Ans

Maximize $z = 4x_1 + 3x_2$ subject to

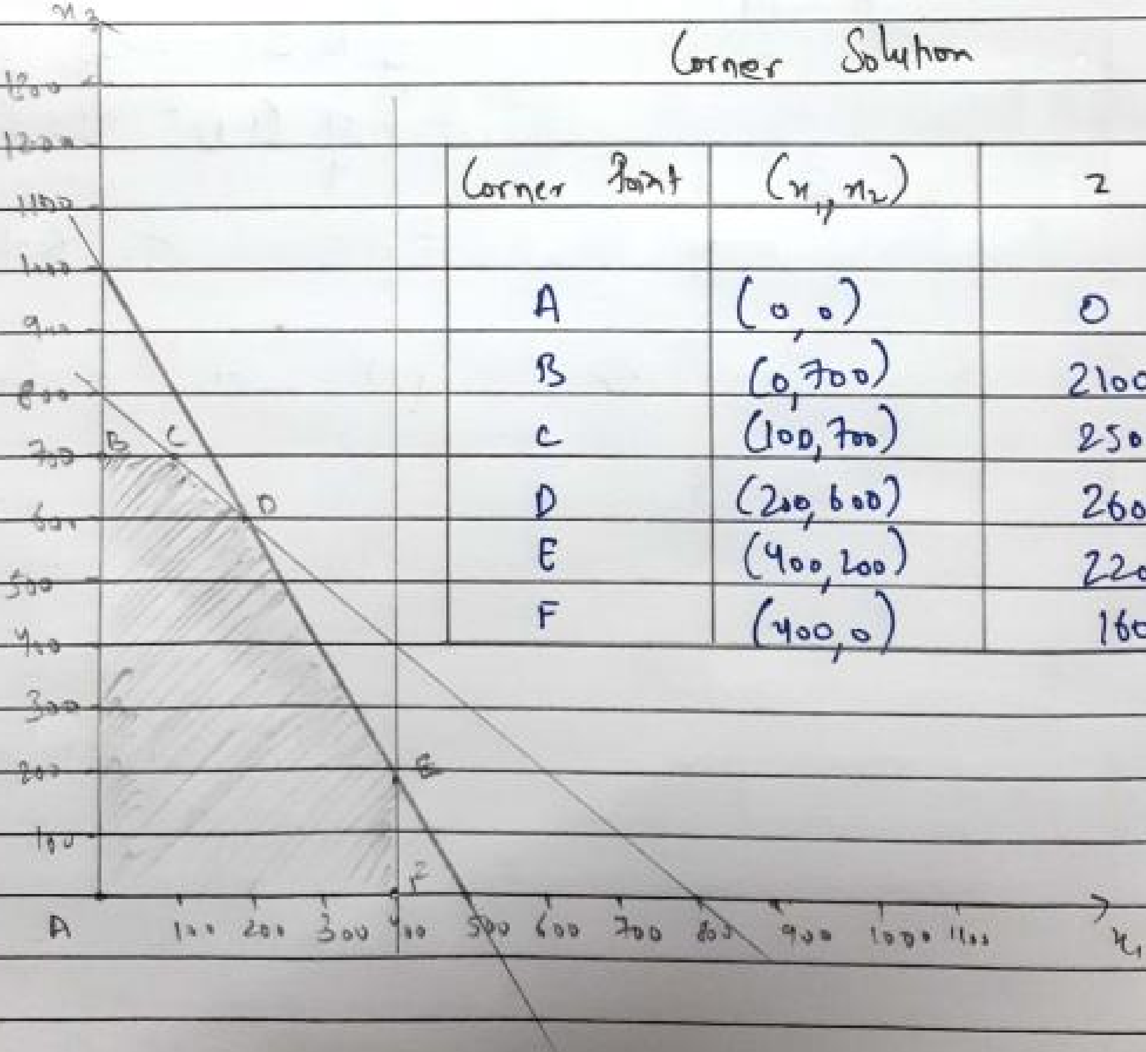
(i) $2x_1 + x_2 \leq 1000$

(ii) $x_1 + x_2 \leq 800$

(iii) $x_1 \leq 400$

(iv) $x_2 \leq 700$, $x_1, x_2 \geq 0$

Graphical solution:



Corner Solution

Corner Point	(x_1, x_2)	z
A	(0, 0)	0
B	(0, 700)	2100
C	(100, 700)	2500
D	(200, 600)	2600
E	(400, 200)	2200
F	(400, 0)	1600

We see that at $D(x_1, x_2) = (200, 600)$, z attains its maximum value

Thus $x_1 = 200$ and $x_2 = 600$ is the optimum solution resulting in $z = 2600$ as per graphical solution

Simplex Method The equations become on introducing slack variables as follows

Maximize $z = 4x_1 + 3x_2 + 0s_1 + 0 \cdot s_2 + 0 \cdot s_3 + 0 \cdot s_4$

subject to (i) $2x_1 + x_2 + s_1 = 1000$

(ii) $x_1 + x_2 + s_2 = 800$

(iii) $x_1 + s_3 = 400$

(iv) $x_2 + s_4 = 700$

such that $x_1, x_2, s_1, s_2, s_3, s_4 \geq 0$

$m = 4$ $n = 6$

Let Basic variables = $\{s_1, s_2, s_3, s_4\}$

Non-Basic variables = $\{x_1, x_2\} = (0, 0)$

Maximize $z - 4x_1 - 3x_2 = 0$

~~First Iteration~~ Initial Simplex Table:

Basic	z	x_1	x_2	s_1	s_2	s_3	s_4	Solution	Ratio
z	1	-4	-3	0	0	0	0	0	
s_1	0	2	1	1	0	0	0	1000	500
s_2	0	1	1	0	1	0	0	800	800
Leaving variable s_3	0	1	0	0	0	1	0	400	400
s_4	0	0	1	0	0	0	1	700	700/0

Entering Variable Pivot Element = 1

After Gauss-Jordon Row Operation:

First iteration:

Basic	Z	x_1	x_2	s_1	s_2	s_3	s_4	Solution	Ratio
Z	1	0	-3	0	0	4	0	1600	
s_1	0	0	1	1	0	-2	0	200	200
s_2	0	0	1	0	1	-1	0	400	400
x_1	0	1	0	0	0	1	0	400	400/0
s_4	0	0	1	0	0	0	1	700	700

Entering variable = x_2 Leaving variable = s_1

Pivot element = 1

After Gauss-Jordon row operations:

Second iteration:

Basic	Z	x_1	x_2	s_1	s_2	s_3	s_4	Solution	Ratio
Z	1	0	0	3	0	-2	0	2200	
x_2	0	0	1	1	0	-2	0	200	-100
s_2	0	0	0	-1	1	1	0	200	200
x_1	0	1	0	0	0	1	0	400	400
s_4	0	0	0	-1	0	2	1	500	250

Leaving base variable = s_2 Entering variable = s_3, Pivot element = 1

After Gauss-Jordan row operations

Third Iteration:

Basic	Z	x_1	x_2	s_1	s_2	s_3	s_4	Solution	Ratio
Z	1	0	0	1	2	0	0	2600	
x_2	0	0	1	-1	2	0	0	600	
s_3	0	0	0	-1	1	1	0	200	
x_1	0	1	0	1	-1	0	0	200	
s_4	0	0	0	1	-2	0	1	100	

After 3 iterations, we have no entering variable.

$x_1 = 200$, $x_2 = 600$, $S_1 = 0$, $S_2 = 0$, $S_3 = 200$, $S_4 = 100$

And $Z_{max} = 2800$

This solution is in agreement with the graphical solution.

Ans 2 Maximize $Z = x_1 + x_2 + x_3$ subject to

(i) $4x_1 + 5x_2 + 3x_3 \le 15$

(ii) $10x_1 + 7x_2 + x_3 \le 12$

$x_1, x_2, x_3 \ge 0$

In simplex method $4x_1 + 5x_2 + 3x_3 + S_1 = 15$

$10x_1 + 7x_2 + x_3 + \cdot S_2 = 12$

$Z - x_1 - x_2 - x_3 = 0$, $x_1, x_2, x_3, S_1, S_2 \ge 0$

$n = 5$ $m = 2$ Basic variables = S_1, S_2

Basic	Z	x_1	x_2	x_3	S_1	S_2	Solution	Ratio
Z	1	-1	-1	-1	0	0	0	
S_1	0	4	5	3	1	0	15	15/4
S_2	0	10	7	1	0	1	12	6/5

Entering variable = x_1, Leaving variable = S_2, Pivot element = 10

First iteration: ~~is complete~~

Basic	Z	x_1	x_2	x_3	S_1	S_2	Solution	Ratio
Z	1	0	-3/10	-7/10	0	1/10	6/5	
S_1	0	0	11/5	13/5	1	-2/5	51/5	51/13
x_1	0	1	7/10	1/10	0	1/10	6/5	12

Entering variable = x_3, Leaving Basic Variable = S_1, Pivot element = 13/5

Second iteration:

Basic	z	x_1	x_2	x_3	s_1	s_2	Solution	Ratio
z	1	0	$6/13$	0	$9/26$	$-1/26$	$123/26$	
x_3	0	0	$11/13$	1	$5/13$	$-2/13$	$51/13$	$-51/2$
x_1	0	1	$8/13$	0	$-1/26$	$3/26$	$21/26$	7

Entering variable $= s_2$, Leaving Basic variable $= x_1$, Pivot element $= 3/26$

Third iteration:

Basic	z	x_1	x_2	x_3	s_1	s_2	Solution
z	1	$1/3$	$2/3$	0	$1/3$	0	5
x_3	0	$+4/3$	$5/3$	1	$1/3$	0	5
s_2	0	$26/3$	$16/3$	0	$-1/3$	1	7

After 3 iterations, we have no entering variable

Thus

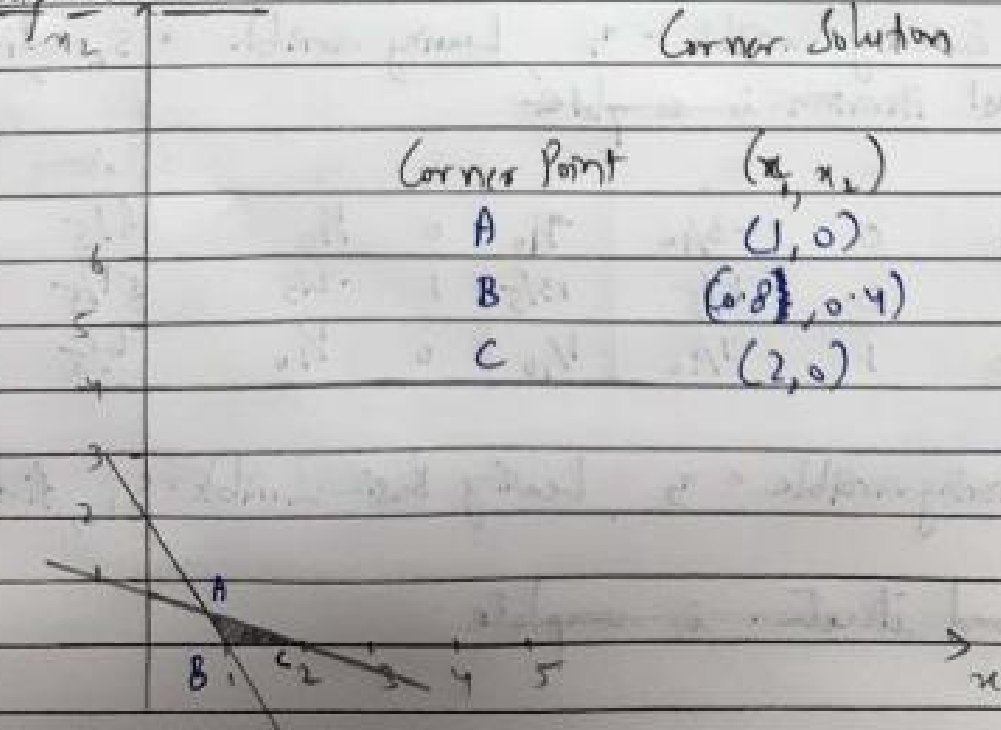
$$\boxed{z_{max} = 5}$$

for $x_1 = 0$, $x_2 = 0$, $x_3 = 5$, $s_1 = 0$, $s_2 = 7$

Ans 3 Minimize ~~$z = 2x_1 +$~~ $z = 3x_1 - x_2$ subject to

(i) $2x_1 + x_2 \geq 2$ (ii) $x_1 + 3x_2 \leq 2$, (iii) $x_2 \leq 4$, $x_1, x_2 \geq 0$

Graphical Method:



Corner Solution

Corner Point	(x_1, x_2)	z
A	$(1, 0)$	3
B	$(0.8, 0.4)$	2
C	$(2, 0)$	6

We observe that for $x_1 = 0.8$, $x_2 = 0.4$, we have minimum value of z, $\underline{\underline{z_{min} = 2}}$

Simplex Method:

Constraints $2x_1 + x_2 - s_1 + R_1 = 2$

$x_1 + 3x_2 + s_2 = 2$

$x_2 + s_3 = 4$

$s_1, s_2, s_3 \rightarrow$ surplus and slack variables, $R_1 \rightarrow$ artificial variable

Minimize $Z = 3x_1 - x_2 + 0 \cdot s_1 + 0 \cdot s_2 + 0 s_3 + MR_1$

$Z - 3x_1 + x_2 + 0 \cdot s_1 + 0 \cdot s_2 + 0 \cdot s_3 - MR_1 = 0$

such that $x_1, x_2, s_1, s_2, s_3, R_1 \geq 0$

$n = 6 \quad m = 3$

Basic variables $= (R_1, s_2, s_3)$

Initial Simplex Table

Basic	x_1	x_2	s_1	s_2	s_3	R_1	Solution	Ratio
z	-3	1	0	0	0	$-M$	0	
R_1	2	1	-1	0	0	1	2	
s_2	1	3	0	1	0	0	2	
s_3	0	1	0	0	1	0	4	

New z-Row = Old z row + $M \times R_1$-row

Basic	x_1	x_2	s_1	s_2	s_3	R_1	Solution	Ratio
z	$-3+2M$	$1+M$	$-M$	0	0	0	$2M$	
R_1	2	1	-1	0	0	1	2	1
s_2	1	3	0	1	0	0	2	2
s_3	0	1	0	0	1	0	4	$4/0$

Leaving Basic Variable $= R_1$, Entering Variable $= x_1$, Pivot Element $= 2$

1st Iteration

Basic	x_1	x_2	s_1	s_2	s_3	R_1	Solution	Ratio
z	0	5/2	-3/2	0	0	3/2-M	3	
x_1	1	1/2	-1/2	0	0	1/2	1	2
S_2	0	5/2	1/2	1	0	-1/2	1	2/5
S_3	0	1	0	0	1	0	4	4

Entering variable = x_2, Leaving variable = S_2, Pivot Element = $5/2$

2nd iteration

Basic	x_1	x_2	s_1	s_2	s_3	R_1	Solution	Ratio
z	0	0	-2	-1	0	2-M	2	
x_1	1	0	-3/5	-1/5	0	3/5	4/5	
x_2	0	1	1/5	2/5	0	-1/5	2/5	
s_3	0	0	-1/5	-2/5	1	1/5	18/5	

After 2nd iteration no entering variable

Thus $\boxed{z_{min} = 2}$

with $x_1 = 4/5 = 0.8$, $x_2 = 2/5 = 0.4$, $s_3 = 18/5 = 3.6$

This solution is in agreement with the graphical solution

Ans 4

Minimize $z = 2x_1 + x_2$

subject to (i) $3x_1 + x_2 = 3$, (ii) $4x_1 + 3x_2 \geq 6$, (iii) $x_1 + 2x_2 \leq 4$

$x_1, x_2 \geq 0$

Simplex method

Constraints:

$$3x_1 + x_2 + R_1 = 3$$
$$4x_1 + 3x_2 - S_1 + R_2 = 6$$
$$x_1 + 2x_2 + S_2 = 4$$
$$x_1, x_2, S_1, S_2, R_1, R_2 \geq 0$$

$R_1, R_2 \rightarrow$ artificial variable

$S_1 \rightarrow$ surplus variable

$S_2 \rightarrow$ slack variable

Minimize $z = 2x_1 + x_2 + MR_1 + MR_2$

$$z - 2x_1 - x_2 - MR_1 - MR_2 = 0$$

$n = 6$, $m = 3$ Basic variable = (R_1, R_2, S_2)

Initial Simplex Table

Basic	x_1	x_2	S_1	S_2	R_1	R_2	Solution	Ratio
z	-2	-1	0	0	$-M$	$-M$	0	
R_1	3	1	0	0	1	0	3	
R_2	4	3	-1	0	0	1	6	
S_2	1	2	0	1	0	0	4	

New z-row = Old z-row + M row-R_1 + M row R_2

Basic	x_1	x_2	S_1	S_2	R_1	R_2	Solution	Ratio
z	$-2+7M$	$-1+4M$	$-M$	0	0	0	$9M$	
R_1	3	1	0	0	1	0	3	1
R_2	4	3	-1	0	0	1	6	$3/2$
S_2	1	2	0	1	0	0	4	4

Entering variable = x_1, Leaving base variable = R_1, Pivot Element = 3

First iteration:

Basic	x_1	x_2	S_1	S_2	R_1	R_2	Solution	Ratio
z	0	$\frac{-1}{3}+5M/3$	$-M$	0	$(-7M+2)/3$	0	$2M+2$	
x_1	1	$1/3$	0	0	$1/3$	0	1	3
R_2	0	$5/3$	-1	0	$-4/3$	1	2	$6/5$
S_2	0	$5/3$	0	1	$-1/3$	0	3	$9/5$

Entering variable = x_2, Leaving base variable = R_2, Pivot Element = $5/3$

Second iteration:

Basic	x_1	x_2	S_1	S_2	R_1	R_2	Solution	Ratio
z	0	0	$-1/5$	0	$-M+2/5$	$1/5-M$	$12/5$	
x_1	1	0	$1/5$	0	$3/5$	$-1/5$	$3/5$	
x_2	0	1	$-3/5$	0	$-4/5$	$3/5$	$6/5$	
S_2	0	0	1	1	1	-1	1	

After 2 iterations, no entering variable is left.

$Z_{min} = \frac{12}{5} = 2.4$ with $x_1 = 3/5$, $x_2 = 6/5$, $s_2 = 1$

and other non-base variables.

i.e. $S_1 = R_1 = R_2 = 0$

Ans5 Decision Variables:

x_1 = Number of 100s of hard cover books

x_2 = Number of 100s of paperback books

Objective function : Minimize $z = 600x_1 + 500x_2$

Constraints $2x_1 + x_2 \geq 80$ {Printing Press hours}

$x_1 + 2x_2 \geq 60$

$x_1, x_2 \geq 0$ {Non negativity}

Simplex Method

Constraints $2x_1 + x_2 - S_1 + R_1 = 80$

$x_1 + 2x_2 - S_2 + R_2 = 60$

$x_1, x_2, S_1, S_2, R_1, R_2 \geq 0$, $S_1, S_2 \rightarrow$ surplus variable, $R_1, R_2 \rightarrow$ artificial variable

Minimize $z = 600x_1 + 500x_2 + MR_1 + MR_2$

$z - 600x_1 - 500x_2 - MR_1 - MR_2 = 0$

Initial Simplex Table Basic variables = R_1, R_2

Basic	x_1	x_2	S_1	S_2	R_1	R_2	Solution	Ratio
z	-600	-500	0	0	-M	-M	0	
R_1	2	1	-1	0	1	0	80	
R_2	1	2	0	-1	0	1	60	

New z-row = old z-row + M×R_1-row + M×R_2-row

Basic	x_1	x_2	S_1	S_2	R_1	R_2	Solution	Ratio
z	3M-600	3M-500	-M	-M	0	0	140M	
R_1	2	1	-1	0	1	0	80	80
R_2	1	2	0	-1	0	1	60	30

Entering variable = x_2, Leaving basic variable = R_2, Pivot Element = 2

1st Iteration

Basic	x_1	x_2	S_1	S_2	R_1	R_2	Solution	Ratio
Z	$(3M-700)/2$	0	$-M$	$M/2-250$	0	$(500-3M)/2$	$50M+15000$	
R_1	$3/2$	0	-1	$1/2$	1	$-1/2$	50	$100/3$
x_2	$1/2$	1	0	$-1/2$	0	$1/2$	30	60

Entering variable = x_1, Leaving Variable = R_1, Pivot Element = $3/2$

2nd Iteration

Basic	x_1	x_2	S_1	S_2	R_1	R_2	Solution
Z	0	0	$-700/3$	$-400/3$	$-M+\frac{700}{3}$	$-M+\frac{400}{3}$	$80000/3$
x_1	1	0	$-2/3$	$1/3$	$2/3$	$-1/3$	$100/3$
x_2	0	1	$1/3$	$-2/3$	$-1/3$	$2/3$	$40/3$

After 2 iterations, we have no more entering variables.

Thus $Z_{min} = 80000/3$

with $x_1 = \frac{100}{3}$ and $x_2 = \frac{40}{3}$

Hard cover books printed in one hour $= \frac{100}{3} \times 100 = \frac{10000}{3}$

Paperback books printed in one hour $= \frac{40}{3} \times 100 = \frac{4000}{3}$

Total minimum cost $=$ INR $\frac{80000}{3}$