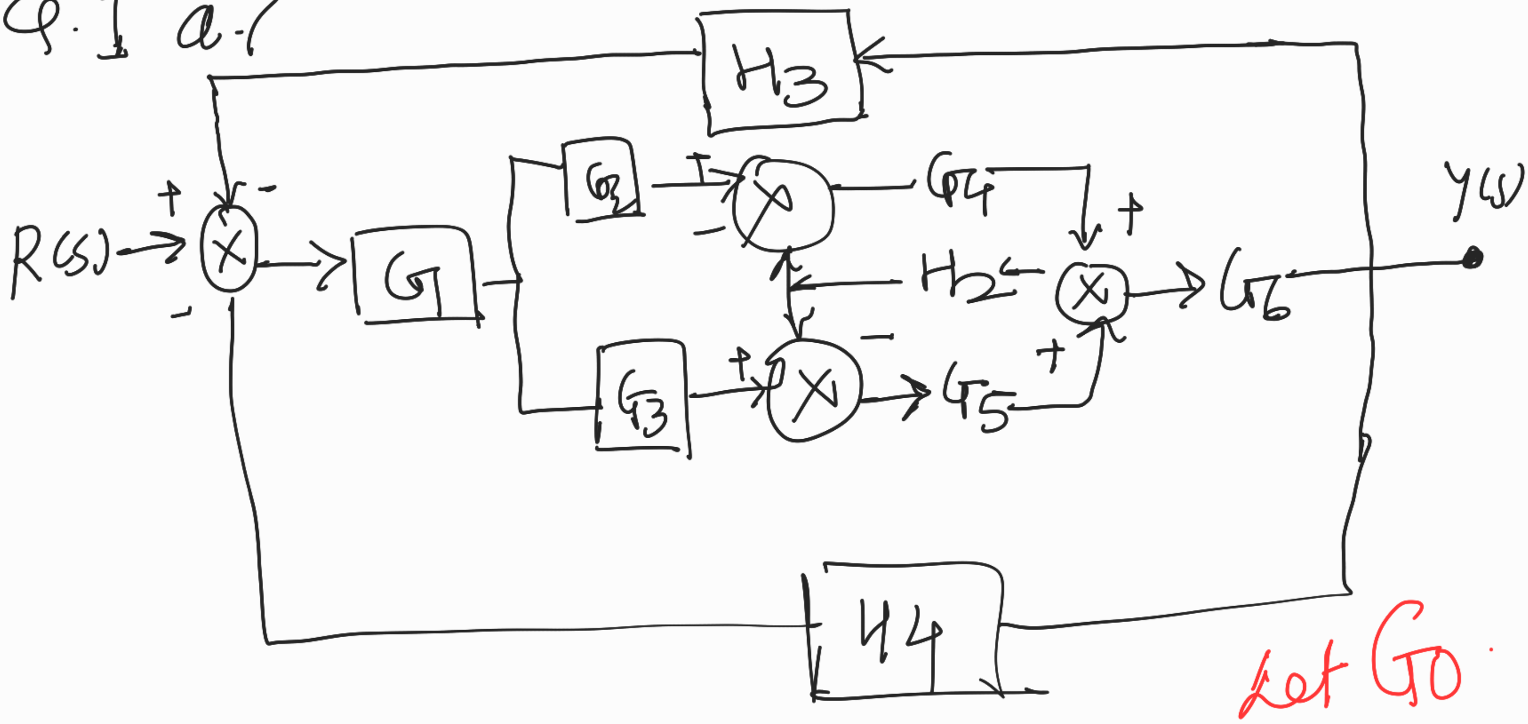
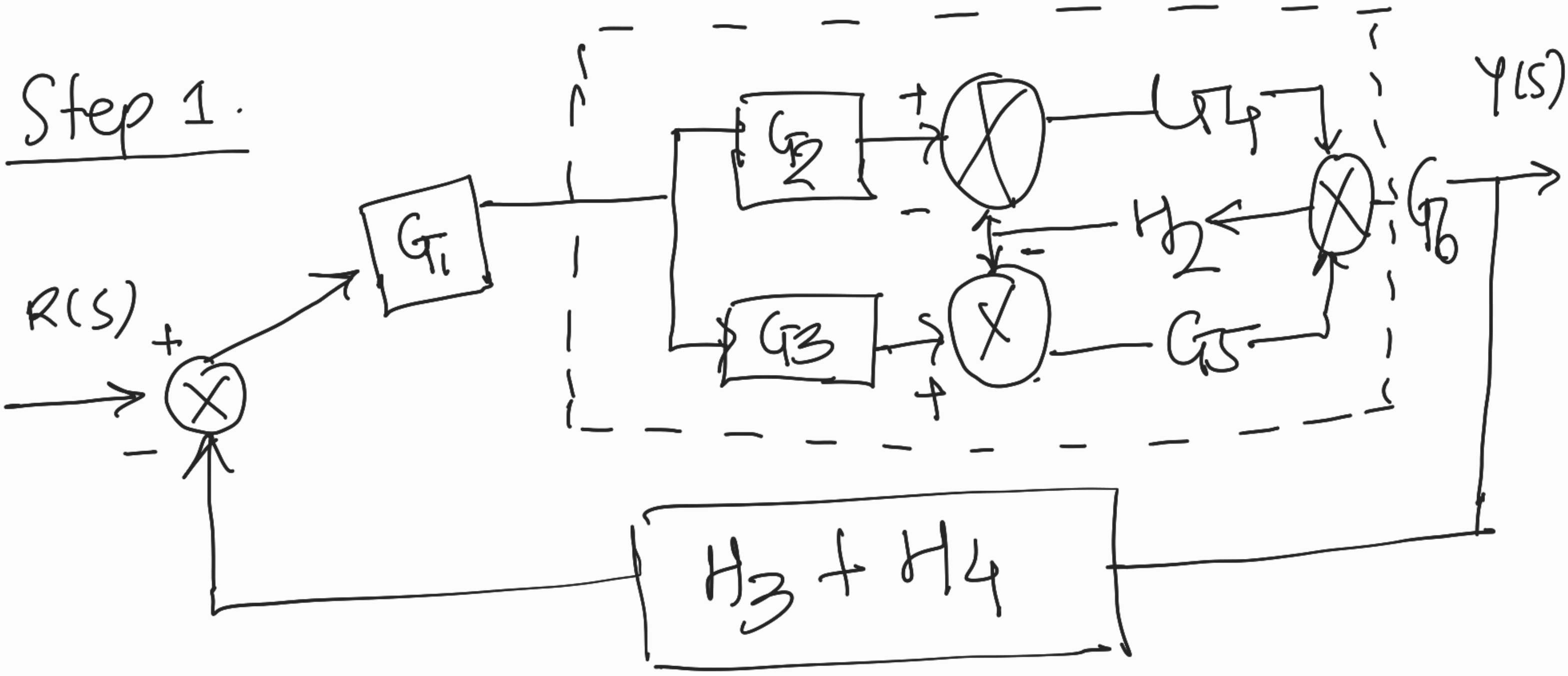
Solutions of Assignment 01

Q.1 a.)

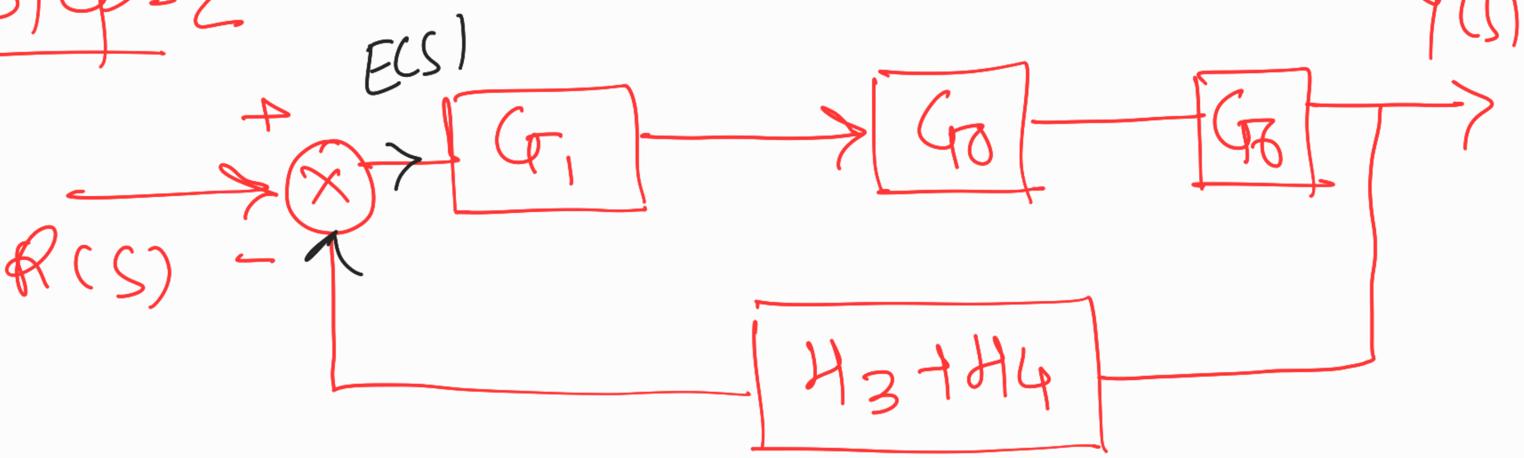


Let G_0

Step 1.



Step-2



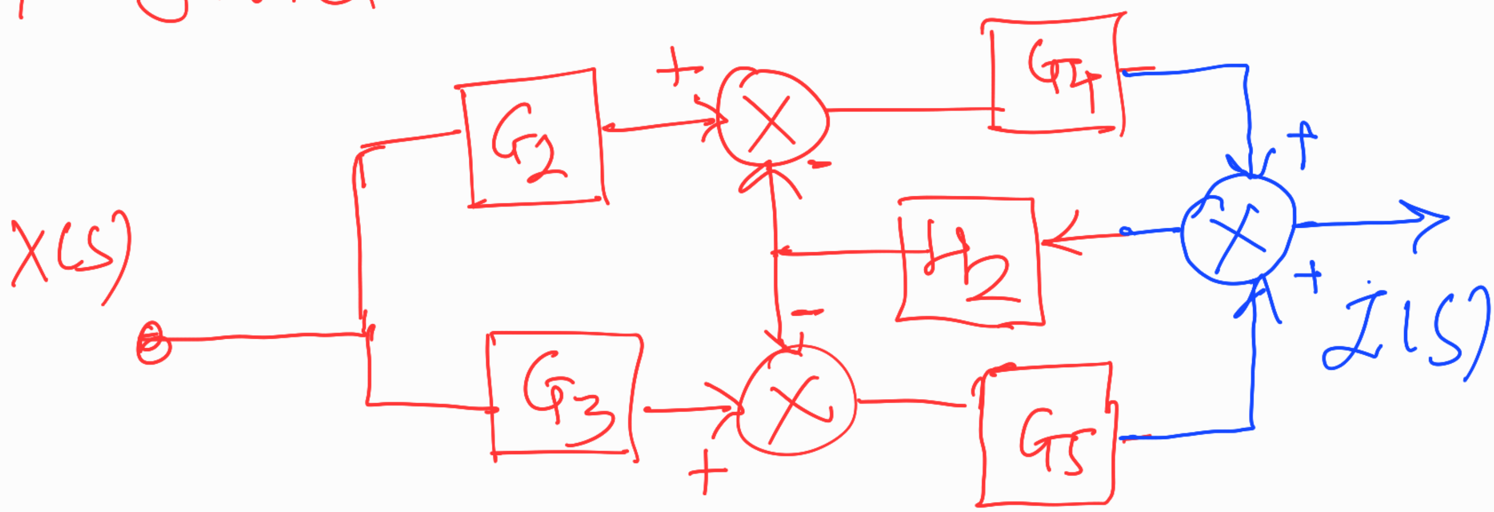
So $R(s) - (H_3 + H_4) Y(s) = E(s)$

$$E(s)\, G_7 G_0 G_6 = Y(s).$$

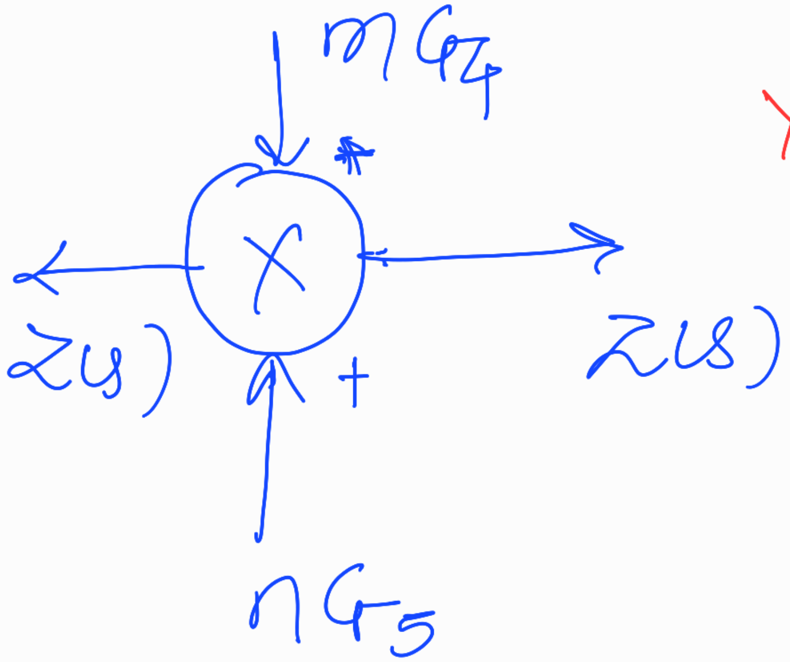
$$R(s) = Y(s)\left[(H_3 + H_4) + \frac{1}{G_7 G_0 G_6}\right]$$

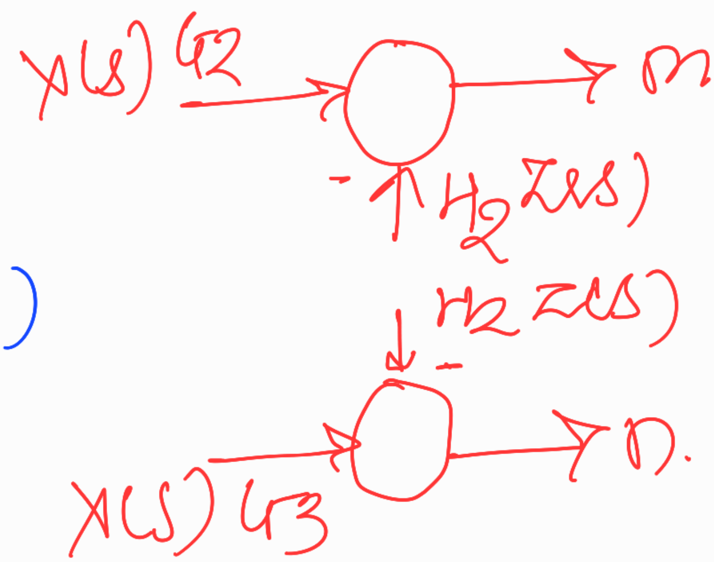
$$\boxed{\frac{Y(s)}{R(s)} = \frac{G_7 G_0 G_6}{1 + (H_3 + H_4)\, G_7 G_0 G_6}}$$

To find $G_0 = Z(s)/X(s)$.



Here catching point is the summing point with 'blue' Has some input but out put is Z(s) so.





So $m = X(s) G_2 - H_2 Z(s)$

$n = X(s) G_3 - H_2 Z(s)$

$Z = m G_4 + n G_5$

$Z(s) = X(s)(G_4 G_2 + G_3 G_5) - H_2 Z(s)(G_4 + G_5)$

$\Rightarrow Z(s)[1 + H_2(G_4 + G_5)] = X(s)[(G_4 G_2 + G_3 G_5)]$

$\therefore G_0 = Z(s)/X(s) = \dfrac{G_2 G_4 + G_3 G_5}{(1 + H_2(G_4 + G_5))}$

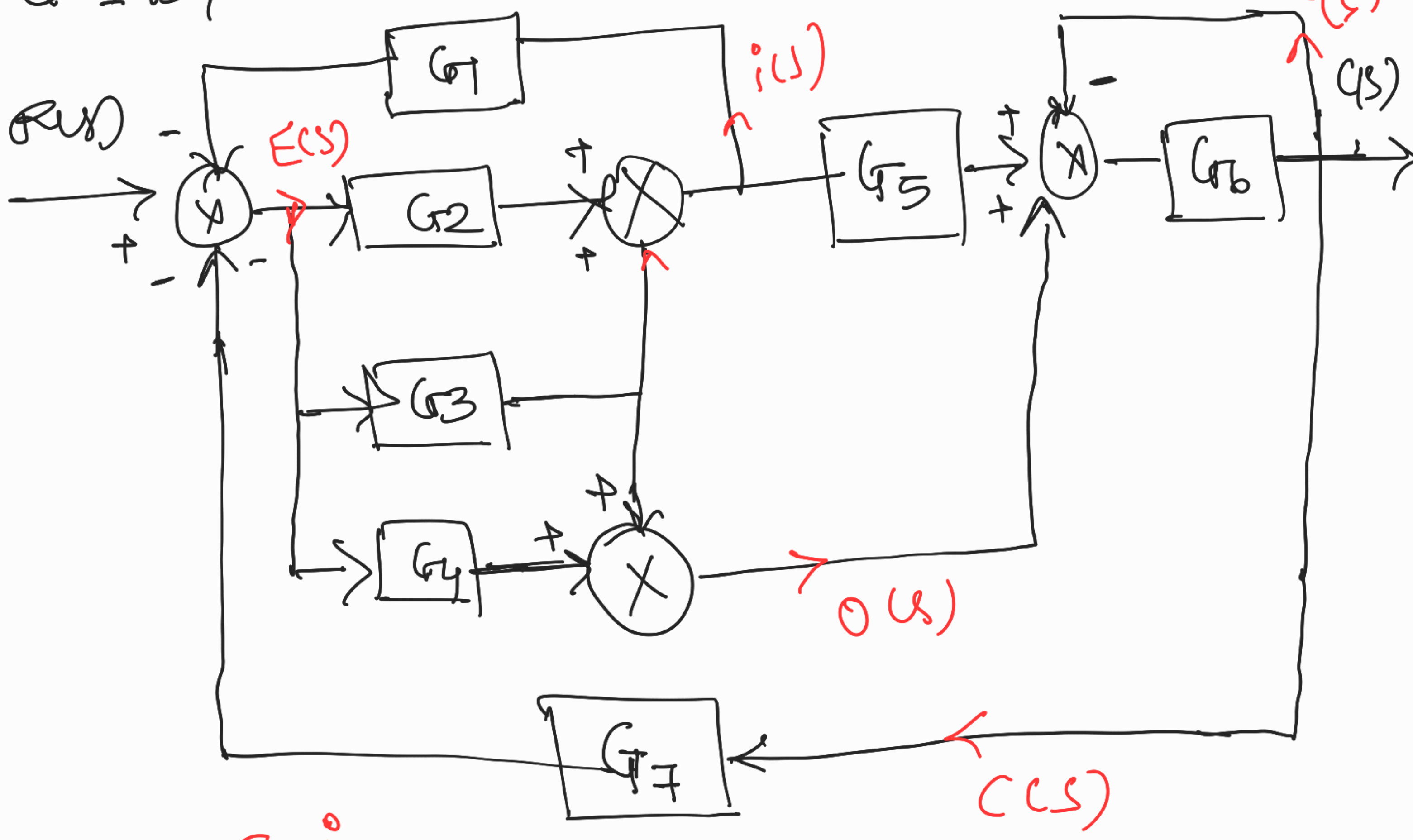
Now putting value of G_0 in $T(s)$
$= Y(s)/R(s)$.

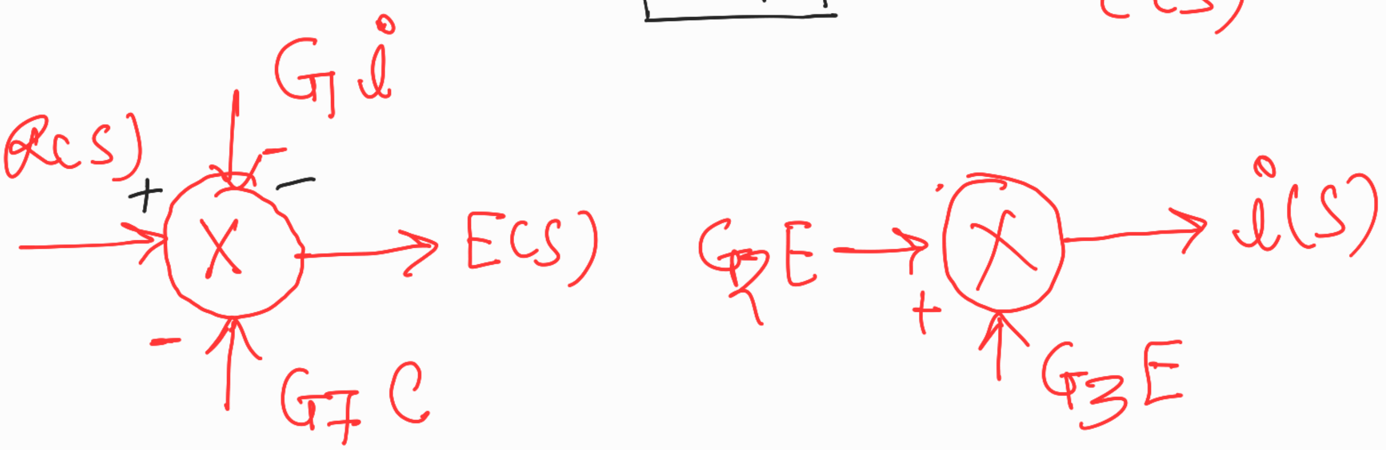
$$T(s) = \frac{G_1 G_6 (G_2 G_4 + G_3 G_5)}{[1 + H_2(G_4 + G_5) + (H_3 + H_4) G_1 G_6 (G_2 G_4 + G_3 G_5)]}$$

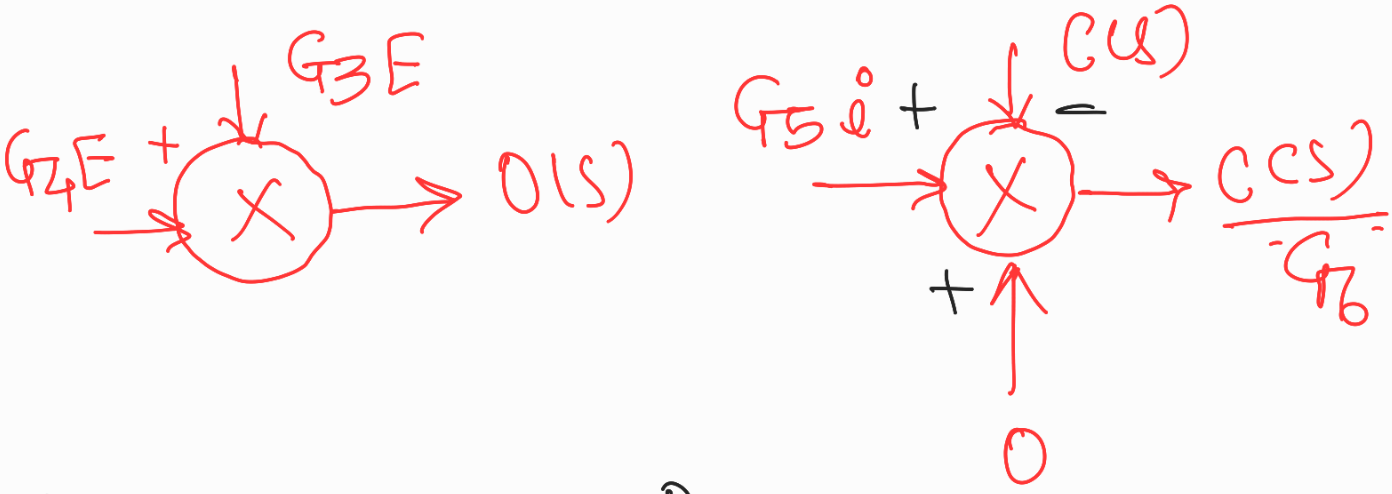
Q.1 (a) Answer:

$$\frac{Y(s)}{R(s)} = \frac{G_1 G_6 (G_2 G_4 + G_3 G_5)}{1 + H_2(G_4 + G_5) + G_1 G_6 (H_3 + H_2)(G_2 G_4 + G_3 G_5)}$$

Q.1 b)







So, $R(s) - G_1 i - G_7 C = E(s)$

$$(G_2+G_3)E = i$$

$$(G_3+G_4)E = O$$

$$G_5 i + O - C = C/G_6$$

$$G_5(G_2+G_3)E + (G_3+G_4)E = \frac{(G_6+1)C}{G_6}$$

$$\left[G_5 G_6 (G_2+G_3) + G_6(G_3+G_4)\right]E = (1+G_6)C.$$

$$E(s) = R(s) - G_1(G_2+G_3)E - G_7 C$$

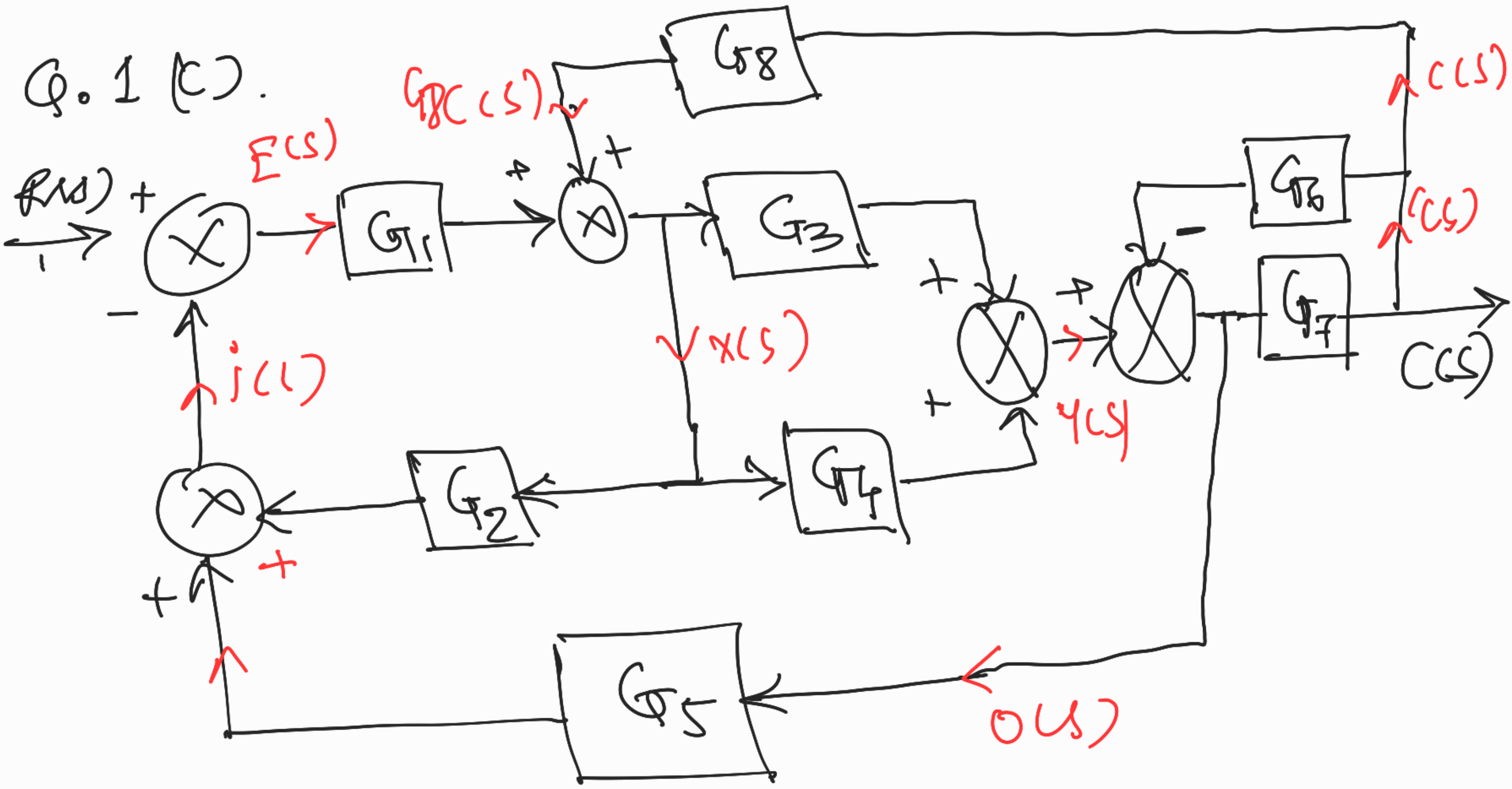
$$R(s) = E(s)\left[1 + G_1(G_2+G_3)\right] + G_7 C.$$

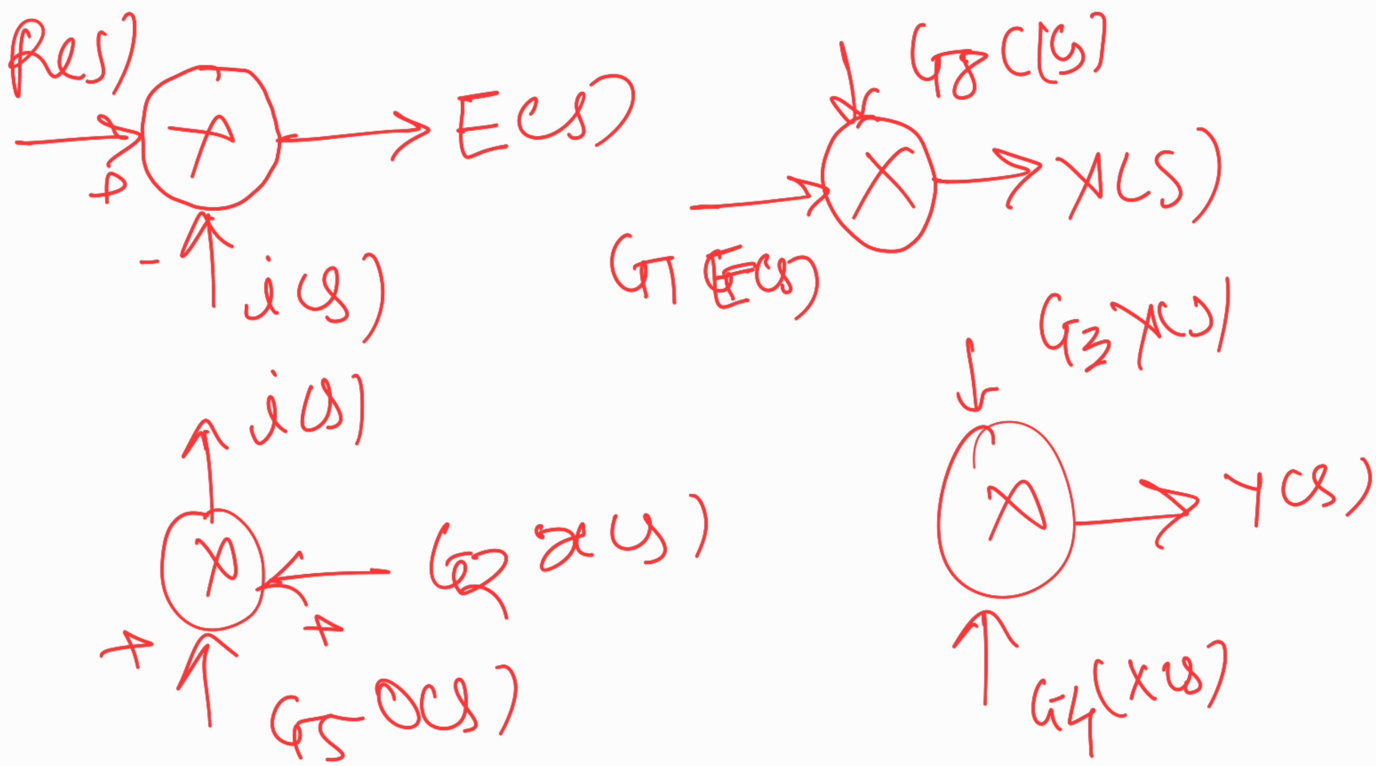
$$R(s) = \frac{(1+G_6)\left[1+G_1(G_2+G_3)\right]C(s)}{G_6\left[G_5(G_2+G_3)+(G_3+G_4)\right]} + G_7 C(s).$$

$$T(s) = \frac{C(s)}{R(s)} = \frac{G_6\left[G_5(G_2+G_3)+(G_3+G_4)\right]}{(1+G_6)\left[1+G_1(G_2+G_3)\right]+G_6G_7\left[G_5(G_2+G_3)+(G_3+G_4)\right]}$$

* you can also solve these problem by block reduction method but your answer should match

Q. 1 (c).





$$O(s) = \frac{C(s)}{G_7}$$

$$R(s) = i(s) + E(s) \; ; \; x(s) = G_8 C + G_1 E$$

$$i(s) = G_2 x(s) + \frac{G_5}{G_7} C(s) \; ; \; Y(s) = (G_4 + G_3)x$$

$$y(s) - G_6 C(s) = \frac{C(s)}{G_7}$$

$$(G_4 + G_3)\, x(s) = \frac{(1 + G_6 G_7) C(s)}{G_7}$$

$$(G_8 C + G_1 E) = \frac{(1 + G_6 G_7)\, C}{G_7 (G_4 + G_3)}$$

$$R(s) = G_2 (G_8 C + G_1 E) + \frac{G_5}{G_7} C(s) + E(s)$$

$$R(s) = \frac{G_2 (1 + G_6 G_7)}{G_7 (G_4 + G_3)} C + \frac{G_5}{G_7} C + E(s)$$

$$R(s) = \frac{G_2(1+G_6G_7) + G_5(G_4+G_3)}{G_7(G_4+G_3)} C(s) + E(s)$$

$$E(s) = \frac{X(s) - G_8 C(s)}{G_1}$$

$$E(s) = \frac{[1+G_6G_7 - G_7G_8(G_4+G_3)]C(s)}{G_1G_7(G_4+G_3)}$$

$$\Rightarrow R(s) = \left[\frac{G_1G_2(1+G_6G_7) + G_5G_1(G_4+G_3) + [1+G_6G_7 - G_7G_8(G_4+G_3)]}{G_1G_7(G_4+G_3)}\right] C(s)$$

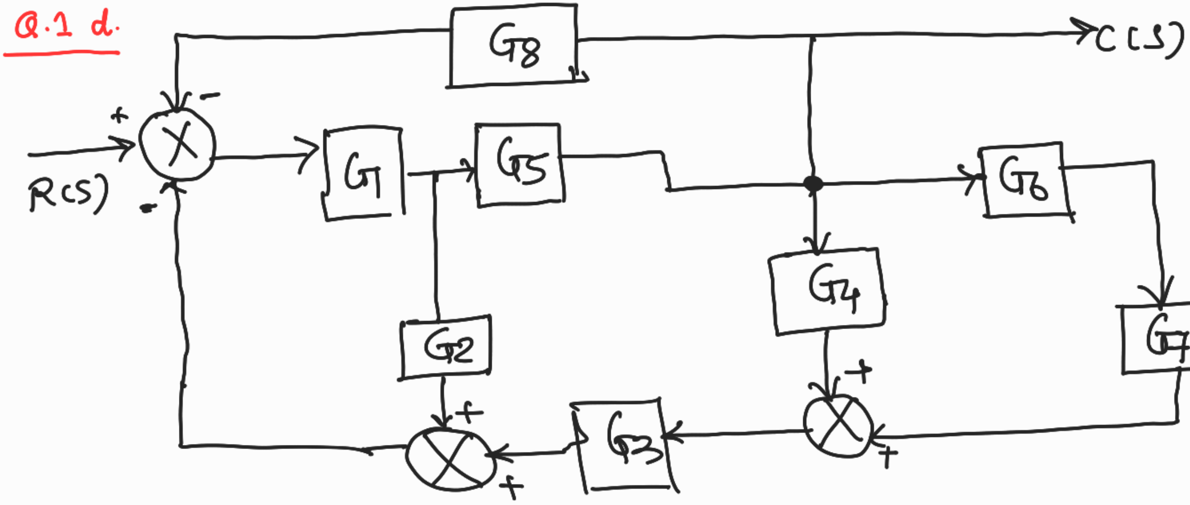
$$\frac{C(s)}{R(s)} = \frac{G_1G_7(G_4+G_3)}{[(1+G_6G_7)(1+G_1G_2) + (G_4+G_3)(G_5G_1 - G_7G_8)]}$$

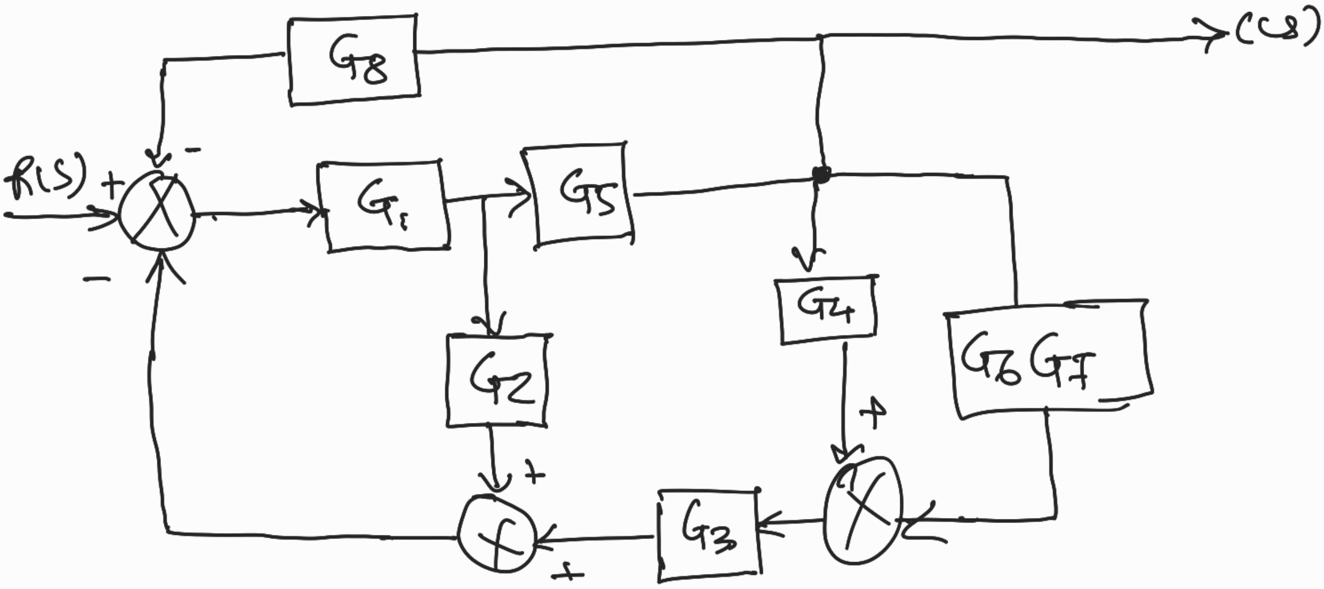
Q.1 c. Ans

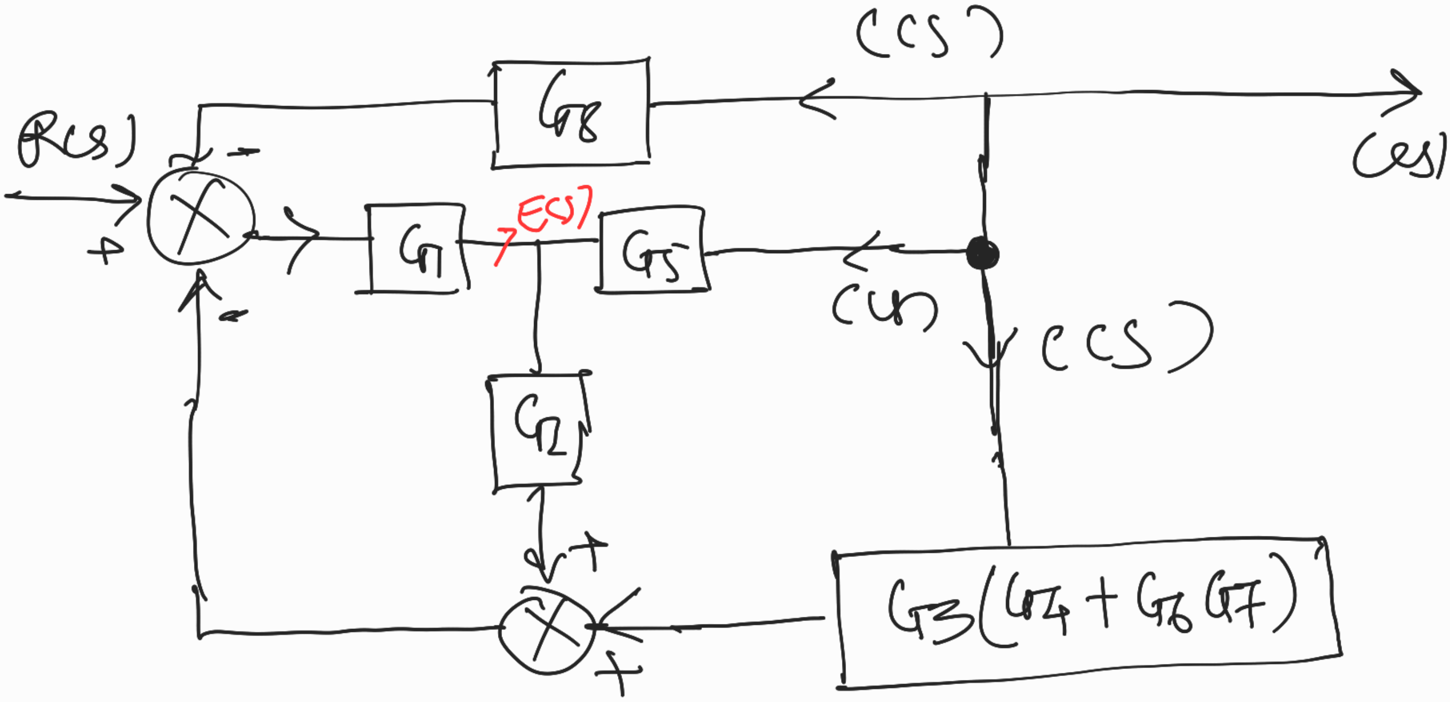
$$\frac{C(s)}{R(s)} = \frac{G_1G_7(G_4+G_3)}{[(1+G_6G_7)(1+G_1G_2) + (G_4+G_3)(G_5G_1 - G_7G_8)]}$$

Let us solve next problem i.e Prob. 1.d by block reduction method.

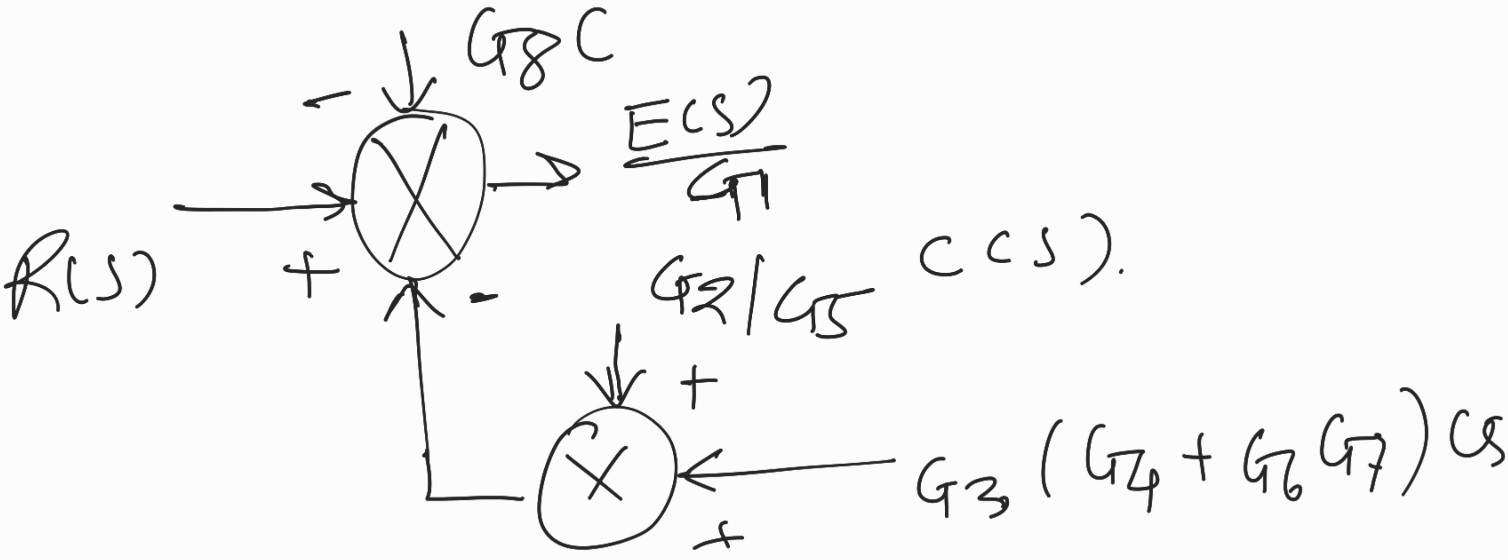
Q.1 d.

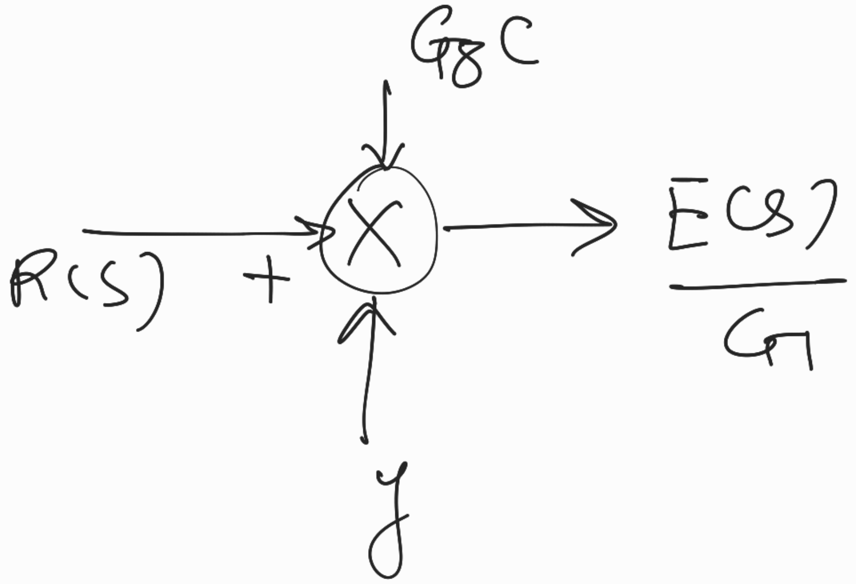

G8
C(s)
R(s)
G1
G5
G6
G2
G4
G7
G3


G8
C(s)
R(s)
G1
G5
G4
G2
G6 G7
G3


C(s)
R(s)
G8
G1
E(s)
G5
C(s)
C(s)
C(s)
G2
G3(G4 + G6 G7)

$$C(s) = G_5 E(s)$$





$$y = \left[\frac{G_2}{G_5} + G_3\left(G_4 + G_6 G_7\right)\right] C_s .$$

$$y = \frac{G_2 + G_5 G_3 \left(G_4 + G_6 G_7\right) C_s}{G_5}$$

$$R(s) - y - G_8 C = \frac{E(s)}{G_1} = \frac{C(s)}{G_1 G_5}$$

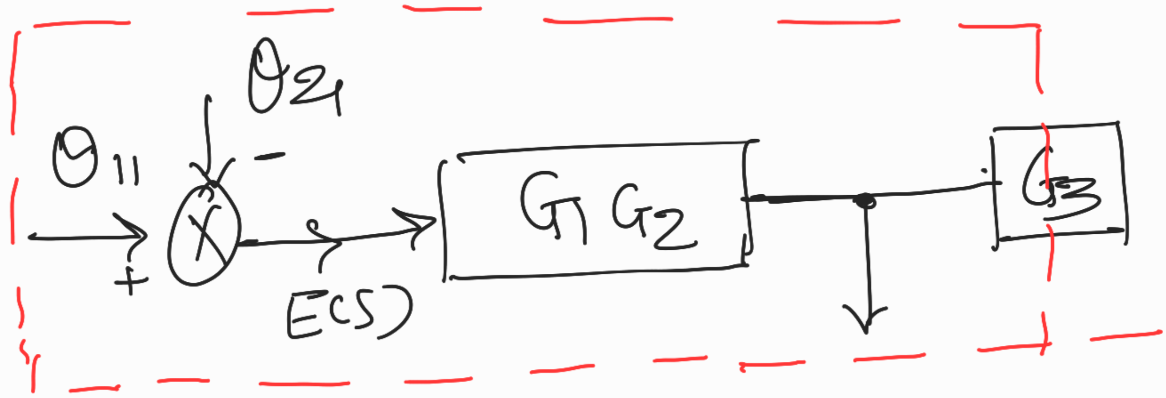
$$R(s) = y + \frac{(G_1 G_5 G_8 + 1)}{G_1 G_5} C(s)$$

$$R(s) = \frac{G_1 G_2 + G_1 G_3 G_5 (G_4 + G_6 G_7) + (1 + G_1 G_5 G_8)}{G_1 G_5} C(s).$$

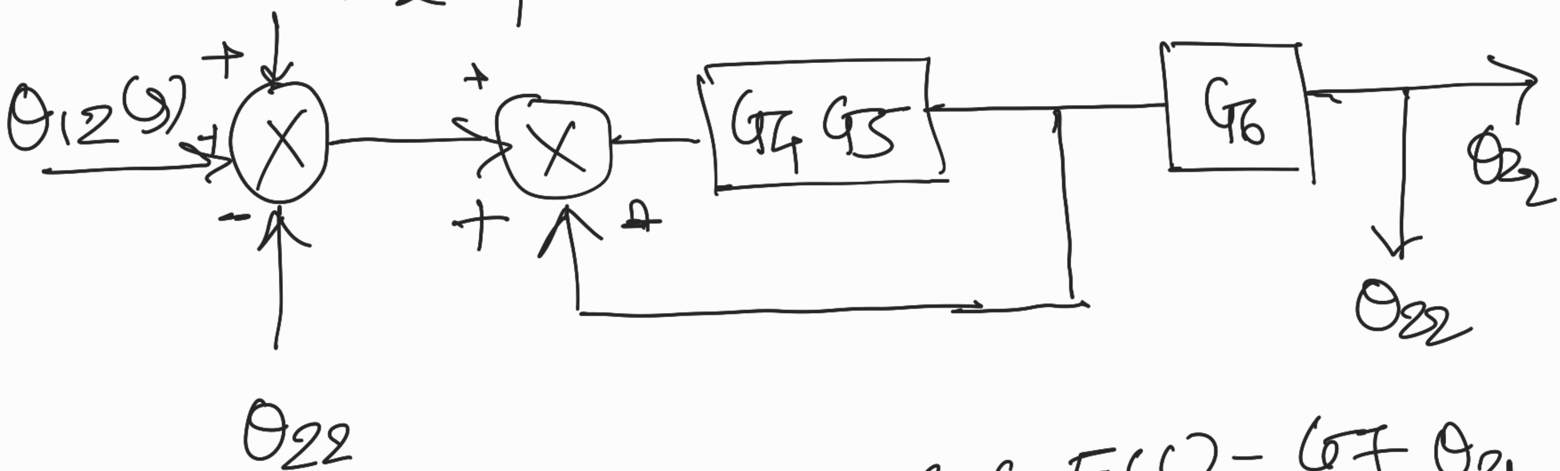
$$\frac{C(s)}{R(s)} = \frac{G_1 G_5}{1 + G_1 G_2 + G_1 G_5 G_8 + G_1 G_3 G_5 (G_4 + G_6 G_7)}$$

Q.2

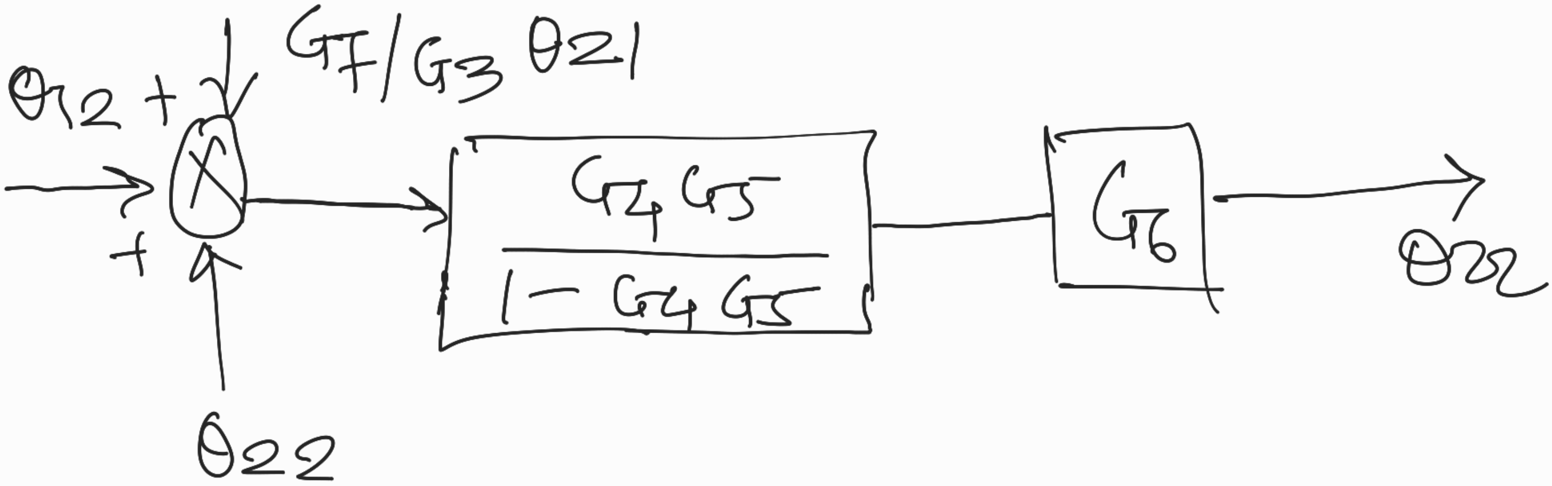
$$\theta_{21}(s) = G_1 G_2 G_3 E(s).$$

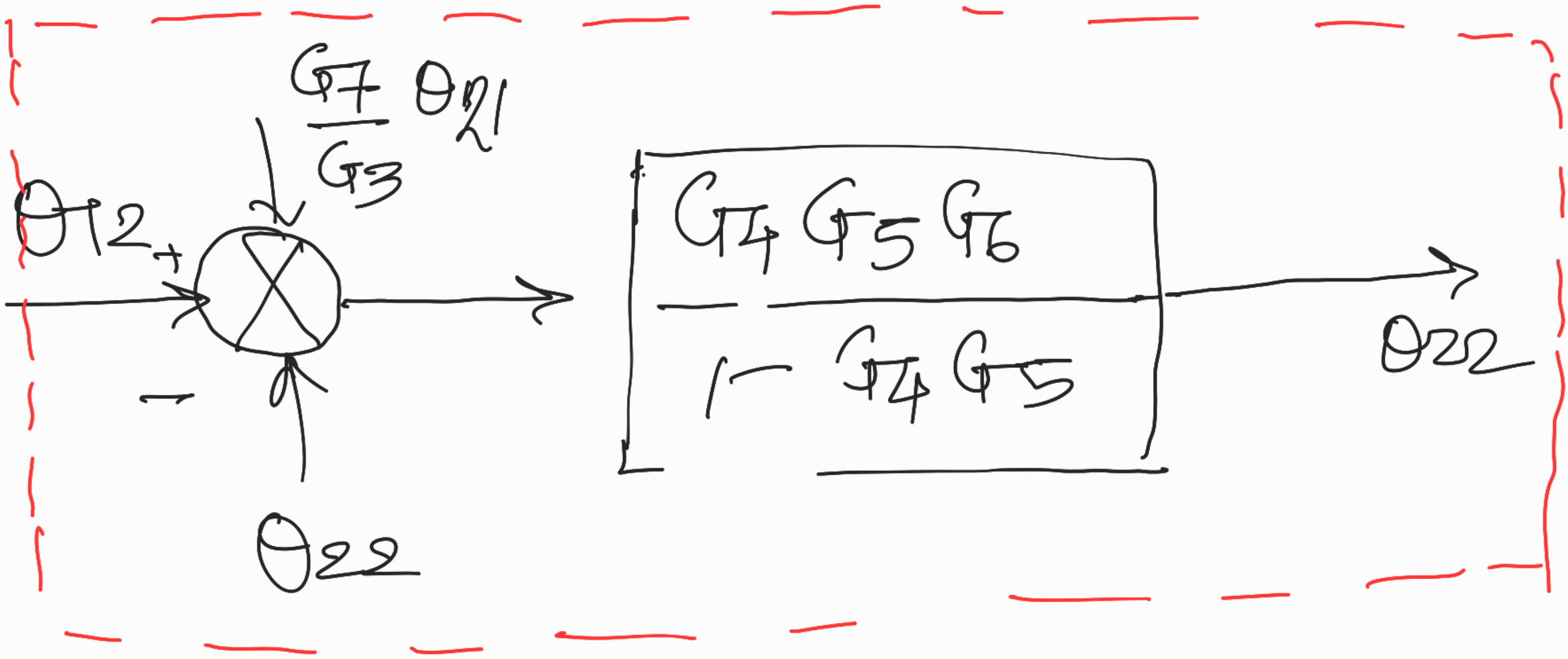


$G_1 G_2 G_7 E(s)$



$$\therefore G_1 G_2 E(s) = \frac{G_7 \theta_{21}}{G_3}$$





$$\theta_{11} - \theta_{21} = E(s).$$

$$\theta_{11} - \theta_{21} = \frac{\theta_{21}}{G_1 G_2 G_3}$$

$$\theta_{11} = \frac{1 + G_1 G_2 G_3}{G_1 G_2 G_3}\,\theta_{21}$$

$$\boxed{\frac{\theta_{21}}{\theta_{11}} = \frac{G_1 G_2 G_3}{1 + G_1 G_2 G_3}}$$

$$\theta_{12} - \theta_{22} + \frac{G_7}{G_3}\theta_{21} = F(s)$$

$$\frac{G_4 G_5 G_6 \, F(s)}{1 - G_4 G_5} = \theta_{22}$$

$$\boxed{\theta_{12} + \frac{G_7}{G_3}\theta_{21} = \frac{\theta_{22}(1 - G_4 G_5 + G_4 G_5 G_6)}{G_4 G_5 G_6}}$$

$$\theta_{12} + \frac{G_1 G_2 G_7 \theta_{11}}{(1 + G_1 G_2 G_3)} = \frac{\theta_{22}(1 - G_4 G_5 + G_4 G_5 G_6)}{G_4 G_5 G_6}$$

Here relation should be given bet^n θ_{12} and θ_{21} but due to symmetry we can take it as $\theta_{12} = \theta_{21}$

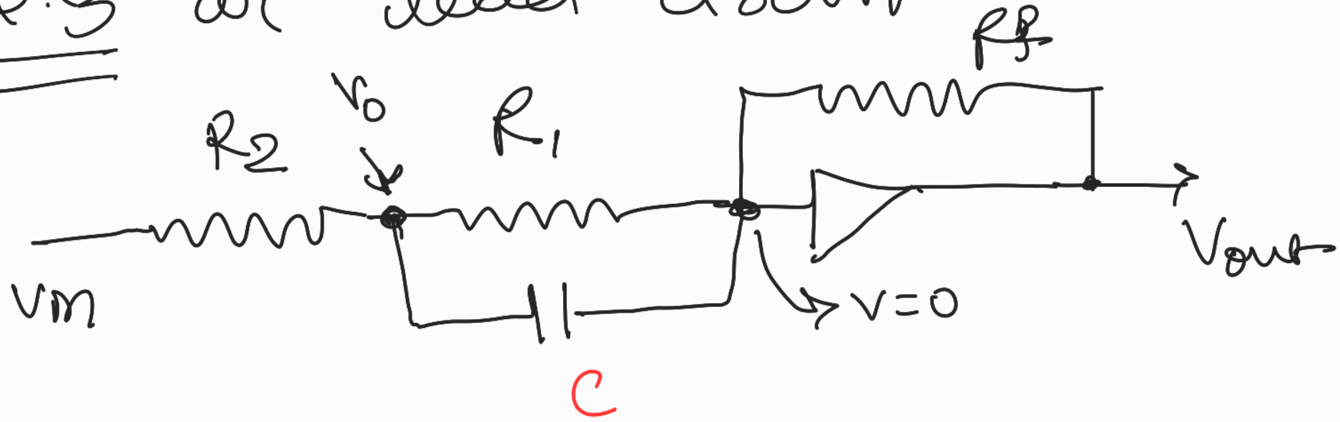
Then

$$\frac{G_1 G_2 (G_3 + G_7)}{(1 + G_1 G_2 G_3)} = \frac{\theta_{22}(1 - G_4 G_5 + G_4 G_5 G_6)}{\theta_{11} \quad G_4 G_5 G_6}$$

$$\frac{\theta_{22}}{\theta_{11}} = \frac{G_1 G_2 G_4 G_5 G_6 (G_3 + G_7)}{1 - G_4 G_5 + G_4 G_5 G_6 + G_1 G_2 G_3 - G_1 G_2 G_3 G_4 G_5 + G_1 G_2 G_3 G_4 G_5 G_6 G_7}$$

This should be answer so please cross check whether it is correct or not solutions given online

Q.3 or lead circuit



$$\frac{V_{in} - V_o}{R_2} - \frac{V_o}{R_1} + C\frac{dV_o}{dt} = 0 \quad \text{— (a)}$$

$$\frac{V_{in} - V_o}{R_2} = \frac{0 - V_{out}}{R_f} \quad \text{— (b)}$$

$$\frac{V_o}{R_2} = \frac{V_{in}}{R_2} + \frac{V_{out}}{R_f}$$

$$\therefore V_o = V_{in} + \frac{R_2}{R_f} V_{out} \quad \text{Putting}$$

(a)

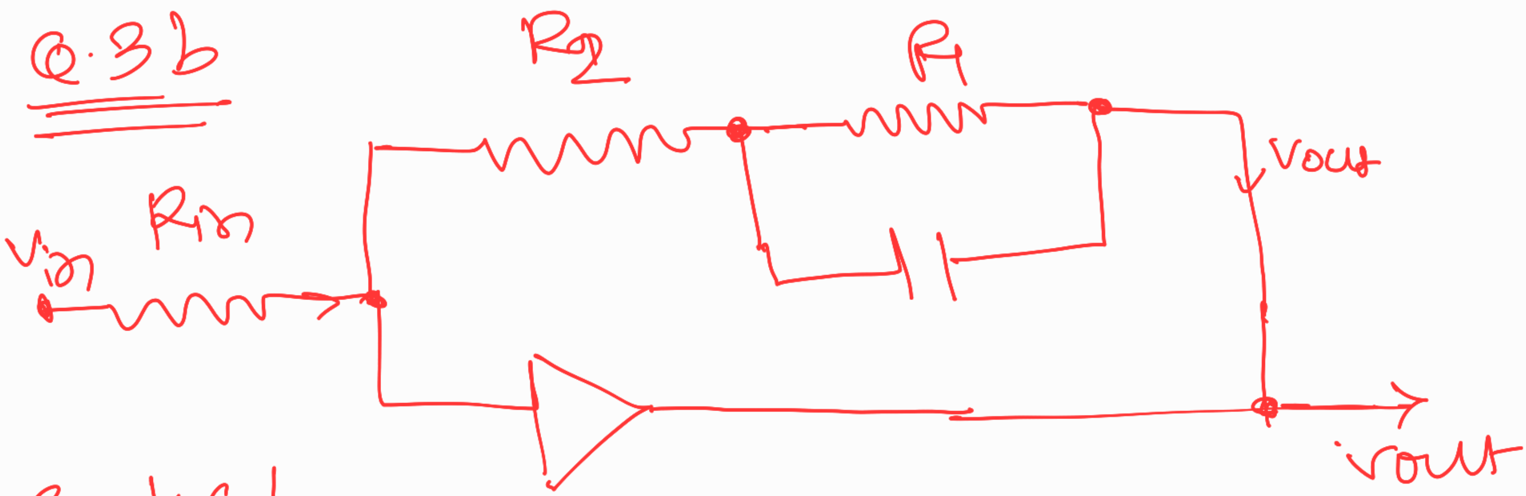
$$\frac{V_{in}}{R_2} - \frac{V_{in}}{R_2} - \frac{V_{out}}{R_f} - \frac{V_{in}}{R_1} - \frac{R_2}{R_1 R_f} V_{out} - C\left(\dot{V}_{in} + \frac{R_2}{R_f}\dot{V}_{out}\right) = 0$$

$$\left(\frac{1 + R_1 C S}{R_1}\right) V_{in} = -\frac{1}{R_f}\left[\left(1 + \frac{R_2}{R_1} + R_2 C S\right] V_{out}$$

After rearranging we have,

$$\frac{V_{out}}{V_{in}} = \frac{-R_f}{R_2}\left[\frac{S + \frac{1}{R_1 C}}{S + \frac{1}{R_1 C} + \frac{1}{R_2 C}}\right].$$

Q.3b



By KCL

$$\frac{V_{in}}{R_{in}} = \frac{-V_o}{R_2} = \frac{V_o - V_{out}}{R_1} + C\frac{d}{dt}(V_o - V_{out})$$

$$\therefore V_o = -\frac{R2}{R_{in}} V_{in}.$$

$$\therefore \frac{-R_2 V_{in}}{R_{in} R_1} + \frac{v_{out}}{R_1} + C\frac{d}{dt}\left(\frac{-R_2}{R_{in}} V_{in} - v_{out}\right) = \frac{V_{in}}{R_{in}}$$

$$\therefore \frac{V_{in}}{R_{in}} = \frac{-1}{R_1}\left(\frac{R2}{R_{in}} V_{in} + V_{out}\right) - c\left(\frac{R2}{R_{in}} \dot{V}_{in} + \dot{V}_{out}\right)$$

$$\therefore \frac{V_{in}}{R_{in}} + \frac{R_2}{R_1 R_{in}} V_{in} + \frac{CR_2}{R_{in}} \dot{V}_{in} = -\frac{V_{out}}{R_1} - C\dot{V}_{out}$$

Taking laplace we have,

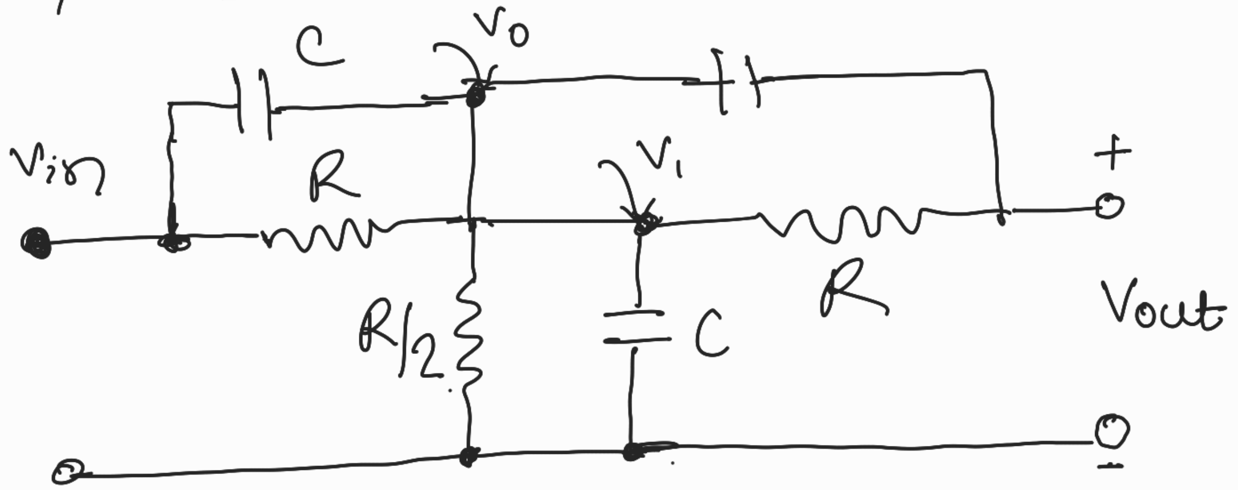
$$V_{in}\left(\frac{1}{R_{in}} + \frac{R2}{R_1 R_{in}} + \frac{CSR2}{R_{in}}\right) = -V_{out}\left(\frac{1}{R_1} + Cs\right)$$

$$\frac{V_{out}}{V_{in}} = -\frac{R_1}{R_{in}}\left(\frac{R_2 Cs + R2/R_1 + 1}{R_1 Cs + 1}\right)$$

*

You can also express this transfer function w.r.t R_2 as we have relation bet^n them so do that by yourself.

c) Notch circuit



The equations that we have,

$$C\frac{d}{dt}(v_{in}-v_0) - \frac{2v_0}{R} + C\frac{d}{dt}(v_{out}-v_0) = 0$$

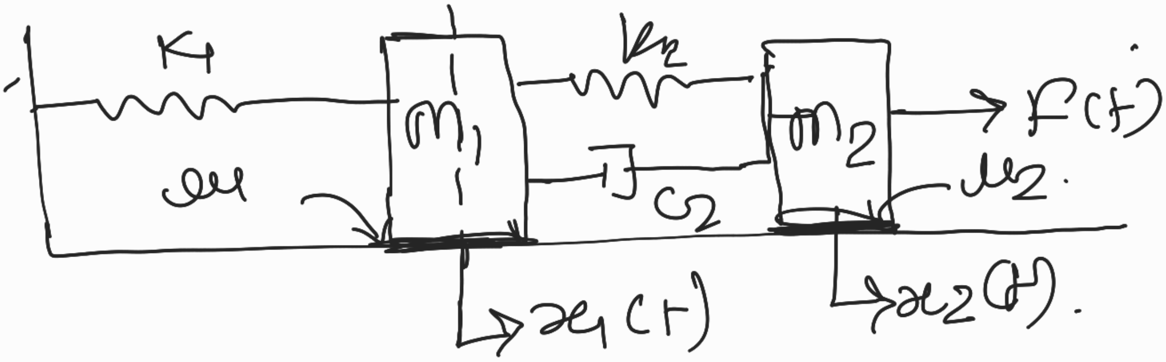
$$\frac{v_{in}-v_1}{R} + 2C\frac{d}{dt}(-v_1) + \frac{v_{out}-v_1}{R} = 0$$

$$C\frac{d}{dt}(v_0 - v_{out}) + \frac{v_1 - v_{out}}{R} = 0$$

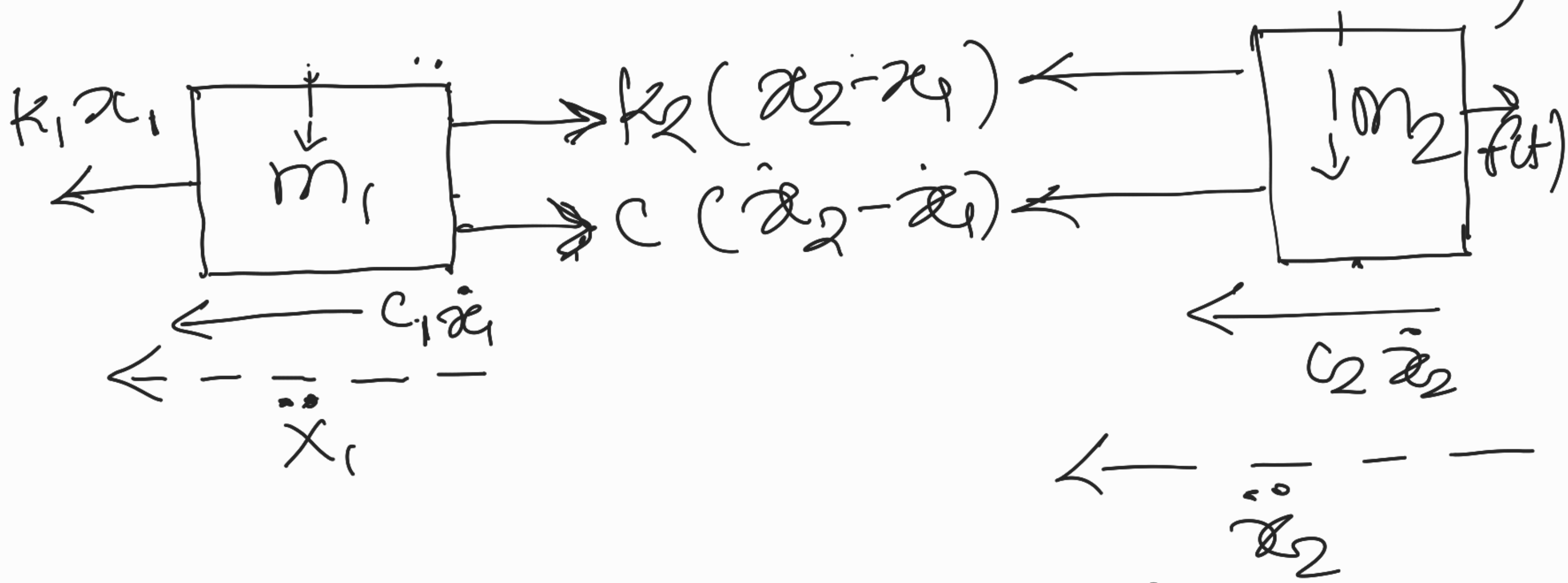
eliminate v_0 and v_1 and express the final equation in terms of Vout and Vin by taking Laplace for each of the equation. The probable answer should be,

$$\boxed{\frac{V_{out}}{V_{in}} = \frac{s^2 + \frac{1}{R^2C^2}}{s^2 + \frac{4s}{RC} + \frac{1}{R^2C^2}}}$$

Q.4 a) :-



Here friction damping is present, we have to write equation of motion by free body diagram (F.B.D).



So equation of motion is

$$m_1 \ddot{x}_1 + k_1 x_1 - k_2(x_2 - x_1) - c(\dot{x}_2 - \dot{x}_1) + c_1 \dot{x}_1 = 0$$

$$\ddot{x}_1 + 9x_1 + 6\dot{x}_1 - 3\dot{x}_2 - 5x_2 = 0$$

$$(s^2 + 6s + 9)X_1(s) - (3s + 5)X_2(s) = 0$$

$$m_2 \ddot{x}_2 + k_2(x_2 - x_1) + c(\dot{x}_2 - \dot{x}_1) + c_2 \dot{x}_2 = f(t)$$

$$2\ddot{x}_2 + 5x_2 + 5\dot{x}_2 - 5x_1 - 3\dot{x}_1 = F(s)$$

Taking laplace,

$$(2s^2+5s+5)X_2(s)-(3s+5)X_1(s)=F(s)$$

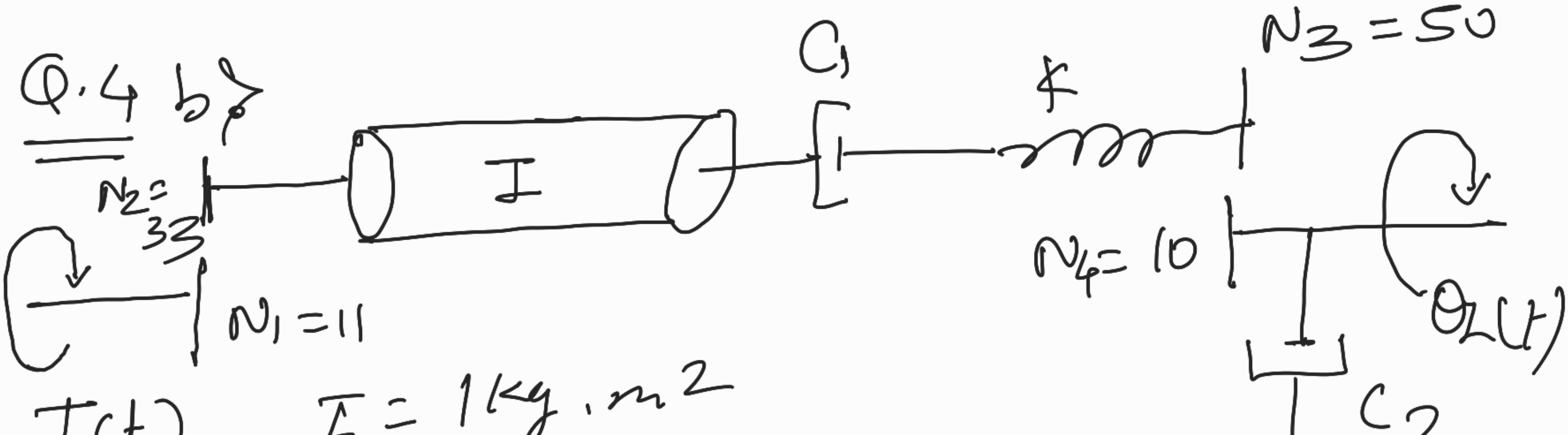
$$X_2(s)=\frac{(s^2+6s+9)}{(3s+5)}X_1(s).$$

$$\left[\frac{(2s^2+5s+5)(s^2+6s+9)}{(3s+5)}-(3s+5)\right]X_1(s)=F(s)$$

$$\frac{X_1(s)}{F_1(s)}=\frac{3s+5}{(2s^4+12s^3+18s^2+5s^3+30s^2+45s+5s^2+30s+45-9s^2-30s-25)}$$

$$\frac{X_1(s)}{F_1(s)}=\frac{3s+5}{2s^4+17s^3+44s^2+45s+20}$$

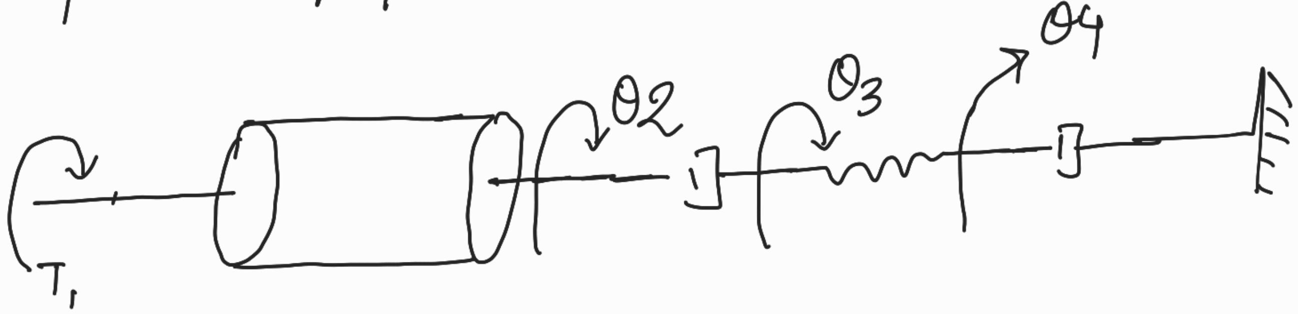
Q.4 b)



$I=1\,kg.m^2$

$C_1=2\,\frac{Nms}{rad}$; $K=3\,\frac{Nm}{rad}$; $C_2=0.04\,\frac{Nms}{rad}$

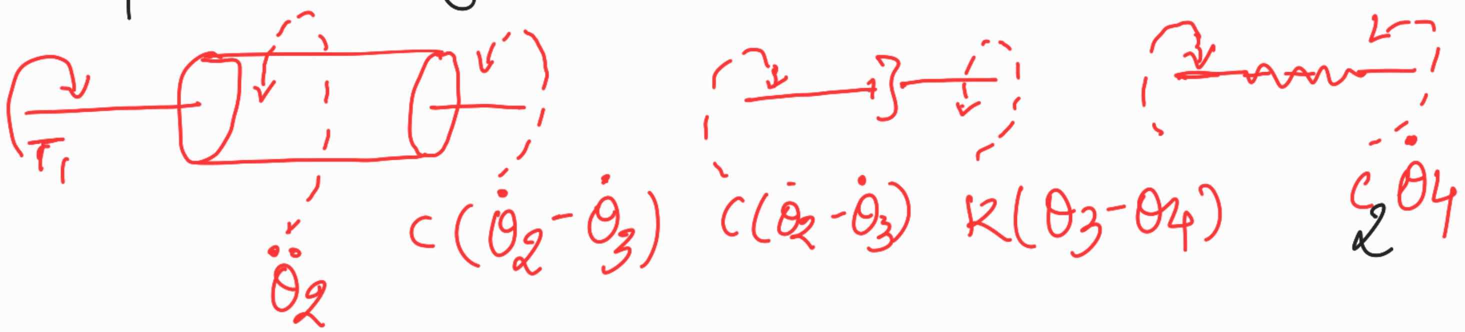
$$\frac{T_1}{T} = \frac{N_2}{N_1} \Rightarrow \boxed{T_1 = 3T}$$



$$N_4 \theta_2 = N_3 \theta_4$$

$$\boxed{\theta_4 = \frac{\theta_2}{5}}$$

So equation of motion can be written as,



$$T_1 = \ddot{\theta}_2 + c(\dot{\theta}_2 - \dot{\theta}_3) \; ; \; \ddot{\theta}_2 + c\dot{\theta}_2 - c\dot{\theta}_3 = 3T \longrightarrow (a)$$

$$c(\dot{\theta}_2 - \dot{\theta}_3) - K\theta_3 + K\theta_4 = 0$$

$$c\dot{\theta}_3 + K\theta_3 - c\dot{\theta}_2 - K\theta_4 = 0 \text{ ——— } (b)$$

$$K(\theta_3 - \theta_4) - c_2\dot{\theta}_4 = 0$$

$$c_2\dot{\theta}_4 + K\theta_4 - K\theta_3 = 0 \text{ ——— } (c).$$

Putting values we have, and taking Laplace for all eqns we have,

$$(s^2+2s)\,\theta_2(s) - 2s\,\theta_3(s) = 3T(s)$$

$$(2s+3)\,\theta_3(s) - 2s\,\theta_2(s) - 3\,\theta_4(s) = 0$$

$$-3\,\theta_3(s) + (0.04s+3)\,\theta_4(s) = 0$$

$$\theta_3(s) = \frac{(0.04s+3)}{3}\,\theta_4(s)$$

$$\left[\frac{(2s+3)(0.04s+3) - 9}{6s}\right]\theta_4(s) = \theta_2(s)$$

$$\left[\frac{0.08s^2 + 6s + 0.12s + 9 - 9}{6s}\right]\theta_4(s) = \theta_2(s)$$

$$\therefore\ \theta_2(s) = \left[\frac{0.08s + 6.12}{6}\right]\theta_4(s)$$

$$\left[\frac{(s^2+2s)(0.08s+6.12) - 6s(0.04s+3)}{6}\right]\theta_4(s) = 3T(s)$$

$$s\left[(s+2)(0.08s+6.12) - 0.24s - 18\right]\theta_4(s) = 18\,T(s)$$

$$s\left[0.08s^2+6.12s+0.16s+12.24-0.24s-18\right]\theta_4(s) = 18\,T(s)$$

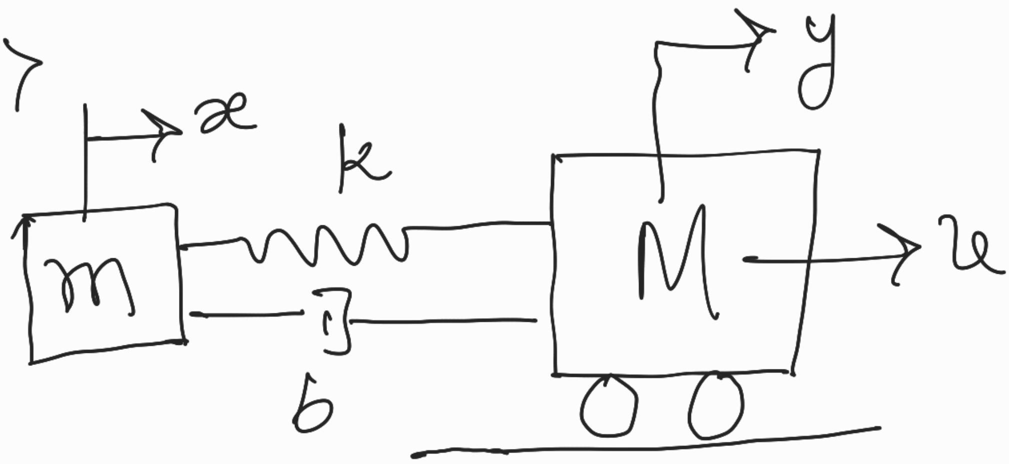
$$s\left[0.08s^2+6.04s-5.76\right]\theta_4(s) = 18\,T(s)$$

$$\boxed{\frac{\theta_L(s)}{T(s)} = \frac{4500}{s(4s^2+302s-288)}}$$

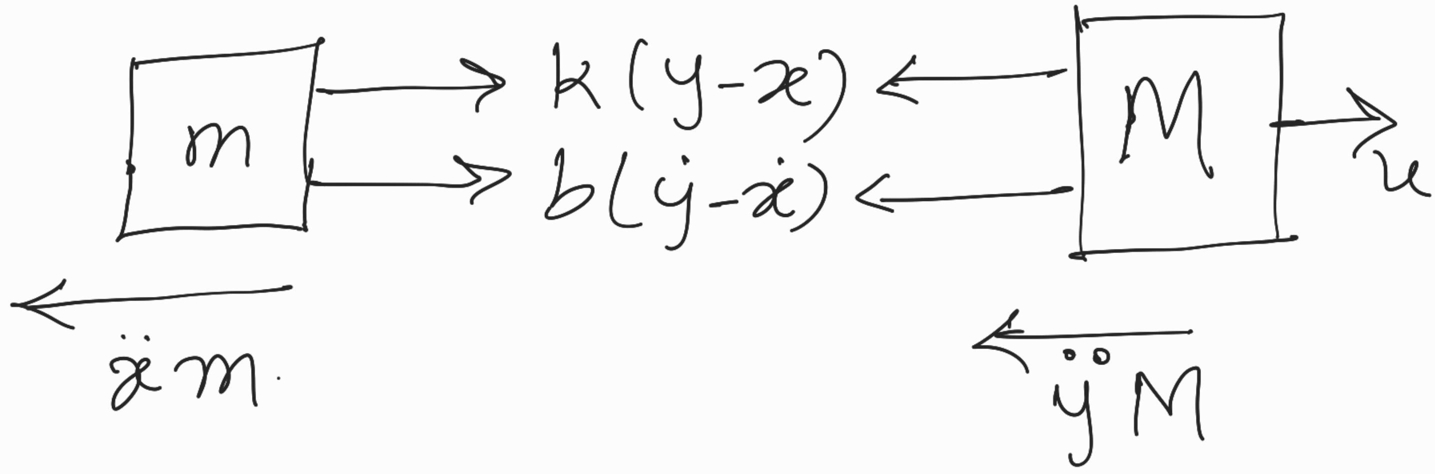
* All of you have copied solution from solutions which is available online. You all have taken $C_2 = 10\,N\cdot m\cdot s/rad$ but given in problem is

$$\boxed{C_2 = 0.04\ \frac{N\cdot m\cdot s}{rad}}$$

Q.5 a)



So by drawing FBD



EOM

$$\ddot{x}m - k(y-x) - b(\dot{y}-\dot{x}) = 0$$

$$m\ddot{x} + b\dot{x} + kx = b\dot{y} + ky \quad \text{— (a)}$$

$$\ddot{y}M + b\dot{y} + ky = u + kx + b\dot{x} \quad \text{— (b)}$$

So taking laplace we have,

$$(ms^2+bs+k)X(s) = (bs+k)Y(s)$$

$$X(s) = \frac{(bs+k)}{(ms^2+bs+k)} Y(s).$$

$$\left[(Ms^2+bs+k)-\frac{(bs+k)^2}{(ms^2+bs+k)}\right]Y(s)=U(s)$$

$$\frac{Y(s)}{U(s)}=\frac{ms^2+bs+k}{(Ms^2+bs+k)(ms^2+bs+k)-(bs+k)^2}$$

So by solving denomenator

we have,

$$T(s)=\frac{Y(s)}{U(s)}=\frac{(ms^2+bs+k)}{mMs^4+(m+M)(bs^3+ks^2)}$$

$$T(s)=\frac{ms^2+bs+k}{s^2\left[mMs^2+(m+M)(bs+k)\right]}$$

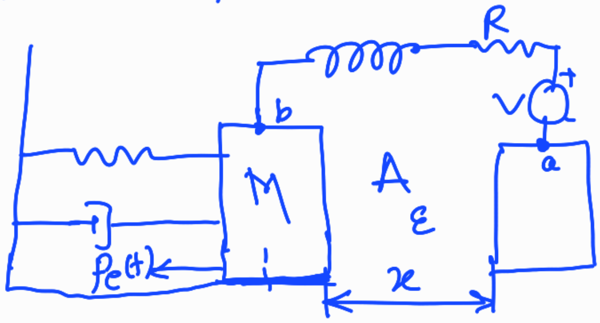
Dear friends next time onwards please read Q. at least & then copy, some of students have copied state space also but in assignment, it is not asked.

Q.6 Electromechanical system having capacitance betⁿ two plates at a distance x is $C(x) = \frac{\varepsilon A}{x}$.

Relation betⁿ charge and voltage is $q = C(x)e$.

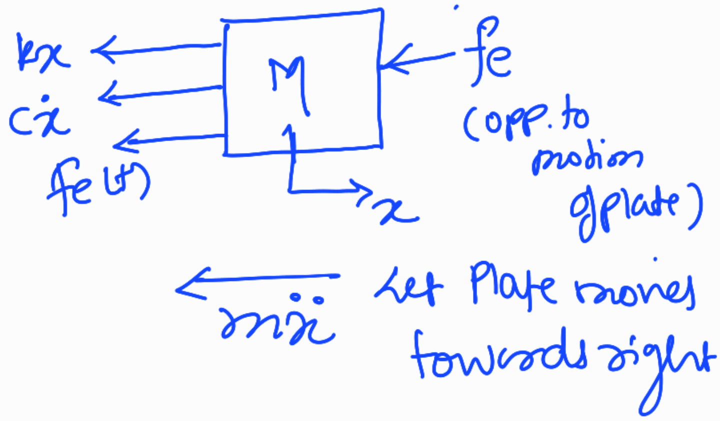
And electrical force is $f_e = \frac{q^2}{2\varepsilon A}$. in opposite to its motion.

So system is represented as



So It is coupled system.

F.B.D

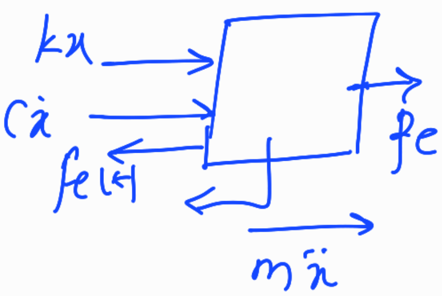


So eqⁿ of motion is

$$\therefore \; m\ddot{x} + \beta\dot{x} + kx + f_e = -f_e(t)$$

$$\therefore \; m\ddot{x} + \beta\dot{x} + kx + \frac{q^2}{2\varepsilon A} = -f_e(t)$$

if plate moves towards left then



So eqⁿ of motion is

$$m\ddot{x} + \beta\dot{x} + kx + \frac{q^2}{2\varepsilon A} = f_e(t)$$

and circuit equation can be written as see the direction of current $i(t)$

$$V = I(t) \cdot R + L\frac{dI(t)}{dt} + \frac{q}{C}$$

$$\therefore \boxed{V = R\dot{q} + L\ddot{q} + \frac{qx}{A\varepsilon}}$$

we have two equations

$$V = L\ddot{q} + R\dot{q} + qx/A\varepsilon$$

$$m\ddot{x} + \beta\dot{x} + kx + q^2/2A\varepsilon = f_e(t)$$

If we write force bet^n capacitor plate in terms of voltage i.e.

$$q = cv \qquad \because c = \frac{\varepsilon A}{x}$$

$$\therefore f_e = \frac{c^2v^2}{2A\varepsilon} = \frac{\varepsilon^2A^2v^2}{2A\varepsilon x^2}$$

$$f_e = \frac{v^2A\varepsilon}{2x^2}$$

So eq^n of motion becomes,

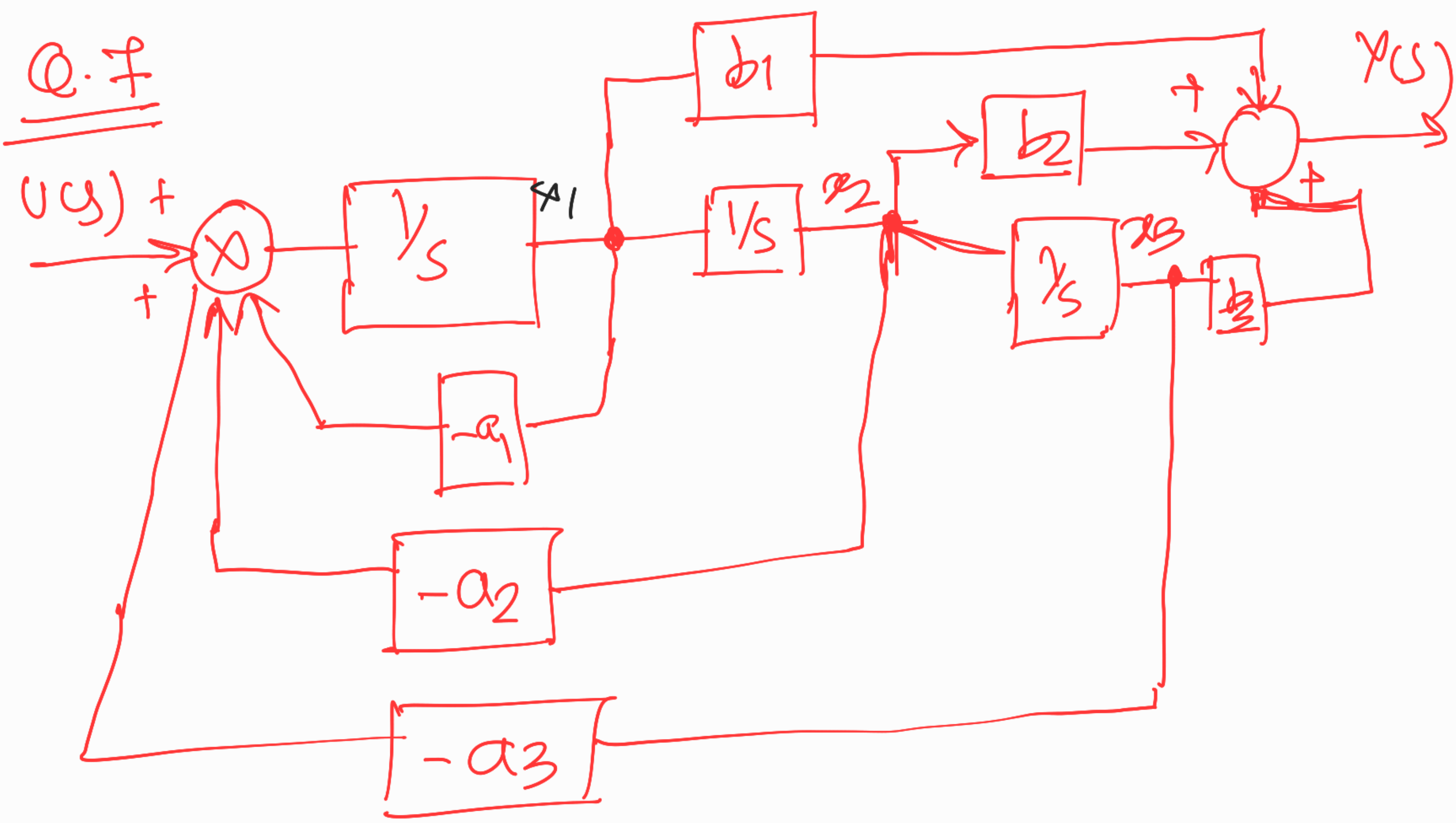
$$m\ddot{x} + B\dot{x} + kx + \frac{v^2A\varepsilon}{2x^2} = f_e(t)$$

And $V = L\ddot{q} + R\dot{q} + \left(\frac{qx}{\varepsilon A}\right)$

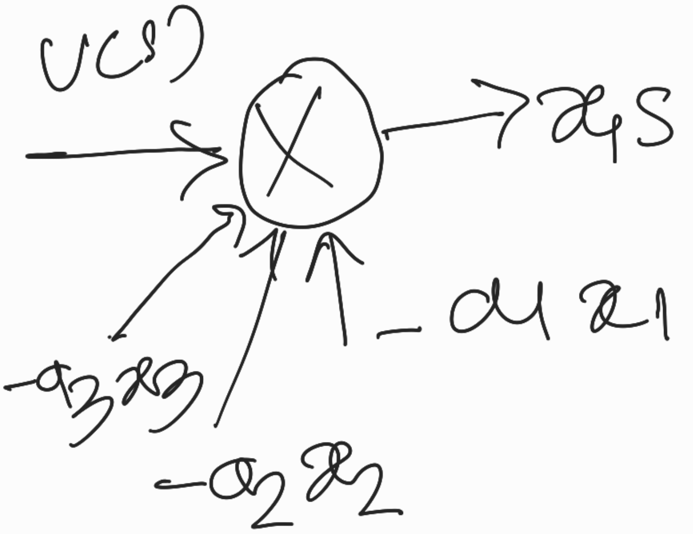
b) Here, terms q^2 & qx or v^2 & $f_e \propto \frac{1}{x^2}$ makes equations impossible to determine the linear model.

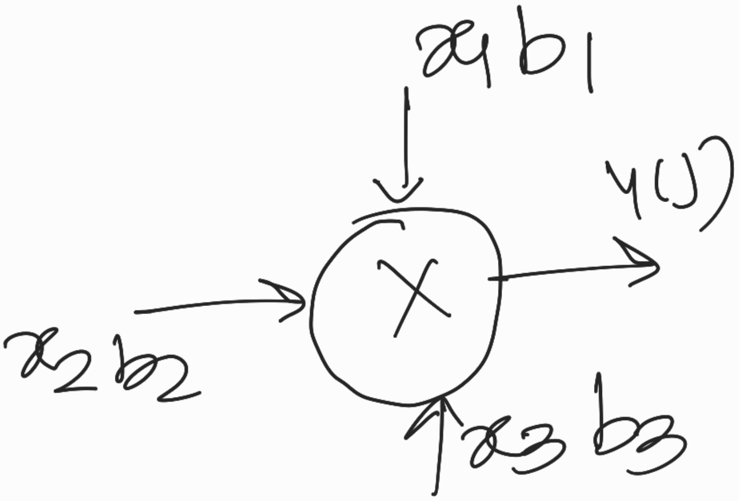
(c) Here $\dot{q}$ is the input to the system so output will be sound.

Q.7



find out the transfer function





$$\frac{x_1}{s} = x_2 \quad \text{(a)} \; ; \qquad \frac{x_2}{s} = x_3 . \quad \text{(b)}$$

$$U(s) = x_1 s + x_1 a_1 + x_2 a_2 + x_3 a_3 . \quad \text{(c)}$$

$$Y(s) = x_1 b_1 + x_2 b_2 + x_3 b_3 . \quad \text{(d)}$$

$$U(s) = x_1 s + x_1 a_1 + \frac{x_1 a_2}{s} + \frac{x_1 a_3}{s^2}$$

$$U(s) = \frac{s^2 (s + a_1) + s a_2 + a_3}{s^2} X_1(s)$$

$$Y(s) = x_1 b_1 + \frac{x_1 b_2}{s} + \frac{x_1 b_3}{s^2}$$

$$Y(s) = \frac{(s^2 b_1 + b_2 s + b_3)}{s^2} X_1(s)$$

$$U(s) = \frac{Y(s)(s^3 + a_1 s^2 + a_2 s + a_3)}{(s^2 b_1 + b_2 s + b_3)}$$

$$T(s) = \frac{Y(s)}{U(s)} = \frac{(s^2 b_1 + b_2 s + b_3)}{(s^3 + a_1 s^2 + a_2 s + a_3)}$$

So differential equation can be,

$$\dddot{y} + a_1 \ddot{y} + a_2 \dot{y} + a_3 = b_1 \ddot{u} + b_2 \dot{u} + b_3$$

So state space variables representation in time domain is given as from equation a, b, c, d taking Laplace inverse we get

$$x_1 = \dot{x}_2 \; ; \quad x_2 = \dot{x}_3$$

$$u = \dot{x}_1 + a_1 x_1 + a_2 x_2 + a_3 x_3$$

$$y = b_1 x_1 + b_2 x_2 + b_3 x_3$$

So $\dot{x}_1 = u - a_1 x_1 - a_2 x_2 - a_3 x_3$.

In matrix form can be written as,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -a_1 & -a_2 & -a_3 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u(t)$$

$$y = \begin{bmatrix} b_1 & b_2 & b_3 \end{bmatrix} x.$$