Assignment 3

Ans 1) Maximize $Z = 3x_1 + 2x_2 + x_3$

subject to the constraint

(i) $2x_1 + 5x_2 + x_3 = 12$ (ii) $3x_1 + 4x_2 = 11$

$x_2, x_3 \geq 0$, x_1 unrestricted

Since x_1 is unrestricted, $x_1 = x_1^- - x_1^+$, where $x_1^- \geq 0$ and $x_1^+ \geq 0$

Equation form for simplex method

Maximize $z = 3(x_1^- - x_1^+) + 2x_2 + x_3 - MR_1 - MR_2$

or $z = 3x_1^- + 2x_2 + x_3 - MR_1 - MR_2 - 3x_1^+$

or $z - 3x^- - 2x_2 - x_3 + MR_1 + MR_2 + 3x_1^+ = 0$

subjected to

(i) $2(x_1^- - x_1^+) + 5x_2 + x_3 + R_1 = 12$

or $2x_1^- + 5x_2 + x_3 + R_1 - 2x_1^+ = 12$

(ii) $3(x_1^- - x_1^+) + 4x_2 + R_2 = 11$

or $3x_1^- + 4x_2 + R_2 - 3x_1^+ = 11$

such that $x_1^-, x_1^+, x_2, x_3, R_1, R_2 \geq 0$

Ans 2) (a) Maximize $Z = 5x_1 + 3x_2$

subject to (i) $x_1 + x_2 \leq 2$

(ii) $5x_1 + 2x_2 \leq 10$

(iii) $3x_1 + 8x_2 \leq 12$ $x_1, x_2 \geq 0$

Simplex Method

constraints : $x_1 + x_2 + S_1 = 2$

$5x_1 + 2x_2 + S_2 = 10$

$3x_1 + 8x_2 + S_3 = 12$ $x_1, x_2, S_1, S_2, S_3 \geq 0$

Maximize $Z = 5x_1 + 3x_2 + 0 \cdot S_1 + 0 \cdot S_2 + 0 \cdot S_3$

$Z - 5x_1 - 3x_2 = 0$

$n = 5$, $m = 3$ Basic variables $= \{S_1, S_2, S_3\}$

Initial Simplex Table

Basic	Z	x_1	x_2	S_1	S_2	S_3	Solution	Ratio	
Z	1	-5	-3	0	0	0	0		
S_1	0	1	1	1	0	0	2	2	} Tie
S_2	0	5	2	0	1	0	10	2	
S_3	0	3	8	0	0	1	12	4	

Entering variable $= x_1$, Leaving variable $= S_1$, Pivot element $= 1$

1st Iteration

Basic	Z	x_1	x_2	S_1	S_2	S_3	Solution	Ratio
Z	1	0	2	5	0	0	10	
~~S_1~~ x_1	0	1	1	1	0	0	2	
S_2	0	0	-3	-5	1	0	0	
S_3	0	0	5	-3	0	1	6	

No entering variable left

$x_1 = 2,\ x_2 = 0,\ z = 5x_1 + 3x_2 = 10 \leftarrow$ Optimum solution

Since there was a tie in ratio when chosing deciding variable, this is a case of degeneracy.

The constraint $5x_1 + 2x_2 \le 10$ is redundant.

(b) Maximize $z = 6x_1 + 3x_2$
subject to (i) $2x_1 + x_2 \le 8$
(ii) $3x_1 + 3x_2 \le 18$
(iii) $x_2 \le 3$, $x_1, x_2 \ge 0$

Simplex Method
Constraint : $2x_1 + x_2 + s_1 = 8$
$3x_1 + 3x_2 + s_2 = 18$
$x_2 + s_3 = 3$, $x_1, x_2, s_1, s_2, s_3 \ge 0$

Maximize
$$z = 6x_1 + 3x_2 + 0 \cdot s_1 + 0 \cdot s_2 + 0 \cdot s_3$$
$$z - 6x_1 - 3x_2 = 0$$
$n = 5,\ m = 3$. Basic Variables $= \{s_1, s_2, s_3\}$

Initial Simplex Table

Basic	z	x_1	x_2	s_1	s_2	s_3	Solution	Ratio
z	1	−6	−3	0	0	0	0	
s_1	0	2	1	1	0	0	8	4
s_2	0	3	3	0	1	0	18	6
s_3	0	0	1	0	0	1	3	3/0

Entering variable $= x_1$, leaving variable $= s_1$, Pivot element $= 2$

1st Iteration

Basic	z	x_1	x_2	s_1	s_2	s_3	solution	Ratio
z	1	0	0	3	0	0	24	
x_1	0	1	1/2	1/2	0	0	4	8
s_2	0	0	3/2	-3/2	1	0	6	4
s_3	0	0	1	0	0	1	3	3

We have already attained optima, $z = 24, x_1 = 4, x_2 = 0$
still, entering variable $= x_2$, leaving variable $= s_3$, Pivot element $= 1$

Basic	z	x_1	x_2	s_1	s_2	s_3	solution	Ratio
z	1	0	0	3	0	0	24	
x_1	0	1	0	1/2	0	-1/2	5/2	
s_2	0	0	0	-3/2	1	-3/2	3/2	
x_2	0	0	1	0	0	1	3	

Alternate optima, $x_1 = 5/2, x_2 = 3, z = 24$

This is the case of alternate optima, where the objective function $(6x_1 + 3x_2)$ is parallel to the binding constraint $(2x_1 + x_2 \leq 8)$ and all solution on that line between $(4,0)$ and $(5/2, 3)$ are optimal.

(C) Maximize $z = -2x_1 + 3x_2$
Subject to (i) $x_1 \leq 5$
(ii) $2x_1 - 3x_2 \leq 6$, $x_1, x_2 \geq 0$

Simplex Method:
Constraints $x_1 + s_1 = 5$
$2x_1 - 3x_2 + s_2 = 6$, $x_1, x_2, s_1, s_2 \geq 0$

Maximize $z = -2x_1 + 3x_2 + 0 \cdot S_1 + 0 \cdot S_2$

$z + 2x_1 - 3x_2 = 0$

~~x~~ $n = 4$, $m = 2$ Basic Variables $= S_1, S_2$

Initial Simplex Table

Basic	z	x_1	x_2	S_1	S_2	Solution	Ratio
z	1	2	-3	0	0	0	
S_1	0	1	0	1	0	5	5/0
S_2	0	2	-3	0	1	6	-2

All constraint coefficients under x_2 are either 0 or negative, meaning no leaving variable and that x_2 can be increased infinitely without violating any constraints.

Thus, this is a case of unbounded solution.

(d) Maximize $z = 2x_1 + 3x_2$

subject to (i) $x_1 - x_2 \geq 4$

(ii) $x_1 + x_2 \leq 6$

(iii) $x_1 \leq 2$, $x_1, x_2 \geq 0$

Simplex Method

Constraint : $x_1 - x_2 - S_1 + R_1 = 4$

$x_1 + x_2 + S_2 = 6$

$x_1 + S_3 = 2$, $x_1, x_2, S_1, S_2, S_3, R_1 \geq 0$

Maximize $z = 2x_1 + 3x_2 + 0 \cdot S_1 + 0 \cdot S_2 + 0 \cdot S_3 - MR_1$

$\Rightarrow z - 2x_1 - 3x_2 + MR_1 = 0$

$n = 6$, $m = 3$, Basic Variable = $\{R_1, S_2, S_3\}$

Initial Simplex Table

Basic	x_1	x_2	S_1	S_2	S_3	R_1	Solution	Ratio
Z	-2	-3	0	0	0	M	0	
R_1	1	-1	-1	0	0	1	4	
S_2	1	1	0	1	0	0	6	
S_3	1	0	0	0	1	0	2	

New z-row = Old z-row − MR_1-row

Basic	x_1	x_2	S_1	S_2	S_3	R_1	Solution	Ratio
Z	-2-M	-3+M	M	0	0	0	-4M	
R_1	1	-1	-1	0	0	1	4	4
S_2	1	1	0	1	0	0	6	6
S_3	1	0	0	0	1	0	2	2

Entering variable = x_1, leaving variable = S_3, Pivot element = 1

Basic	x_1	x_2	S_1	S_2	S_3	R_1	Solution	Ratio
Z	0	-3+M	M	0	2+M	0	4-2M	
R_1	0	-1	-1	0	-1	1	2	
S_2	0	1	0	1	-1	0	4	
x_1	1	0	0	0	1	0	2	

This is pseudo-optimum solution

In this optimum iteration, artificial variable R is non zero (=2), thus this is a case of infeasible solution.

Ans 3) Maximize, $z = 4x_1 + 6x_2 + 2x_3$

subject to the constraints

(i) $x_1 + x_2 + x_3 \le 3$

(ii) $x_1 + 4x_2 + 7x_3 \le 9$, $x_1, x_2, x_3 \ge 0$

where x_1, x_2, x_3 = number of units produced of A, B and C, respectively.

(a) Simplex Method: Constraint

$x_1 + x_2 + x_3 + s_1 = 3$

$x_1 + 4x_2 + 7x_3 + s_2 = 9$, $x_1, x_2, x_3, s_1, s_2 \ge 0$

Maximize $z = 4x_1 + 6x_2 + 2x_3 + 0.s_1 + 0.s_2$

$\longrightarrow z - 4x_1 - 6x_2 - 2x_3 = 0$ Basic Variable $\{s_1, s_2\}$

Initial Simplex Table

Basic	z	x_1	x_2	x_3	s_1	s_2	solution	Ratio
z	1	-4	-6	-2	0	0	0	
s_1	0	1	1	1	1	0	3	3
s_2	0	1	4	7	0	1	9	$9/4$

Entering Variable = x_2, leaving variable = s_2, Pivot element = 4

1st Iteration

Basic	z	x_1	x_2	x_3	s_1	s_2	solution	Ratio
z	1	$-5/2$	0	$17/2$	0	$3/2$	$27/2$	
s_1	0	$3/4$	0	$-3/4$	1	$-1/4$	$3/4$	1
x_2	0	$1/4$	1	$7/4$	0	$1/4$	$9/4$	9

Entering variable = x_1, leaving variable = s_1, Pivot element = $3/4$

2nd Iteration

Basic	z	x_1	x_2	x_3	s_1	s_2	solution	Ratio
z	1	0	0	6	10/3	2/3	16	
x_1	0	1	0	-1	4/3	-1/3	1	
x_2	0	0	1	2	-1/3	1/3	2	

We have no more entering variable, thus we have an optimum solution.

$x_1 = 1,\ x_2 = 2,\ x_3 = 0,\ z = 16$

Number of units of products of A ; $x_1 = \underline{\underline{1}}$
Number of units of product of B, $x_2 = \underline{\underline{2}}$
Number of units of product of C, $x_3 = \underline{\underline{0}}$

Corresponding profit $z = \underline{\underline{₹16}}$

(b) The z - row in optimal simplex table is:

Basic	x_1	x_2	x_3	s_1	s_2	solution
z	0	0	6	10/3	2/3	

→ Dual Price

The dual price for constraint 1 = 10/3
dual price for constraint 2 = 2/3

Therefore constraint 1 ($x_1 + x_2 + x_3 \leq 3$) should be given the priority to increase z since it has higher dual price.

(C) $x_1 + x_2 + x_3 \leq 3 + D_1$

$x_1 + 4x_2 + 7x_3 \leq 9 + D_2$

The optimal simplex Table

Basic	Z	x_1	x_2	x_3	D_1 S_1	D_2 S_2	Solution
Z	1	0	0	6	10/3	2/3	16
x_1	0	1	0	-1	4/3	-1/3	1
x_2	0	0	1	2	-1/3	1/3	2

$$Z = 16 + 10/3 D_1 + 2/3 D_2$$
$$x_1 = 1 + 4/3 D_1 - 1/3 D_2$$
$$x_2 = 2 - 1/3 D_1 + 1/3 D_2$$

$$x_1 = 1 + 4/3 D_1 - 1/3 D_2 \geq 0$$

$$x_2 = 2 - 1/3 D_1 + 1/3 D_2 \geq 0$$

When we change RHS of constraint 1 keeping other constraint same, then $D_2 = 0$

$\Rightarrow \quad 1 + \frac{4}{3} D_1 \geq 0 \Rightarrow D_1 \geq -3/4$

and $\quad 2 - 1/3 D_1 \geq 0 \Rightarrow D_1 \leq 6$

$\Rightarrow \quad -3/4 \leq D_1 \leq 6$

$\Rightarrow$ RHS of equation 1 : $3 + D_1$

$$9/4 \leq 3 + D_1 \leq 9$$

Minimum value of RHS of constraint $1 = 9/4$
Maximum value of RHS of constraint $1 = 9$

$$\text{Range} = 9 - \frac{9}{4} = \frac{27}{4}$$

for which dual price of the preferred constraint remain same.

(d) Maximize $z = (4+d_1)x_1 + (6+d_2)x_3 + (2+d_3)x_3$

optimal Simplex Table

		d_1	d_2	d_3	0	0	
	Basic	x_1	x_2	x_3	S_1	S_2	solution
1	Z	0	0	6	10/3	2/3	16
d_1	x_1	1	0	-1	4/3	-1/3	1
d_2	x_2	0	1	2	-1/3	1/3	2

Reduced cost for $x_3 = 6 - d_1 + 2d_2 - d_3 \geq 0$

Reduced cost for $S_1 = \frac{10}{3} + \frac{4}{3}d_1 - \frac{1}{3}d_2 \geq 0$

Reduced cost for $S_2 = \frac{2}{3} - \frac{1}{3}d_1 + \frac{1}{3}d_2 \geq 0$

Set $d_1 = d_2 = 0$

$$\Rightarrow 6 - d_3 \geq 0 \Rightarrow d_3 \leq 6$$

$$-\infty < d_3 \leq 6$$

$$\Rightarrow -\infty < 2 + d_3 \leq 8$$

The range of profit contribution of product C in the objective function is $(-\infty, 8]$ given profit contribution of A and B remain unchanged.

(e) Set $d_2 = d_3 = 0$

$\Rightarrow 6 - d_1 \geq 0 \Rightarrow d_1 \leq 6$

also $\frac{10}{3} + \frac{4}{3} d_1 \geq 0 \Rightarrow d_1 \geq -5/2$

also $\frac{2}{3} - \frac{1}{3} d_1 \geq 0 \Rightarrow d_1 \leq 2$

$\Rightarrow -5/2 \leq d_1 \leq 2$

$\Rightarrow 3/2 \leq 4 + d_1 \leq 6$

The range of profit contribution of product A in the objective function is $[3/2, 6]$, given profit contribution of B and C remain unchanged.