ME 421 IEOR: Queuing Model-2

Customers arrive at a one window drive-in bank according to Poisson distribution with mean 10 per hour. Service time per customer is exponential with mean 5 minutes. The space in front of window, including that for the serviced car can accommodate a maximum of three cars. Other cars can wait outside this space. (a) What is the probability that an arriving customer can drive directly to the space in front of the window? (b) What is the probability that an arriving customer will have to wait outside the indicated space? (c) How long is an arriving customer expected to wait before being served?

$(M/M/1) : GD/\infty/\infty)$

$\lambda = 10$ per hour

$\mu = \frac{60}{5} = 12$ per hour

since $\rho = \frac{\lambda}{\mu} = \frac{10}{12} < 1$, the system can operate under steady-s

$n = 3$

(a) $\cancel{p_a} = \cancel{\frac{\rho}{1-\rho}}$ $\quad \cancel{\rho = \frac{10}{12}}$ $p = p_0 + p_1 + p_2 = 0.421$

(b) $\cancel{p_3} = \cancel{(1-\rho)\rho^3}$ $\quad \cancel{1-(p+p_3)}$ $[1-(p_0+p_1+p_2 \cancel{+p_3})]$

$1 - 0.4213 = 0.57$

(c) $W_q = W_s - \frac{1}{\mu}$ $\quad W_s = \frac{1}{\mu-\lambda} = 0.5$

$W_q = 0.5 - \frac{1}{12} = 0.417$ hour. or 25 mins.

according to exponential distribution

ME 421 IEOR: Queuing Model-3
Trains arrive at the yard every 15 minutes and the service time is 33 minutes. If the line capacity of the yard is limited to 4 trains find (a) the probability that the yard is empty and (b) the average number of trains in the system (c) facility utilization.

$\lambda = \frac{60}{15} = 4$ per hr $\quad M/M/1 : GD/4/\infty$

$\mu = \frac{60}{33} = 1.82$ per hr.

$N = 4$

$c = 1 \qquad \rho = \lambda/\mu = 2.2$

(a) $p_0 = \frac{1-\rho}{1-\rho^{N+1}} = \frac{1-2.2}{1-(2.2)^5} = 0.024$

(b) $L_s = \frac{2.2\left[1-5(2.2)^4 + 4(2.2)^5\right]}{(1-2.2)\left(1-(2.2)^5\right)}$

$= \frac{2.2\,(90.17)}{60.648} = 3.265$

(c) $\bar{c} = L_s - L_q = \frac{\lambda_{eff}}{\mu}$

$\lambda_{eff} = \lambda p_N = 4 \times \frac{(1-2.2)(2.2)^4}{(1-(2.2)^5)} = 4 \times 0.556 = 2.22$

$\bar{c} = \frac{2.225}{1.82} = 1.223$

utilization $= \bar{c}/c = \frac{1.223}{4} \times 100 = 30.56\%$