Inventory Control

What Is Inventory ?

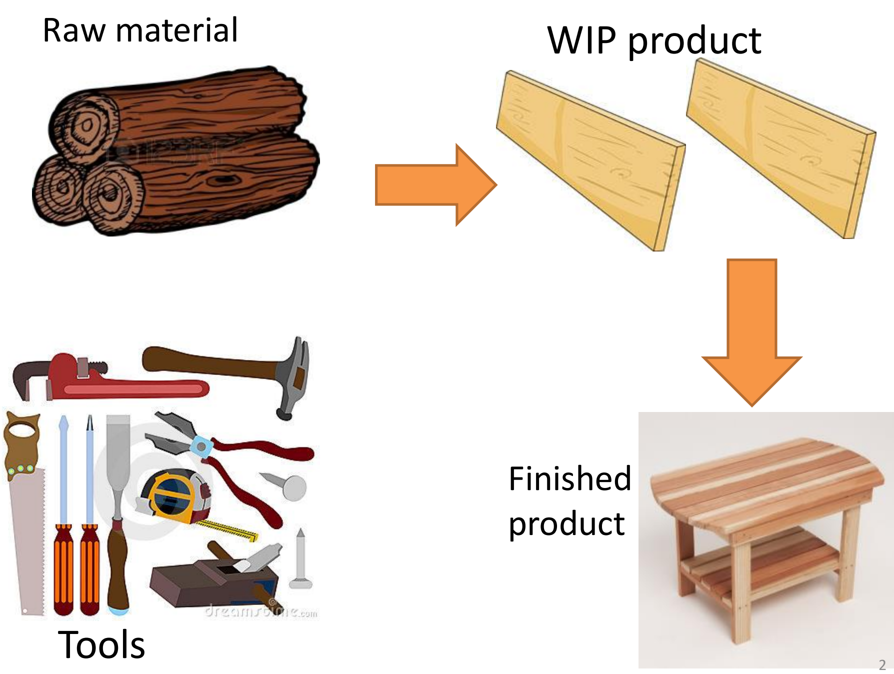
Inventory is a stock of items kept to meet future demand.

Material that has been purchased from a supplier, may have been partially or completely converted, but not yet sold to the customer (we "own" it - if it gets stolen, it is our loss).

Inventory, in production context, is an idle resource. Resource is idle does not mean it is serving no purpose. It is available when needed.

Types of Inventory

- **Raw materials & purchased parts**: Materials and components required for making a product.
- **Partially completed goods called *work in progress product* (WIP):** Materials and components that have begun their transformation to finished goods.
- **Finished-goods inventories**: Goods ready for sale to customers i.e, Items being transported and stored in ware houses.
- **Tools and equipment**.



Inventory Control

Why Hold Inventory ?

- To balance against uncertainty (safety)
- To smooth production requirements. Material buffers are used to avoid uncertainty in material deliveries
- To meet time varying demand or supply patterns. (To meet seasonal demand)
- To ensure a high level of customer service
- Economies of scale in production or purchasing
- To take advantage of quantity discounts
- To hedge against price increases

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Inventory Costs

Holding or carrying costs (H): cost to carry an item in inventory for a length of time.

Holding cost = Storage cost + Handling cost + Depreciation cost + Insurance + Taxes

> **Holding cost increases as the order size increases because larger orders mean higher inventory levels.**

Ordering costs (O): costs of ordering and receiving inventory. Primarily the staff costs associated with processing the order.

Shortage costs: costs when demand exceeds supply; temporary or permanent loss of sales when demand cannot be met.

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Inventory Control Systems

Continuous system (fixed-order-quantity)

constant amount ordered when inventory declines to predetermined level

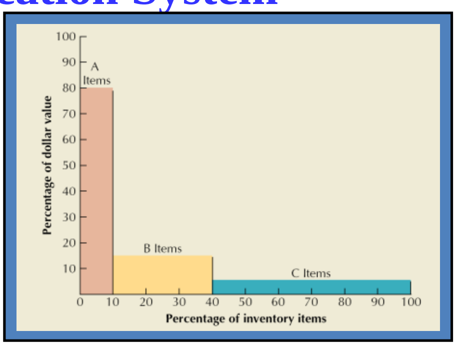
Periodic system (fixed-time-period)

order placed for variable amount after fixed passage of time

ABC Classification System

Classifying inventory according to some measure of importance and allocating control efforts accordingly.

- Class A – Very important
  - 5 – 15 % of units
  - 70 – 80 % of value
- Class B – Mod. important
  - 30 % of units
  - 15 % of value
- Class C – least important
  - 50 – 60 % of units
  - 5 – 10 % of value



ABC Classification: Example

Part	Unit cost ($)	Annual usage	Total cost ($)
1	60	90	5400
2	350	40	14000
3	30	130	3900
4	80	60	4800
5	30	100	3000
6	20	180	3600
7	10	170	1700
8	320	50	16000
9	510	60	30600
10	20	120	2400
Total:		1000	85400

Sort parts in descending order of total cost

ABC Classification: Example (cont.)

PART	TOTAL VALUE ($)	% OF TOTAL VALUE	% OF TOTAL QUANTITY		% CUMMULATIVE
9	30,600	35.9	6.0		6.0
8	16,000	18.7	5.0	A	11.0
2	14,000	16.4	4.0		15.0
1	5,400	6.3	9.0		24.0
4	4,800	5.6	6.0	B	30.0
3	3,900	4.6	10.0		40.0
6	3,600	4.2	18.0		58.0
5	3,000	3.5	13.0		71.0
10	2,400	2.8	12.0	C	83.0
7	1,700	2.0	17.0		100.0
	85,400				

ABC Classification: Example (cont.)

PART	TOTAL VALUE ($)	% OF TOTAL VALUE	% OF TOTAL QUANTITY		% CUMMULATIVE
9	30,600	35.9	6.0		6.0
8	16,000	18.7	5.0	A	11.0
2	14,000	16.4	4.0		15.0
1	5,400	6.3	9.0		24.0
4	4,800	5.6	6.0	B	30.0

CLASS	ITEMS	% OF TOTAL VALUE	% OF TOTAL QUANTITY
A	9, 8, 2	71.0	15.0
B	1, 4, 3	16.5	25.0
C	6, 5, 10, 7	12.5	60.0

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Inventory models

Inventory models quantify the relationship to identify the order size that minimized total cost.

- ❑ Economic order quantity model (EOQ)
- ❑ Economic production quantity model (EPQ)

Assumptions of EOQ Model

- Only one product is involved
- Annual demand requirement is known
- Demand is even throughout the year
- Lead time does not vary
- Each order is received in a single delivery
- There are no quantity discounts

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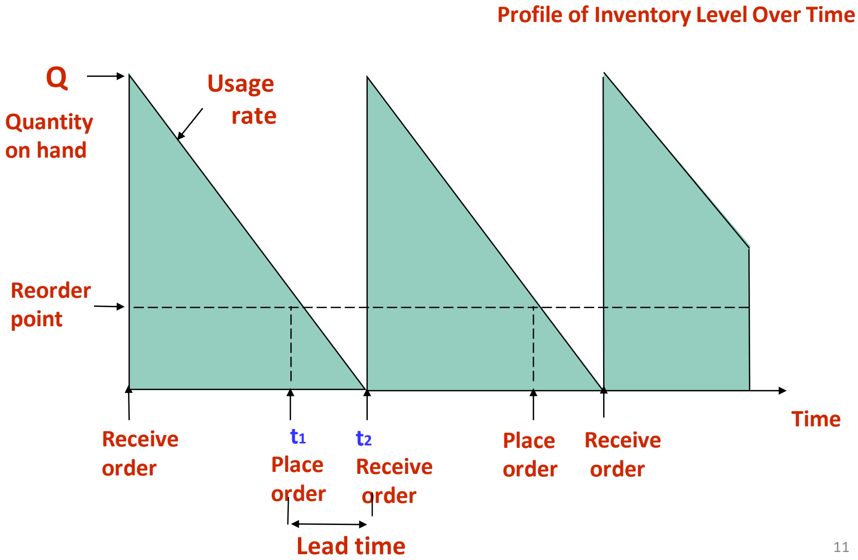
EOQ Model



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EOQ Model

***C_o* - cost of placing order** **_D_ - annual demand**

***C_h* - annual per-unit carrying cost** **_Q_ - order quantity**

Annual holding cost (H) = (average number of inventory)×(holding cost/unit/year)

- Average number of inventory = (Q+0)/2 = Q/2

  Annual carrying cost (H) = (Q/2) C_h

- Annual ordering cost (O) = Cost per order × No. of order/year

  O = C_o × D/Q

Total Cost = Holding cost + Ordering cost

$$TC = (Q/2) \times C_h + C_o \times D/Q$$

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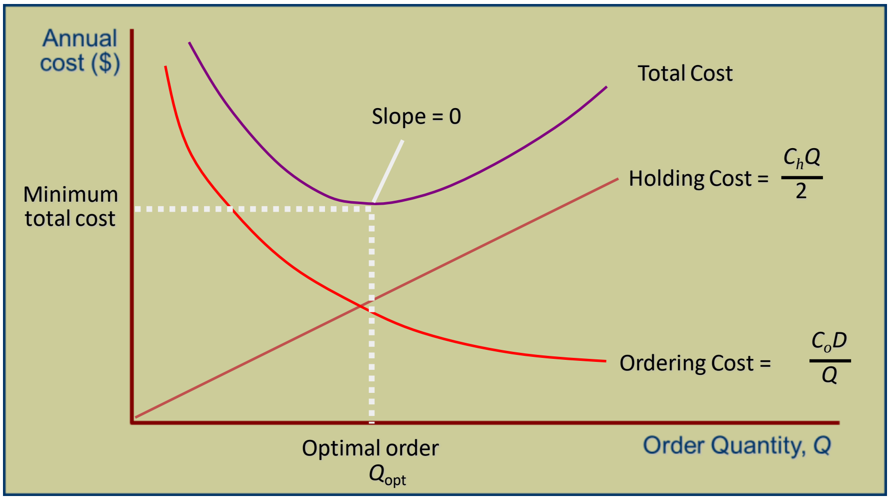
EOQ Model



EOQ Model

Example:

Deriving Q_{opt}

$$TC = \frac{C_o D}{Q} + \frac{C_h Q}{2}$$

$$\frac{\partial TC}{\partial Q} = -\frac{C_o D}{Q^2} + \frac{C_h}{2}$$

$$Q_{opt} = \sqrt{\frac{2 C_o D}{C_h}}$$

A toy manufacturer uses approximately 32,000 silicon chips annually. The chips are used at a steady rate during the 240 days a year that the plant operates. Annual holding cost is $3 per chip, and ordering cost is $120. Determine

- The optimal order quantity.
- Holding cost, ordering cost, and the total cost.
- The number of workdays in an order cycle.

D= 32, 000 $\quad$ C_o = 120 $\quad$ C_h = 3

$$Q_{opt} = \sqrt{\frac{2 \times 120 \times 32000}{3}} = 1600$$

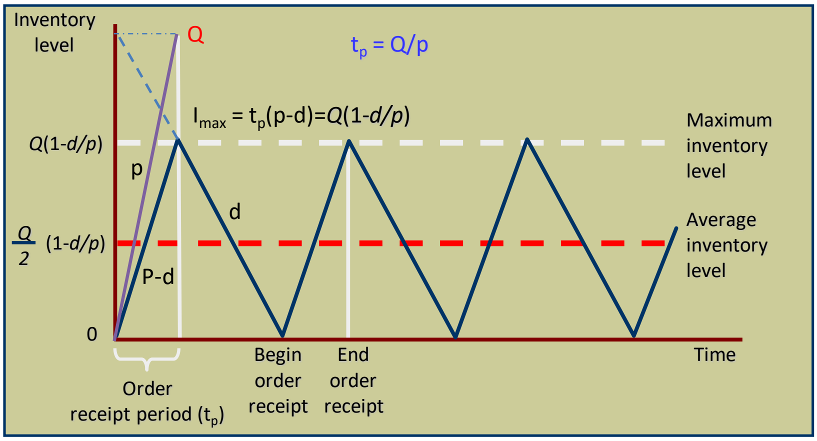
HC= C_hQ/2= 3*800= 2400 $\quad$ OC= C_oD/Q = 120*32000/1600 = 2400

No. of Workdays = Q/Daily consumption rate = 12

Daily consumption rate= D/240 = 400/3

Economic Production Quantity Model (EPQ)

- An inventory system in which an order is received gradually, as inventory is simultaneously being depleted
- *p* - daily rate at which an order is received over time, or *production rate*
- *d* - daily rate at which inventory is demanded



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EPQ Model

$$\text{Maximum inventory level} = Q - \frac{Q}{p}d$$

$$= Q\left(1 - \frac{d}{p}\right)$$

$$\text{Average inventory level} = \frac{Q}{2}\left(1 - \frac{d}{p}\right)$$

$$TC = \frac{C_oD}{Q} + \frac{C_hQ}{2}\left(1 - \frac{d}{p}\right)$$

$$Q_{opt} = \sqrt{\frac{2C_oD}{C_h\left(1 - \frac{d}{p}\right)}}$$

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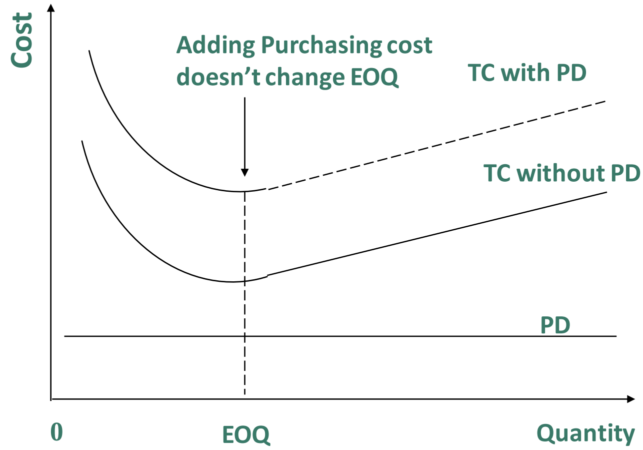
Quantity Discount Model

$$TC = \frac{C_o D}{Q} + \frac{C_h Q}{2} + PD$$

P: per unit price of item
D: annual demand



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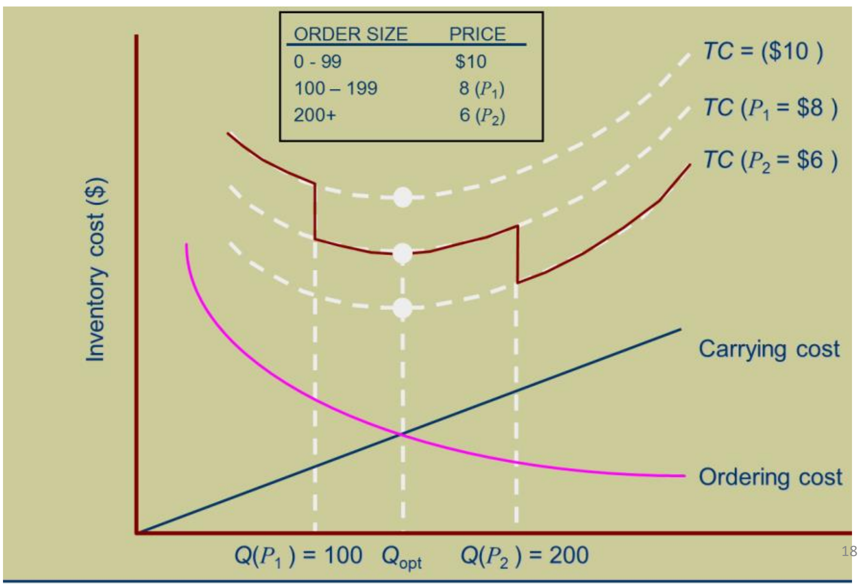
Quantity Discount Model

Price per unit decreases as order quantity increases



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Quantity Discount: Example

QUANTITY	PRICE
1 - 49	$ 1,400
50 - 89	1,100
90+	900

$C_o =$ $2,500
$C_h =$ $190 per computer per year
$D =$ 200

Step 1: First determine the optimal order size and total cost with the basic EOQ model.

$$Q_{opt} = \sqrt{\frac{2C_oD}{C_h}} = \sqrt{\frac{2(2500)(200)}{190}} = 72.5 \text{ PCs}$$

For $Q = 72.5$

$$TC = \frac{C_oD}{Q_{opt}} + \frac{C_hQ_{opt}}{2} + PD = \$233,784$$

Step 2: Compute Q using the lowest unit price

For $Q = 90$

$$TC = \frac{C_oD}{Q} + \frac{C_cQ}{2} + PD = \$194,105$$

Optimum order quantity = 90

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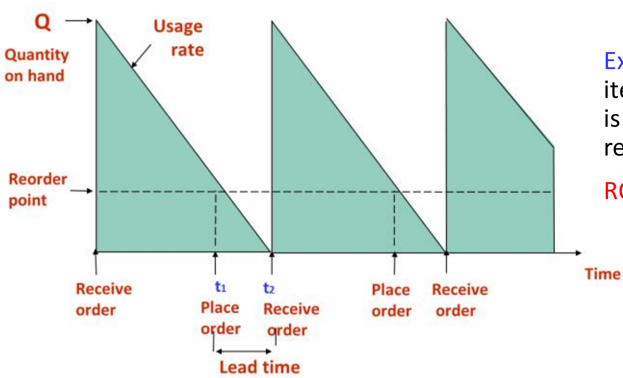
Inventory Control

Reorder Point: *Level of inventory at which a new order is placed*

$ROP = d\,L$ where,
d = demand rate per period
L = lead time



Example: Demand is 20 items per day and lead time is one week. What is the reorder point (ROP)?

ROP = 20*7=140

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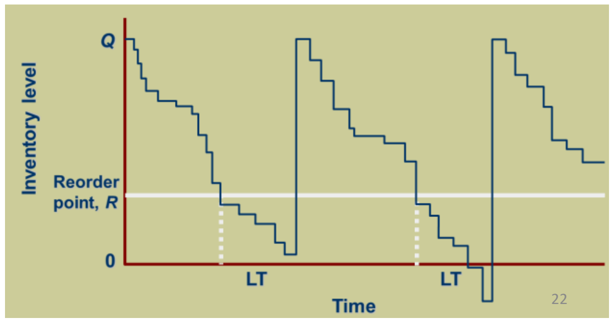
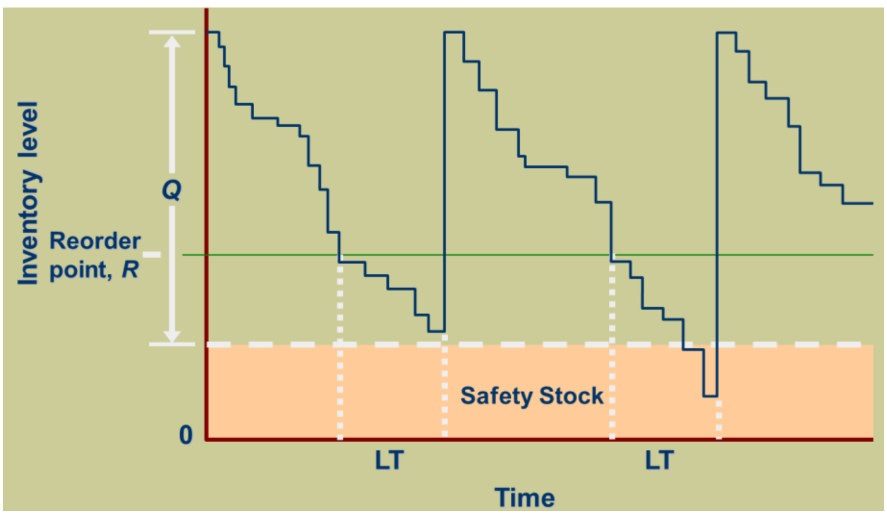
Inventory Control

- *Reorder Point* - When the quantity on hand of an item drops to this amount, the item is reordered.
- *Safety Stock* - Stock that is held in excess of expected demand due to variable demand rate and/or lead time.
- *Stockout* – *An inventory shortage*
- *Service Level* - Probability that demand will not exceed supply during lead time.

Variable Demand with a Reorder Point



Reorder Point with a Safety Stock



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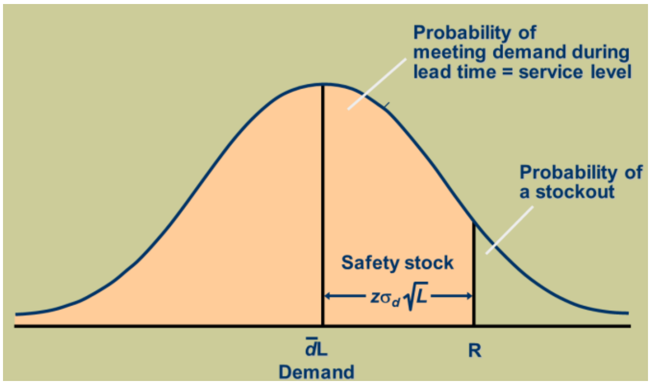
Reorder Point With Variable Demand

Normal distribution

PDF



ROP = Expected demand during lead time + safety stocks

ROP for variable demand and constant lead time

$$ROP = \bar{d} \times LT + z\sqrt{LT}\sigma_d$$

ROP for variable lead time and constant demand

$$ROP = d \times \overline{LT} + zd\sigma_{LT}$$

z – standard normal

Reorder Point With Variable Demand

The carpet store wants a reorder point with a 95% service level and a 5% stockout probability

$\bar{d}$ = 30 yards per day
LT = 10 days
σ_d = 5 yards per day

For a 95% service level, z = 1.65

$ROP = \bar{d}\,LT + z\,\sigma_d\sqrt{LT}$
$= 30(10) + (1.65)(5)(\sqrt{10})$
$= 326.1$ yards

Safety stock $= z\,\sigma_d\sqrt{LT}$
$= (1.65)(5)(\sqrt{10})$
$= 26.1$ yards

ROP for variable lead time and constant demand

The housekeeping department of a Hotel uses approximately 600 bars of soap per day, and this tends to be fairly constant. Lead time for soap delivery is normally distributed with a mean of six days and standard deviation of two days. A service lead of 98% is desired. Find the ROP.

$$ROP = d \times \overline{LT} + zd\sigma_{LT}$$

Case of: ROP for variable lead time and constant demand

Given data:
d = 600 bars/day
$\overline{LT}$ = 6 days
$\sigma_{LT} = 2\ days$
z = 1.96

ROP = 600*6 + 1.96*600*2 = 5952

ROP for variable demand and variable lead time

The Hotel replaces broken glasses at a rate of 25 per day. In the past, this quantity has tend to vary normally and have a standard deviation of 3 glasses per day. Glasses are ordered from a Cleveland supplier. Lead time is normally distributed with an average of 10 days and a standard deviation of 2 days. What ROP should be used to achieve a service level of 95%?

$$ROP = \bar{d} \times \overline{LT} + z\sqrt{\overline{LT}\sigma_d^2 + \bar{d}^2\sigma_{LT}^2}$$

Case of: variable lead time and variable demand

Given data:
$\bar{d}$ = 25 glasses/day
$\sigma_d = 3\ glasses/day$
$\overline{LT}$ = 10 days
$\sigma_{LT} = 2\ days$
Z = 1.65

ROP = 25*10+ 1.65$\sqrt{10 * (3 * 3) + (25 * 25) * (2 * 2)}$
= 315.371