1. Write the dual of the following LP problems:
   a. $Minimize\ z = x_1 - x_2 + 3x_3,$
      $subject\ to\ (i)\ x_1 + x_2 + x_3 \le 10; (ii)\ 2x_1 - x_2 - x_3 \le 2;$
      $(iii)\ 2x_1 - 2x_2 - 3x_3 \le 6, and\ x_1, x_2, x_3 \ge 0.$

   b. $Maximize\ z = 3x_1 - 2x_2 + 4x_3,$
      $subject\ to\ (i)\ 3x_1 + 5x_2 + 4x_3 \ge 7; (ii)\ 6x_1 + x_2 + 3x_3 \ge 4;$
      $(iii)\ 7x_1 - 2x_2 - x_3 \le 10; (iv)\ x_1 - 2x_2 + 5x_3 \ge 3;$
      $(v)\ 4x_1 + 7x_2 - 2x_3 \ge 2, and\ x_1, x_2, x_3 \ge 0.$

   c. $Minimize\ z = x_1 - 3x_2 - 3x_3,$
      $subject\ to\ (i)\ 3x_1 - x_2 + 2x_3 \le 7; (ii)\ 2x_1 - 4x_2 \ge 12;$
      $(iii) -4x_1 + 3x_2 + 8x_3 = 10, and\ x_1, x_2 \ge 0;\ x_3\ unrestricted.$

2. Use the dual simplex method to solve the following problems
   a. $Maximize\ z = -3x_1 - 2x_2,$
      $subject\ to\ (i)\ x_1 + x_2 \ge 1; (ii)\ x_1 + x_2 \le 7; (iii)\ x_1 + 2x_2 \ge 10;$
      $(iv)\ x_2 \le 3, and\ x_1, x_2 \ge 0.$

   b. $Minimize\ z = 3x_1 + x_2$
      $subject\ to\ (i)\ x_1 + x_2 \ge 1; (ii)\ 2x_1 + 3x_2 \ge 2, and\ x_1, x_2 \ge 0.$

   c. $Minimize\ z = -2x_1 - 2x_2 - 4x_3,$
      $subject\ to\ (i)\ 2x_1 + 3x_2 + 5x_3 \ge 2; (ii)\ 3x_1 + x_2 + 7x_3 \le 3;$
      $(iii)\ x_1 + 4x_2 + 6x_3 \le 5; and\ x_{1,}\ x_{2,} x_3 \ge 0$