ME 324 :- INDUSTRY & RESEARCH

Inventry :- it is a stock of items kept to meet further demand.

Supplier : Raw material — Inprocess — Final Product — Distributor

$\underbrace{\text{Raw material} \quad \text{Inprocess} \quad \text{Final Product}}_{\text{Inventry}}$

Types of Inventry

(1) Raw material and Purchased parts :- Materials ~~are~~ and components required for making a product

(2) Partially completed goods :- called workin process products (WIP) Materials and components that have begun this transformation to finished goods.

(3) Finished goods :- Goods ready for sale to customers, i.e., items being transported and stored in warehouses.

(4) Tools and Equipments

Purpose of Inventry

(1) To balance against uncertainty → demand, → material delivery

(2) Smooth production (3) to ensure high level of customer service

(4) Economics in scale in Production or Purchasing

- → Optimum production can lead to minimum cost
- → Buying in bulk can reduce purchasing cost
- → Quantity discount
- → To tackle against price increase.

Two Extreme Cases

(1) Large Inventry :- can increase cost of capital and storage

(2) Small Inventry :- disrupt production and/or sale

So inventry control deals with optimum size of inventry, hence, by minimising appropriate cost.

Inventry Policy

(1) How much to order?

(2) When to order?

Inventry Cost

$$\rightarrow \text{Total Inventry Cost} = \begin{pmatrix}\text{purchase}\\ \text{cost}\end{pmatrix} + \begin{pmatrix}\text{holding}\\ \text{cost}\end{pmatrix} + \begin{pmatrix}\text{ordering}\\ \text{cost}\end{pmatrix} + \begin{pmatrix}\text{shortage \&}\\ \text{penalty}\\ \text{cost}\end{pmatrix}$$

Purchasing cost = price per unit of an inventry item = $(P)\times(D)$ = price x demand
- discound for buying large volume / order.

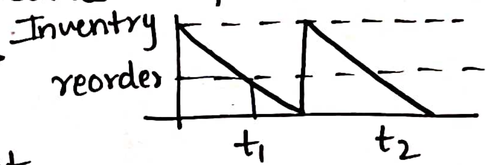
Holding cost = storage cost + handling cost + deprication cost + insurance & taxes.
→ cost to carry an item in inventry.

Ordering cost = fixed cost when an order is placed
- cost of ordering & receiving inventry

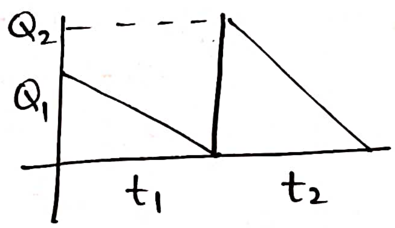
Shortage cost = penalty cost for temporary or permanent loss & sales when demand can't be made.

Inventry Control of System :-

(1) Continuous System (fixed-order quantity)
* constant amount order when inventry declines to predetermined level.
* perpetual inventry.

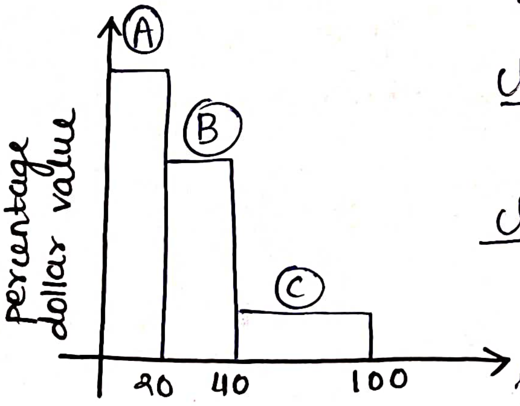


(2) Periodic System (fixed time period)
* orders get placed for variable amount after fixed passage of time.
* Weekly or daily, etc.



$(t_1 = t_2)$; but $(Q_1 \neq Q_2)$

→ ABC Classification System :→



class A :→ very important
* 5-15% of units
* 70-80% of value

class B :→ moderate important
* 30% of units
* 15% of value

class C :→ least imp.
* 50-60% of units
* 5-10% of value.

ABC classification System :

:- classify points according to ABC classification

(Part)	(unit cost)(P)	(Annual Usage) (D)	(Total Cost) (PD)
(1)	60	90	5400
(2)	350	40	14000
(3)	3~~6~~0	130	3900
(4)	80	60	4800
(5)	30	100	3000
(6)	20	180	3600
(7)	10	170	1700
(8)	320	50	16000
(9)	510	60	30600
(10)	20	120	2400
	Total	= (1000)	= (85400)

(Part)		Total cost value	% of total value		% total quantity		Cumulative Q (%)
(9)	A	30,600	35.9	71%	6		6
(8)		16,000	18.7		5		11
(2)		14,000	16.4		4		15
(1)	B	5400	6.3	16.5%	9	25%	24
(4)		4800	5.8		6		30
(3)		3900	4.6		10		40
(6)	C	3600	4.2	12.5%	18	60%	58
(5)		3000	3.5		13		71
(10)		2400	2.8		12		83
(7)		1700	2.0		17		100

$\xrightarrow[\text{EOQ}]{S}$ Plant $\longrightarrow$ customer.

Inventry models

Find the order size so that total inventry cost is minimum.

(1). Economic order Quantity (EOQ) model

(2). Economic production Quantity model (EPQ)

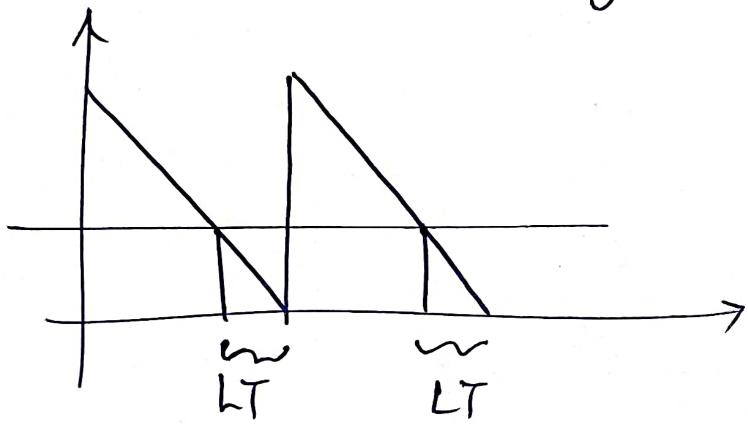
Assumptions for EOQ model

(1). Only one item is involved

(2). Annual demand requirement is known (deterministic model)

(3). Demand is even throughout the year.

(4). Lead time does not vary.

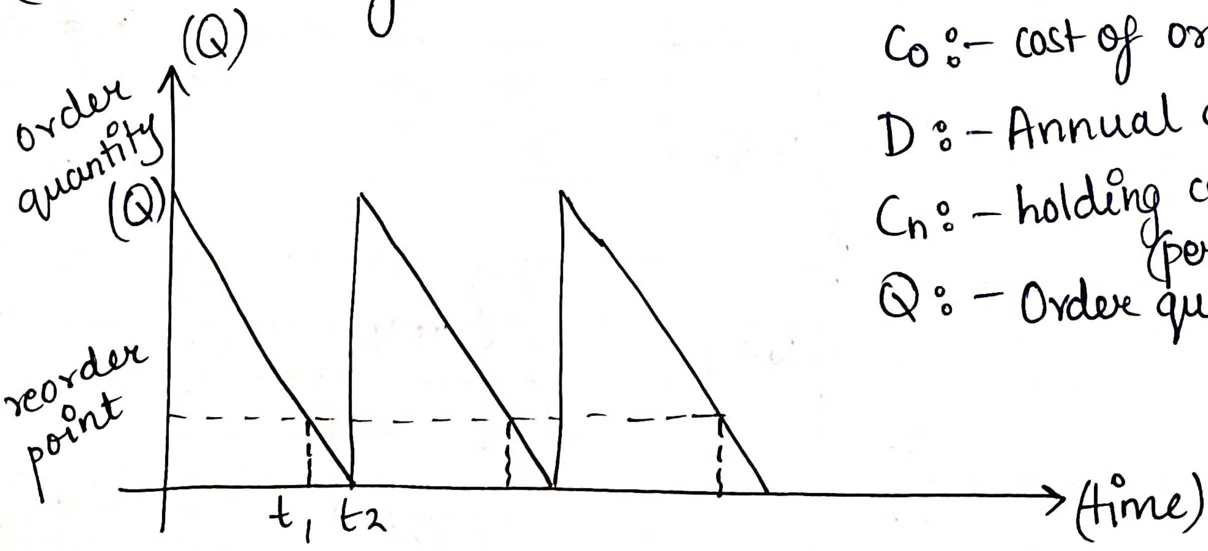


(LT ⇒ lead time)

(5). Each order is received in a single delivery

(6). There is no quatity discounts

(7). No shortage.



C_o :- cost of ordering

D :- Annual demand

C_h :- holding cost / carrying (per unit) cost

Q :- Order quantity.

(1) Annual holding cost (H)

$= (\text{average no. of inventry}) \times (\text{holding cost per unit per year})$

$$= \left(\frac{Q}{2}\right) \times (C_h) = \frac{Q C_h}{2}$$

(2) Annual ordering cost

= cost per order x no. of order per year

$$= C_o \times \left(\frac{D}{Q}\right) = \frac{C_o D}{Q}$$

D → demand

Q → order quantity

(3) Total Inventry cost

$$TC \Rightarrow \left(\frac{Q C_h}{2} + \frac{C_o D}{Q}\right)$$

for getting optimum

$$\frac{d(TC)}{d(Q)} = \frac{C_h}{2} + \left(-\frac{C_o D}{Q^2}\right) = 0$$

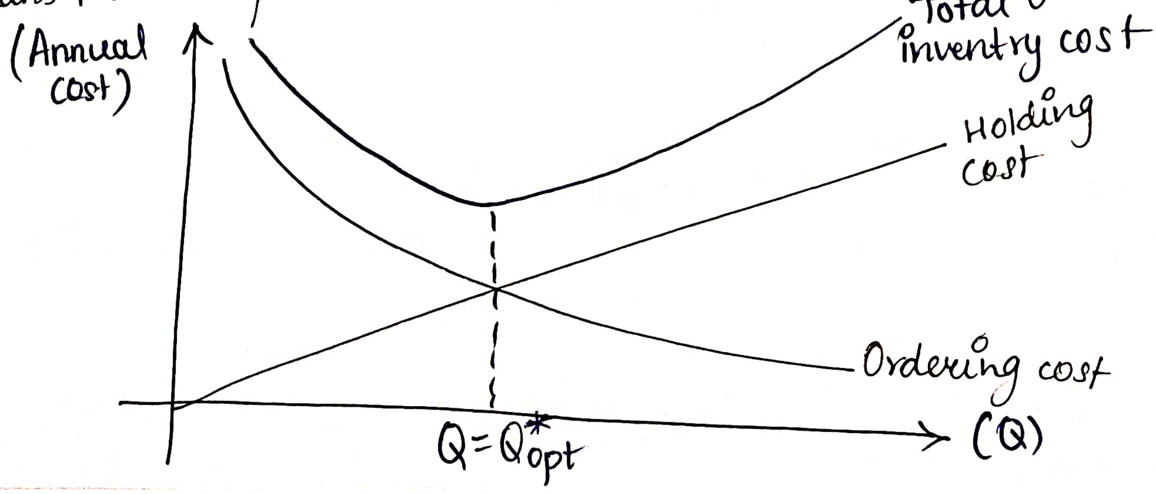
$$\therefore \boxed{Q^* = \sqrt{\frac{2 C_o D}{C_h}}}$$

(necessary condition for optimality)

$$\frac{d^2(TC)}{dQ^2} = +\frac{2 C_o D}{Q^3} > 0 \quad \text{at } Q = Q_{opt}$$

for minimising.

It means that Q_{opt} will minimise the total inventry cost.



Remarks :→

1> Optimal length of inventry replenishment cycle time (t^*) $\left(t^* = \frac{Q^*}{D}\right)$

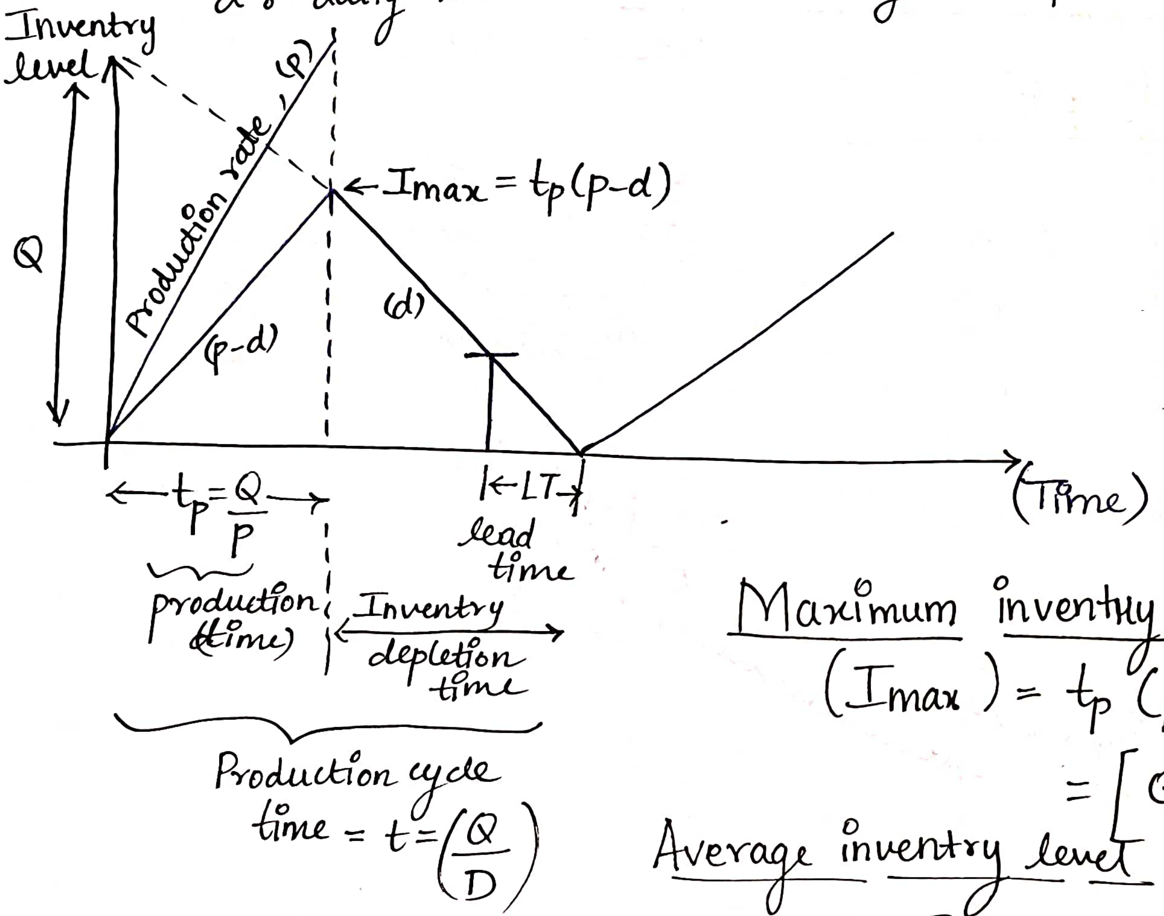
2> optimum no. of order $N^* = \left(\frac{D}{Q^*}\right)$

3> If (r) is the inventry carrying rate & "C" is the unit cost of item $\therefore$ $\left(C_h = r \times c\right)$

Economic Production Quantity (EPQ) model :-

- An inventry system in which an order is received gradually, as inventry simultaneously being depleted.

- P: daily rate at which order is received over a time.
- d :- daily rate at which inventry is depleted.



Maximum inventry level

$$(I_{max}) = t_p(p-d) = \frac{Q}{P}(p-d)$$
$$= \left[Q\left(1-\frac{d}{P}\right)\right]$$

Average inventry level

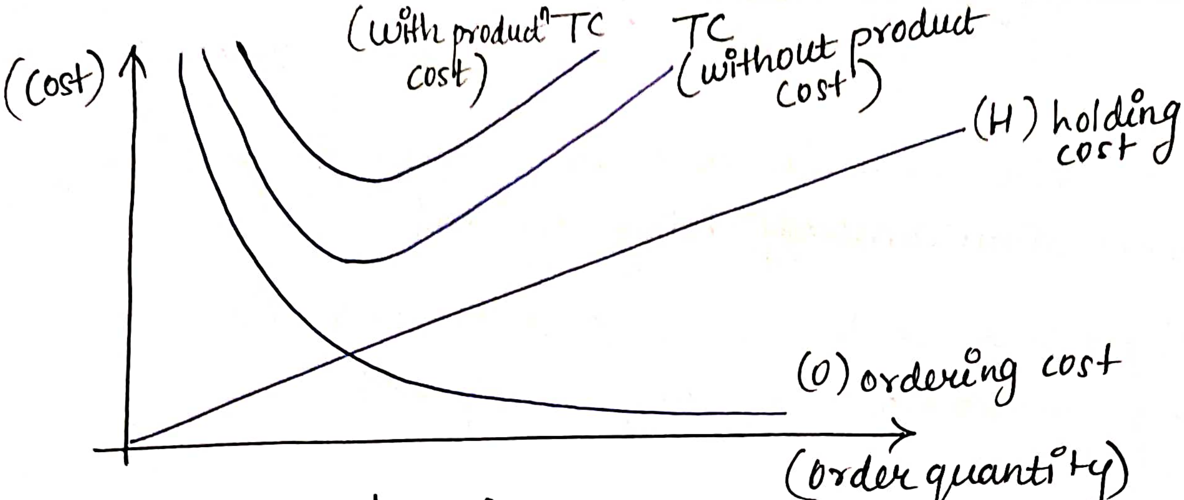
$$= \left[\frac{Q}{2}\left(1-\frac{d}{P}\right)\right]$$

Total cost of inventry

$$(TC) = C_o\left(\frac{D}{Q}\right) + C_h\frac{Q}{2}\left(1-\frac{d}{P}\right)$$

$$\frac{d(TC)}{dQ} = 0 \Rightarrow \frac{-C_oD}{(Q)^2} + \frac{1}{2}\left(1-\frac{d}{P}\right)C_h = 0$$

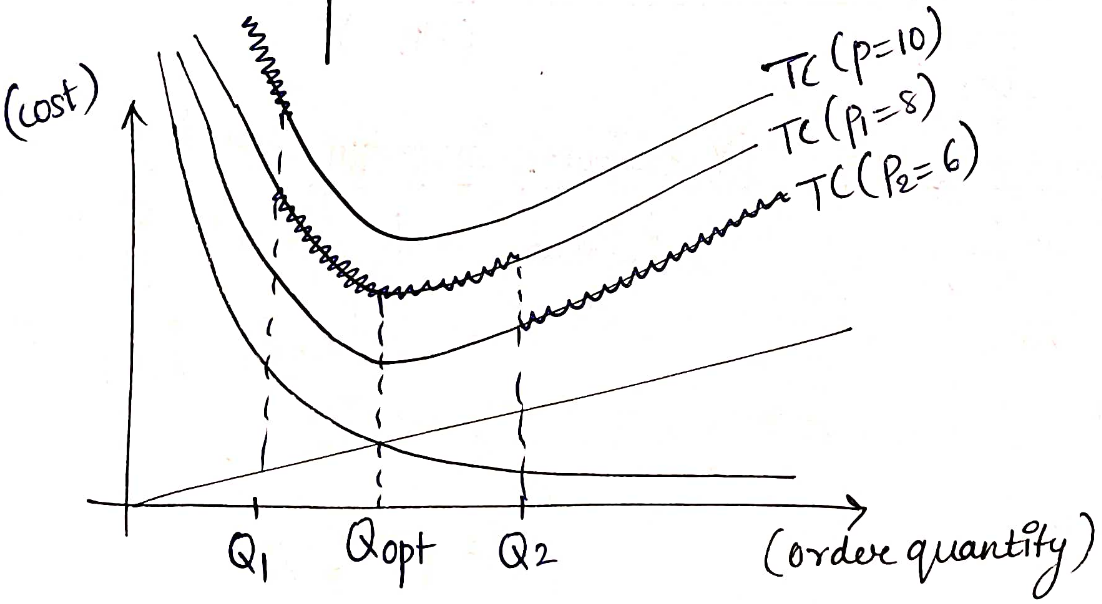
$$\therefore \boxed{Q = \sqrt{\frac{2C_oD}{C_h(1-d/p)}}}$$



Order size	price
0-99	10
100-199	8 (P_1)
200 +	6 (P_2)

Q_1 100 ↓ Q_2 200

Qoptimum (in between) $= \sqrt{\dfrac{2 C_o D}{C_h}}$



(1) ⇨ A Toy manufacturer uses approximately 32,000 silicon chips anually. The chips are used at a steady rate during the 240 days a year that the plant operates. Anually holding cost is \$3 per chip, ordering cost \$120. Determine optimum order quantity, holding cost, ordering cost, & total cost, the number of workdays in an order cycle.

(2) Determine the optimum order size for quantity discount model; Given $C_o =$ \$2500 ; $C_h =$ \$190 per computer per year $D = 200$.

Quantity	price / unit
1-49	$1400
50-89	$1100
90+	$900

Answer :→ $D = 32000$, $C_o = 120$

$C_h = 3$ ∴ $Q_{opt} = \sqrt{\frac{2C_oD}{C_h}} = 1600$

$H = C_h\left(\frac{Q}{2}\right) = 3 \times \frac{1600}{2}$ (holding)

$= 2400$

$O = \frac{C_oD}{Q} = \frac{120 \times 32000}{1600} = 2400$ (ordering)

$t^* = \frac{Q_{opt}}{D} = \frac{1600}{32{,}000} = \frac{1}{20} \times 240 = 12 \text{ days}$

240 → no. of work days.

$TC = O + H = 4800$

Answer :→ Step 1 :- First determine Q_{opt}

$Q_{opt} = \sqrt{\frac{2C_oD}{C_h}} = \sqrt{\frac{2 \times 2500 \times 200}{(190)}} = 72.5$

$TC = (pD) + \left(\frac{C_oD}{Q_{opt}}\right) + \left(\frac{C_h Q_{opt}}{2}\right)$

{p=1100} least one selected

$= (1100 \times 200) + \left(\frac{2500 \times 32000}{72.5}\right) + \left(\frac{190 \times 72.5}{2}\right)$

$= 2{,}33{,}784\ \$$

:→ Step 2 :- Compute the total inventry cost using lower unit price. $Q = 90$, $P = 900$

Q_1	Q_{opt}	Q_2
50	↓ 72.5	90

$TC = 900 \times 200 + \frac{2500 \times 900}{90} + \frac{190 \times 90}{2}$

$= 1{,}94{,}105\ \$$

∴ we will choose P_2 only !! cause TC is lesser in this case.

→ Reorder level with constant discount

EOQ model → how much should be ordered?

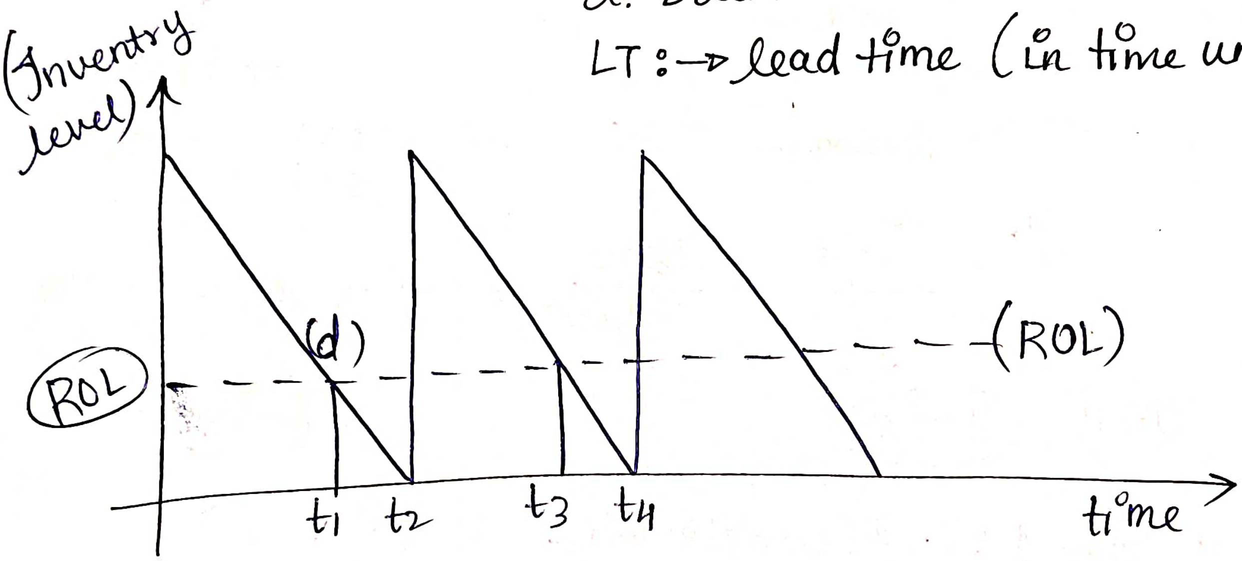
Now - when should an order be placed for replenishment.

Reorder point or reorder level (ROL); level of inventory at which order is placed.

* When both the demand and the lead time are constant & known. $(ROL = d \times LT)$

d:→ demand (in units) rate per time period

LT:→ lead time (in time units).



Example :→

$d = 20$ units per day

$LT = 1$ week $= 7$ days

$ROL = d \times LT = 20 \times 7 = 140$ units

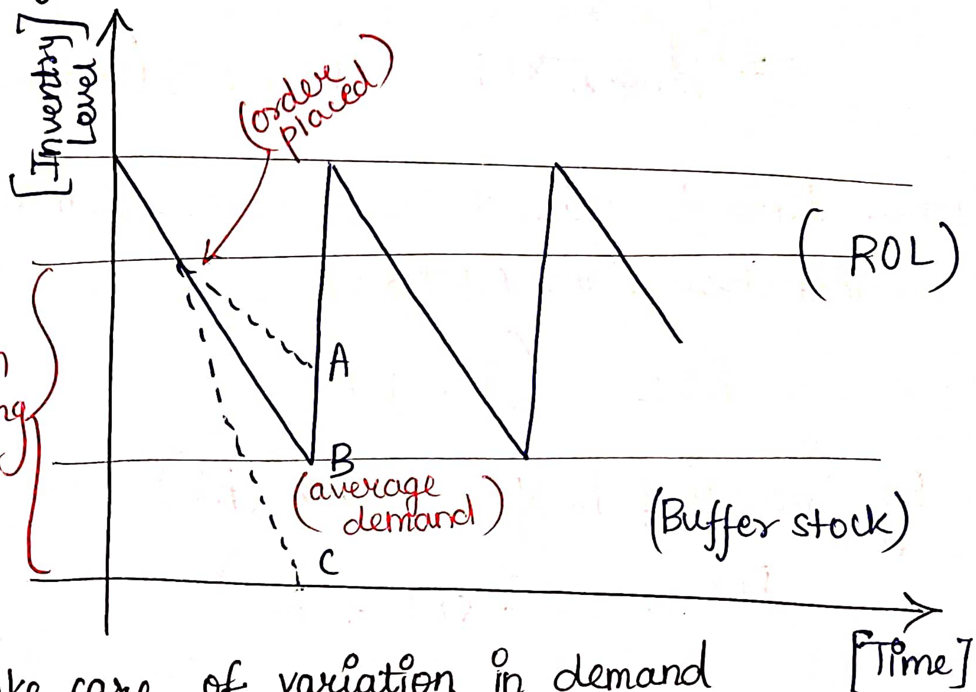
Note :→ No shortage

→ Additional Stocks

When the demand rate and/or the lead time are not known with certainty, then additional stocks are maintained.

Types of Stocks

(1) **Buffer Stocks:-**
Is maintained based on average demand during the average lead time.



(2) **Reserve Stocks :-**
is maintained to take care of variation in demand during reorder period.

(3) **Safety stock :-** is maintained to take care of variation in lead time.

→ Advantages of keeping Stock :→

* To maintain stock outs
* to provide better customer service.

→ Limitations :→

* Increase in total inventry cost

Buffer stock (BS) = (Average demand × average lead time)

Exp. If demand rate is 100 units per month and the normal & maximum lead time are 10 & 30 days, then

BS = average demant × avg lead time

$= \left(\frac{100}{30}\right) \times \left(\frac{10+30}{2}\right) \Rightarrow \frac{100}{30} \times \frac{40}{2} \Rightarrow \frac{200}{3}$

= 66.6 units.

→ when no stockouts are desired

$$BS = (\text{Maximum demand during LT}) - (\text{Average demand during LT})$$

$$= d_{max} \times LT - \bar{d} \times LT = (d_{max} - \bar{d})(LT)$$

when demand around the average demand ($\bar{d}$) during constant lead time (LT)

$$ROL = (\bar{d} \times LT)$$

By including buffer stock (BS)

$$ROL = (\bar{d} \times LT + BS)$$

C_o ordering cost   (D) $0.1 \times 10 = 1$

<u>Example</u> :→ The annual demand is 12,000 units, the ordering cost is Rs 60 per order, the carrying cost is 10% of unit price, the unit cost of the item is Rs 10, & the lead time is 10 days. There are 300 woking days in a year. Determine the (EOQ). & number of orders per year. In the past two years the demand rate has gone as high as 70 units per day. For a reordering system, based on inventry level, what should be the buffer stock? What should be the reorder level at this buffer stock? What would be the carrying cost for a year?

$D = 12000$ units

$C_o = ₹60$ per order

$C_h = 10\% \, P = 0.1 \times 10 = $ Rs (1) per unit per year.

$P = $ Rs 10 ; LT (10 days)

Working days in a year = 300 days

(i) $EOQ = \sqrt{\frac{2C_oD}{C_h}} = \sqrt{\frac{2\times60\times12000}{1}} = 1200 \text{ units}$

(ii) No. of orders $= \frac{D}{Q_{opt}} = \frac{12,000}{1200} = 10$ orders (per year)

(iii) $d_{max} = 70$ units/day

Average ~~com~~ consumption / demand per day $= \frac{12,000}{300}$

$= 40$ units/days

$BS \Rightarrow (d_{max} - \bar{d}) \times LT = (70 - 40) \times (10)$

$= 300$ units

(iv) ROL = Average demand during the LT + buffer stock.

$= \bar{d} \times LT + \underbrace{300}_{BS} = 40 \times 10 + 300 = 700$ units

(v)

Average inventry stock in a year $= \left(BS + \frac{Q^*}{2}\right)$

$= \left(300 + \frac{1200}{2}\right) = 300 + 600 = 900$ units

Annual carrying / holding cost $= 900 \times C_h = 900 \times (1)$ per unit per year

$=$ ₹900

→ Continuous System (Q-system) with uncertainty demand

* If the demand during lead time is varying unexpectedly in a way that the amount of buffer stock is not enough to meet it, then there would be stockout.

It can be avoided by

→ 1) Increasing BS, meaning, increasing the average inventry ($BS + Q^*/2$) [optimum size of BS should be maintained]

→ 2) raising level of ROL above average demand and adjusting order quantity (Q^*)

In Q-system, the probability of variation in demand during lead time (DDLT). is controlled by raising or lowering (ROL).

Step 1 :- Calculate $Q^*(EOQ) = \sqrt{\frac{2 C_o D}{C_e}}$

Step 2 :- Determine ROL to trade off between shortage cost and carrying cost for creating additional inventry (also called reserve stock). This reserve stock is to no. of units by which ROL is raised above expected (or average) demand during lead time to balance shortage cost.

→ If DDLT is probablistic and normally distributed, then

ROL = average demand during average lead time

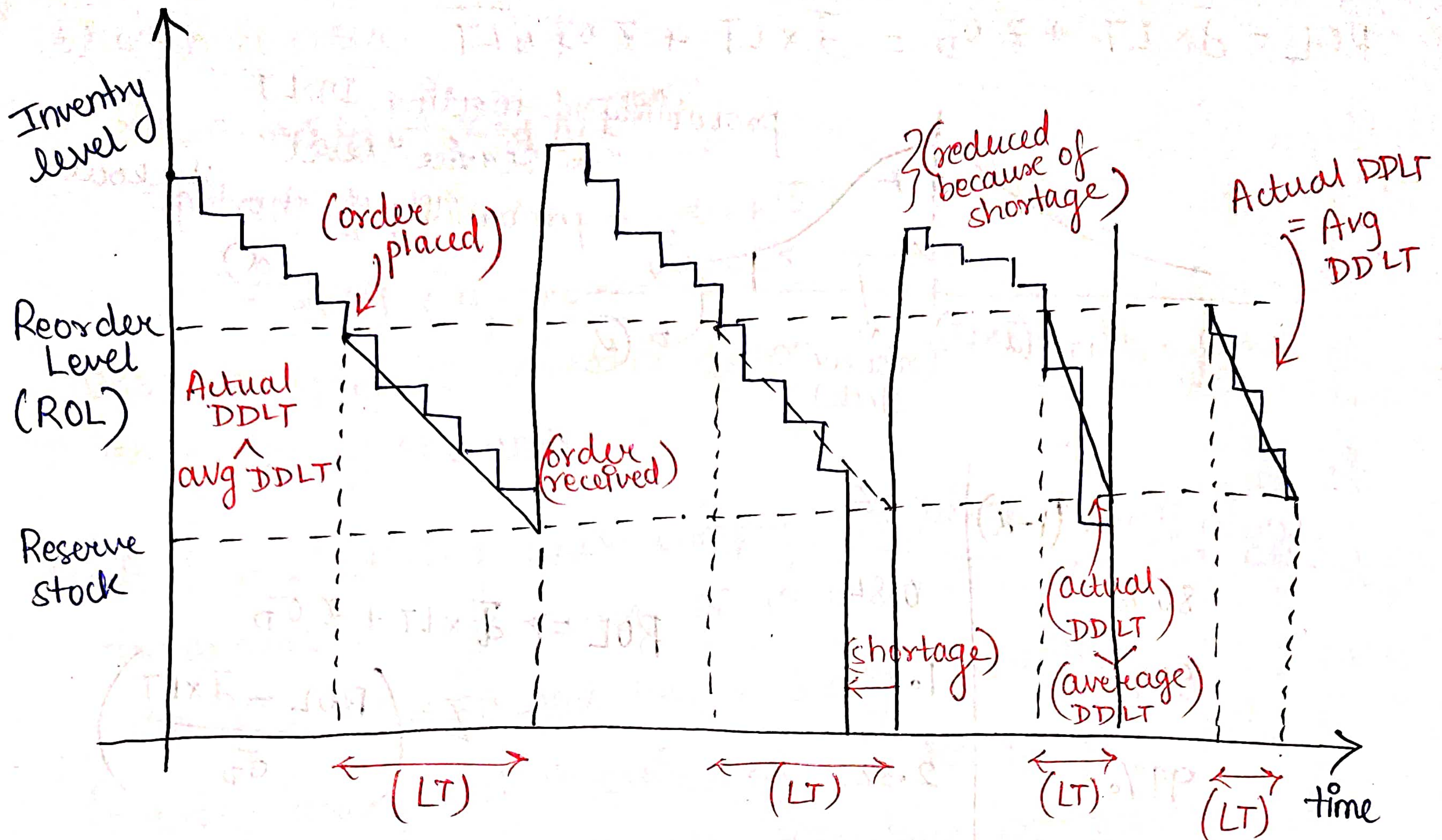
+

Reserve stock + Safety stock.

→ Reserve stock (RS) = service level factor × standard deviation of demand during lead time.

→ Safety stock (SS) = Average demand during maximum delay in lead time × probability of such delay.

→ If DDLT is described by a probablity density function per unit time (eg. per day or min).



→ Assumptions during (DDLT) :-

(1) $\bar{D}$:- average demand during LT ; σ_D :- standard deviation described by normal probability distribution.

(2) Demand in one period is independent of another periods.

n :- number of periods in the lead time.

$\bar{d}$:- average demand for items per unit time (periods)

Z :- number of standard deviations from average of DDLT distribution redured for a specific service level.

$(\sigma_d)^2$: variance of demand for items per unit time.

(Adding variance)

$\left[(\sigma)^2 = v\right]$ (std deviation) (variance)

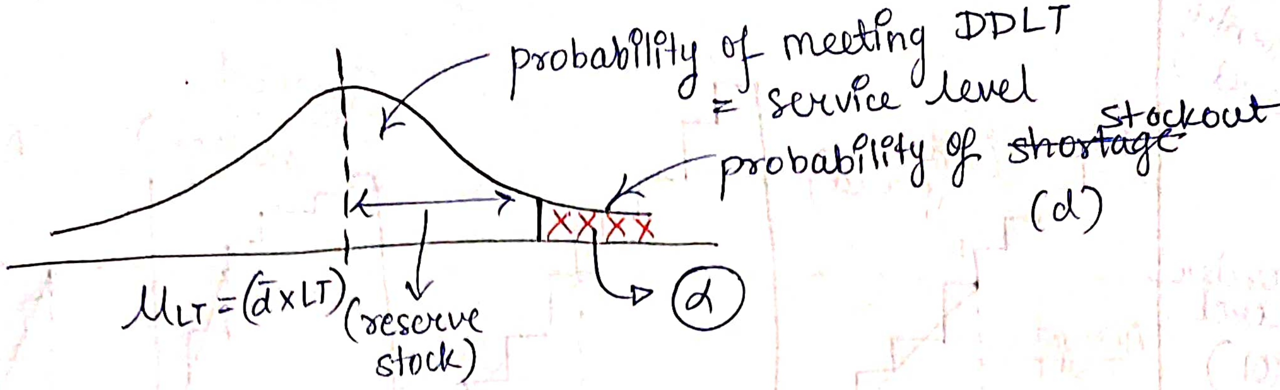
$$(\sigma_D)^2 = n_1(\sigma_d)^2 + n_2(\sigma_d)^2 + \dots n_n(\sigma_d)^2$$

$$= \sum n_i(\sigma_d)^2 = (\sigma_d)^2 (LT)$$

$$\sigma_D = \sigma_d \sqrt{LT}$$

Reserve stock $= Z\sigma_D = \left(Z\sigma_d\sqrt{LT}\right)$

$$ROL = \bar{d} \times LT + Z\sigma_D = \bar{d} \times LT + Z\sigma_{\bar{d}}\sqrt{LT}$$



service level $(1-\alpha)$	Z
80%	0.84
90%	1.28
99%	2.32

$$ROL \Rightarrow \bar{d} \times LT + Z\sigma_D$$

$$\Rightarrow Z = \left(\frac{ROL - \bar{d} \times LT}{\sigma_D}\right)$$

(Q1) The following ~~it~~ have been collected for an item ; Annual demand (D) = 1800 units , ordering costs Rs 100 per order. cost of item is Rs 5 per unit ; carrying cost is 20% of unit cost per year per item . Replenishment lead time is 2 days , mean demand during replenishment time is 100 units with standard deviation of 30 units & normally distributed . The stockout probability during lead time is 25% . When & how much to order for the Q-system?

Given :→ $D = 1800$; $C_o = 100$

$p = $ Rs 5/unit ; $C_h = 20\%$ of $P = \frac{20}{100} \times 5 = $ Rs 1 per unit per year.

$LT = 2$ days ; $\bar{d} = 100$; $\sigma_{\bar{d}} = 30$

$Z(75\%) = 0.67$

(a) $Q^* = \sqrt{\frac{2C_oD}{C_h}} = 600$

(b) When to order

$ROL = \bar{d} \times LT + Z\sigma_d \sqrt{LT}$

$= 100 \times 2 + 0.67 \times 30 \times \sqrt{2}$

$= 229$ (approx)

when an effective inventry drops to 229 units, place an order of 600 units.

(10) ⇒ A manufacturing unit company requires a component at the annual average value of 1000 units. Placing an order costs Rs 480 and has a 5 day lead time. Inventry holding cost is estimated as Rs 15 per unit per year. The plant operates 250 days per year. It is assumed that daily demand is normally distributed with an average of 4 units; with a standard deviation of 1.2. Suggest an inventry policy to control inventry of the item based on a 95% service level.

$D = 1000$, $C_o =$ Rs 480, $LT = 5$ days

$C_h =$ Rs 15 per unit per year.

DDLT : $\bar{d} = 4$; $\sigma_d = 1.2$

service level $(1-\alpha) = 95\%$ $Z(95\%) = 1.65$

$ROL = \bar{d} \times LT + Z\sigma_d \sqrt{LT} = 4 \times 5 + 1.65 \times 1.2 \times \sqrt{5}$

$= 25$ units (approx)

Inventry policy is to keep

$RS = ROL - \bar{d} \times LT = 25 - 20 = 5$ units & place an order when effective inventory drops to 25 units.

Forecasting :-

(1) Forecast is a prediction of what will occur in the future.
(2) The process of analysing current and historical data to determine future trends.
(3) Forecasting is an art of specifying meaningful information about future.

Example :-

- Metereologists forecast the weather
- Gamblers predict the winner of a football game
- Managers of business firms attempts to predict how much of their product will be desired in the future.
- A fore case of product demand is the basis for most important management planning decisions
  - scheduling, inventry control, process control, facility layout, work force, material purchasing, etc.

Characteristics of forecast :-

(1) A good forecast gives some measure of error.
(2) forcasting aggregate units is easier than for individual units.
(3) Forecasting technique should be used with known information.

Forecasting horizons :-

Short-range forecast :-

- time frame from one day to three months
- used for day to day production :- scheduling, inventry planning, workforce planning, etc

Medium-range forecast :-

- time frame from 3 months to three years.
- used for production and layout planning, sales & marketting planning, cash budget, planning, capital budget planning, etc.

Long-range forecast :-

- Time frame more than 3 years.
- Used for strategic planning in terms of capacity planning, new product planning, expansion planning, etc.

Forecasting Methods :-

(1) Quantitive Methods :-

- ~~Lowe~~ based on human judgement
- If historical data is not available.
- used for long-range forecasting

(2) Casual forecasting method :-

- uses some explanatory variables to predict the future.

(3) Time series methods / analysis :-

- Statistical techniques based on historical data.

Qualitative Methods

(1) Sales force composites

- sales force or customer service area has direct contact with consumers.
- They are in good position to see changes in markets.
- Sales managers can predict demand.

(2) Customer survey or market survey

- organised approach using surveys to determine customer needs and wants for the products / services.
- It can provide quite accurate and useful forecast.
- Needs skills, should be conducted accurately.
- It can be expensive
- conducted through mailing, telephone or personal interview.
- Needs sufficient responses for forecast.
- Design of questionnaries, no. of responses and ~~satisfying~~ & sampling plane or scheme should be accurate.

(3) The Delphi Method :-

- Useful for forecasting for technological change & advances.
- Expert opinions are collected for preparing forecast.

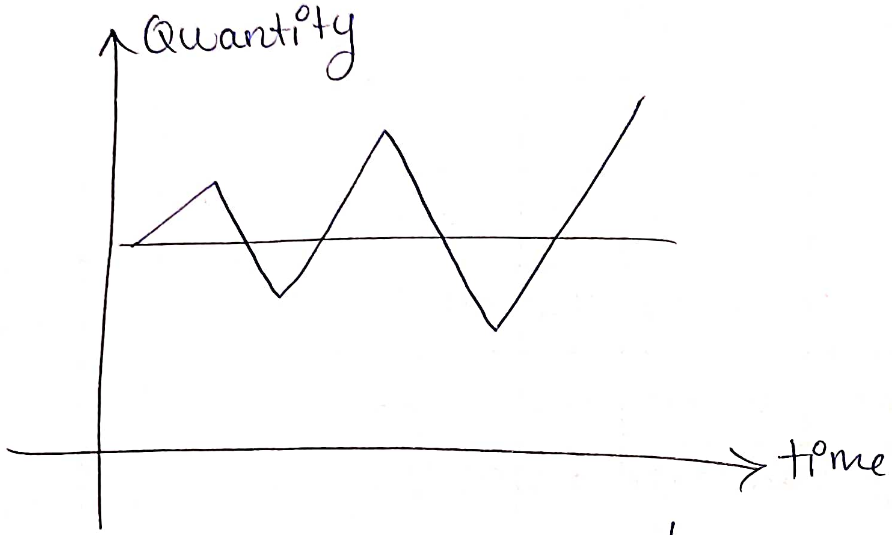
~~Time series~~

Time Series Method :.

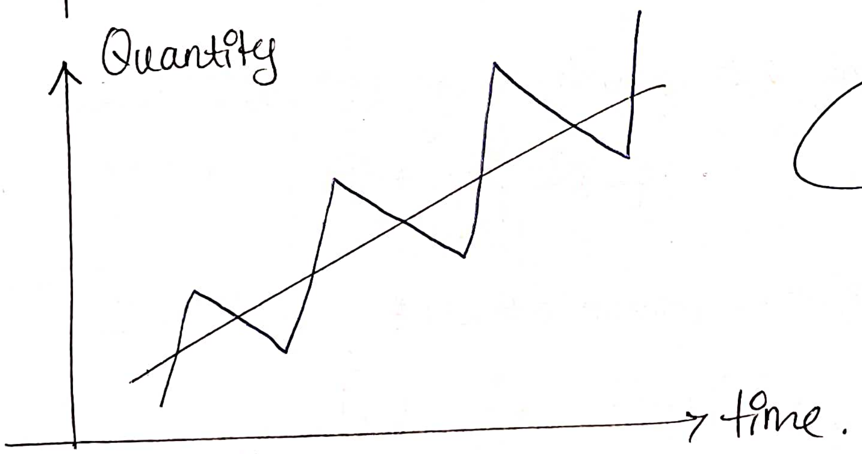
→ Statistical techniques using historical data.

→ Assumption :- past trend will continue in future.

Demand behaviour

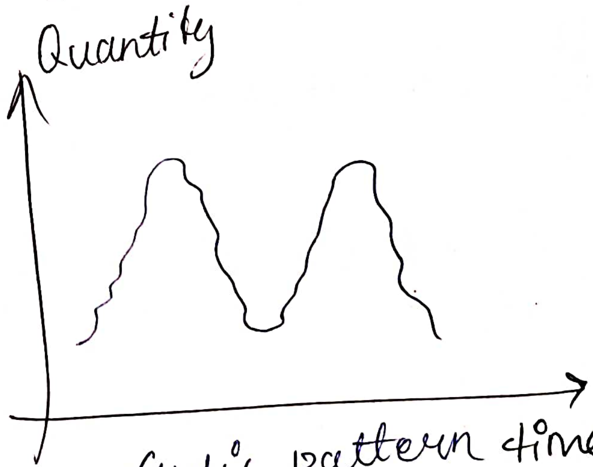


Horizontal pattern /
historical pattern /
Stationary pattern.



Trend pattern

↳ Gradual long term up ordown movement.



Seasonal pattern

↳ short term regular variation in data.

Cyclic pattern time

↳ length of a single cycle is longer than a year.

Months	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec	Jan
Demand	450	440	460	510	520	495	475	560	510	520	540	550	
1-3 months moving avg	—	—	—	450	470	496.7	508.3	496.7	510	515	530	523.3	536.7
mean-absolute error				60	50	1.7	33.3	63.3	0	5	10	26.7	$\frac{\text{sum}}{9} = 27.8$
six month moving avg	—	—	—	—	—	—	479	483.33	503.3	511.7	513.3	516.7	525.8
mean-absolute error							4	76.7	6.7	8.3	26.7	33.3	26
1-3 months weighted moving avg	—	—	—	452.6	480	502.5	505	491.3	522.5	513.8	527.5	527.5	540
mean abs error				57.5	40	7.5	30	68.75	12.5	6.25	12.5	22.5	28.6
Exponential smoothning	450	450	442	457.6	432.1	518.5	490.8	475.6	543.6	506.8	515.4	532.3	541.9
mean abs error		10	18	52.4	21.9	13.5	15.8	84.4	37.5	13.2	24.6	17.7	25.04

→ Simple moving average :- Average data for desired number of periods $F_t = \sum_{j=1}^{n} D_{t-j}$

for example :- 3 months moving average , $n=3$

$$F_t = \frac{D_{t-1} + D_{t-2} + D_{t-3}}{3}$$

$$F_{april} = \frac{D_{march} + D_{Feb} + D_{Jan}}{3} = \frac{460+440+450}{3} = 450$$

$$F_{may} = \frac{D_{april} + D_{march} + D_{Feb}}{3} = \frac{510+460+440}{3} = 470$$

$$F_{june} = \frac{520+510+460}{3} = 496.7$$

Forecasting performance error

(1) Mean absolute error / derivative = $\frac{\sum \left| \text{Actual demand} - \text{Forecasted value} \right|}{W}$

MAD

(2) Cumulative forecast Error (CFE)

$$CFE = \sum (\text{Actual demand} - \text{forecasted demand})$$

→ measures any bias in the forecast

(3) Mean square error (MSE)

$$MSE = \frac{\sum(\text{Actual demand} - \text{forecasted demand})^2}{W}$$

→ penalise large error.

MAD for 3 months moving average

$$MAD_{April} = |510-450| = 60$$

$$MAD_{may} = |520-470| = 50$$

6 months moving average.

$$F_{july} = \frac{D_{Jan} + D_{Feb} + D_{mar} + D_{april} + D_{may} + D_{june}}{6}$$

$$= 479.107 \approx 479$$

$$F_{aug} = \frac{D_{Feb} + D_{mar} + D_{april} + D_{may} + D_{june} + D_{july}}{6}$$

$$= 483.33$$

$$MAD_{aug} = |560 - 483.33| = 76.7$$

weighted moving average

→ higher weight is assigned to most recent dates

$$F_t = \sum_{j=1}^{n} W_j D_j \quad \text{when} \quad \sum_{j=1}^{n} W_j = 1$$

Exp :- $F_{april} = 0.5 D_{mar} + 0.25 D_{feb} + 0.25 D_{jan} = (452.6)$

$$MAD_{april} = |510 - 452.6| = 57.6$$

Exponential smoothing

$$F_{t+1} = \alpha D_t + (1-\alpha) F_t$$

α = smoothing constant, $0 \le \alpha \le 1$

→ most frequently used in time series method because it is simple to use and minimal amount of data needed for F_t, D_t, α

⇒ $F_1 = D_1$; $F_2 =$

let $\alpha = 0.8$, $F_{Feb} = \alpha D_{Jan} + (1-\alpha) F_{Jan}$

$= 0.8 \times 450 + (1-0.8) \times 450$

$F_2 = 450$

$F_{march} = \alpha D_{Feb} + (1-\alpha) F_{Feb}$

$= (0.8)(440) + (1-0.8)(450)$

$F_{march} = 442$

$MAD = |440-450| = 10$

, $MAD = |460-442| = 18$

→ Double exponential Smoothing or Holt trend

$$F_{t+1} = \alpha D_t + (1-\alpha)(F_t + T_t)$$

$$T_t = \beta (F_t + F_{t-1}) + (1-\beta) T_{t-1}$$

$0 \le \beta \le 1$

T_t : Trend

Initial values :-
1> $F_1 = D_1$
2> $T_1 = D_2 - D_1$ } method 1
3> $T_1 = [(D_2 - D_1) + (D_3 - D_2) + (D_4 - D_3)] / 3$
$= (D_4 - D_1)/3$

$$\therefore \; T_1 = \frac{D_n - D_1}{n-1}$$

Exp $\Rightarrow$ $\alpha = 0.2$; $\beta = 0.2$

$$F_1 = D_1 \; ; \; T_1 = \frac{D_{Dec} - D_{Jan}}{11} = \frac{550 - 450}{11}$$

$$= 9.1$$

$$F_2 = 0.2 \times D_{Jan} + 0.8\,(F_{Jan} + T_{Jan})$$

$$= 0.2 \times 450 + 0.8\,(450 + 9.1)$$

$$= 457.3$$

$$T_2 = \beta\,(457.3 - 450) + 0.8\,(9.1)$$

$$= 0.2\,(7.3) + 0.8\,(9.1) = 8.7$$

Double Exponent smooth	450	457.3	460.8										549.8
Trend (T_t)	9.1	8.7	7.7										~~549.8~~ 8.2
MAD	0	17.3	0.8										18.3

Quarter ↓ / Year →	1	2	3	4	Average ~~seasonal~~ ~~index~~.	seasonal Index
1	72	76	78	81	76.75	
2	110	112	119	134	118.75	
3	172	194	201	216	195.75	
4	117	130	128	141	129	
Average	117.75	128	131.5	143	130.06	

Step 4 $\xrightarrow{SE_1}$ $\dfrac{76.75}{130.06} = (0.5901)$

$SE_2 \rightarrow \dfrac{118.75}{130} = (0.9130)$

Step 5 ⇒ Forecast for the 5th year

$Q_1 = 149.875 \times SI_1 = 149.875 \times 0.5901$
$= 88.44$

$Q_2 = 149.875 \times SI_2 = ✓$

$Q_3 = 149.875 \times SI_3 = ✓$

$Q_4 = 149.875 \times SI_4 = ✓$

2nd Quize → (30th April Saturday)

syllabus; all after midsem.

Step (1)

→ divide the data seasonwise

Step (2)

→ linear regression for trend.

Year	Avg (Y)	xY
1	117.75	117.75
2	128	256
3	131.5	394.5
4	143	572
$\overline{\text{Year}} = 2.5$	$\bar{Y} = 130.06$	$\Sigma XY = 1340.25$

$\bar{X} =$

$$b = \left(\frac{1340.25 - 4 \times 2.5 \times 130.06}{30 - 4(2.5)^2} \right)$$

$$= 7.93$$

$$a = \bar{Y} - b\bar{X} = 130.06 - 7.93(2.5)$$

$$= 110.235$$

$$Y(5) = 110.235 + 7.93(5) = 149.885$$

Step 3

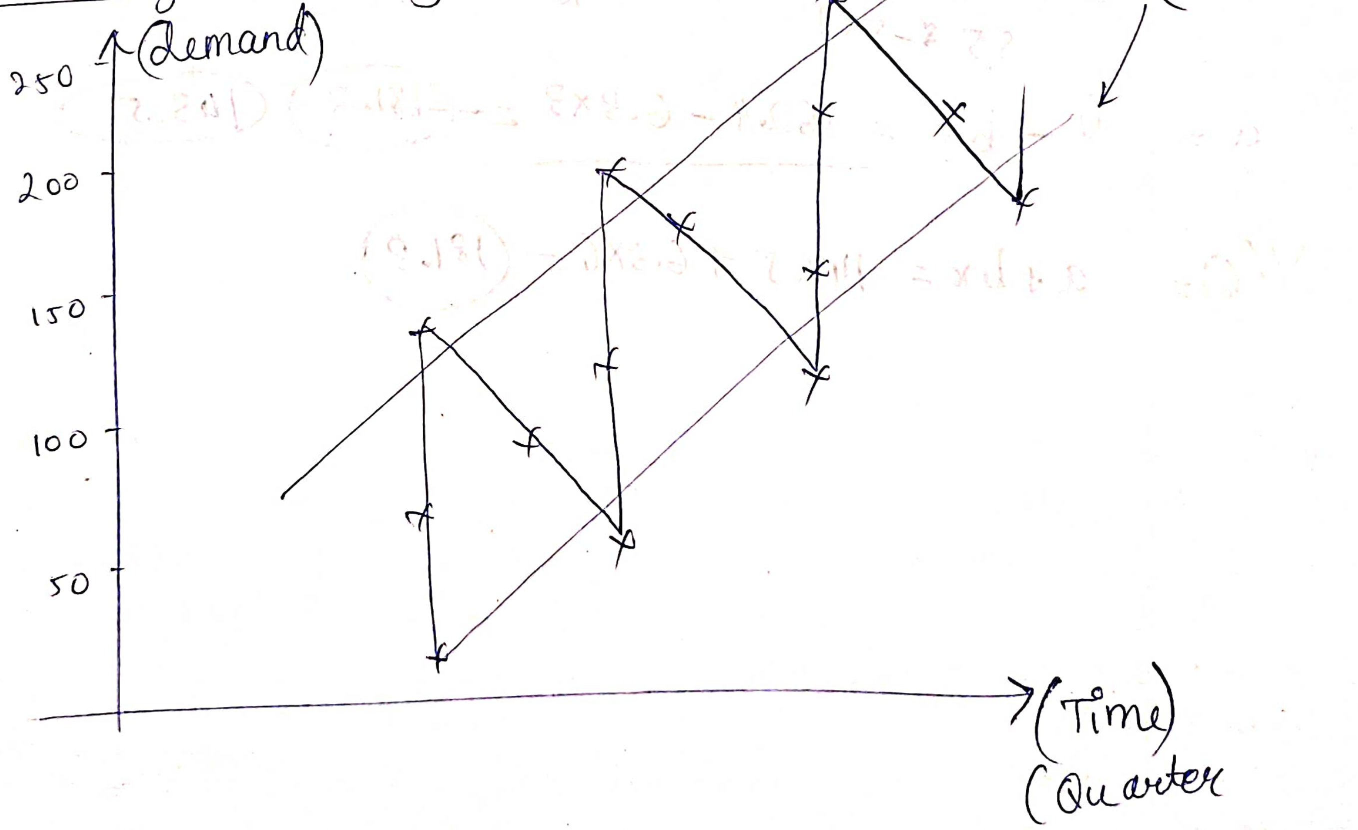
→ Average of each ~~sensor~~ season

Step 4

→ seasonal index $= \dfrac{\text{Average of each season}}{\text{total average}}$.

Year	1				2				3				4				5			
Quarter	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
demand	72	110	172	117	76	112	194	730	78	119	201	128	81 ~~201~~	134 ~~128~~	216 ~~81~~	141 ~~134~~		149.875		
seasonal index																	0.5901	0.913	1.575	0.9918
forecast																	88.94	136.83	222.53	148.65

Forecasting seasonality



Trend projection by linear regression

$Y = a + bx$ X : time
Y : Forecasted value / demand.

$$b = \frac{\sum xy - n\bar{x}\bar{y}}{\sum x^2 - n\bar{x}^2}$$

$$a = \bar{y} - b\bar{x}$$

Exp.

Weeks (X)	Sales (Y)	XY	X×X
(1)	150	150	1
(2)	157	314	4
(3)	162	486	9
(4)	166	~~486~~ 664	16
(5)	177	885	25
$\bar{X} = \frac{15}{5} = (3)$	$\bar{Y} = 162.4$	$\sum XY = 2499$	$\sum X \cdot X = 55$

$$b = \frac{2499 - 5\times3\times162.4}{55 - 5\times9} = \frac{63}{10} = 6.3$$

$$a = \bar{y} - b\bar{x} = 162.4 - 6.3\times3 = \cancel{181.3}\ 143.5$$

$$Y(6) = a + bx = 143.5 + 6.3\times6 = 181.3$$