Rectangle Dissection

Find all values of **n > 1** for which one can dissect a rectangle into **n** right triangles, and outline an algorithm for doing such a dissection.

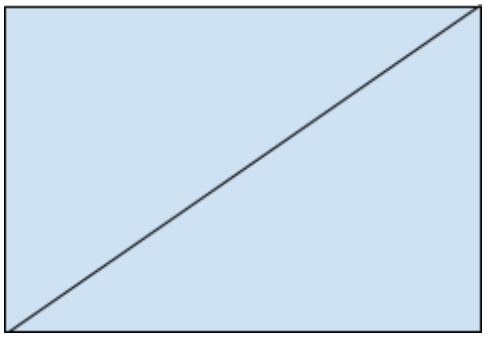
Hint: Triangles in question need not be of the same size.

Solution:

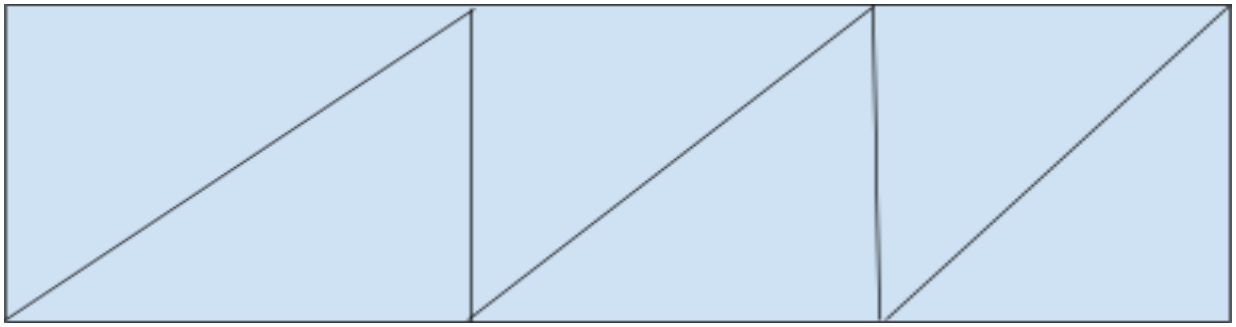
Any number larger than 1.

For n = even number:
We first divide it into n/2 small rectangles,
And then we cut the small rectangles diagonally in half.
E.g. n=2

n/2 = 2/2 = 1
So we have one rectangle, no need to divide it into smaller rectangles.
And we cut it diagonally in half, we get two right angled triangles.

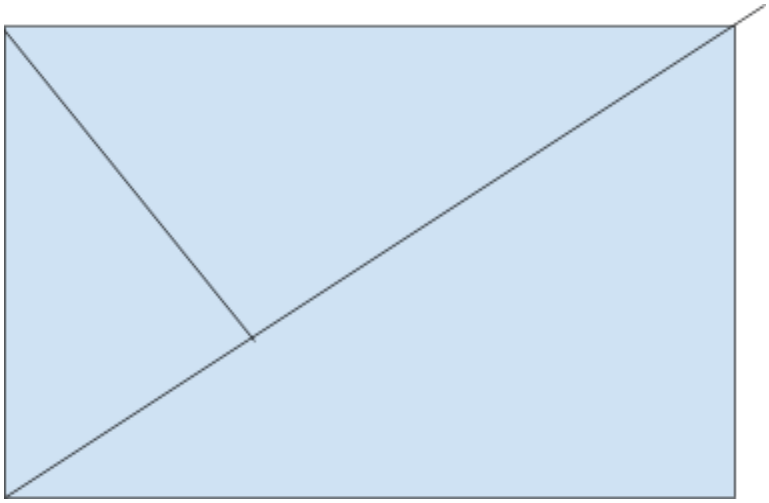


E.g n=6
n/2 = 6/2 = 3
So we have 3 smaller rectangles in the rectangle given.
We cut each rectangle diagonally in half, we get 6 rectangles.

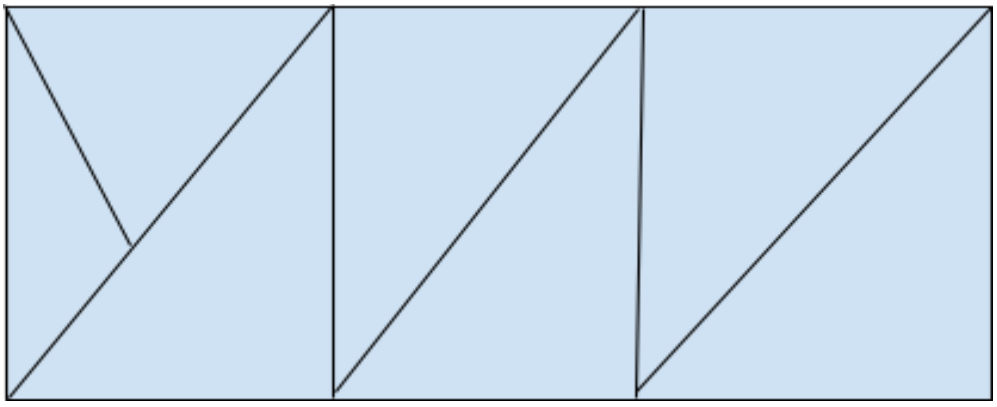


For n = odd number:
Eg. n = 3

We can use the same approach as even number and add one cut to one of the right angled triangles to make two right angled triangles.

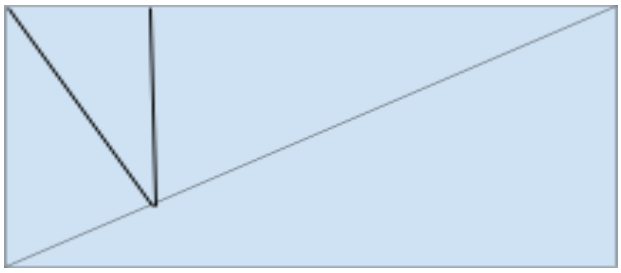


Eg. n = 7
We can use the same approach as even number and add one cut to one of the right angled triangles to make two right angled triangles.

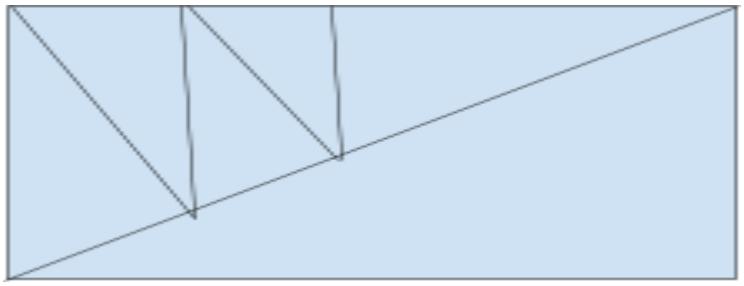


Method 2 for cutting n = even number:

E.g. n = 4
after dissecting the rectangle in half diagonally,
We cut (n-2) cuts to make three right angled triangles.



E.g. n = 6
Cutting the rectangle diagonally
And cut one of the right angled triangles with (n-2) cuts

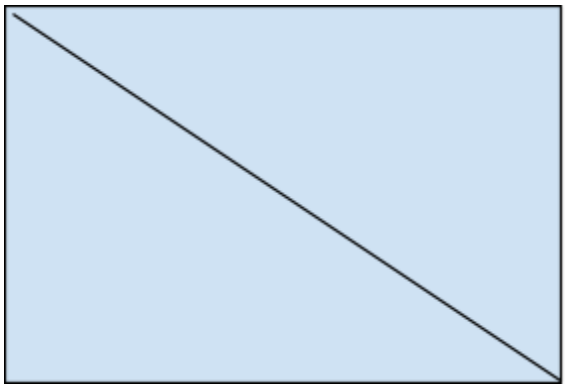


PSEUDO CODE

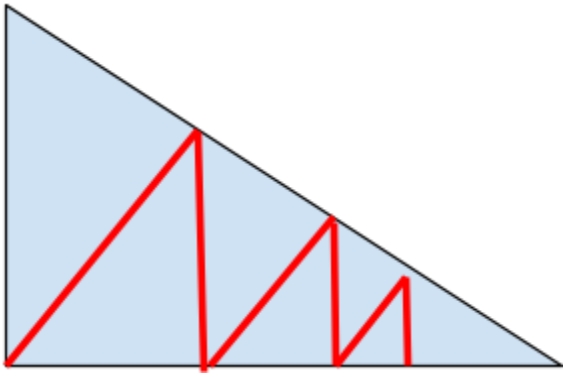
Calculate how many right triangles we can make in a rectangle

Let n = number of right triangles in a rectangle

1. Define the width and length of the rectangle
2. <u>Rectangle Dissection:</u> Cut the rectangle diagonally
    a. n=2
3. To cut additional triangles we will only cut 1 triangle but will mirror on another triangle. Therefore, every cut in this triangle is **$n = x^2$**
    a. Let x = number of right-triangle in the triangle
4. <u>Triangle Dissection:</u> Cut the Triangle diagonally
    a. Let y = number of slices in a triangle. Each slice produce 1 additional right-triangle
    b. Loop until satisfied
    c. Let **x = (y+1)**, where y is the number of slices performed.



Rectangle Dissection : **$n = x^2$**



Triangle Dissection: **x = (y+1)**

In this example we have y = 6

Therefore: **x = (6+1)**
The number of the right triangles in this triangle is 7
Therefore: **$n = (7)^2$**
The number of the right-triangles in this square is 14