1.

	D_1	D_2	D_3	D_4	supply (Availability)
S_1	19	30	50	10	7
S_2	70	30	40	60	9
S_3	40	8	70	20	18
Demand (Requirement)	5	8	7	14	34

(a) Model Formulation : Let x_{ij} = number of units of product to be transported from a production facility i (i = 1, 2, 3) to warehouse j (j = 1, 2, 3, 4).

The transportation problem is stated as an LP model as follows:

Minimize total transportation cost, Z

$$Z = 19x_{11} + 30x_{12} + 50x_{13} + 10x_{14} + 70x_{21} + 30x_{22} + 40x_{23} + 60x_{24} + 40x_{31} + 8x_{32} + 70x_{33} + 20x_{34}$$

subject to the constraints,

$$\left.\begin{aligned} x_{11} + x_{12} + x_{13} + x_{14} &= 7 \\ x_{21} + x_{22} + x_{23} + x_{24} &= 9 \\ x_{31} + x_{32} + x_{33} + x_{34} &= 18 \end{aligned}\right\} \text{supply}$$

$$\left.\begin{array}{l} x_{11} + x_{21} + x_{31} = 5 \\ x_{12} + x_{22} + x_{32} = 8 \\ x_{13} + x_{23} + x_{33} = 7 \\ x_{14} + x_{24} + x_{34} = 14 \end{array}\right\} \text{Demand}$$

and $x_{ij} \geq 0$ for $i = 1, 2, 3$ and $j = 1, 2, 3$ and 4.

In the above LP model, there are $m \times n = 3 \times 4 = 12$ decision variables, x_{ij} and $m + n = 7$ constraints, where m are number of rows and n are the number of columns in a general transportation table.

(b) Using North - West corner method

The cell (S_1, D_1) is the north-west corner cell in the given transportation table. The rim values for row S_1 and column D_1 are compared. The smaller of the two values i.e 5 is assigned as the first allocation; otherwise it will violate the feasibility condition. This means that 5 units of a commodity are to be transported from source S_1 to destination D_1. However, this allocation leaves a supply of $7 - 5 = 2$ units of commodity at S_1.

Move horizontally and allocate as much as possible to (S_1, D_2). The rim value for row S_1 is 2 and for column D_2 is 8. The smaller of the two i.e. 2, is placed in the cell. Proceeding to row S_2, since the demand of D_1 is fulfilled. The unfulfilled demand of D_2 is $8 - 2 = 6$ units. This can be fulfilled by S_2 with capacity of 9 units. So, 6 units are allocated to cell (S_2, D_2). The demand of D_2 is satisfied and a balance of 3 units remains with S_2.

	D_1	D_2	D_3	D_4	Supply
S_1	19 5	30 2	50	10	7
S_2	70	30 6	40 3	60	9
S_3	40	8	70 4	30 14	18
Demand	5	8	7	14	34

Continue to move horizontally and vertically in the same manner to make desired allocations. Once the procedure is over, count the number of positive allocations. These allocations should be equal to $m+n-1 = 6$. If yes, then solution is non-degenerate feasible solution. Otherwise degenerate solution.

The total transportation cost of the initial solution obtained by multiplying the quantity x_{ij} in the occupied cells with the corresponding unit cost c_{ij} and adding all values together. Thus, total transportation cost of the solution is

$$\text{Total cost} = 5\times 19 + 2\times 30 + 6\times 30 + 3\times 40 + 4\times 70 + 14\times 20$$
$$= ₹\,1015$$

(c) Least cost method

The cell with lowest cost i.e. 8 is (S_3, D_2). The max. units which can be allocated to this cell is 8. This meets the complete demand of D_2 and leave 10 units with S_3.

In the reduced table with column D_2 the next smallest unit transportation cost is 10 in cell (S_1, D_4). The max. which can be allocated to this cell is 7. This exhausts the capacity of S_1 and leaves 7 units with D_4 as unsatisfied demand.

	D_1	D_2	D_3	D_4	supply
S_1	19	30	50	10 7	7
S_2	70	30	40	60	9
S_3	40	8 8	70	20	18
Demand	5	8	7	14	34

The next smallest cost is 20 in cell (S_3, D_4). The max. units that can be allocated to this cell is 7 units. This satisfies the entire demand of D_4 and leaves 3 units with S_3, as the remaining supply.

The next smallest unit cost cell is not unique i.e there are two cells - (S_2, D_3) and (S_3, D_1) - that have the same unit transportation cost of 40. Allocate 7 units in cell (S_2, D_3) first because it can accomodate more units as compared to cell (S_3, D_1). Then allocate 3 units to cell (S_3, D_1). The remaining demand of 2 units of D_1 is fulfilled from S_2.

Since supply and demand at each supply centre and demand centre is exhausted, the initial solution is arrived at

	D_1	D_2	D_3	D_4	supply
S_1	19	30	50	10 (7)	7
S_2	70 (2)	30	40 (7)	60	9
S_3	40 (3)	8 (8)	70	20 (7)	18
Demand	5	8	7	14	34

The total transportation cost of the initial solution by LCM is calculated as:

Total cost $= 7(10) + 2(70) + 7(40) + 3(40) + 8(8) + 7(20)$
$= ₹ 814$

The total cost obtained by LCM is less than the cost obtained by ~~NE~~ NWCM.

(d) Vogel's Approximation Method (VAM)

The differences for each row and column have been calculated. In the first round, max penalty = 22 occurs ~~to~~ in column D_2. Thus the cell (S_3, D_2) having the least transportation cost is chosen for allocation. The max possible allocation in this cell is 8 units and it satisfies demand in column D_2. Adjust the supply of S_3 from 18 to 10.

	D_1	D_2	D_3	D_4	supply	Row differences			
S_1	19 5	30	50	10 2	7	9	9	40	40
S_2	70	30	40 7	60 2	9	10	20	20	20
S_3	40	8 8	70	20 10	18	12	20	50	-
Demand	5	8	7	14	34				
Column differences	21	22	10	10					
	21	-	10	10					
	-	-	10	10					
	-	-	10	50					

The new row and column penalties are calculated except column D_2 because its demand has already been satisfied. In the second round, the largest penalty = 21 appears at column D_1. Thus, cell (S_1, D_1) having the least transportation cost is chosen for allocating 5 units. After adjusting the supply and demand in the table, we move to the third round of penalty calculations.

In the third round, the max. penalty = 50 appears at row S_3. The max possible allocation of 10 units is made in cell (S_3, D_4) that has the least transportation cost of 20.

The process is continued with new allocations till a complete solution is obtained. The initial solution using VAM is done by this method and total transportation cost is :

$$\text{Total cost} = 5(19) + 2(10) + 7(40) + 2(60) + 8(8) + 10(20)$$
$$= 2779.$$

(e) Since number of occupied cells are $m + n - 1 = 6$.
$\therefore$ this initial solution is non-degenerable. Thus, an optimal solution can be obtained.

In order to calculate the values of u_i's and v_j's for each occupied cell, assigning arbitrarily $v_4 = 0$ in order to simplify the calculations.
Given $v_4 = 0$, u_1, u_2, u_3 can be computed using the relation $c_{ij} = u_i + v_j$ for occupied cells.

	D_1	D_2	D_3	D_4	supply	u_i
S_1	19	30	50	10	7	$u_1 = 10$
S_2	70	30	40	60	9	$u_2 = 60$
S_3	40	8	70	20	18	$u_3 = 20$
demand	5	8	7	14	34	
v_j	$v_1 = 9$	$v_2 = -12$	$v_3 = -20$	$v_4 = 0$		

$c_{14} = u_1 + v_4 \Rightarrow u_1 = 10$

$c_{24} = u_2 + v_4 \Rightarrow u_2 = 60$

$c_{34} = u_3 + v_4 \Rightarrow u_3 = 20$

$c_{11} = u_1 + v_1 \Rightarrow v_1 + 10 = 19 \Rightarrow v_1 = 9.$

$c_{23} = u_2 + v_3 \Rightarrow v_3 + 60 = 40 \Rightarrow v_3 = -20$

$c_{32} = u_3 + v_2 \Rightarrow 20 + v_2 = 8 \Rightarrow v_2 = -12.$

Now, the opportunity cost for each of the occupied cell is determined.

$$d_{ij} = c_{ij} - (u_i + v_j)$$

$d_{12} = c_{12} - (u_1 + v_2) = 30 - (10 - 12) = 32.$

$d_{13} = 60, \ d_{21} = 1, \ d_{22} = -18, \ d_{31} = 11, \ d_{33} = 70$

Since the opportunity cost of the unoccupied cells are not all zero or positive, the current solution is not optimal. The value $d_{22} = -18$ in cell (S_2, D_3) is indicating that the total transportation cost can be reduced in multiple of 18 by shifting an allocation to this cell.

	D_1	D_2	D_3	D_4	supply	u_i
S_1	19 (5)	30	50	10 (2)	7	$u_1 = 10$
S_2	70	30 (2)	40 (7)	60	9	$u_2 = 60$
S_3	40	8 (8)	70	20 (12)	19	$u_3 = 20$
demand	5	8	7	14	34	
v_j	$v_1 = 9$	$v_2 = -12$	$v_3 = -20$	$v_4 = 0$		

Total cost $= 5(19) + 2(10) + 2(30) + 7(4) + 6(9) + 12(20)$

$= ₹ 743$

2. The total plant availability of 235 truckloads exceeds the total requirements of 215 truckloads by 20 truckloads. The excess truckload capacity of 20 is handled by adding a dummy project location, D_{excess} with a requirement of 20 truckloads. We assign unit transportation costs to the dummy project location.

	A	B	C	D_{excess}	Supply
W	4	8 (35)	8 (41)	0	76
X	16	24 (62)	16	0 (26)	82
Y	8 (72)	16 (5)	24	0	77
Demand	72	102	41	20	235

The initial solution is obtained by using Vogel's approximation method. It may be noted that 20 units are allocated ~~to~~ from pit X to dummy project location D. This means pit X is short by 20 units. To test the optimality of the initial solution, calculate u_i s and v_j s corresponding to rows and columns respectively.

	A	B	C	D_{excess}	supply	u_i
W	4 +4	8 (+) 35	8 41 (-)	0 +16	76	$u_1 = 8$
X	16	24 (-) 62	16 +-8	0 20	82	$u_2 = 24$
Y	8 72	16 5	24 +8	0 +9	77	$u_3 = 16$
demand	72	102	41	20	235	
v_j	$v_1 = -8$	$v_2 = 0$	$v_3 = 0$	$v_4 = -24$		

Opportunity cost as shown in the cell (X, C) is negative, the current solution is not optimal. Thus the unoccupied cell (X, C), where $d_{23} = -8$ must enter into the basis and cell (W, C) must leave the basis. The new solution is :

	A	B	C	D excess	supply	u_i
W	4 +4	8 (76)	8 +8	0 +16	76	
X	16	24 (21)	16 (41)	0 (26)	82	
Y	8 (72)	16 (5)	24 +16	0 +8	77	
Demand	72	102	41	26	235	
v_j	$v_1 = 16$	$v_2 = 24$	$v_3 = 16$	$v_4 = 0$		

Since all opportunity costs, d_{ij} are non-negative, the current solution is optimal. The total minimum transportation cost associated with this solution is :

Total cost $= 8(76) + 24(21) + 16(41) + 0(26) + 8(72) + 16(5)$

$= ₹\ 2424.$

3. Applying Hungarian algorithm, the reduced opportunity time matrix is

	I	II	III	IV	V
A	5	0	8	10	11
B	0	6	15	10	3
C	8	5	0	0	0
D	0	4	2	0	5
E	3	5	6	0	8

Examine all rows, until a row containing only single zero element is found. Rows A, B and E are such rows. Make an assignment in these cells and cross off all zeros in assigned columns.

	I	II	III	IV	V
A	5	$\boxed{0}$	8	10	11
B	$\boxed{0}$	6	15	10	3
C	8	5	$\boxed{0}$	$\cancel{0}$	$\cancel{0}$
D	$\cancel{0}$	4	2	$\cancel{0}$	5
E	3	5	6	$\boxed{0}$	8

Examine each column starting with I. There is one zero in column III, assignment is made in this cell. The cell (C, V) is crossed off. All zeroes are either crossed off or assigned. The solution is not optimal as only four assignments are made.

	I	II	III	IV	V
A	7	0	8	12	11
B	0	4	13	10	1
C	10	5	0	2	0
D	0	2	0	0	3
E	3	3	4	0	6

Cover the zeros with min. number of lines = 4 as explained.
Mark (✓) row D where there is no assignment.
Mark (✓) columns I and IV since D has zero element in these columns.
Mark (✓) rows B and E since columns I and IV have assignment in row B and E.
Since no other rows or columns can be marked, draw straight lines through unmarked rows A and C and marked columns I and IV.

Develop the revised matrix by selecting the smallest element among all uncovered elements by the lines.
Repeat the steps to find new solution.

	I	II	III	IV	V
A	7	$\boxed{0}$	8	12	11
B	$\boxed{0}$	4	13	10	1
C	10	5	$\cancel{0}$	2	$\boxed{0}$
D	$\cancel{0}$	2	$\boxed{0}$	$\cancel{0}$	3
E	3	3	4	$\boxed{0}$	6

Since the number of assignments = number of rows (or columns), solution is optimal.
The pattern of assignments among jobs and employees with their respective time is

Job	Employee	Time (in hours)
A	II	5
B	I	3
C	V	2
D	III	9
E	IV	4
		23

4. $t_e = \frac{1}{6}(t_o + 4t_m + t_p)$, $\sigma_i^2 = \left\{\frac{1}{6}(t_p - t_o)\right\}^2$

The earliest and latest expected completion time for all events considering the expected completion time of each activity is :

Activity	t_o	t_p	t_m	t_e	σ^2
1 - 2	5	10	8	7.8	0.696
1 - 3	18	22	20	20.0	0.444
1 - 4	26	40	33	33.0	5.429
2 - 5	16	20	18	18.0	0.443
2 - 6	15	25	20	20.0	2.780
3 - 6	6	12	9	9.0	1.000
4 - 7	7	12	10	9.8	0.694
5 - 7	7	9	8	8.0	0.111
6 - 7	3	5	4	4.0	0.111

Forward Pass method

$E_1 = 0$ $\qquad E_2 = E_1 + t_{1,2} = 7.8$

$E_3 = E_1 + t_{1,3} = 20$ $\qquad E_4 = E_1 + t_{1,4} = 33$

$E_5 = E_2 + t_{2,5} = 25.8$ $\qquad E_6 = \max\{E_i + t_{i,6}\}$

$= \max\{E_2 + t_{2,6} ; E_3 + t_{3,6}\}$

$= 29$

$E_7 = \max\{E_i + t_{i,7}\} = \max\{E_4 + t_{4,7} ; E_5 + t_{5,7} ; E_6 + t_{6,7}$

$= 42.8$

Backward Pass method

$L_7 = E_7 = 42.8$ $\quad L_6 = L_7 - t_{6,7} = 38.8$

$L_5 = L_7 - t_{5,7} = 34.8$ $\quad L_4 = L_7 - t_{4,7} = 33$

$L_3 = L_6 - t_{3,6} = 29.8$ $\quad L_1 = \min\{L_j - t_{1,j}\}$

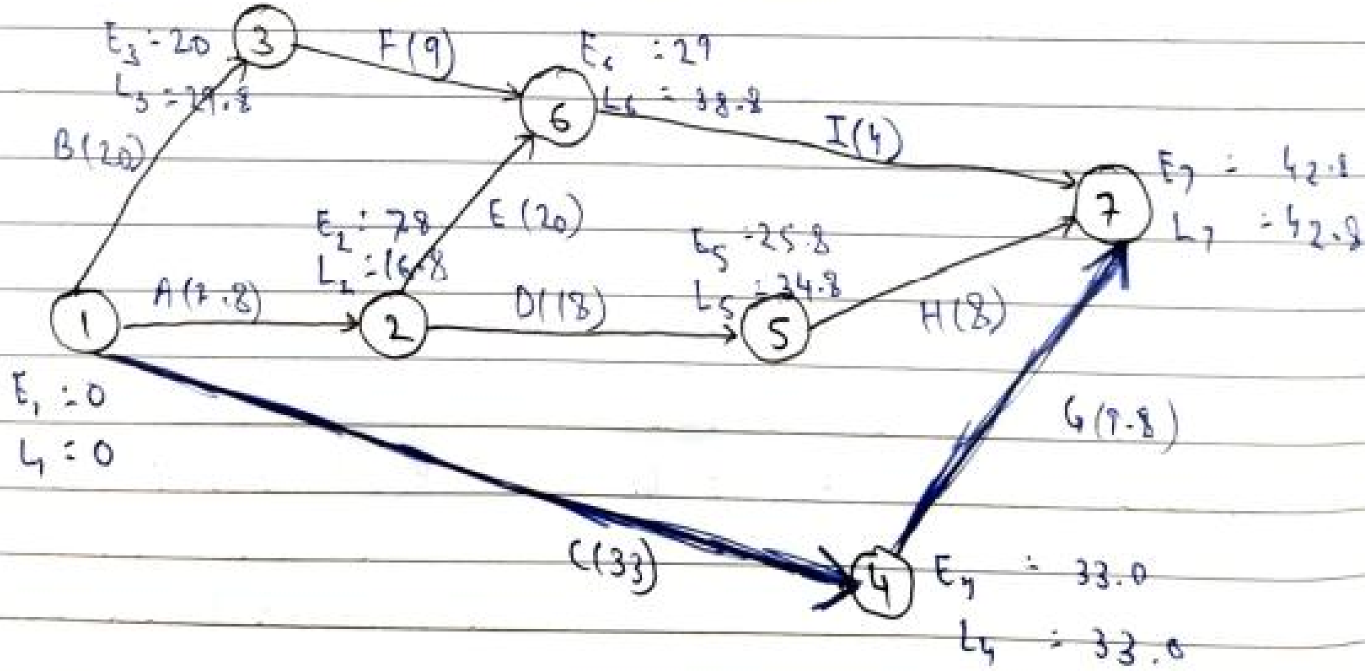
$= \min\{L_2 - t_{1,2}, L_3 - t_{1,3}, L_4 - t_{1,4}\}$

$= 0$

$L_2 = \min\{L_j - t_{2,j}\}$

$= \min\{L_5 - t_{2,5}, L_6 - t_{2,6}\} = 16.8.$

E values and L values are shown.



(a) The critical path is shown by thick line where E-values and L-values are same. Critical path is 1-4-7 and the expected time ~~to~~ for completion is 42.8 weeks.

(b) Expected length of critical path, $T_e = t_c + t_g = 33 + 9.8$

project duration $= 42.8$ weeks

Variance of critical path length, $\sigma^2 = \sigma_c^2 + \sigma_g^2 = 5.429 + 0.694$

$= 6.123$ weeks.

Since $T_s = 41.5$, $T_e = 42.8$, and $\sigma = \sqrt{6.123} = 2.475$, the probability of meeting the schedule time is given by :

$$\text{Prob}\left(Z \le \frac{T_s - T_e}{\sigma}\right) = P\left(Z \le \frac{41.5 - 42.8}{2.474}\right)$$

$$= \text{Prob}(Z \le -0.52)$$

$$= 0.5 - 0.1952 = 0.3048 \text{ (from normal distribution table)}$$

Thus the probability that the project can be completed in less than or equal to 41.5 weeks is 0.3048. In other words, the probability that the project will be delayed beyond 41.5 weeks is 0.6952.

(c) Given that $P\left(Z \le \frac{T_s - T_e}{\sigma}\right) = 0.95$

But $Z_{0.95} = 1.64$, from normal distribution table.

Thus, $1.64 = \frac{T_s - 42.8}{2.474}$.

$\Rightarrow T_s = 46.85$ weeks