{x\to 0} (\sec^2 x)^{\cot^2 x}$

$\lim_{k\to 0} (1+\tan^2 x)^{\frac{1}{\tan^2 x}}$

we, know that

$\lim_{x\to 0} (1+px)^{\frac{1}{px}} = e$

$= e$

$\therefore k = e$

iii) $f(x) = \dfrac{\sqrt{3} - \tan x}{\pi - 3x}$ $\quad x = \pi/3$ $\quad x = \pi/3$ $\Big\}$ at $x - \frac{\pi}{3}$

$x - \frac{\pi}{3} = h$

$x = h + \frac{\pi}{3}$

where $h \to 0$

$f(\pi/3 + h) = \dfrac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3\left(\frac{\pi}{3} + h\right)}$

$\cancel{\lim_{h\to 0} \dfrac{\sqrt{3} - \tan(\pi/3 + h)}{\pi/3\ (\pi/3 + h)}}$

using

$\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

$\lim_{h\to 0} \sqrt{3} - \dfrac{\tan\frac{\pi}{3} + \tan h}{1 - \tan\frac{\pi}{3}\tan h}$

$\lim_{h\to 0} \dfrac{\left(\dfrac{\sqrt{3} - 3\tan h - \sqrt{3} - \tan h}{1 - \sqrt{3}\tan h}\right)}{-3h}$

$\lim_{h\to 0} \dfrac{-4\tan h}{-3h(1 - \sqrt{3}\tan h)}$

$\lim_{h\to 0} \dfrac{4\tan h}{3h(1 - \sqrt{3}\tan h)}$

$\frac{4}{3} \lim_{h\to 0} \frac{\tan h}{h} \lim_{h\to 0} \frac{1}{(1 - \sqrt{3}\tan h)}$ $\quad \frac{\tan h}{h} = 1$

$= \frac{4}{3}\left(\frac{1}{1 - \sqrt{3}(0)}\right)$

$= \frac{4}{3}\left(\frac{1}{1}\right) = \frac{4}{3}$

7.] $f(x) = \dfrac{1 - \cos 3x}{x \tan x}$ $\quad x \neq 0$ $\Big\}$ at $x = 0$

$= 9$ $\quad x = 0$

$f(x) = \dfrac{1 - \cos 3x}{x \tan x}$

$\lim_{x\to 0} = \dfrac{2\sin^2 3/2\, x}{x \tan x}$

$\lim_{x\to 0} = \dfrac{\dfrac{2\sin^2\frac{3x}{2}}{x^2} \times x^2}{\dfrac{x \tan x}{x^2} \times x^2}$

$= 2 \lim_{x\to 0} \dfrac{\left(\frac{3}{2}\right)^2}{1} = \dfrac{2 \times 9}{4} = \dfrac{9}{2}$

$\lim_{x\to 0} f(x) = \frac{9}{2} \neq f(0)$.

$\therefore f$ is not continuous at $x=0$

Redefine function

SC

$$f(x) = \begin{cases} \dfrac{1-\cos 3x}{x\tan x} & x \neq 0 \\ \dfrac{9}{2} & x = 0 \end{cases}$$

Now $\lim\limits_{x\to 0} f(x) = f(0)$

f has removable discontinuity at $x=0$

8] $f(x) = \dfrac{e^{x^2} - \cos x}{x^2}$ $\quad x=0$

is continuous at $x=0$

$\lim\limits_{x\to 0} f(x) = f(0)$

$= \lim\limits_{x\to 0} \dfrac{e^{x^2} - \cos x - 1 + 1}{x^2} = f(0)$

$= \lim\limits_{x\to 0} \dfrac{e^{x^2} - \cos x - 1 + 1}{x^2}$

$\lim\limits_{x\to 0} \dfrac{(e^{x^2} - 1) + (1-\cos x)}{x^2}$

$\lim\limits_{x\to 0} \dfrac{e^{x^2}-1}{x^2} + \lim\limits_{x\to 0} \dfrac{1-\cos x}{x^2}$

lose $+ \lim\limits_{x\to 0} \dfrac{2\sin^2 x/2}{x^2}$

lose $+ \lim\limits_{x\to 0} 2\left(\dfrac{\sin x/2}{x}\right)^2$

Multiply with 2 on Num & Den.

$= 1 + 2 \times \dfrac{1}{4/2} = \dfrac{3}{2} = f(0)$

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9] $f(x) = \dfrac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x}$ $\quad x \neq \pi/2$

$f(0)$ is continuous at $x = \pi/2$

$\lim\limits_{x\to \pi/2} \dfrac{\sqrt{2} - \sqrt{1-\sin x}}{\cos^2 x} \times \dfrac{\sqrt{2} + \sqrt{1+\sin x}}{\sqrt{2} + \sqrt{1+\sin x}}$

$\lim\limits_{x\to \pi/2} \dfrac{2 - 1 + \sin x}{\cos^2 x(\sqrt{2} + \sqrt{1+\sin x})}$

$= \lim\limits_{x\to \pi/2} \dfrac{1+\sin x}{1-\sin^2 x(\sqrt{2} + \sqrt{1+\sin x})}$

$= \lim\limits_{x\to \pi/2} \dfrac{1}{(1-\sin x)(\sqrt{2} + \sqrt{1+\sin x})}$

$= \dfrac{1}{2(\sqrt{2})} = \dfrac{1}{4\sqrt{2}}$

$\therefore f(\pi/2) = \dfrac{1}{4\sqrt{2}}$

No.

PRACTICAL NO: 2

DERIVATIVE

Q1) Show that the foll. function defined from R to R are differentiable.

i) $\cot x$

$f(x) = \cot x$

$Df(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$

$= \lim_{x\to a} \frac{\cot x - \cot a}{x-a}$

$= \lim_{x\to a} \frac{\frac{1}{\tan x} - \frac{1}{\tan a}}{x-a}$

$= \lim_{x\to a} \frac{\tan a - \tan x}{(x-a)\tan x \tan a}$

put $x-a = h$

$x = a+h$

as $x \to a$, $h \to 0$

$Df(h) = \lim_{h\to 0} \frac{\tan a - \tan(a+h)}{(a+h-a)\tan(a+h)\tan a}$

$= \lim_{h\to 0} \frac{\tan a - \tan(a+h)}{h - \tan(a+h)\tan a}$

$= \lim_{h\to 0} \frac{(a-a-h) - (1+\tan a \tan(a+h))}{h - \tan(a+h)\tan a}$

$= \lim_{h\to 0} \frac{\tan h}{h} \times \frac{1+\tan a \tan(a+h)}{\tan(a+h)\tan a}$

$= -1 \times \frac{1+\tan^2 a}{\tan^2 a}$

$= -\frac{\sec^2 a}{\tan^2 a}$

$= \frac{-1}{\cos^2 a} \times \frac{\cos^2 a}{\tan \sin^2 a}$

$= -\operatorname{cosec}^2 a$

$\therefore Df(a) = -\cos^2 a$

$\therefore f$ is differentiable $\forall a \in R$

ii) $\operatorname{cosec} x$

$f(x) = \operatorname{cosec} x$

$Df(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$

$= \lim_{x\to a} \frac{\operatorname{cosec} x - \operatorname{cosec} a}{x-a}$

$= \lim_{x\to a} \frac{\frac{1}{\sin x} - \frac{1}{\sin a}}{x-a}$

$= \lim_{x\to a} \frac{\sin a - \sin x}{(x-a)\sin a \sin x}$

Put $x-a=h$

$x=a+h$

as $x\to a$, $h\to 0$

$Df(h) = \lim_{h\to 0} \frac{\sin a - \sin(a+h)}{(a+h-a)\sin a \cdot \sin(a+h)}$

$= \lim_{h\to 0} \frac{2\cos\left(\frac{a+a+h}{2}\right)\sin\left(\frac{a-a-h}{2}\right)}{h \times \sin a \sin(a+h)}$

$= \lim_{h\to 0} \frac{-\sin\frac{h}{2}}{\frac{h}{2}} \times \frac{1}{2} \times \frac{2\cos\left(\frac{2a+h}{2}\right)}{\sin a \sin(a+h)}$

$= -\frac{1}{2} \times \frac{2\cos\left(\frac{2a+0}{2}\right)}{\sin(a+0)}$

$$= \frac{-\cos a}{\sin^2 a} = -\cot a \operatorname{cosec} a$$

(Q2) If $f(x) = 4x+1$, $x \le 2$
$= x^2+5$, $x>0$, at $x=2$ then find function is differentiable or not

Sol:
LHD:
$$Df(2^-) = \lim_{x\to2^-} \frac{f(x)-f(2)}{x-2}$$
$$= \lim_{x\to2^-} \frac{4x+1-(4\times2+1)}{x-2}$$
$$= \lim_{x\to2^-} \frac{4x+1-9}{x-2}$$
$$= \lim_{x\to2^-} \frac{4x-8}{x-2}$$
$$= \lim_{x\to2^-} \frac{4(x-2)}{(x-2)} = 4$$
$Df(2^-) = 4$

RHD:
$$Df(2^+) = \lim_{x\to2^+} \frac{x^2+5-9}{x-2}$$
$$= \lim_{x\to2^+} \frac{x^2-4}{x-2}$$
$$= \lim_{x\to2^+} \frac{(x+2)(\cancel{x-2})}{\cancel{x-2}}$$
$$= 2+2 = 4$$
$Df(2^+) = 4$

RHD = LHD

f is differentiable at $x=2$

(Q3) If $f(x) = 4x+7$, $x<3$
x^2+3x+1, $x \ge 3$ at $x=3$ then find f is diff or not?

Sol: RHD:
$$Df(3^+) = \lim_{x\to3^+} \frac{f(x)-f(3)}{x-3}$$
$$= \lim_{x\to3^+} \frac{x^2+3x+1-(3^2+3\times3+1)}{x-3}$$
$$= \lim_{x\to3^+} \frac{x^2+3x+1-19}{x-3}$$
$$= \lim_{x\to3^+} \frac{x^2+3x-18}{x-3}$$
$$= \lim_{x\to3^+} \frac{x(x+6)-3(x+6)}{x-3}$$
$$= \lim_{x\to3^+} \frac{(x+6)(x-3)}{(x-3)} = 3+6 = 9$$
$Df(3^+) = 9$

LHD: $Df(3^-)$
$$= \lim_{x\to3^-} \frac{f(x)-f(3)}{x-3}$$
$$= \lim_{x\to3^-} \frac{4x+7-19}{x-3}$$
$$= \lim_{x\to3^-} \frac{4x-12}{x-3}$$
$$= \lim_{x\to3^-} \frac{4(x-3)}{(x-3)}$$
$Df(3^-) = 4$

RHD $\ne$ LHD $\therefore$ f is not diff at $x=3$

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B.S.

$= \frac{-\cos a}{\sin^2 a} = -\cot a \operatorname{cosec} a$

Q2) If $f(x) = 4x+1 \quad , x \leq 2$

$\quad = x^2+5 \quad x>0$, at $x=2$ then

find function is differentiable or not

Sol:

LHD:

$Df(2^-) = \lim_{x\to 2^-} \frac{f(x)-f(2)}{x-2}$

$= \lim_{x\to 2^-} \frac{4x+1-(4\times 2+1)}{x-2}$

$= \lim_{x\to 2^-} \frac{4x+1-9}{x-2}$

$= \lim_{x\to 2^-} \frac{4x-8}{x-2}$

$= \lim_{x\to 2^-} \frac{4(x-2)}{(x-2)} = 4$

$Df(2^-) = 4$

RHD:

$Df(2^+) = \lim_{x\to 2^+} \frac{x^2+5-9}{x-2}$

$= \lim_{x\to 2^+} \frac{x^2-4}{x-2}$

$= \lim_{x\to 2^+} \frac{(x+2)(x-2)}{x-2}$

$= 2+2 = 4$

$Df(2^+) = 4$

RHD = LHD

f is differentiable at $x=2$

Q3) If $f(x) = 4x+7 \quad , x<3$

$\quad = x^2+3x+1 \quad , x\geq 3$ at $x=3$ then

find f is diff or not ?

Sol: RHD:

$Df(3^+) = \lim_{x\to 3^+} \frac{f(x)-f(3)}{x-3}$

$= \lim_{x\to 3^+} \frac{x^2+3x+1-(3^2+3\times 3+1)}{x-3}$

$= \lim_{x\to 3^+} \frac{x^2+3x+1-19}{x-3}$

$= \lim_{x\to 3^+} \frac{x^2+3x-18}{x-3}$

$= \lim_{x\to 3^+} \frac{x(x+6)-3(x+6)}{x-3}$

$= \lim_{x\to 3^+} \frac{(x+6)(x-3)}{(x-3)} = 3+6 = 9$

$Df(3^+) = 9$

LHD: $Df(3^-)$

$= \lim_{x\to 3^-} \frac{f(x)-f(3)}{x-3}$

$= \lim_{x\to 3^-} \frac{4x+7-19}{x-3}$

$= \lim_{x\to 3^-} \frac{4x-12}{x-3}$

$= \lim_{x\to 3^-} \frac{4(x-3)}{(x-3)}$

$Df(3^-) = 4$

RHD ≠ LHD $\therefore$ f is not diff at $x=3$

Q4) If $f(x) = 8x - 5 \quad , x < 2$

$= 3x^2 - 4x + 7 \quad , x > 2$ at $x = 2$ then

find f is diff or not

Sol:

$f(2) = 8 \times 2 - 5 = 16 - 5 = 11$

RHD:

$$Df(2^+) = \lim_{x \to 2^+} \frac{f(x) - f(2)}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{3x^2 - 4x + 7 - 11}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{3x^2 - 4x - 4}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{3x^2 - 6x + 2x - 4}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{3x(x - 2) + 2(x - 2)}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{(3x + 2)\cancel{(x - 2)}}{\cancel{(x - 2)}}$$

$= 3 \times 2 + 2 = 8$

$Df(2^+) = 8$

LHD:

$$Df(2^-) = \lim_{x \to 2} \frac{f(x) - f(2)}{x - 2}$$

$$= \lim_{x \to 2} \frac{8x - 5 - 11}{x - 2}$$

$$= \lim_{x \to 2} \frac{8x - 16}{x - 2}$$

$$= \lim_{x \to 2^-} \frac{8(x - 2)}{(x - 2)}$$

$= 8$

$Df(2^-) = 8$

LHD = RHD

f is differentiable at $x = 2$

22/01/2020

APPLICATION OF DERIVATIVES

) Find the intervals in which function is increasing or decreasing

). $f(x) = x^3 - 5x - 11$

Sol:

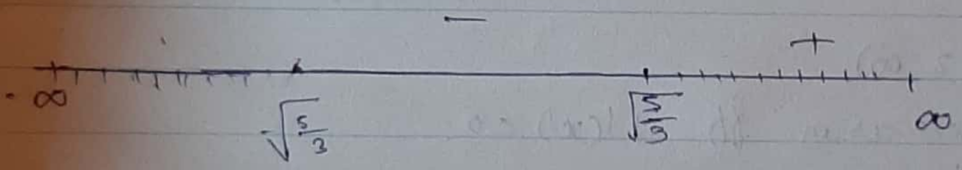
f is increasing iff $f'(x) = 0$

$\therefore f(x) = x^3 - 5x - 11$

$f'(x) = 3x^2 - 5$

$\therefore 3x^2 - 5 > 0$

$\therefore x = \pm\sqrt{\frac{5}{3}}$

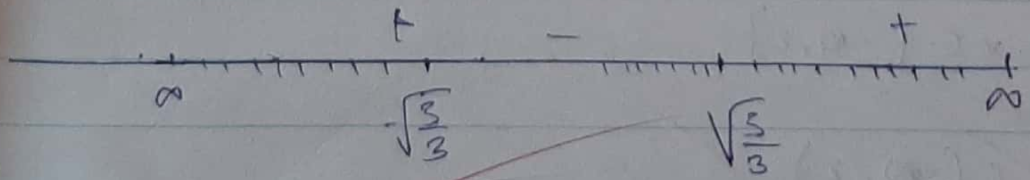


$\therefore x \in \left(-\infty, \sqrt{\frac{5}{3}}\right) \cup \left(\sqrt{\frac{5}{3}}, \infty\right)$

Now f is decreasing iff $f'(x) = 0$

$\therefore 3x^2 - 5 < 0$

$x = \pm\sqrt{\frac{5}{3}}$



$\therefore x \in \left(-\sqrt{\frac{5}{3}}, \sqrt{\frac{5}{3}}\right)$

b) $F(x) = x^2 - 4x$

Sol: f is increasing iff $f'(x) \geq 0$

$\therefore F(x) = x^2 - 4x$

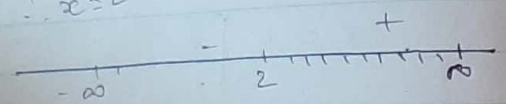
$f'(x) = 2x - 4$

$\therefore 2x - 4 > 0$

$2(x-2) > 0$

$x - 2 > 0$

$\therefore x = 2$



$\therefore x \in (2, \infty)$

Now f is decreasing iff $F'(x) < 0$

$\therefore 2x - 4 < 0$

$\therefore 2(x-2) < 0$

$\therefore x - 2 < 0$

$\therefore \underline{\underline{x < 2}}$

$\therefore x \in [-\infty, 2]$

$\therefore x \in (-\infty, 2)$

c) $F(x) = 2x^3 + x^2 - 20x + 4$

Sol: f is increasing iff $F'(x) \geq 0$

$\therefore F(x) = 2x^3 + x^2 - 20x + 4$

$\therefore F'(x) = 6x^2 + 2x - 20$

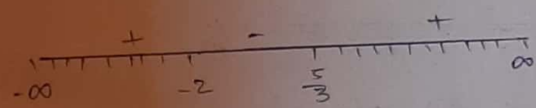
$\therefore 6x^2 + 2x - 20 > 0$

$\therefore 6x^2 + 12x - 10x - 20 > 0$

$\therefore 6x(x+2) - 10(x+2) > 0$

$\therefore (x+2)(6x-10) > 0$

$\therefore x = -2, \frac{5}{3}$



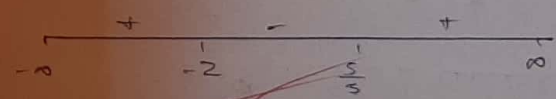
$\therefore x \in (-\infty, -2) \cup (5/3, \infty)$

Now f is decreasing iff $F'(x) < 0$

$\therefore 6x^2 + 2x - 20 < 0$

$\therefore (x+2)(6x-10) < 0$

$x = -2, 5/3$



$\therefore x \in (-2, \frac{5}{3})$

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d) $f(x) = x^3 - 27x + 5$

Sol: f is increasing iff $f'(x) > 0$

$\therefore f(x) = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27 > 0$

$\therefore 3x^2 - 27 > 0$

$\therefore 3(x^2 - 9) > 0$

$x^2 - 9 > 0$

$\therefore x = 3, -3$

+ − +
$-\infty$ -3 3 ∞

$\therefore x \in (-\infty, -3) \cup (3, \infty)$

Now f is decreasing iff

$f'(x) < 0$

$\therefore 3x^2 - 27 < 0$

$\therefore 3(x^2 - 9) < 0$

$x^2 - 9 < 0$

$x = 3, -3$

+ − +
$-\infty$ 3 3 ∞

$\therefore x \in (-3, 3)$

Q7) Find the intervals in which function is concave upwards & concave downwards

a) $y = 3x^2 - 2x^3$

Sol: $f(x) = 3x^2 - 2x^3$

$f'(x) = 6x - 6x^2$

$f''(x) = 6 - 12x$

f is concave upwards iff $f''(x) > 0$

$\therefore 6 - 12x > 0$

$6(1 - 2x) > 0$

$(1 - 2x) > 0$

$-(2x - 1) > 0$

$-\infty$ $\frac{1}{2}$ ∞

$x \in (-\infty, \frac{1}{2})$

$\therefore f$ is concave downwards iff $f''(x) < 0$

$\therefore 6(1 - 2x) < 0$

$-2(2x - 1) < 0$

+
$-\infty$ $\frac{1}{2}$ ∞

$x \in (\frac{1}{2}, \infty)$

Scanned with CamScanner

$y = x^4 - 6x^3 + 12x^2 + 5x + 7$

Sol:

$\because y = f(x)$

$\therefore f(x) = x^4 - 6x^3 + 12x^2 + 5x + 7$

$\therefore f'(x) = 4x^3 - 18x^2 + 24x + 5$

$f''(x) = 12x^2 - 36x + 24$

$\therefore$ f is concave upwards iff

$\therefore 12x^2 - 36x + 24 > 0$

$\therefore 12(x^2 - 3x + 2) > 0$

$\therefore x^2 - 3x + 2 > 0$

$(x-2)(x-1) > 0$

$\therefore x = 2, 1$

$-\infty \quad 1 \quad 2 \quad \infty$

$\therefore$ f is concave downward iff

$f''(x) < 0$

$\therefore 12x^2 - 36x + 24 < 0$

$\therefore 12(x^2 - 3x + 2) < 0$

$x^2 - 3x + 2 < 0$

$(x-2)(x-1) < 0$

$\therefore x = 2, 1$

$+ \quad - \quad +$

$-\infty \quad 1 \quad 2 \quad \infty$

$x \in (1, 2)$

c) $y = x^3 - 27x + 5$

Sol:

$\because y = f(x)$

$\therefore f(x) = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

$\therefore f''(x) = 6x$

$\therefore$ f is concave upward if & only if

$f''(x) > 0$

$\therefore 6x > 0$

$\therefore x > 0$

$\therefore x = 0$

$\therefore x \in (0, \infty)$

$\therefore$ f is concave downward iff

$f''(x) < 0$

$\therefore 6x < 0$

$\therefore x < 0$

$\therefore x = 0$

$\therefore x \in (-\infty, 0)$

d) $y = 69 - 24x - 9x^2 + 2x^3$

Sol:

$\because y = f(x)$

$\therefore f(x) = 69 - 24x - 9x^2 + 2x^3$

$\therefore f'(x) = -24 - 18x + 6x^2$

$\therefore f''(x) = -18 + 12x$

$\therefore$ f is concave upward iff

$f''(x) > 0$

$\therefore -18 + 12x > 0$

$\therefore 6(2x-3) > 0$

$\therefore 2x-3 > 0$

$\therefore x = 3/2$

$\therefore x \in (+3/2, \infty)$

$\therefore f$ is concave downward iff

$f''(x) < 0$

$\therefore -18 + 12x < 0$

$\therefore 6(2x-3) < 0$

$\therefore 2x-3 < 0$

$\therefore x < 3/2$

$\therefore x \in (-\infty, 3/2)$

e) $y = 2x^3 + x^2 - 20x + 4$

Sol:

$\because y = f(x)$

$\therefore f(x) = 2x^3 + x^2 - 20x + 4$

$\therefore f'(x) = 6x^2 + 2x - 20$

$\therefore f''(x) = 12x + 2$

$\therefore f$ is concave upword iff

$f''(x) > 0$

$\therefore 12x + 2 > 0$

$\therefore 2(6x+1) > 0$

$\therefore 6x + 1 > 0$

$\therefore x < -1/6$

$\therefore x \in (-1/6, \infty)$

$\therefore f$ is concave downward iff

$f''(x) < 0$

$\therefore 12x + 2 < 0$

$\therefore 2(6x+1) < 0$

$\therefore 6x + 1 < 0$

$\therefore x < -1/6$

$x \in (-\infty, -1/6)$

18/12/19

PRACTICAL NO:4

Q1)

i) $f(x) = x^2 + \frac{16}{x^2}$

$f'(x) = 2x - \frac{32}{x^3}$

for maximum/minimum

$f'(x) = 0$

$\therefore 2x - \frac{32}{x^3} = 0$

$\therefore 2x = \frac{32}{x^3}$

$\therefore x^4 = 16$

$x = \pm 2$

$f''(x) = 2 + \frac{96}{x^4}$

$\therefore f''(2) = f''(-2) = 2 + \frac{96}{(\pm 2)^4} = 2 + \frac{96}{16} = 8 > 0$

$\therefore f(x)$ is minimum at $x = \pm 2$

$\therefore f(2) = 8$ is the minimum value

ii) $f(x) = 3 - 5x^3 + 3x^5$

$f'(x) = 15x^4 - 15x^2$

for maximum/minimum

$f'(x) = 15x^4 - 15x^2 = 0$

$\therefore x^4 - x^2 = 0$

$\therefore x^2(x^2 - 1) = 0$

$\therefore x = 0, -1, 1$

$f''(x) = 60x^3 - 30x$

$f''(0) = 0$

$f''(-1) = -60 + 30 = -30 < 0$

$f''(1) = 60 - 30 = 30 > 0$

$\therefore f(x)$ is maximum at -1 & minimum at 1

$f(-1) = 3 + 5 - 3 = 5$

$f(1) = 3 - 5 + 3 = 1$

iii) $f(x) = x^3 - 3x^2 + 1$

$f'(x) = 3x^2 - 6x$

for maximum / minimum

$f'(x) = 0$

$3x^2 - 6x = 0$

$\therefore x^2 - 2x = 0$

$\therefore x(x-2) = 0$

$x = 0, 2$

$f''(x) = 6x - 6$

$f''(0) = -6 < 0$

$f''(2) = 6 > 0$

$f(x)$ is maximum at $x=0$ & minimum at $x=2$

$\therefore f(0) = 1$

$f(2) = -3$

Q2]

i) $f(x) = x^3 - 3x^2 - 55x + 9.5$

$f'(x) = 3x^2 - 6x - 55$

$x_1 = 0$

$f(x_0) = 9.5$

$f'(x_0) = -55$

$x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)}$

$= 0 - \dfrac{9.5}{-55}$

$x_1 = 0.1727$

$f(x_1) = -0.0828$

$f'(x_1) = -55.9667$

$x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)}$

$= 0.1727 - \dfrac{-0.0828}{-55.9667}$

$x_2 = 0.1712$

$f(x_2) = 0.0011$

$f'(x_2) = -55.9393$

$x_3 = x_2 - \dfrac{f(x_2)}{f'(x_2)} = 0.1712 - \dfrac{0.0011}{-55.9373}$

M

ii) $f(x) = 2x^3 - 3x^2 - 12x + 1$

$f'(x) = 6x^2 - 6x - 12$

for maximum / minimum

$\therefore f'(x) = 0$

$\therefore 6x^2 - 6x - 12 = 0$

$\therefore x^2 - x - 2 = 0$

$\therefore x^2 - 2x + x - 2 = 0$

$\therefore x(x-2) + 1(x-2) = 0$

$\therefore (x+1)(x-2) = 0$

$\therefore x = -1, 2$

$f''(x) = 12x - 6$

$f''(-1) = -12 - 6 = -18 < 0$

$f''(7) = 24 - 6 = 18 > 0$

$\therefore f(x)$ is maximum at $x=1$ & minimum at $x=2$

$\therefore f(-1) = 8$

$\therefore f(2) = -19$

iii) $f(x) = x^3 - 4x - 9$

$f'(x) = 3x^2 - 4$

$f(2) = -9$

$f(3) = 6$

$\therefore$ 6 is lower to 0 in the number lis,

$x_0 = 3$

$f(x_0) = 6$

$f'(x_0) = 23$

$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$x_1 = 2.7391$

$f(x_1) = 0.5942$

$f'(x_1) = 18.508$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$= 0.5942/18.508$

$x_2 = 2.707$

$\therefore f(x_2) = 0.0085$

$\therefore f'(x_2) = 17.9835$

$\therefore x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$= 2.707 - \frac{0.0085}{17.9835}$

$x_3 = 2.7065$

$f(x_3) = -0.0005$

$f'(x_3) = 17.9757$

$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$

$= 2.7065 - \frac{-0.0001}{17.9757}$

$\underline{\underline{x_4 = 2.7065}}$

$\therefore$ 2.7065 is the root of the given fun'

iii) $f(x) = x^3 - 1.8x^2 - 10x + 17$ $\quad [1, 2]$

$f'(x) = 3x^2 - 3.6x - 10$

$f(1) = 1 - 1.8 - 10 + 17 = 6.2$

$f(2) = -2.2$

$\therefore$ -2.2 is close to 0 on the number line

$\therefore x_0 = 2$

$f(x_0) = -2.2$

$f'(x_0) = -5.2$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$= 2 - \left(\frac{-2.2}{-5.2}\right)$

$x_1 = 1.5769$

$f(x_1) = (1.5769)^3 - 1.8(1.5769)^2 - 10(1.5769) + 17$

$= 0.6762$

$f'(x_1) = -8.217$

$\therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$= 1.5769 - \frac{0.6762}{-8.277}$

$x_2 = 1.6592$

$\therefore f(x_2) = 0.0204$

$f'(x_2) = -7.7143$

$\therefore x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$= 1.6592 - \frac{0.0205}{-7.7143}$

$x_3 = 1.6618$

$\therefore f(x_3) = (1.6618)^3 - 1.8(1.6618)^2 - 16(1.6618) + 17$

$\therefore f(x_3) = 0$

$\therefore$ 1.6618 is the root of f the function

PRACTICAL NO:5

i) $I = \int \frac{1}{\sqrt{x^2+2x-3}} \, dx$

$= \int \frac{1}{\sqrt{x^2+2x+1-4}} \, dx$

$= \int \frac{1}{\sqrt{(x+1)^2-2^2}} \, dx$

$I = \ln \left| x+1 + \sqrt{x^2+2x-3} \right| + C$

ii) $I = \int (4e^{3x}+1) \, dx$

$= 4 \int e^{3x} \, dx + \int 1 \, dx$

$I = \frac{4e^{3x}}{3} + x + C$

iii) $I = \int (2x^2 - 3\sin x + 5\sqrt{x}) \, dx$

$= 2\int x^2 \, dx - 3\int \sin x \, dx + 5\int x^{1/2} \, dx$

$= \frac{2x^3}{3} + 3\cos x + \frac{5x^{3/2}}{3/2} + C$

$I = \frac{2x^3}{3} + 3\cos x + \frac{10}{3} x\sqrt{x} + C$

iv) $\int \frac{x^3+3x+4}{\sqrt{x}} \, dx$

put $\sqrt{x} = t$

$\therefore \frac{1}{2\sqrt{x}} = \frac{dt}{dx}$ $\therefore \frac{dx}{\sqrt{x}} = 2dt$

$I = \int \frac{(\sqrt{x})^6 + 3(\sqrt{x})^2 + 4}{\sqrt{x}} \, dx$

$= 2\int t^6 + 3t^2 + 4 \, dt$

$= 2\left[\frac{t^7}{7} + \frac{3t^3}{3} + 4t\right] + C$

$= 2\left[\frac{x^{7/2}}{7} + x^{3/2} + 4x^{1/2}\right] + C$

v) $I = \int t^7 \sin(2t^4) \, dt$

$\int t^4 \sin(2t^4) \, t^3 \, dt$

put $t^4 = x$

$\therefore 4t^3 = \frac{dx}{dt}$

$\therefore t^3 \, dt = \frac{1}{4} dx$

$\therefore I = \frac{1}{4} \int x \cdot \sin(2x) \, dx$

$\therefore I = \frac{1}{4} \left[x \int \sin 2x \, dx - \int \left(\frac{dx}{dx} \int \sin 2x \, dx \right) dx \right]$

$= \frac{1}{4} \left[-\frac{x \cdot \cos 2x}{2} + \frac{1}{2} \int \cos 2x \, dx \right]$

$= \frac{1}{16}\sin 2x - \frac{t.\cos 2x}{8} + c$

$I = \frac{1}{16}\sin 2t^{4} - t^{4}\frac{-\cos 2t^{5}}{8} + c$

vi) $I = \int \sqrt{x}\,(x^2-1)\,dx$

$\therefore I = \int x^2\sqrt{x}\,dx - 1\int\sqrt{x}\,dx$

$= \int x^{5/2}\,dx - \int x^{1/2}\,dx$

$= \frac{x^{7/2}}{7/2} - \frac{x^{3/2}}{3/2} + c$

$= \frac{2}{7}x^3\sqrt{x} - \frac{2}{3}x\sqrt{x} + c$

viii) $I \int \frac{1}{x^3}\sin\left(\frac{1}{x^2}\right)dx$

$\frac{1}{x^2} = t$

$\therefore \frac{-2}{x^3} = \frac{dt}{dx}$

$\therefore \frac{dx}{x^3} = \frac{-1}{2}\,dt$

$\therefore I = \frac{-1}{2}\int \sin t\,dt$

$= \frac{-1}{2}\,c - \cos t + c$

$= \frac{\cos t}{2} + c$

$I = \cos\frac{(1/x^2)}{2} + c$

ix) $I = \int \frac{\cos x}{\sqrt[3]{\sin^2 x}}\,dx$

put $\sin x = t$

$\therefore \cos x\,dx = dt$

$\therefore I = \int \frac{1}{t^{2/3}}\,dt$

$= \int t^{-2/3}\,dt$

$= \frac{t^{-2/3+1}}{-2/3+1} + c$

$= 3t^{1/3} + c$

$I = 3\sqrt[3]{\sin x} + c$

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i) $I = \int e^{\cos^2 x} \cdot \sin 2x \, dx$

put $\cos^2 x = t$

$\therefore -2\cos x \sin x = \frac{dt}{dx}$

$\therefore \sin 2x \, dx = -dt$

$\therefore I = -\int e^t \, dt$

$= -e^t + c$

$\therefore I = -e^{\cos^2 x} + C$

PRACTICAL NO: 6.

i) $x = \sin t$ $\qquad$ $y = 1 - \cos t$

$\int \frac{dx}{dt} = + d\sin t + \sin t \, dt$ $\qquad$ $\frac{dy}{dt} = \sin t$

$\therefore \frac{dx}{dt} = -\cos t$

$L = \int_0^{2\pi} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \, dt$

$= \int_0^{2\pi} \sqrt{(1-\cos t)^2 + \sin^2 t} \, dt$

$= \int_0^{2\pi} \sqrt{1 - 2\cos t + \cos^2 t + \sin^2 t}$

$= \int_0^{2\pi} \sqrt{2 - 2\cos t} \, dt$

$\int_0^{2\pi} \sqrt{2} \sqrt{1 - \cos t} \, dt$

$= \int_0 \sqrt{2} \times \sqrt{2} \sqrt{\sin^2 t/2} \, dt$

$= 2 \int_0^{2\pi} \sin t/2 \, dt$

$= -2 \left[\frac{\cos t/2}{1/2} \right]_0^{2\pi}$

$= -4 (\cos 2\pi/2 - \cos 0)$

$= -4(-1 - 1)$

$L = 8$ units

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2) $y = \sqrt{4-x^2}$

$\therefore \frac{dy}{dx} = \frac{1}{2\sqrt{4-x^2}} \times (-2x)$

$= \frac{-x}{\sqrt{4-x^2}}$

$L = \int_0^2 \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$

$= \int_{-2}^{2} \sqrt{1+\frac{x^2}{4-x^2}}\, dx$

$= \int_{-2}^{2} \sqrt{\frac{4-x^2+x^2}{4-x^2}}\, dx$

$= \int_{-2}^{2} \sqrt{\frac{4}{4-x^2}}\, dx$

$= 2\int_{-2}^{2} \frac{1}{\sqrt{2^2-x^2}}\, dx$

$= 2\left[\sin^{-1}\left(\frac{x}{2}\right)\right]_{-2}^{2}$

$= 2\left[\sin^{-1}(1) - \sin^{-1}(-1)\right]$

$= 2\left[\frac{\pi}{2} - \left(\frac{-\pi}{2}\right)\right]$

$L = 2\pi$

3) $y = x^{3/2}$ $\quad x \in (0,4)$

Sol. $f'(x) = \frac{3}{2} x^{1/2}$

$[f'(x)]^2 = \frac{9}{4}x$

$L = \int_0^4 \sqrt{1+[f'(x)]^2}\, d$

$= \int_0^4 \sqrt{1+\frac{9}{4}x}\, dx$

put $u = 1+\frac{9}{4}x$, $du = \frac{9}{4}dx$

$L = \int \frac{4}{9}\sqrt{u}\, du = \left[\frac{4}{9}\cdot\frac{2}{3}\left(u^{3/2}\right)\right]_1 \quad \frac{-9}{4}x$

$= \frac{8}{27}\left[\left(1+\frac{9x}{4}\right)-1\right]$

(4) $x = 3\sin t$, $y = 3\cos t$ $\quad t \in (0, 2\pi)$

$\frac{dx}{dt} = 3\cos t \quad \frac{dy}{dt} = -3\sin t$

$L = \int_0^{2\pi} \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}\, dt$

$= \int_0^{2\pi} \sqrt{(3\cos t)^2 + (-3\sin t)^2}\, dt$

Hfi.

$= \int_0^{2\pi} 3\sqrt{x}\, dt$

$= \int_0^{2\pi} dt$

$= 3(2\pi - 0)$

$= l = 6\pi$ units.

(5) $x = \frac{1}{6}y^3 + \frac{1}{2y}$ on $y \in (1,2)$

$\therefore \frac{dx}{dy} = \frac{y^2}{2} - \frac{1}{2y^2}$

$\frac{dx}{dt} = \frac{y^2-1}{2y^2}$

$l = \int_1^{y} \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$

$= \int_1^{2} \sqrt{1 + \frac{(y^2-1)}{4y^2}}\, dy$

$= \int^{2} \frac{y^4+1}{2y^2}\, dy$

$= \frac{1}{2}\int_1^2 y^2 dy + \frac{1}{2} \cdot \int_1^2 y^{-2} dy$

$= \frac{1}{2}\left[\frac{y^3}{3} - \frac{y^{-2}}{1}\right]^2$

$= \frac{1}{2}\left[\frac{8}{3} - \frac{1}{2} - \frac{1}{3} + 1\right]$

$= \frac{1}{2}\left[\frac{7}{3} - \frac{1}{2}\right]$

$= \frac{1}{2}\left[\frac{17}{6}\right]$

$L = \frac{17}{12}$ units

08/01/2020

A7

PRACTICAL NO: 7

1) Solve the foll. differential equ:

i) $x\frac{dy}{dx} + y = e^x$

ii) $e^x\frac{dy}{dx} + 2e^x y = 1$

iii) $x\frac{dy}{dx} = \frac{\cos x}{x} - 2y$

iv) $x\frac{dy}{dx} + 3y = \frac{\sin x}{x^2}$

v) $e^{2x}\frac{dy}{dx} + 2e^{2x}y = 2x$

vi) $\sec^2 x + \tan y\, dx + \sec^2 y \tan x\, dy = 0$

vii) $\frac{dy}{dx} = \sin^2(x-y+1)$

viii) $\frac{dy}{dx} = \frac{2x - 3y - 1}{6x + 9y + 6}$

i) Solution:

$x\frac{dy}{dx} + y = e^x$

$\therefore \frac{dy}{dx} + \frac{1}{x}y = \frac{e^x}{x}$

$P(x) = 1/x \qquad Q(x) = e^x/x$

$IF = e^{\int P(x)dx}$

$= e^{\int 1/x\, dx}$

$= e^{\ln x}$

$IF = x$

$y \cdot (I.F) = \int Q\,(IF)\,dx + c$

$\therefore yx = \int \frac{e^x}{x} \times x\, dx + c$

$= \int e^x\, dx + c$

$\therefore xy = \underline{\underline{e^x + c}}$

ii) Solution:

$$e^{x}\frac{dy}{dx} + 2e^{x}y = 1$$

$$\therefore \frac{dy}{dx} + 2y = e^{-x}$$

$$p(x) = 2 \qquad Q(x) = e^{-x}$$

$$IF = e^{\int p(x)\,dx}$$

$$= e^{\int 2dx}$$

$$= e^{2x}$$

$$y(IF) = \int Q(x)(I.F)\,dx + c$$

$$y \cdot e^{2x} = \int e^{-x} \times e^{2x}\,dx + c$$

$$= \int e^{-x+2x}\,dx + c$$

$$= \int e^{x}\,dx + c$$

$$y e^{2x} = e^{x} + c$$

iii) Solution:

$$x\frac{dy}{dx} = \frac{\cos x}{x} - 2y$$

$$\therefore \frac{dy}{dx} + \frac{2}{x}y = \frac{\cos x}{x^2}$$

$$P(x) = 2/x \qquad Q(x) = \cos x/x^2$$

$$IF = e^{\int P(x)\,dx}$$

$$= e^{\int 2/x\,dx}$$

$$= e^{2\ln x}$$

$$= e^{\ln x^2}$$

$$IF = x^2$$

$$y(IF) = \int Q(x)(IF)\,dx + c$$

$$x^2y = \int \frac{\cos x}{x^2} \times x^2\,dx + c$$

$$= \int \cos x + c$$

$$x^2y = \sin x + c$$

iv) solution:

$$x\frac{dy}{dx} + 3y = \frac{\sin x}{x^2}$$

$$\therefore \frac{dy}{dx} + \frac{3}{x}y = \frac{\sin x}{x^3}$$

$$p(x) = 3/x \qquad Q(x) = \sin x/x^3$$

$$IF = e^{\int p(x)\,dx}$$

$$= e^{\int 3/x\,dx}$$

$$= e^{3\ln x}$$

$$= e^{\ln x^3}$$

$$IF = x^3$$

$$y(I.F) = \int Q(x)(IF)\,dx + c$$

$$x^3y = \int \frac{\sin x}{x^3} \times x^3\,dx + c$$

$$= \int \sin x\,dx + c$$

$$x^3y = \cos x + c$$

v) Soultion:

$$e^{2x}\frac{dy}{dx} + 2e^{2x}y = 2x$$

$$\therefore \frac{dy}{dx} = 2y = \frac{2x}{e^{2x}}$$

$$P(x) = 2 \qquad Q(x) = 2xe^{-2x}$$

$$IF = e^{\int P(x)\,dx}$$

$$= e^{\int 2dx}$$

$$= e^{2x}$$

$$y(IF) \int Q(x)(IF)\,dx + c$$

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ii) Solution:

$$e^{x}\frac{dy}{dx} + 2e^{x}y = 1$$

$$\therefore \frac{dy}{dx} + 2y = e^{-x}$$

$$p(x) = 2 \qquad Q(x) = e^{-x}$$

$$IF = e^{\int p(x)\,dx}$$

$$= e^{\int 2dx}$$

$$= e^{2x}$$

$$y\,(IF) = \int Q(x)(I.F)\,dx + c$$

$$y \cdot e^{2x} = \int e^{-x}\, x^{2x \cdot 4/2} + c$$

$$= \int e^{-x+2x}\,dx + c$$

$$= \int e^{x}\,dx + c$$

$$ye^{2x} = e^{x} + c$$

iii) Solution:

$$x\frac{dy}{dx} = \frac{\cos x}{x} - 2y$$

$$\therefore \frac{dy}{dx} + \frac{2}{x}y = \frac{\cos x}{x^2}$$

$$p(x) = 2/x \qquad Q(x) = \cos\, 2/x^2$$

$$IF = e^{\int P(x)\,dx}$$

$$= e^{\int 2/x\,dx}$$

$$= e^{2\ln x}$$

$$= e^{\ln x^2}$$

$$IF = x^2$$

$$y\,(IF) = \int Q(x)\,(IF)\,dx + c$$

$$x^2y = \int \frac{\cos^2}{x^2} \times x^2\,dx + c$$

$$= \int \cos x + c$$

$$x^2y = \sin x + c$$

iv) solution:

$$x\frac{dy}{dx} + 3y = \frac{\sin x}{x^2}$$

$$\therefore \frac{dy}{dx} + \frac{3}{x}y = \frac{\sin x}{x^3}$$

$$p(x) = 3/x \qquad Q(x) = \sin x/x^3$$

$$IF = e^{\int p(x)\,dx}$$

$$= e^{\int 3/x\,dx}$$

$$= e^{3\ln x}$$

$$= e^{\ln x^3}$$

$$IF = x^3$$

$$y\,(I.F) = \int Q(x)\,(IF)\,dx + c$$

$$x^3y = \int \frac{\sin x}{x^3} \times x^3\,dx + c$$

$$= \int \sin x\,dx + c$$

$$x^3y = \cos x + c$$

v) Soultion:

$$e^{2x}\frac{dy}{dx} + 2e^{2x}y = 2x$$

$$\therefore \frac{dy}{dx} = 2y = \frac{2x}{e^{2x}}$$

$$P(x) = 2 \qquad Q(x) = 2xe^{-2x}$$

$$IF = e^{\int P(x)\,dx}$$

$$= e^{\int 2dx}$$

$$= e^{2x}$$

$$y\,(IF) = \int Q(x)\,(IF)\,dx + c$$

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$\therefore 4e^{2x} = \int 2x\,dx + c$

$\therefore 4e^{2x} = x^2 + c$

vi) Solution:

$\sec^2 x \tan y\,dx + \sec^2 y \tan x\,dy = 0$

$\sec^2 x \tan y\,dx = -\sec^2 y \tan x\,dy$

$\frac{\sec^2 x\,dx}{\tan x} = -\frac{\sec^2 y\,dy}{\tan y}$

$\int \frac{\sec^2 x\,dx}{\tan x} = -\int \frac{\sec^2 y\,dy}{\tan y}$

$\therefore \log|\tan x| = -\log|\tan y| + c$

$\therefore \log|\tan x \cdot \tan y| = c$

$\tan x \cdot \tan y = e^c$

vii) Solution:

$\frac{dy}{dx} = \sin^2(x-y+1)$

put $x-y+1 = v$

Diff both sides

$x-y+1 = v$

$\therefore 1 - \frac{dy}{dx} = \frac{dv}{dx}$

$\therefore 1 - \frac{dv}{dx} = \frac{dv}{dx}$

$\therefore 1 - \frac{dv}{dx} = \sin^2 v$

PRACTICAL NO: 8

TOPIC: Eulers method

1) $\frac{dy}{dx} = y + e^x - 2 \quad y(0) = 2 \quad h = 0.5$ find $y(2) = ?$

$\rightarrow f(x) = y + e^x - 2 \quad x_0 = 0 \quad y(0) = 2 \quad h = 0.5$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	2	1	25
1	0.5	25	2.1487	3.5743
2	1	3.5743	4.2925	5.7205
3	1.5	5.7205	8.2021	9.8215
4	2	9.8215		

$y(2) = 9.8215$

2) $\frac{dy}{dx} = 1 + y^2, \quad y(0) = 1 \quad h = 0.2$ find $y(1) = ?$

$\rightarrow f(x) = 1 + y^2, \quad x_0 = 0, \quad y_0 = 0 \quad h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	0	1	0.2
1	0.2	0.2	1.04	0.408
2	0.4	0.0408	1.664	0.6412
3	0.6	0.6412	1.4111	0.9234
4	0.8	0.9234	1.8526	1.2989
5	0.1	1.2939		

$y(1) = 1.2939$

3] $\frac{dy}{dx} = \sqrt{\frac{x}{4}}$ $y(0)=1$ $h=0.2$ find $y(1)=?$

$f(x) = \sqrt{x/4}$ $x_0 = 0$ $y(0)=1$ $h=0.2$

n	x_n	y_n	$f(x_n, x_n)$	y_{n+1}
			0	1
0	0	1	0.4472	1.0894
1	0.2	1	0.8039	1.2105
2	0.4	1.0894	0.7040	1.3393
3	0.6	1.2105	0.7696	1.5051
4	0.8	1.3512		
5	1	1.505		

$\therefore y(1) = 1.505$

4] $\frac{dy}{dx} = 3x^2+1$ $y(1)=2$ find $y(2)$ $h=0.5$

$\rightarrow f(x) = 3x^2+1$, $y_0=2$ $x_0=1$ $h=0.5$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	4
1	1.5	4	7.75	7.875
2	2	7.875		

$y(2) = 7.875$

5] $\frac{dy}{dx} = \sqrt{xy} + 2$ $y(1)=1$ $h=0.2$

$\rightarrow f x = \sqrt{xy}+2$ $x_0=1$ $y_0=1$ $h=0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	1	2	3.6
1	1.2	3.6		

$y(1.2) = 3.6$

PRACTICAL NO: 9

TOPIC: Limits & partial order derivatives

Q1] Evaluate the foll limits

1) $\lim_{(x,y)\to(-4,-1)} \frac{x^3-3x+y^2-1}{xy+5}$

$= \lim_{(x,y)\to(-4,-1)} \frac{x^3-3x+y^2-1}{xy+5}$

Apply limit

$= \frac{(-4)^3-3(-4)+(-1)^2-1}{(-4)(-1)+5} = \frac{-64+12+01-1}{4+5} = -\frac{52}{9}$

2) $\lim_{(x,y)\to(2,0)} \frac{(y+1)(x^2+y^2-4x)}{x+3y}$

$= \lim_{(x,y)\to(2,0)} \frac{(y+1)(x^2+y^2-4x)}{x+3y}$

Apply limit

$= \frac{(0+1)\left[(2)^2+(0)^2-4(2)\right]}{2+3(0)} = \frac{1(4+0-8)}{2} = \frac{4-8}{2} = \frac{-4}{2} = -2$

3] $\lim_{(x,y,z)\to(1,1,1)} \frac{x^3-y^2z^2}{x^3-x^2-y^2}$

$= \lim_{(x,y,z)\to(1,1,1)} \frac{x^2-y^2-z^2}{x^2x^2y^2}$

Apply limit

$= \frac{(1)^2-(1)^2(1)^2}{(1)^3-(1)^2(1)(1)} = \frac{1-1}{1-1} = \frac{0}{0}$ $\therefore$ limit does not exist

Q2] Find f_x, f_y for each of the foll. f

i) $f(x,y) = xye^{x^2+y^2}$

$f_x = y\left(1e^{x}+y^2\right) + xy\left(e^{x^2+y^2}\cdot 2x\right)$

$= ye^{x^2+y^2} + 2x^2y\, e^{x^2+y^2}$

$f_y = x\left(1\cdot e^{x^2+y^2}\right) + xy\left(e^{x^2+y^2}\cdot 2y\right)$

$= x\cdot e^{x^2+y^2} + 2xy^2\, e^{x^2+y^2}$

$f_x = ye^{x^2+y^2} + 2x^2y\cdot e^{x^2+y^2}$

$f_y = xe^{x^2+y^2} + 2xy^2 e^{x^2+y^2}$

ii] $f(x,y) = e^x\cos y$

$f_x = \cos y e^x$

$f_y = e^x - \sin y$

$\therefore f_y = -\sin y e^x$

iii) $f(x,y) = x^3y^2 - 3x^2y + y^3 + 1$

$f_x = y^2 3x^2 - 3y^2x + 6 + 0$

$= 3x^2y^2 - 6xy$

$f_y = x^3 2y - 3x^2 + 3y^2$

$= 2x^3y - 3x^2 + 3y^2$

Q3] Using definition find value of f_x, f_y at $(0,0)$ for

$f(x,y) = \frac{2x}{1+y^2}$

$f_x(a,b) = \lim_{h\to 0} \frac{f(a+h,b) - f(a,b)}{h}$

$f_y(a,b) = \lim_{h\to 0} \frac{f(a,b+h) - f(a,b)}{h}$

According to given $(a,b) = (0,0)$

$f_x(0,0) = \lim_{h\to 0} \frac{f(h,0) - f(0,0)}{h}$

$= \lim_{h\to 0} \frac{2h-0}{h} = 2$

$f_y(0,0) = \lim_{h\to 0} \frac{f(0,h) - f(0,0)}{h}$

$= \lim_{h\to 0} \frac{0-0}{h} = 0$

$\therefore f_x = 2, \ f_y = 0$

$f_x(0,0) = \lim_{h\to 0} \frac{f(h,0) - f(0,0)}{h}$

$= \lim_{h\to 0} \frac{2h-0}{h} = 2$

$f_y(0,0) = \lim_{h\to 0} \frac{f(0,h) - f(0,0)}{h}$

$= \lim_{h\to 0} \frac{0-0}{h} = 0$

$\therefore f_x = 2, \ f_y = 0$

Q4] Find all second order partial derivatives of f also verify whether $f_{xy} = f_{yx}$

1] $f(x,y) = \frac{y^2 - xy}{x^2}$

$f_{xx} = \frac{d^2 f}{dx^2}$, $f_{yy} = \frac{d^2 f}{dy^2}$

Applying $\frac{u}{v}$ rule

$f_x = \frac{x^2(0-y) - (y^2 - xy)2x}{x^4}$

$= \frac{-x^2y - 2xy^2 + 2x^2y}{x^4}$

$\therefore f_x = \frac{x^2y - 2xy^2}{x^4}$

$f_{xx} = \frac{x^4(2xy - 2y^2) - (x^2y - 2xy^2)(4x^3)}{x^8}$

$= \frac{2x^5y - 2x^4y^2 - (4x^5y - 8x^4y^2)}{x^8}$

$= \frac{2x^5y - 2x^4y^2 - (4x^5y - 8x^4y^2)}{x^8}$

$= \frac{2x^5y - 2x^4y^2 - 4x^5y + 8x^4y^2}{x^8}$

$= \cancel{\frac{2x^5y + 6x^4y^2}{x^8}}$

$f_{xx} = \frac{6y^2 - 2xy}{x^4}$

$f_y = \frac{1}{x^2}[2y - x] = \frac{2y - x}{x^2}$

$f_{xy} = \frac{2y - x}{x^2}$

$= \frac{x^2(-1) - (2y-x)(2x)}{}$

$$= \frac{-x^2 \cdot 4xy + 2x^2}{x^4}$$

$$= \frac{x^2 - 4xy}{x^4}$$

$$= \frac{x - 4y}{x^3}$$

$$\therefore f_{xy} = \frac{x-4y}{x^3}$$

$$f_{yx} = \frac{x^2y - 2xy}{x^4}$$

$$= \frac{x^2 - 4xy}{x^4}$$

$$= \frac{x-4y}{x^3}$$

$$= f_{xy}$$

$\therefore f_{yx} = f_{xy}$

Hence, ~~move~~ verified

Q5] Find the linearization of $f(x,y)$ at given point

i) $f(x,y) = 1 - x + y\sin x$ at $\frac{\pi}{2}$

$$f(\pi/2, 0) = 1 - \frac{\pi}{2} + 0 + \sin\frac{\pi}{2}$$

$$f\left(\frac{\pi}{2}, 0\right) = \frac{2+\pi}{2}$$

$$f_x = -1 + y\cos x$$

$$f_x\left(\frac{\pi}{2}, 0\right) = -1 + 0 \cdot \cos\frac{\pi}{2}$$

$$= -1$$

$$f_y\left(\frac{\pi}{2}, 0\right) = \sin\frac{\pi}{2}$$

$$= 1$$

$$L(x,y) = f\left(\frac{\pi}{2}, 0\right) + f_x\left(\frac{\pi}{2}, 0\right)(x - \pi/2) + f_y\left(\frac{\pi}{2}, 0\right)(y - 0)$$

$$= \frac{2-\pi}{2} + (-1)\left(x - \frac{\pi}{2}\right) + 1(y)$$

$$= 1 - \frac{\pi}{2} - x + \frac{\pi}{2} + y$$

$$L(x,y) = 1 - x + y$$

iii) $f(x,y) = \log x + \log y$

$$f(1,1) = \log 1 + \log 1$$

$$= 0$$

$$f_x = \frac{1}{x} \qquad f_y = \frac{1}{y}$$

$$f_x(1,1) = 1 \qquad f_y(1,1) = 1$$

$$\therefore L(x,y) = f(1,1) + f_x(1,1)(x-1) + f_y(1,1)(y-1)$$

$$= 0 + 1(x-1) + 1(y-1)$$

$$L(x,y) = x + y - 2$$

22/01/2020

PRACTICAL NO-10

Aim: Directional derivative, gradient vector & maxima, minima & tangent & normal.

Q1) Find directional derivative of the following function at given points & in the direction of given vector.

i) $f(x,y) = x + 2y - 3$ $\quad a = (1,-1)$ $\quad u = 3i - j$

Here $u = (3i - j)$

$|u| = \sqrt{9+1} = \sqrt{10}$

unit vector along $\frac{u}{|u|} = \frac{1}{\sqrt{10}}(3,-1)$

$= \frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}$

$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right) - 3$

$= f(1,-1) + \left(\frac{3h}{\sqrt{10}}, \frac{-h}{\sqrt{10}}\right) - 3$

$= f\left(1 + \frac{3h}{\sqrt{10}}, -1 - \frac{h}{\sqrt{10}}\right) - 3$

$= 1 + \frac{3h}{\sqrt{10}} - 2 - \frac{2h}{\sqrt{10}} - 3$

$f(a+hu) = -4 + \frac{h}{\sqrt{10}}$

$Df(a) = \lim_{h \to 0} \frac{f(a+hu) - f(a)}{h}$ $\qquad f(a) = (1,-1)$

$= 1 - 2 - 3$

$= 1 - 5 = -4$

$= \lim_{h \to 0} \frac{-4 + h/\sqrt{10} + 4}{h}$

$= \frac{1}{\sqrt{10}}$

ii) $f(x,y) = y^2 - 4x + 1$ $\quad a = (3,4)$ $\quad u = i + 5j$

Here, $u = (1,5)$

$|u| = \sqrt{1+25} = \sqrt{26}$

$= \frac{u}{|u|} = \frac{1}{\sqrt{26}}(1,5)$

$f(a) = (4)^2 - 4(3) + 1$

$= 16 - 12 + 1$

$= 4 + 1 = 5$

$f(a+hu) = f(3,4) + h\left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$

$= f\left(3 + \frac{h}{\sqrt{26}}, 4 + \frac{5h}{\sqrt{26}}\right)$

$= \left(4 + \frac{5h}{\sqrt{26}}\right)^2 - 4\left(3 + \frac{h}{\sqrt{26}}\right) + 1$

$= 16 + \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - 12 - \frac{4h}{\sqrt{26}} + 1$

$= \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - \frac{4h}{\sqrt{26}} + 5$

$= \frac{25h^2}{26} + \frac{40h - 4h}{\sqrt{26}} + 5$

$= \frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5$

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iii) $f(x,y) = 2x + 3y$ $a = (1,2)$ $u = 3i + 4j$

$a = (3,4)$

$|u| = \sqrt{9+16} = \sqrt{25} = 5$

$\frac{u}{|u|} = \frac{1}{5}(3,4)$

$= \frac{3}{5}, \frac{4}{5}$

$f(a+hu) = f(1,2) + h\left(\frac{3}{5}, \frac{4}{5}\right)$

$= f\left(1 + \frac{3h}{5}, 2 + \frac{4h}{5}\right)$

$= 2\left(1 + \frac{3h}{5}\right) + 3\left(2 + \frac{4h}{5}\right)$

$= 2 + \frac{6h}{5} + 6 + \frac{12h}{5} = 8 + \frac{18h}{5}$

$f(a) = 2(1) + 3(2) = 2 + 6 = 8$

$\frac{f(a+hu) - f(a)}{h} = \frac{8 + \frac{18h}{5} - 8}{h} = 18/5$

Q) Find gradient vector for foll function at given point:

i) $f(x,y) = x^2 + y^x$, $a = (1,1)$

$f_x = y \cdot x^{y-1} + y^x \log y$

$f_y = x^y \log x + x y^{x-1}$

$\nabla f(x,y) = (y \cdot x^{y-1} + y^x \log y, x^y \log x + x y^{x-1})$

$f(1,1) = (1+0, 1+0)$

$= (1,1)$

ii) $f(x,y) = (\tan^{-1}x) \cdot y^2$ $a = (1,-1)$

$f_x = y^2 \left(\frac{1}{1+x^2}\right)$

$f_y = \tan^{-1}x \, 2y$

$f(x,y) = \left(\frac{y^2}{1+x^2}, \tan^{-1}x \, 2y\right)$

$f(1,-1) = \left[\frac{1}{2}, \tan^{-1}(1)(-2)\right)$

$= \frac{1}{2}, \frac{\pi}{4}(-2))$

$= \left(\frac{1}{2}, -\frac{\pi}{2}\right)$

iii) $f(x,y) = xyz - e^{x+y+z}$ at $(1,-1,0)$

$f_x = yz - e^{x+y+z}$

$f_y = xz - e^{x+y+z}$

$f_z = xy - e^{x+y+z}$

$f(x,y,z) = (yz - e^{x+y+z}, xz - e^{x+y+z}, xy - e^{x+y+z})$

$f(1,-1,0) = (1(0), e^{1-1+0}, (1)(0) - e^{1-1+0}, (1)(-1) - e^{1-1+0})$

$= (0-1, 0-1, (-1)-1)$

$= (-1, -1, -2)$

Q3) Find the equation of tangent & normal to each of the foll curve at given point

i) $x^2\cos y + e^{xy} = 2$ at $(1,0)$

$f_x = \cos y \, 2x + e^{xy} y$

$f_y = x^2(-\sin y) + e^{xy} x$

$(x_0, y_0) = (1,0)$

Eq. of tangent

$f_x(x - x_0) + f_y(y - y_0) = 0$

$x_0, y_0 = \cos 0 \; 2(1) + e^0(0)$

$= 1(2) + 0$

$= 2$

$f_y(x_0, y_0) = (1)^2(-\sin 0) + e^0(1)$

$= 0 + 1$

$= 1$

$2(x-1) + 1(y-0) = 0$

$2x - 2 + y = 0$

$2x + y - 2 = 0$

It is required Eq. of tangent

Eq. of Normal

$= ax + by + c = 0$

$= bx + ay + d = 0$

$1(1) + 2(y) + d = 0$

$1 + 2y + d = 0$ at $(1,0)$

$= 1 + 2(0) + d = 0$

$d + 1 = 0$ $d = -1$

ii) $x^2 + y^2 - 2x + 3y + 2 = 0$ at $(2,-2)$

$f(x) = 2x + 0 - 2 + 0 + 0$

$= 2x - 2$

$f_y = 0 + 2y - 0 + 3 + 0$

$= 2y + 3$

$(x_0, y_0) = (2,-2)$ $\therefore x_0 = 2, y_0 = -2$

$f(x)(x_0, y_0) = 2(2) - 2 = 2$

$f_y(x_0, y_0) = 2(-2) + 3 = -1$

Eq. of tangent

$f(x)(x - x_0) + f_y(y - y_0) = 0$

$2(x-2)(1(y+2)) = 0$

$2x - 2 - y - 2 = 0$

$2x - y - 4 = 0$ It is required eq.

Equation of normal

$= ax + by + c = 0$

$bx + ay + d = 0$

$= (1)(x) + 2(y) + d = 0$

$-x + 2y + d = 0$ at $(2,-2)$

$-2 + 2(-2) + d = 0$

$-2 - 4 + d = 0$

$-6 + d = 0$

$\therefore d = 6$

Q4] find the Eq. of tangent & normal are the each of foll. surface

i) $x^2 - 2yz + 3y + xz = 7$ at $(2,1,0)$

$f_x = 2x - 0 + 0 + z$

$f_x = 2x + z$

$f_y = b - 2z + 3 + 0$

$= 2z + 3$

$f_z = 0 - 2y + 0 + x$

$= -2y + x$

$(x_0, y_0, z_0) = (2,1,0)$

$\therefore x_0 = 2,\ y_0 = 1,\ z_0 = 0$

$f_x(x_0, y_0, z_0) = 2(2) + 0 = 4$

$f_y(x_0, y_0, z_0) = 2(0) + 3 = 3$

$f_z(x_0, y_0, z_0) = -2(1) + 2 = 0$

Eq. of tangent

$f_x(x_0 - x_0) + f_y(y_0 - y_0) + f_z(z_0 - z_0) = 0$

$= 4(x-2) + 3(y-1) + 0 \cdot (z-0) = 0$

$= 4x - 8 + 3y - 3 = 0$

$= 4x + 3y - 11 = 0$

required Equation of tangent

Equation of normal at $(4,3,-11)$

$\frac{x - x_0}{f_x} = \frac{y - y_0}{f_y} = \frac{z - z_0}{f_z}$

$= \frac{x-2}{4} = \frac{y-1}{3} = \frac{z+11}{0}$

Q5] Find the local maxima & minima for foll

i) $f(x,y) = 3x^2 + y^2 - 3xy + 6x - 4y$

$f_x = 6x + 0 - 3y + 6 - 0$

$= 6x - 3y + 6$

$f_y = 0 + 2y - 3x + 0 - 4$

$= 2y - 3x - 4$

$f_x = 0$

$6x - 3y + 6 = 0$

$3(2x - y + 2) = 0$

$2x - y + 2 = 0$

$2x - y = -2$ —— (1)

$f_y = 0$

$2y - 3x - 4 = 0$

$2y - 3x = 4$ —— (2)

Multiply Eq (1) with 2

$4x - 2y = -4$

$2y - 3x = 4$

$\boxed{x = 0}$

Substitute value of x in Eq(1)

$2(0) - y = -2$

$= y = -2$

$\therefore \boxed{y = 2}$

$\therefore$ critical point are $(0,2)$

$r = f_{xx} = 6$
$t = f_{yy} = 2$
$s = f_{xy} = -3$
thus $r > 0$
$= rt - s^2$
$= 6(2) - (3)^2$
$= 12 - 9$
$= 3 > 0$

$\therefore f$ has maximum at $(0,2)$
$\therefore 3x^2 + y^2 - 3xy + 6x - 4y$ at $(0,2)$
$3(0)^2 + (2)^2 - 8(0)(2) + 6(0) - 4(2)$
$= 0 + 4 - 0 + 0 - 8 = -4$

ii) $f(x,y) = 2x^4 + 3x^2y - y^2$
$f_x = 8x^3 + 6xy$
$f_y = 3x^2 - 2y$
$f_x = 0$
$\therefore 8x^3 + 6xy = 0$
$\therefore 2x(4x^2 + 3y) = 0$
$\therefore 4x^2 + 3y = 0$ — ①
$f_y = 0$
$3x^2 - 2y = 0$ — ②
Multiply Eq ① with 3 & Eq. 2 with 4
$12x^2 + 9y = 0$
$-12x^2 - 8y = 0$
$\boxed{y = 0}$
Substitute value of y in Eq. ①
$4x^2 + 3(0) = 0$
$4x^2 = 0$
$\boxed{x = 0}$

Critical point is $(0,0)$
$r = f_{xx} = 24x^2 + 6x$
$t = f_{yy} = 0 \cdot 2 = -2$
$s = f_{xy} = 6x - 0 = 6x = 6(0) = 0$

r at $(0,0)$
$= 24(0) + 6(0) = 0$
$\therefore r = 0$
$rt - s^2 = 0(-2) - (0)^2$
$0 - 0 = 0$
$r = 0$, & $rt - s^2 = 0$
$f(x,y)$ at $(0,0)$ nothing to say.
$2(0)^4 + 3(0)^2(0) - 0$
$0 + 0 - 0 = 0$

iii) $f(x,y)$ $x^2 - y^2 + 2x + 8y - 70$
$f_x = 2x + 2$
$f_y = -2y + 8$
$f_x = 0$ $\therefore 2x + 2 = 0$
$x = \frac{-2}{2} = -1$
$f_y = 0$ $-2y + 8 = 0$
$y = \frac{-8}{-2} = 4$
Critical point is $(-1, 4)$
$r = f_{xx} = 2$
$t = f_{yy} = -2$
$s = f_{xy} = 0$
$rt - s^2 = 2(-2) - (0)^2$
$= -4 - 0$
$= -4 < 0$

f_{xy} at $(-1,4)$
$(-1)^2 - (4)^2 + 2(-1) + 8(4) - 70$
$1 + 16 - 2 + 32 - 70$
$= 17 + 30 - 70$
$= 37 - 70$
$= -33$

AV
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