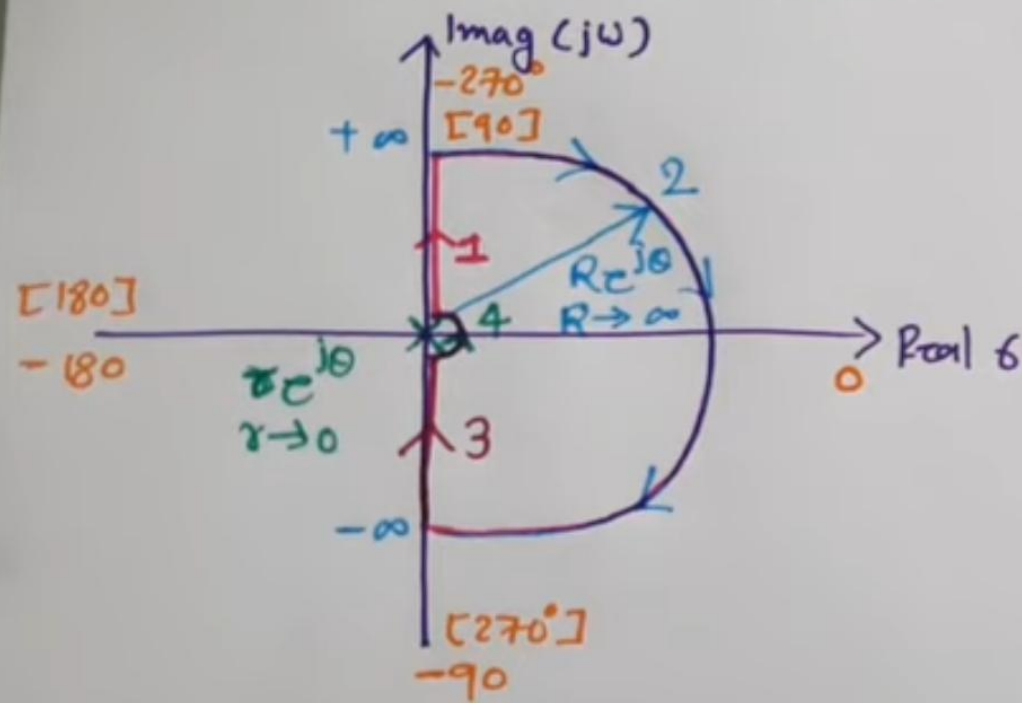


It contains three major steps

- 1) Polar plot
- 2) Inverse Polar plot
- 3) Nyquist Contour



1]  $s = j\omega \rightarrow$  Polar plot  
 $G(j\omega) H(j\omega)$ .

2]  $s = \lim_{R \rightarrow \infty} R e^{j\theta}$ ,  $\theta: 90$  to  $-90$

Find,  $G(s) H(s) = \lim_{R \rightarrow \infty} G(R e^{j\theta}) H(R e^{j\theta})$

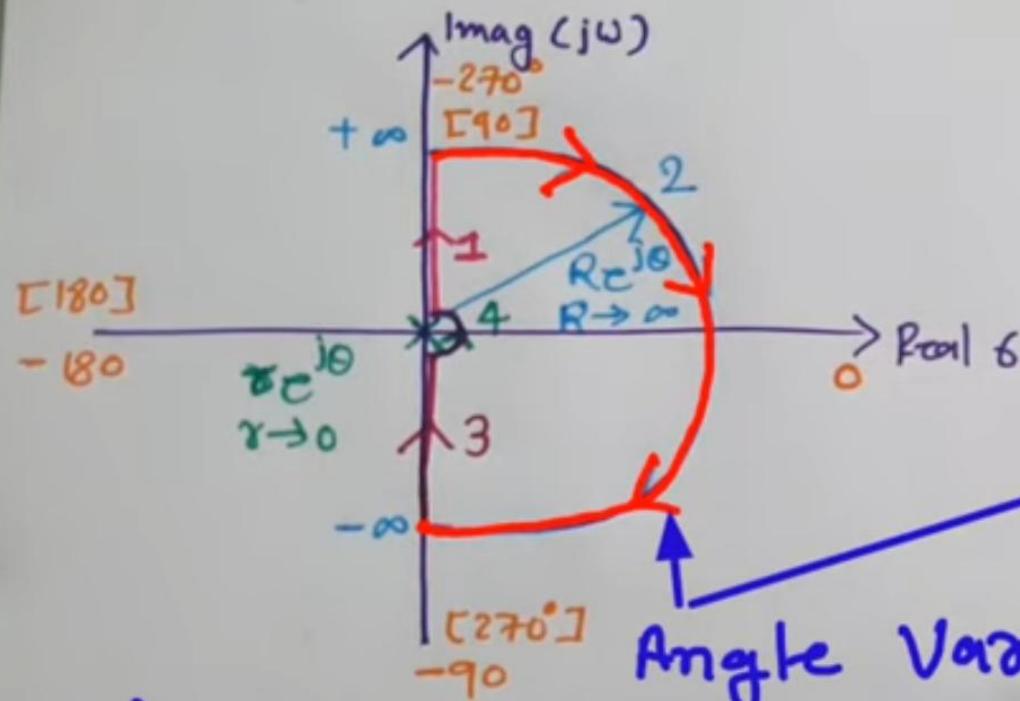
3]  $s = -j\omega \rightarrow$  Inverse polar plot

4]  $s = \lim_{r \rightarrow 0} r e^{j\theta}$ ,  $\theta: -90$  to  $90$

Find,  $G(s) H(s) = \lim_{r \rightarrow 0} G(r e^{j\theta}) H(r e^{j\theta})$

It contains three major steps

- 1] Polar plot
- 2] Inverse Polar plot
- 3] Nyquist Contour



Angle Varies  
from +ve to -ve means  
Clockwise motion of circle

1]  $S = j\omega \rightarrow$  Polar plot  
 $G(j\omega) H(j\omega)$ .

2]  $S = \lim_{R \rightarrow \infty} R e^{j\theta}$

Find,  $G(S) H(S) = \lim_{R \rightarrow \infty} G(R e^{j\theta}) H(R e^{j\theta})$

3]  $S = -j\omega$

Inverse polar plot

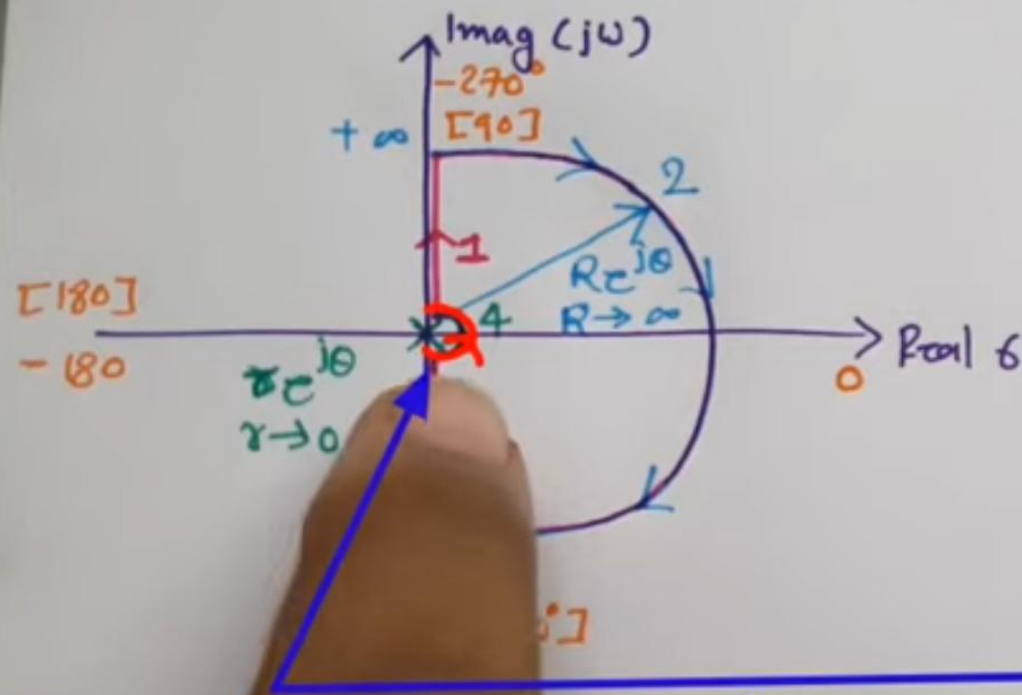
4]  $S = \lim_{r \rightarrow 0} r e^{j\theta}$

Find

$G(r e^{j\theta}) H(r e^{j\theta})$

It contains three major steps

- 1] Polar plot
- 2] Inverse Polar plot
- 3] Nyquist Contour



1]  $s = j\omega \rightarrow$  Polar plot  
 $G(j\omega) H(j\omega)$ .

2]  $s = \lim_{R \rightarrow \infty} R e^{j\theta}$ ,  $\theta: 90$  to  $-90$

Find,  $G(s) H(s) = \lim_{R \rightarrow \infty} G(R e^{j\theta}) H(R e^{j\theta})$

3]  $s = -j\omega \rightarrow$  Inverse polar plot

4]  $s = \lim_{r \rightarrow 0} r e^{j\theta}$ ,  $\theta: -90$  to  $90$

Find,  $G(s) H(s) = \lim_{r \rightarrow 0} G(r e^{j\theta}) H(r e^{j\theta})$

When angle varies from -ve to +ve, circle is in Anticlockwise direction.

## Stability using Nyquist plot

$$\Rightarrow N = P - Z$$

- $N$  is positive for Anti clockwise encirclement around  $(-1, 0)$
- $N$  is negative for clockwise encirclement around  $(-1, 0)$ .

$P$  is Open loop Poles of System on RHP

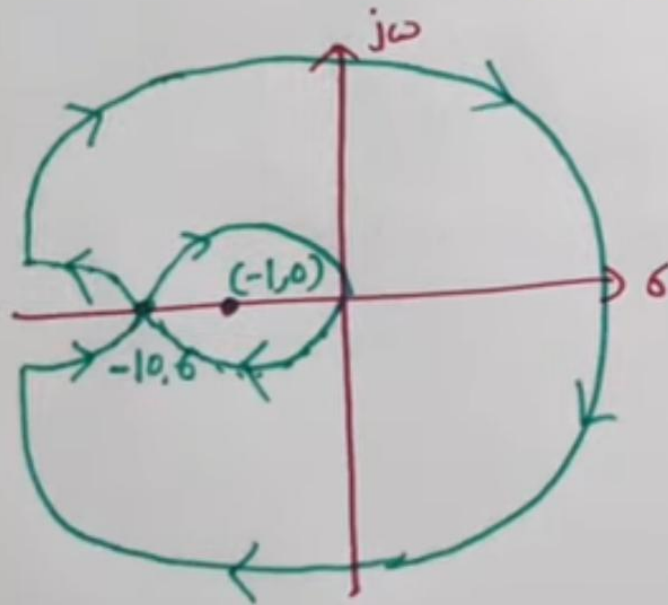
$Z$  is Close loop Poles of System on RHP

Note: Nyquist plot stability identification is possible for Open loop and Close loop system, Also It is applicable for minimum phase and Non minimum phase system.



for Open loop and Close loop system, Also  
It is applicable for minimum phase and Non  
minimum phase system.

Example  $G(s)H(s) = \frac{(4s+1)}{s^2(s+1)(2s+1)}$



$$-N = P - Z$$

$$P = 0$$

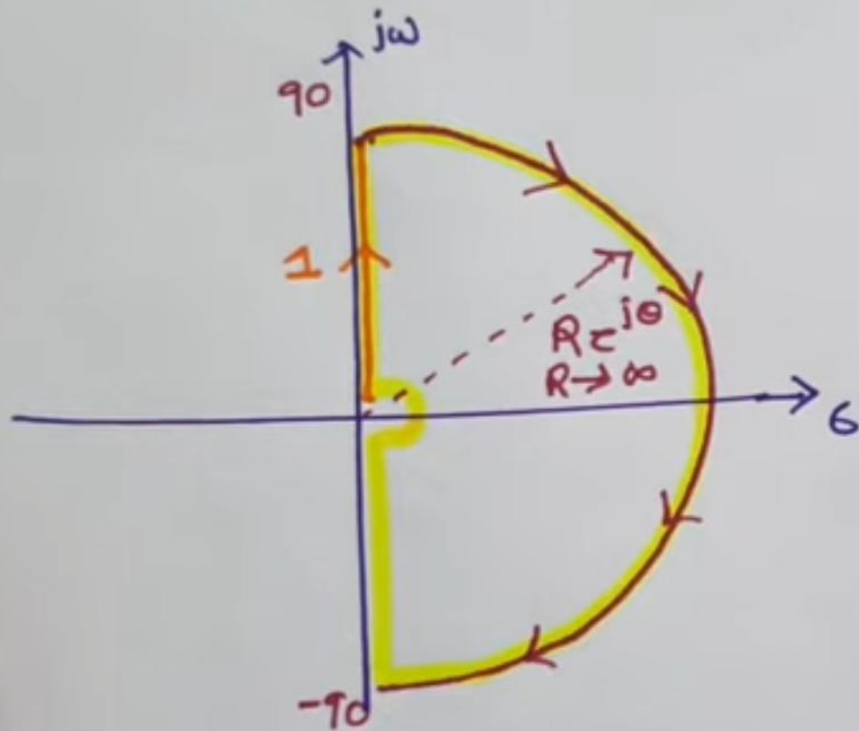
$$N = -2$$

$$\rightarrow -2 = 0 - Z$$

$$\Rightarrow Z = 2$$

- So, there are two poles of  
close loop system on RHP.  
So, system is unstable.

Draw a Nyquist plot for  $G(s) = \frac{1}{s^2(1+s)(1+2s)}$



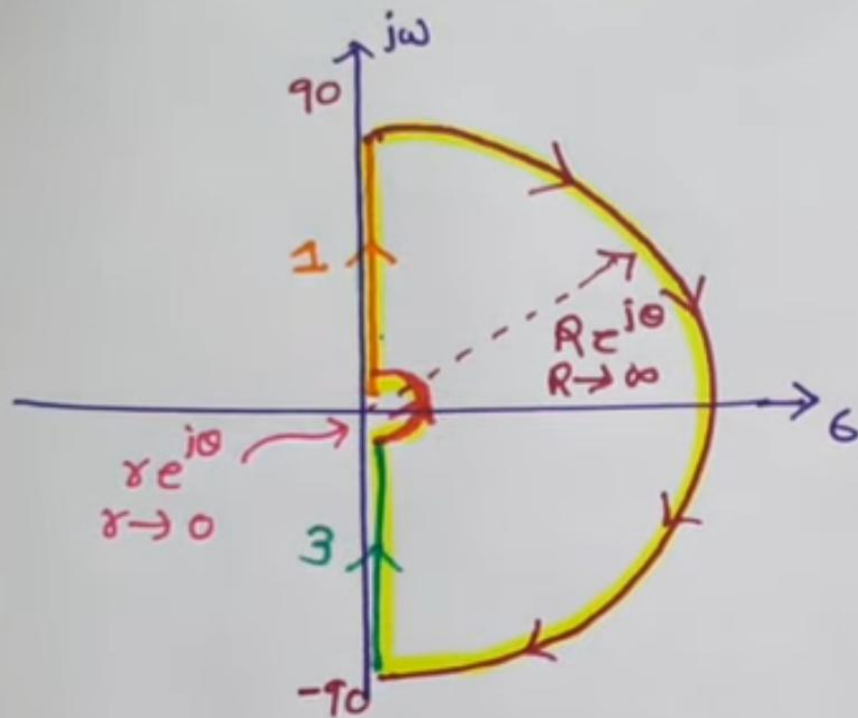
1)  $s = j\omega \rightarrow$  polar plot

2)  $s = \lim_{R \rightarrow \infty} R e^{j\theta}$ ,  $\theta: 90$  to  $-90$

$$G(s) = \lim_{R \rightarrow \infty} \frac{1}{R^2 e^{2j\theta} (1 + R e^{j\theta}) (1 + 2R e^{j\theta})}$$

$$= \lim_{R \rightarrow \infty} \frac{1}{2R^4 e^{4j\theta} (1 + \frac{1}{R} e^{j\theta}) (1 + \frac{1}{2R} e^{j\theta})}$$

$$= \lim_{R \rightarrow \infty} \frac{1}{2R^4} e^{-4j\theta}$$



3)  $s = -j\omega \rightarrow$  Inverse polar plot

4)  $s = \lim_{\sigma \rightarrow 0} \sigma e^{j\theta}, \theta: -90 \text{ to } 90$

$$s^2(1+s)(1+2s)$$

1)  $s = j\omega \rightarrow$  polar plot

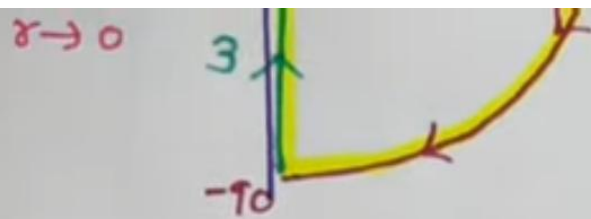
2)  $s = \lim_{R \rightarrow \infty} R e^{j\theta}, \theta: 90 \text{ to } -90$

$$K(s) = \lim_{R \rightarrow \infty} \frac{1}{R^2 e^{2j\theta} (1 + R e^{j\theta}) (1 + 2R e^{j\theta})}$$

$$= \lim_{R \rightarrow \infty} \frac{1}{2R^4 e^{4j\theta} (1 + \frac{1}{2} e^{j\theta}) (1 + \frac{1}{2} e^{j\theta})}$$

$$= \lim_{R \rightarrow \infty} \frac{1}{2R^4} e^{-4j\theta}$$

Mag = 0      phase: -360 to 360  
(Two circle ACW)



3)  $s = -j\omega \rightarrow$  Inverse polar plot

4)  $s = \lim_{r \rightarrow 0} r e^{j\theta}, \theta: -90 \text{ to } 90$

$$G(s) = \lim_{r \rightarrow 0} \frac{1}{r^2 e^{2j\theta} (1 + r e^{j\theta}) (1 + 2r e^{j\theta})}$$

$$= \lim_{r \rightarrow 0} \frac{1}{r^2} \frac{e^{-2j\theta}}{\text{mag} = \infty} \quad \text{phase} \rightarrow 180 \text{ to } -180$$

(One circle cw)

$$= \lim_{R \rightarrow \infty} \frac{1}{2R^4 e^{4j\theta} (1 + \frac{1}{2} e^{j\theta}) (1 + \frac{1}{2} e^{j\theta})}$$

$$= \lim_{R \rightarrow \infty} \frac{1}{2R^4} \frac{e^{-4j\theta}}{\text{mag} = 0} \quad \text{phase: } -360 \text{ to } 360$$

(Two circle ACW)



(One (100k)  $\omega$ )

→ Polar plot,  $s = j\omega$ .

~~$G(j\omega) = \frac{1}{(j\omega)^2 (1+j\omega) (1+2j\omega)}$~~

$$G(j\omega) = \frac{1}{(j\omega)^2 (1+j\omega) (1+2j\omega)}$$

Step 2 - Write polar plot in standard form

-  $G(j\omega) = |G(j\omega)| \angle G(j\omega)$

-  $|G(j\omega)| = \frac{1}{\omega^2 \sqrt{1+\omega^2} \sqrt{1+4\omega^2}}$

-  $\angle G(j\omega) = -180 - \tan^{-1} \omega - \tan^{-1} 2\omega$

~~$$G(j\omega) = \frac{1}{(j\omega)^2 \sqrt{1+\omega^2} \sqrt{1+4\omega^2}}$$~~

$$G(j\omega) = \frac{1}{(j\omega)^2 (1+j\omega)(1+2j\omega)}$$

Step 2 - write polar plot in standard form

$$- G(j\omega) = |G(j\omega)| \angle G(j\omega)$$

$$- |G(j\omega)| = \frac{1}{\omega^2 \sqrt{1+\omega^2} \sqrt{1+4\omega^2}}$$

$$- \angle G(j\omega) = -180 - \tan^{-1} \omega - \tan^{-1} 2\omega$$

Step 3 Find  $\omega = 0$ ,  $\omega = \infty$ .

$$\text{at } \omega = 0, |G(j\omega)| = \infty, \angle G(j\omega) = -180$$

$$\text{at } \omega = \infty, |G(j\omega)| = 0, \angle G(j\omega) = -360$$

Step-4 - Separate real and Imag. parts.

$$G(j\omega) = \frac{1}{(j\omega)^2(1+j\omega)(1+2j\omega)} \times \frac{(1-j\omega)(1-2j\omega)}{(1-j\omega)(1-2j\omega)}$$

$$= \frac{-1 \times (1 - 2\omega^2 - 3j\omega)}{\omega^2(1+\omega^2)(1+4\omega^2)}$$

$$= \frac{(2\omega^2 - 1)}{\omega^2(1+\omega^2)(1+4\omega^2)} + \frac{j3\omega}{\omega^2(1+\omega^2)(1+4\omega^2)}$$

$$= \frac{(2\omega^2 - 1)}{\omega^2(1+\omega^2)(1+4\omega^2)} + \frac{j3}{\omega(1+\omega^2)(1+4\omega^2)}$$

Steps - For Intersection to real axis,  $\text{Imag}(G(j\omega)) = 0$

Steps

$$(j\omega) = (1+j\omega)(1+2j\omega) \quad (1-j\omega)(1-2j\omega)$$

$$= \frac{-1 \times (1 - 2\omega^2 - 3j\omega)}{\omega^2(1+\omega^2)(1+4\omega^2)}$$

$$= \frac{2\omega^2 - 1}{\omega^2(1+\omega^2)(1+4\omega^2)} + \frac{j3\omega}{\omega^2(1+\omega^2)(1+4\omega^2)}$$

$$= \frac{(2\omega^2 - 1)}{\omega^2(1+\omega^2)(1+4\omega^2)} + \frac{j3}{\omega(1+\omega^2)(1+4\omega^2)}$$

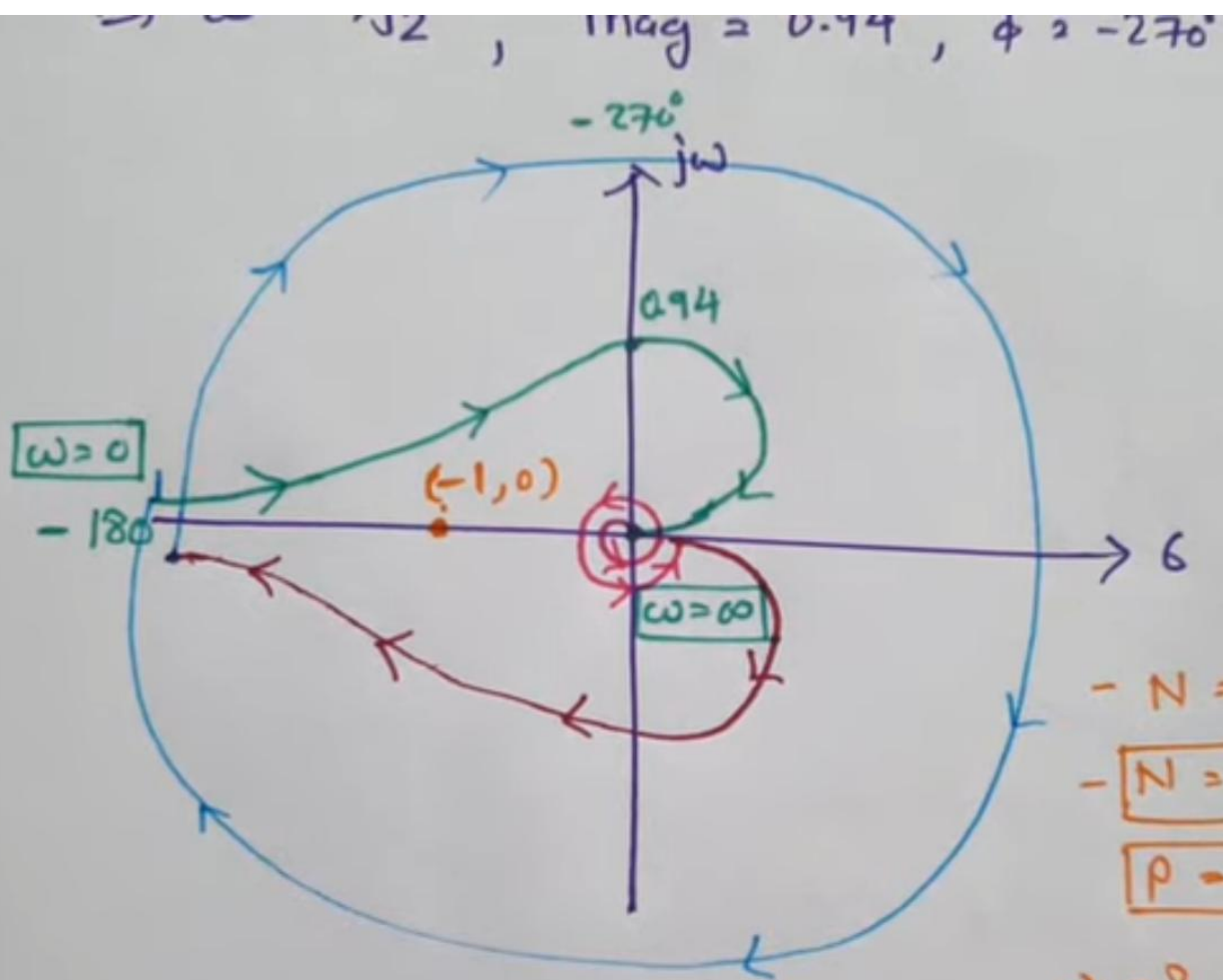
Steps - For Intersection to real axis,  $\text{Imag}(K(j\omega)) = 0$

Steps - For Intersection to imag axis,  $\text{Real}(K(j\omega)) = 0$

$$\Rightarrow 2\omega^2 - 1 = 0$$

$$\Rightarrow \omega = 1/\sqrt{2}, \quad \text{mag} = 0.94, \quad \phi = -270^\circ$$





$$-N = P - Z$$

$$-N = -2$$

$$P = 0$$

$$\Rightarrow -2 = 0 - Z$$

$$\Rightarrow Z = 2$$