DISCRETE-TIME PROCESSING OF C.T. SIGNALS :-

\# Sampling converts any C.T. signal to a discrete time signal corresponding to a sequence of sample values

↳ provides a basis for storing, coding or transmitting C.T. signals.

↳ also offers the possibility ~~of~~ for discrete-time processing of a C.T. signal

↓

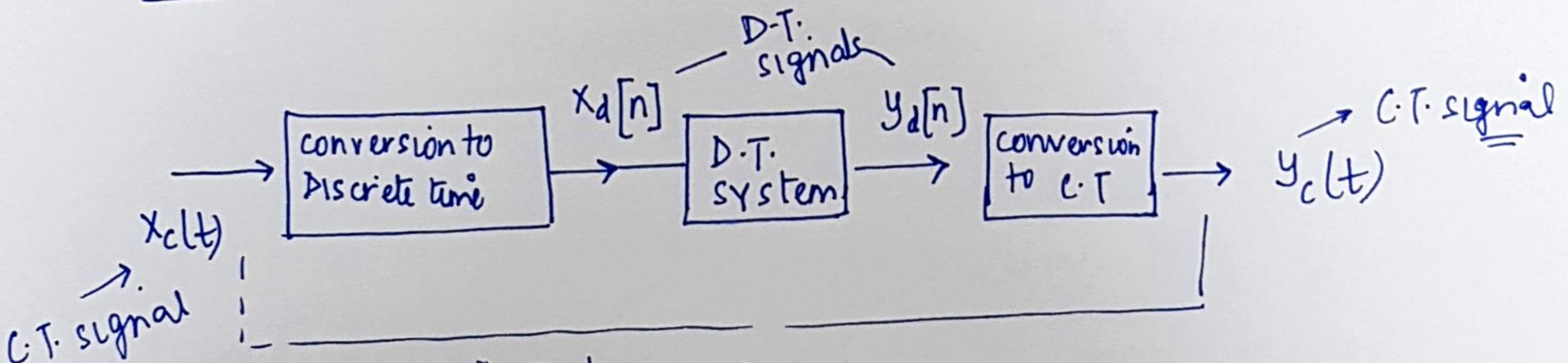
such processing is highly advantageous.

↓ can be done using digital processors

↓

extremely flexible & efficient.!

\# This approach of D.T. processing of a C.T. signal can be viewed as the cascade of the following three operations :→



D.T. processing of C.T. signals

[Sampling Theorem provides the theoretical basis for converting a C.T. signal to a D.T. signal and for reconstructing a C.T. signal from its D.T. representation.

Provided the conditions mandated by sampling Theorem are satisfied

- the C.T. signal $x_c(t)$ is exactly represented by a sequence of instantaneous samples $x_c(nT)$

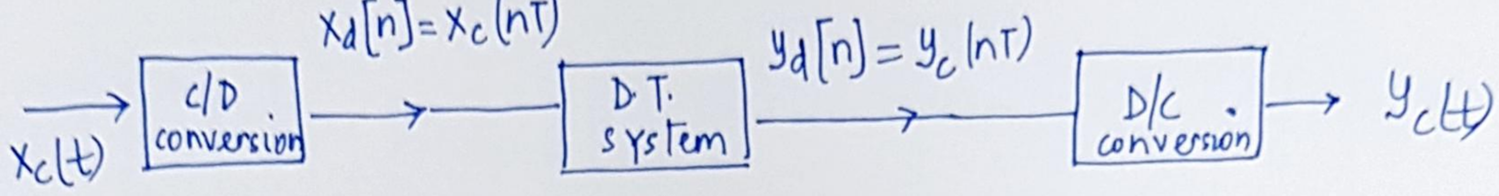
  i.e. the discrete-time sequence

$$x_d[n] = x_c[nT]$$

This transformation from $x_c(t)$ to $x_d[n]$ is referred to as ~~CT to~~ continuous to discrete time conversion [C/D]

- The reverse operation, discrete-time to C.T. conversion, $y_d[n]$ to $y_c(t)$ performs an interpolation between the sample values provided to it as the i/p.

  i.e. D/C conversion produces a C.T. signal $y_c(t)$ that is related to D.T. signal $y_d[n]$ as $\boxed{y_d[n] = y_c(nT)}$

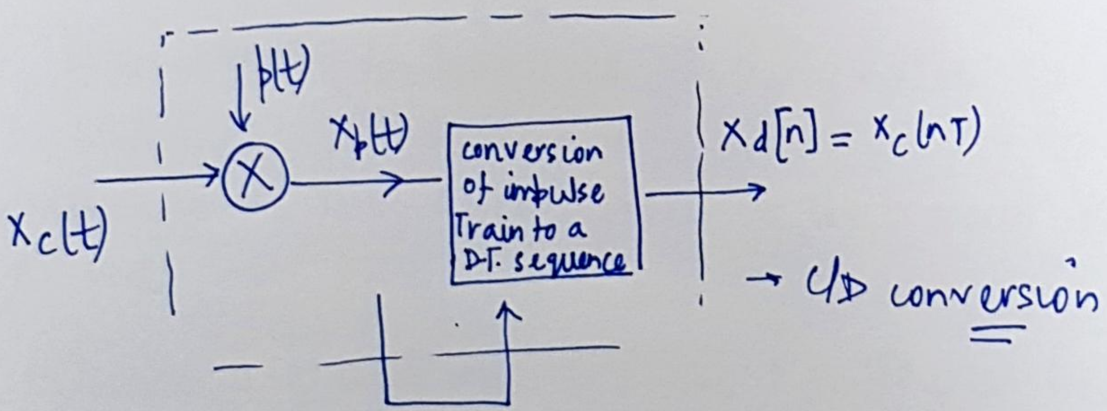


Here T: sampling period!

To further understand the relation between the C.T. signal $x_c(t)$ and its D.T. representation $x_d[n]$, one can view the C/D process as

(i) periodic sampling followed by

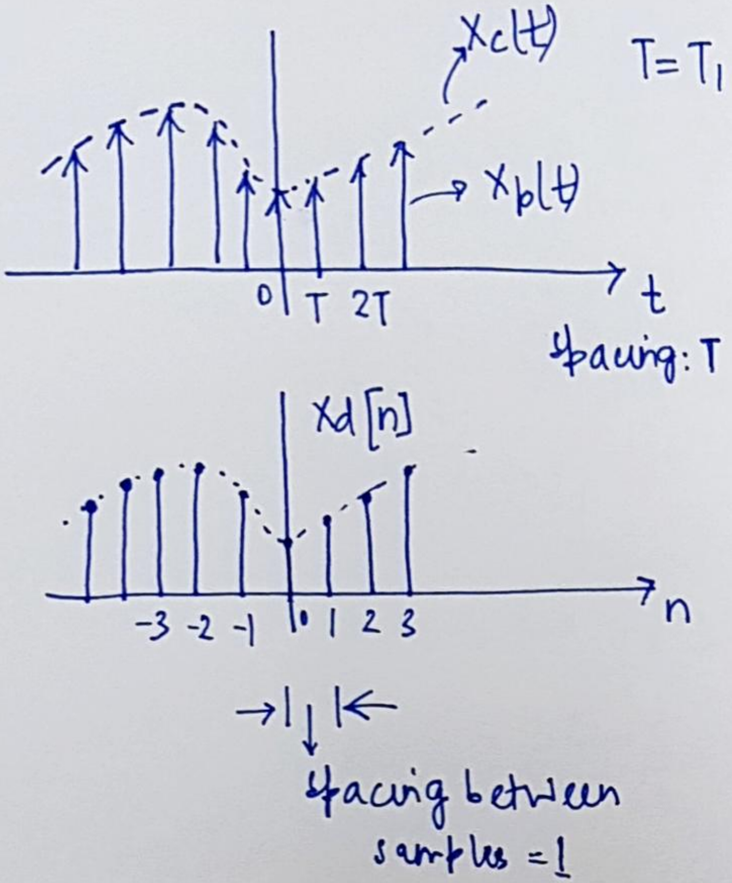
(ii) mapping of impulse train to a sequence

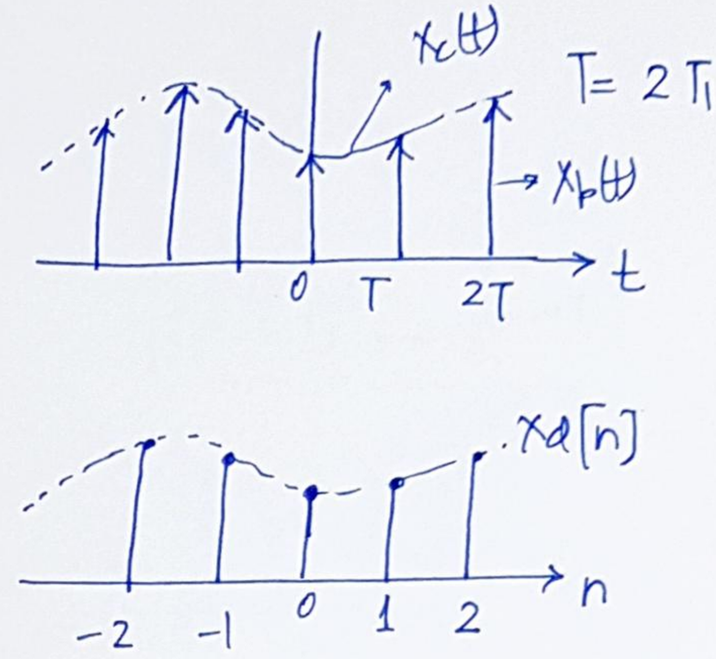


overall system

→ (sampling with a periodic impulse train followed by conversion to a DT sequence)

this conversion can be thought of as a normalization in time





$x_p(t)$ and $x_d[n]$ for two diff. sampling rates

→ [Let us examine this overall process in the frequency-domain as well

since, we will now talk about Fourier Transforms in both C.T. and D.T., let us denote C.T. frequency using (ω) & D.T frequency using (Ω)

$$x_c(t) \overset{\mathcal{F}}{\longleftrightarrow} X_c(j\omega) \qquad x_d[n] \overset{\mathcal{F}}{\longleftrightarrow} X_d(e^{j\Omega})$$

$$y_c(t) \longleftrightarrow Y_c(j\omega) \qquad y_d[n] \overset{\mathcal{F}}{\longleftrightarrow} Y_d(e^{j\Omega})$$

↑ CTFT ↘(DTFT)

We know that

$$x_p(t) = \sum_{n=-\infty}^{\infty} x_c(nT)\,\delta(t-nT)$$

$$\therefore \underset{\substack{\downarrow \\ \mathcal{F}\{x_p(t)\}}}{X_p(j\omega)} = \sum_{n=-\infty}^{\infty} x_c(nT)\, e^{-j\omega nT} \quad -\circledast \qquad \because \boxed{\delta(t-nT) \overset{\mathcal{F}}{\longleftrightarrow} e^{-j\omega nT}}$$

NOW, consider the DTFT of $X_d[n]$

i.e. $$X_d(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x_d[n]\, e^{-j\Omega n}$$

$$\because x_d[n] = x_c(nT)$$

$$\therefore X_d(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x_c(nT)\, e^{-j\Omega n} \qquad \circledast\,\circledast$$

comparing eqns. $\circledast$ and $\circledast\circledast$, we obtain

$$X_d(e^{j\Omega}) = X_p(j\Omega/T)$$

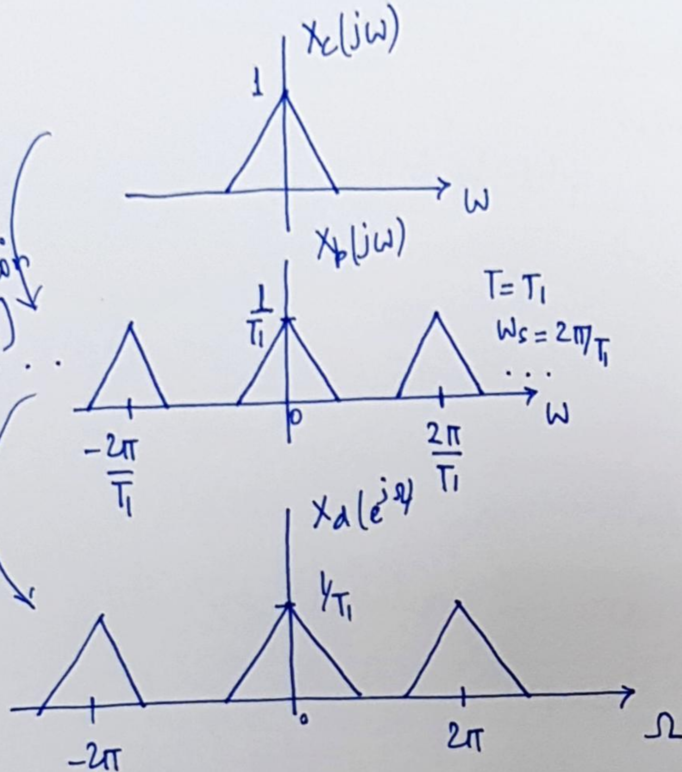
Furthermore, $X_p(j\omega) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X_c(j(\omega-k\omega_s))$, $(\omega_s = 2\pi/T)$

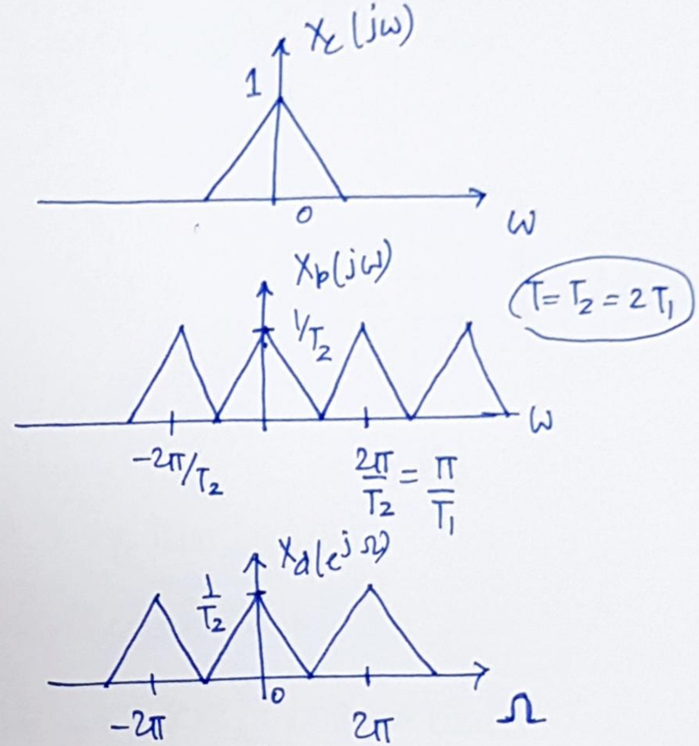
$$\therefore X_d(e^{j\Omega}) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X_c(j(\Omega-2\pi k)/T) \quad [= X_p(j\Omega/T)]$$

→ [Relation between $X_c(j\omega)$, $X_p(j\omega)$ and $X_d(e^{j\Omega})$ for two different sampling rates :→

periodic replication [sampling]

Linear Frequency scaling





$$X_d(e^{j\Omega}) = X_p(j\Omega/T)$$

$X_d(e^{j\Omega})$ can be thought of as a frequency-scaled version of $X_p(j\omega)$

Also, $X_d(e^{j\Omega})$ is periodic in (Ω) with period (2π).

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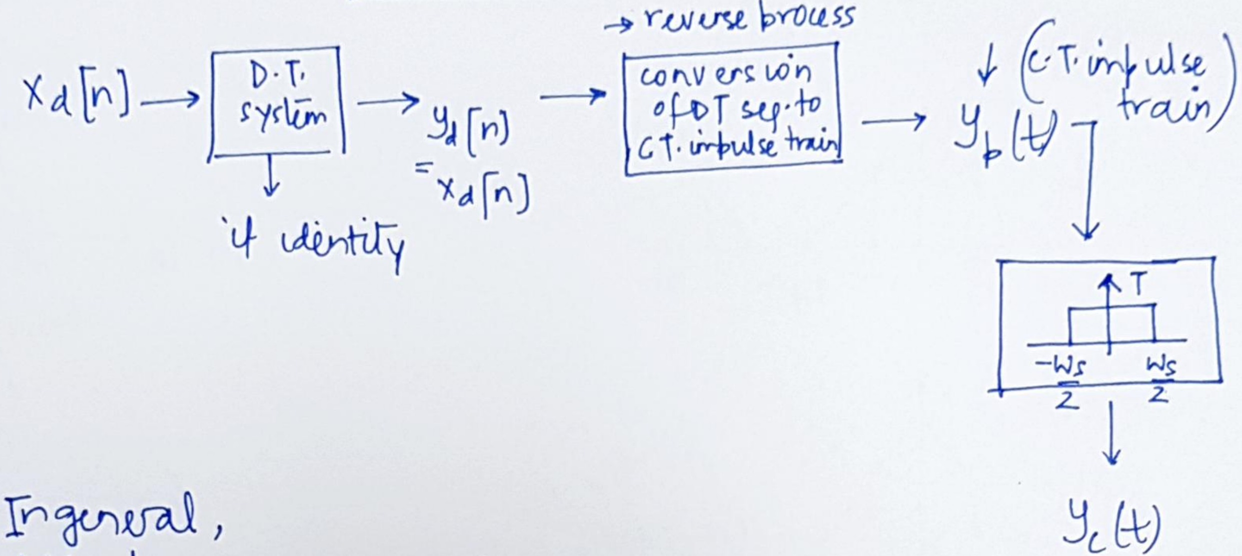
Characteristic of any (DTFT)!

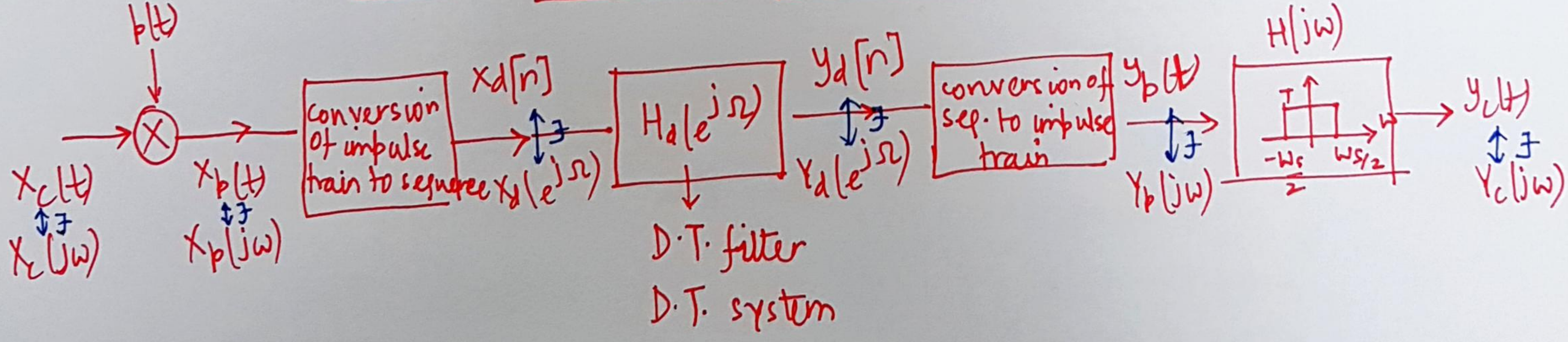
Conversion of a D.T. sequence to a C.T. signal



In general,

After processing $x_d[n]$ with a D.T. system, the resulting sequence $y_d[n]$ is converted back to a C.T. impulse train $y_p(t)$ → LPF (reconstruction/interpolation filter) to obtain $y_c(t)$.

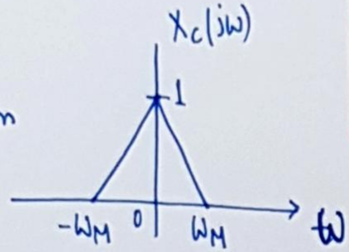
Overall system for filtering a C.T. signal using a D.T. filter



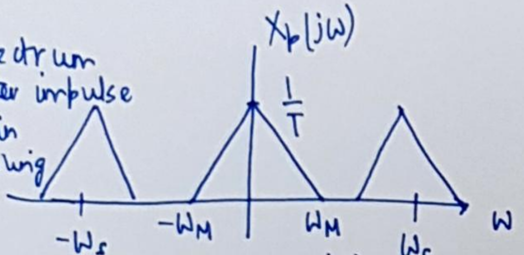
Frequency-domain Illustration of the overall system

Assume $\omega_M < \frac{\omega_s}{2}$

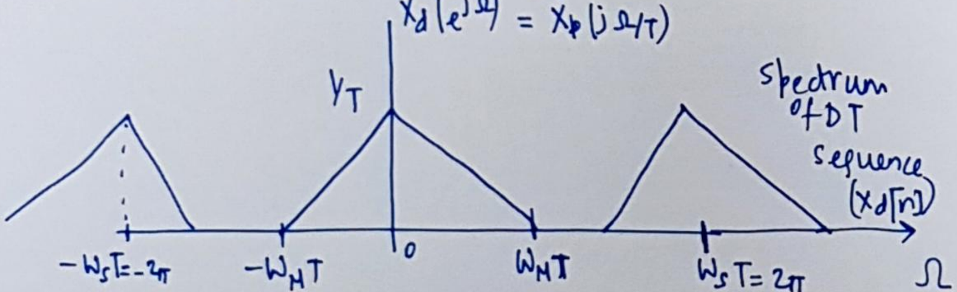
C.T. spectrum



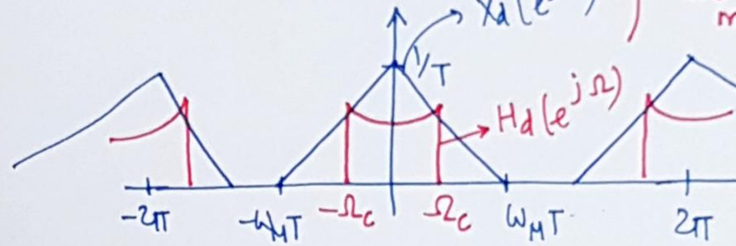
Spectrum after impulse train sampling



$X_d(e^{j\Omega}) = X_p(j\Omega/T)$

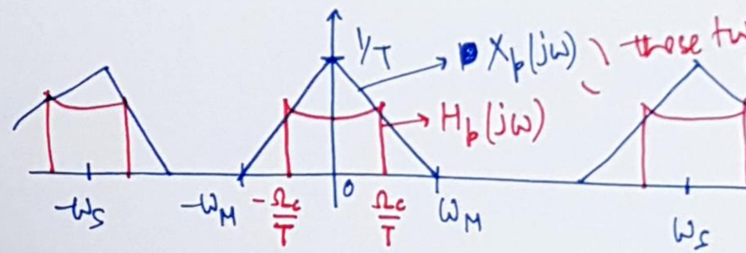


Spectrum of DT sequence ($x_d[n]$)



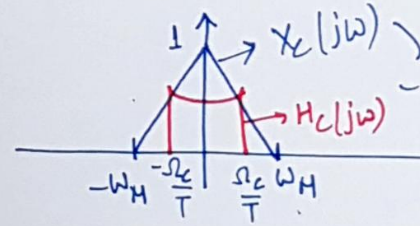
These two are multiplied to obtain $Y_d(e^{j\Omega})$

(scale freq.)

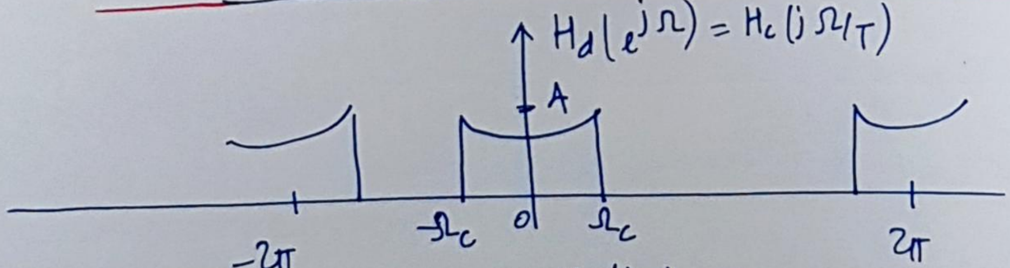


these two are multiplied to obtain $Y_p(j\omega)$

LPF



$\times$ to obtain $Y_c(j\omega)$



Discrete time frequency response of D.T. filter

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Equiv. C.T freq. response