Assignment 4

Ans 1 a) Minimize $z = x_1 - x_2 + 3x_3$

Subject to:

(i) $x_1 + x_2 + x_3 \leq 10$

(ii) $2x_1 - x_2 - x_3 \leq 2$

(iii) $2x_1 - 2x_2 - 3x_3 \leq 6$

$x_1, x_2, x_3 \geq 0$

Solution

Converting into equation form:

Minimize $z = x_1 - x_2 + 3x_3 + 0x_4 + 0x_5 + 0x_6$

Subject to:

(i) $x_1 + x_2 + x_3 + x_4 = 10$

(ii) $2x_1 - x_2 - x_3 + x_5 = 2$

(iii) $2x_1 - 2x_2 - 3x_3 + x_6 = 6$

∴ dual form

Maximize $w = 10y_1 + 2y_2 + 6y_3$

Subject to: (i) $y_1 + 2y_2 + 2y_3 \leq 1$

(ii) $y_1 - y_2 - 2y_3 \leq -1$

(iii) $y_1 - y_2 - 3y_3 \leq 3$

$$\left.\begin{array}{ll} \text{(iv)} & y_1 + 0y_2 + 0y_3 \leq 0 \\ \text{(v)} & 0y_1 + y_2 + 0y_3 \leq 0 \\ \text{(vi)} & 0y_1 + 0y_2 + y_3 \leq 0 \end{array}\right\} \Rightarrow y_1, y_2, y_3 \leq 0$$

y_1, y_2, y_3 are unrestricted

(b) Maximize $z = 3x_1 - 2x_2 + 4x_3$

Subject to:

(i) $3x_1 + 5x_2 + 4x_3 \geq 7$

(ii) $6x_1 + x_2 + 3x_3 \geq 4$

(iii) $7x_1 - 2x_2 - x_3 \leq 10$

(iv) $x_1 - 2x_2 + 5x_3 \geq 3$

(v) $4x_1 + 7x_2 - 2x_3 \geq 2$

$x_1, x_2, x_3 \geq 0$

<u>solution</u> Converting into eq^n form (not considering artificial variables)

Maximize $Z = 3x_1 - 2x_2 + 4x_3 + 0x_4 + 0x_5 + 0x_6 + 0x_7 + 0x_8$

subject to:

(i) $3x_1 + 5x_2 + 4x_3 - x_4 = 7$

(ii) $6x_1 + x_2 + 3x_3 - x_5 = 4$

(iii) $7x_1 - 2x_2 - x_3 + x_6 = 10$

(iv) $x_1 - 2x_2 + 5x_3 - x_7 = 3$

(v) $4x_1 + 7x_2 - 2x_3 - x_8 = 2$

$x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8 \geq 0$

$\therefore$ Dual form: Minimize $W = 7y_1 + 4y_2 + 10y_3 + 3y_4 + 2y_5$

Subject to:

(i) $3y_1 + 6y_2 + 7y_3 + y_4 + 4y_5 \geq 3$

(ii) $5y_1 + y_2 - 2y_3 - 2y_4 + 7y_5 \geq -2$

(iii) $4y_1 + 3y_2 - y_3 + 5y_4 - 2y_5 \geq 4$

(iv) $-y_1 \geq 0$

(v) $-y_2 \geq 0$

(vi) $y_3 \geq 0$

(vii) $-y_4 \geq 0$

(viii) $-y_5 \geq 0$

$\Rightarrow y_1, y_2, y_4, y_5 \leq 0$, $y_3 \geq 0$

y_1, y_2, y_3, y_4, y_5 unrestricted

Spiral

(c) Minimize $Z = x_1 - 3x_2 - 3x_3$

Subject to:

(i) $3x_1 - x_2 + 2x_3 \leq 7$

(ii) $2x_1 - 4x_2 \geq 12$

(iii) $-4x_1 + 3x_2 + 8x_3 = 10$

$x_1, x_2 \geq 0$, x_3 unrestricted.

Solution converting into eq^n form:

Since x_3 is unrestricted, we write $x_3 = x_3^- - x_3^+$

we do not consider artificial variables here, as they do not affect the dual form.

Minimize $Z = x_1 - 3x_2 - 3x_3^- + 3x_3^+ + 0x_4 + 0x_5$

Subject to

(i) $3x_1 - x_2 + 2x_3^- - 2x_3^+ + x_4 = 7$

(ii) $2x_1 - 4x_2 - x_5 = 12$

(iii) $-4x_1 + 3x_2 + 8x_3^- - 8x_3^+ = 10$

$x_1, x_2, x_3^-, x_3^+, x_4, x_5 \geq 0$

∴ Dual form: Maximize $W = 7y_1 + 12y_2 + 10y_3$

Subject to

(i) $3y_1 + 2y_2 - 4y_3 \leq 1$

(ii) $-y_1 - 4y_2 + 3y_3 \leq -3$

(iii) $2y_1 + 8y_3 \leq -3$

(iv) $-2y_1 - 8y_3 \leq 3$

$\Rightarrow 2y_1 + 8y_3 = -3$

(v) $y_1 \leq 0$

(vi) $-y_2 \leq 0$

$\Rightarrow y_1 \leq 0$, $y_2 \geq 0$

y_3 unrestricted

~~y_1, y_2, y_3 unrestricted~~

Ans 2

(a) Maximize $z = -3x_1 - 2x_2$

(i) $x_1 + x_2 \geq 1$

(ii) $x_1 + x_2 \leq 7$

(iii) $x_1 + 2x_2 \geq 10$

(iv) $x_2 \leq 3$

$x_1, x_2 \geq 0$

Solution: Converting all constraints to $\leq$ type and then into equation form.

Maximize $z = -3x_1 - 2x_2$

(i) $-x_1 - x_2 + x_3 = -1$

(ii) $x_1 + x_2 + x_4 = 7$

(iii) $-x_1 - 2x_2 + x_5 = -10$

(iv) $x_2 + x_6 = 3$

$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$

Basic	x_1	x_2	x_3	x_4	x_5	x_6	solution
z	3	2	0	0	0	0	0
x_3	-1	-1	1	0	0	0	-1
x_4	1	1	0	1	0	0	7
x_5	-1	-2	0	0	1	0	-10
x_6	0	1	0	0	0	1	3
Ratio	3	1	-	-	-	-	

Basic	x_1	x_2	x_3	x_4	x_5	x_6	Solution
z	2	0	0	0	1	0	-10
x_3	$-1/2$	0	1	0	$-1/2$	0	4
x_4	$1/2$	0	0	1	$1/2$	0	2
x_2	$1/2$	1	0	0	$-1/2$	0	5
x_6	$-1/2$	0	0	0	$1/2$	1	-2
Ratio	4	-	-	-	-	-	

Basic	x_1	x_2	x_3	x_4	x_5	x_6	Solution
z	0	0	0	0	3	4	-18
x_3	0	0	1	0	-1	-1	6
x_4	0	0	0	1	1	1	0
x_2	0	1	0	0	0	1	3
x_1	1	0	0	0	-1	-2	4
Ratio							

No leaving variable

$\therefore$ Optimum Solution: $\boxed{x_1 = 4}$ $\boxed{x_2 = 3}$

$x_3 = 6$
$x_4 = 0$
$x_5 = 0$
$x_6 = 0$

(b) Minimize $z = 3x_1 + x_2$

Subject to

(i) $x_1 + x_2 \geq 1$

(ii) $2x_1 + 3x_2 \geq 2$

$x_1, x_2 \geq 0$

Solution: Converting all constraints to $\leq$ type and then into equation form.

Minimize $z = 3x_1 + x_2$

(i) $-x_1 - x_2 + x_3 = -1$

(ii) $-2x_1 - 3x_2 + x_4 = -2$

$x_1, x_2, x_3, x_4 \geq 0$

Basic	x_1	x_2	x_3	x_4	solution
z	-3	-1	0	0	0
x_3	-1	-1	1	0	-1
x_4	-2	-3	0	1	-2
Ratio	3/2	1/3	-	-	

Basic	x_1	x_2	x_3	x_4	Solution
z	-7/3	0	0	-1/3	2/3
x_3	-1/3	0	1	-1/3	-1/3
x_2	2/3	1	0	-1/3	2/3
Ratio	7	-	-	1	

Basic	x_1	x_2	x_3	x_4	solution
z	-2	0	-1	0	1
x_4	1	0	-3	1	1
x_2	1	1	-1	0	1
Ratio					

No leaving variable

∴ optimum solution: $\boxed{x_1 = 0 \quad x_2 = 1}$ $x_3 = 0$ $x_4 = 1$

(c) Minimize $z = -2x_1 - 2x_2 - 4x_3$

Subject to:

(i) $2x_1 + 3x_2 + 5x_3 \geq 2$

(ii) $3x_1 + x_2 + 7x_3 \leq 3$

(iii) $x_1 + 4x_2 + 6x_3 \leq 5$

$x_1, x_2, x_3 \geq 0$

Solution: Converting all constraints to $\leq$ type and then into eq^n form.

Minimize $z = -2x_1 - 2x_2 - 4x_3$

(i) $-2x_1 - 3x_2 - 5x_3 + x_4 = -2$

(ii) $3x_1 + x_2 + 7x_3 + x_5 = 3$

(iii) $x_1 + 4x_2 + 6x_3 + x_6 = 5$

$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$

Basic	x_1	x_2	x_3	x_4	x_5	x_6	Solution
z	2	2	4	0	0	0	0
x_4	-2	-3	-5	1	0	0	-2
x_5	3	1	7	0	1	0	3
x_6	1	4	6	0	0	1	5
Ratio	1	2/3	4/5	-	-	-	

First we remove infeasibility using dual simplex method

Basic	x_1	x_2	x_3	x_4	x_5	x_6	Solution	Ratio
z	2/3	0	2/3	2/3	0	0	-4/3	
x_2	2/3	1	5/3	-1/3	0	0	2/3	-2
x_5	7/3	0	16/3	1/3	1	0	7/3	-7
x_6	-5/3	0	-2/3	4/3	0	1	7/3	7/4
Ratio								

Since the solution is feasible, we can now use primal simplex method to find optimal solution.

Basic	x_1	x_2	x_3	x_4	x_5	x_6	Solution	Ratio
z	$3/2$	0	1	0	0	$-1/4$	$-5/2$	
x_2	$1/4$	1	$3/2$	0	0	$1/4$	$5/4$	5
x_5	$11/4$	0	$11/2$	0	1	$-1/4$	$7/4$	$7/11$
x_4	$-5/4$	0	$-2/4$	1	0	$3/4$	$7/4$	$-7/5$

Basic	x_1	x_2	x_3	x_4	x_5	x_6	Solution	Ratio
z	0	0	-2	0	$-6/11$	$-5/44$	$-38/11$	
x_2	0	1	1	0	$-1/11$	$3/11$	$12/11$	
x_1	1	0	2	0	$4/11$	$-1/11$	$7/11$	
x_4	0	0	2	1	$5/11$	$7/11$	$28/11$	

No entering variable

$\therefore$ optimum solution: $\boxed{x_1 = 7/11 \quad x_2 = 12/11 \quad x_3 = 0}$ $\quad x_4 = 28/11, \quad x_5 = 0, \quad x_6 = 0$

$$\Rightarrow x_1 = 7/11$$
$$x_2 = 12/11$$
$$x_3 = 0$$

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