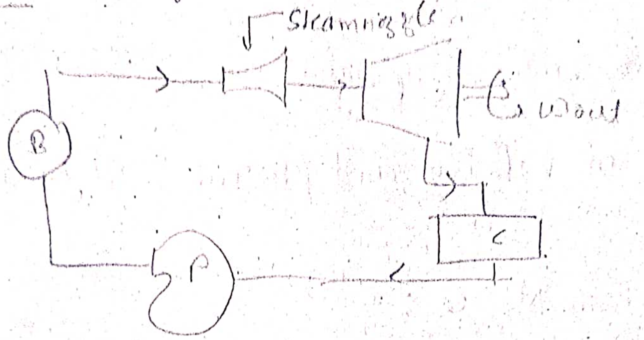
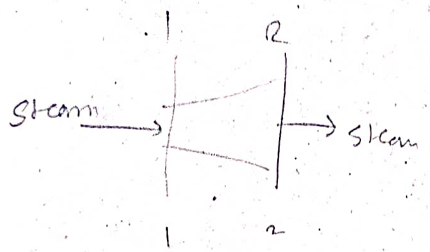
Flow nozzles

- No heat loss
- Negligible friction loss

Steam Nozzle : Analysis and Efficiency





{ thermodynamic properties of ~~noz~~ steam changes when passing through nozzles.

h_1 = enthalpy at inlet
h_2 = enthalpy at outlet

$(h_2 < h_1) \rightarrow$ enthalpy drop of steam

$[h = u + pv]$

So, $h_1 = u_1 + p_1 v_1$, $h_2 = u_2 + p_2 v_2$

Since we don't want to reduce the temperature of steam nozzles are insulated

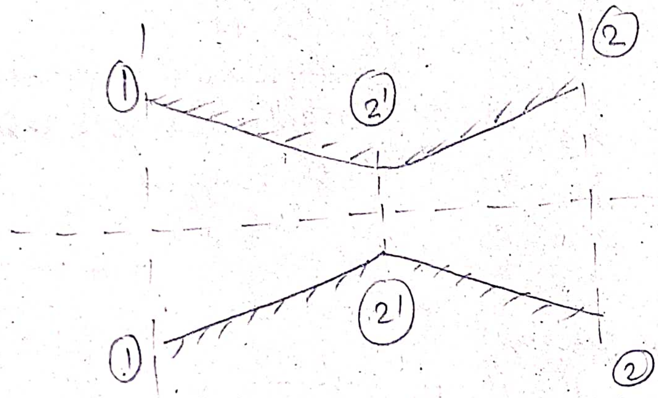
Convergent nozzle (Diffuser) ((Convergent divergent type nozzle)

To get high KE at the exit of nozzle (to get desired velocity), we go for analysis of nozzle.

We will not play with temperature.

So, only changes in velocity and pressure will be interpreted.

Flow through Steam Nozzle :-



Assumptions:

→ No heat transfer (insulated)
→ Frictional effect is neglegible
→ Isentropic flow of steam

$$\Downarrow$$

$$PV^K = \text{constant}$$

K = index of expansion for isentropic process.

or $\boxed{\dfrac{P}{\rho^K} = C}$

→ Flow is one dimensional
→ The system is at steady state

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from continuity equation,

$$\rho_1 A_1 C_1 = \rho_2 A_2 C_2 = \text{constant}$$

on differenciating.

$$\boxed{\frac{dA}{A} + \frac{d\rho}{\rho} + \frac{dC}{C} = 0} \quad \text{---} \quad (1)$$

Now, we have, $\frac{P}{\rho^{k}}$ = constant {isentropic flow}

$\Rightarrow$ on differentiating we get

$$\frac{dP}{P} = k\frac{d\rho}{\rho}$$

$$\Rightarrow \boxed{\frac{d\rho}{\rho} = \frac{1}{k}\frac{dP}{P}} \quad -(2)$$

$$P\rho^{-k} = c_0$$

$$P(-k\rho^{-(k+1)}) + \rho^{-k}dP = 0$$

$$dP\rho^{-k} = kP\rho^{-(k+1)}$$

$$\Rightarrow \frac{dP}{\rho^{k}} = \frac{kP\,d\rho}{\rho^{k+1}}$$

$$\Rightarrow \boxed{\frac{dP}{P} = k\frac{d\rho}{\rho}}$$

Steady state steady flow energy equation (SSSF)

$$\Rightarrow h_1 + \frac{C_1^2}{2} + gz_1 = h_2 + \frac{C_2^2}{2} + gz_2$$ {valid for the considered assumptions}

Length of the fluidic confinement is small such that

$$z_1 \approx z_2$$

then, $h_2 + \frac{C_2^2}{2} = h_1 + \frac{C_1^2}{2}$

$$\Rightarrow \boxed{(h_2 - h_1) + \frac{C_2^2 - C_1^2}{2} = 0} \quad -(3)$$

writing in differential form:

$$\boxed{dh + C\,dC = 0} \quad -(3)$$

For any thermodynamic process, at any state point

$$T\,ds = dh - v\,dP$$

$\downarrow$ 0 {since the process is isentropic}

$$\Rightarrow \boxed{dh = v\,dP = \frac{dP}{\rho}} \quad -(4) \quad \left\{v = \frac{1}{\rho}\right\}$$

Substituting eq (4) in eq (3)

$$\frac{dP}{\rho} + C\,dC = 0$$

$$\text{or} \quad \boxed{\frac{dP}{\rho C^2} + \frac{dC}{C} = 0} \quad -(5)$$

$M = \frac{c}{a} \Rightarrow M^2 = \frac{c^2}{a^2}$

Sonic velocity, $a = \sqrt{KPv}$

$$a = \sqrt{\frac{KP}{\rho}}$$

$$\Rightarrow a^2 = \frac{KP}{\rho}$$

$$\Rightarrow c^2 = \frac{M^2 KP}{\rho}$$

$$\Rightarrow \boxed{\rho c^2 = M^2 K P} \quad - (6)$$

{K = index of insentropic process}

using eq (6), eq (5) can be written as.

$$\frac{dP}{KPM^2} + \frac{dc}{c} = 0$$

$$\Rightarrow \boxed{\frac{dc}{c} = -\frac{1}{K M^2}\frac{dP}{P}} \quad - (7)$$

Now, by using eq. (7) and eq (2) in eq (1) we get

$$\frac{dA}{A} + \frac{1}{K}\frac{dP}{P} - \frac{1}{KM^2}\frac{dP}{P} = 0$$

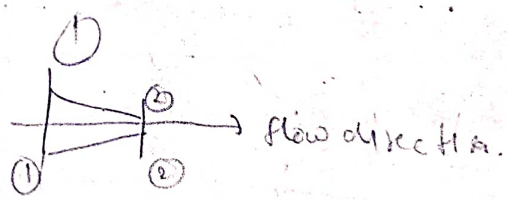
$$\Rightarrow \boxed{\frac{dA}{A} = \frac{1}{K}\frac{dP}{P}\left(\frac{1-M^2}{M^2}\right)} \quad - (8)$$

Area velocity equation for nozzle

Physical explanation of eq. (8)

$$\frac{dA}{A} = \frac{1}{K}\frac{dP}{P}\left(\frac{1-M^2}{M^2}\right)$$

Case I Accelerated flow.



$\Rightarrow P_2 < P_1$

$\Rightarrow dP = P_2 - P_1 < 0$

$\Rightarrow \frac{dP}{P}$ is negative. {called flow through nozzle}

(a) when $M < 1$ {Subsonic flow}

$\Rightarrow \frac{dA}{A} = -ve$ {flow through the convergent part of duct}

(b) when $M = 1$

$\Rightarrow$ constant area

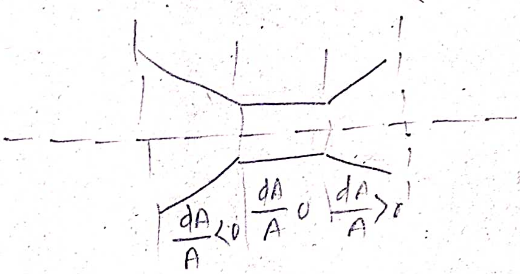
$\Rightarrow$ The throat of the nozzle is reached

(c) when $M > 1$ (supersonic flow)

A ~~= const~~

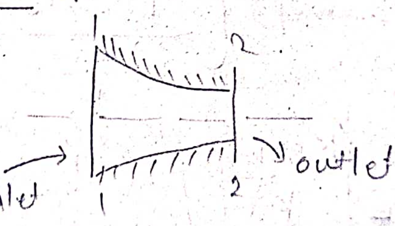
$\frac{dp}{p} < 0$ also $\frac{1-M^2}{M^2} < 0$

$\Rightarrow \frac{dA}{A} = +ve$ [this corresponds the diverging part of duct]



Steam Nozzle analysis and efficiency :-

mass flow rate of steam



Applying (SSSF)

$$h_1 + \frac{C_1^2}{2} + gz_1 = h_2 + \frac{C_2^2}{2} + gz_2$$

To maintain the flow, we need to supply work known as flow work

$z_1 \simeq z_2$.

$$\Rightarrow \frac{C_2^2 - C_1^2}{2} + h_2 - h_1 = 0$$

$$\Rightarrow dh + CdC = 0$$

For any process

$Tds = dh - v\,dp$

$\Downarrow = 0$ {isentropic}.

$$\boxed{dh = \frac{dp}{\rho}} \quad -② $$

Substituting eq ② in eq ①

$$\boxed{\frac{1}{\rho} = \left(\frac{\text{constant}}{P}\right)^{1/k}} \quad - 4$$

$\Rightarrow$ OR,

$$\frac{dp}{\rho} + c\,dc = 0$$

$$\Rightarrow \boxed{\frac{c_2^2 - c_1^2}{2} = -\int_{p_1}^{p} \frac{dp}{\rho}} \quad -③$$

$$\Rightarrow \frac{c_2^2 - c_1^2}{2} = -\int_{p_1}^{p_2} \left(\frac{\text{constant}}{p}\right)^{1/k} dp$$

$$\frac{c_2^2 - c_1^2}{2} = (\text{constan})^{1/k} \left[\frac{p_1^{\frac{k-1}{k}} - p_2^{\frac{k-1}{k}}}{\frac{k-1}{k}}\right]$$

$$\frac{c_2^2 - c_1^2}{2} = \left(\frac{p_1}{\rho_1^k}\right)^{1/k} \left(\frac{k}{k-1}\right) \left[p_1^{k-1/k} - p_2^{k-1/2}\right] \qquad \left\{\frac{p}{\rho^k} = \text{constan}\right.$$

$$\Rightarrow \frac{c_2^2 - c_1^2}{2} = \left(\frac{k}{k-1}\right)\left(\frac{p_1}{\rho_1}\right)\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}\right]$$

$c_1 << c_2$

$$\Rightarrow \frac{c_2^2}{2} = \left(\frac{k}{k-1}\right)\left(\frac{p_1}{\rho_1}\right)\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}\right]$$

$\dot{m}_2 = \rho_2 A_2 C_2$ {ρ_2 is changing}.

$$C_2 = \sqrt{\left(\frac{2K}{K-1}\right)\left(\frac{P_1}{\rho_1}\right)\left[1-\left(\frac{P_2}{P_1}\right)^{\frac{K-1}{K}}\right]}$$

Now,

$$\frac{P}{\rho^K} = \text{constant}$$

$$\Rightarrow \frac{P_1}{\rho_1^K} = \frac{P_2}{\rho_2^K}$$

$$\Rightarrow \boxed{\rho_2 = \rho_1\left(\frac{P_2}{P_1}\right)^{1/K}}$$

$$\dot{m}_2 = A_2 \rho_1 \left(\frac{P_2}{P_1}\right)^{1/K} \sqrt{\left[\frac{2K}{K-1}\left(\frac{P_1}{\rho_1}\right)\left(1-\left(\frac{P_2}{P_1}\right)^{\frac{K-1}{K}}\right)\right]}$$

$$\boxed{\frac{\dot{m}_2}{A_2} = \sqrt{\left(\frac{2K}{K-1}\right)\left(P_1\rho_1\left[\left(\frac{P_2}{P_1}\right)^{2/K} - \left(\frac{P_2}{P_1}\right)^{\frac{K+1}{K}}\right]\right)}}$$

→ mass flow rate at the exit of nozzle.

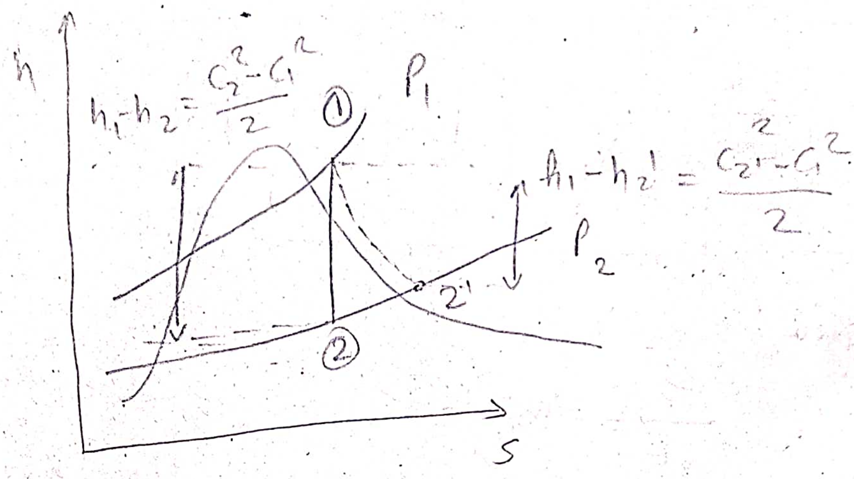
Since K is constant and also P_1, ρ_1 are constant.

then, $\frac{\dot{m}_2}{A_2} = f\left(\frac{P_2}{P_1}\right)$

$\pi = \frac{P_2}{P_1}$ → Critical pressure Ratio for which the quantity $\frac{\dot{m}_2}{A_2}$ will be maximum.

$$\boxed{\pi_{crit} = \left(\frac{P_2}{P_1}\right)_{critical} = \left(\frac{2}{K+1}\right)^{\frac{K}{K-1}}}$$

Nozzle efficiency :-



nozzle efficiency = it is defined as the ratio of actual heat drop to that due to isentropic expansion.

OR

It is the ratio of actual gain in KE to that of isentropic expansion.

$$\boxed{\eta_n = \frac{h_1 - h_2'}{h_1 - h_2} = \frac{C_{2'}^2 - C_1^2}{C_2^2 - C_1^2}}$$

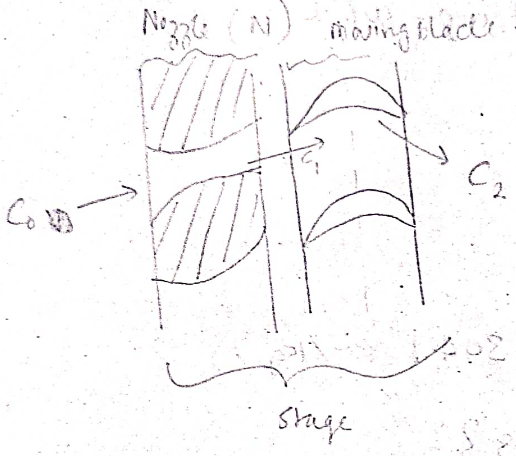
if $C_1 << C_2, C_{2'}$

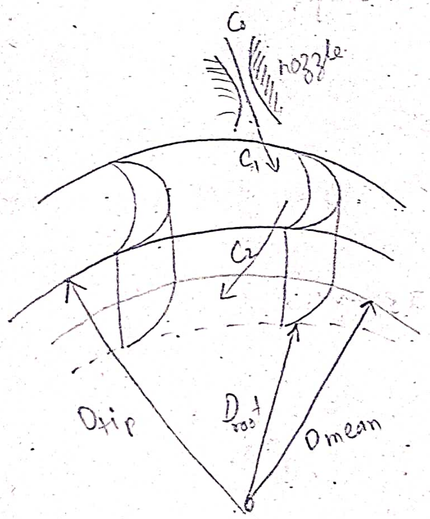
$$\eta_n = \frac{C_{2'}^2}{C_2^2}$$

Impulse

Steam turbine

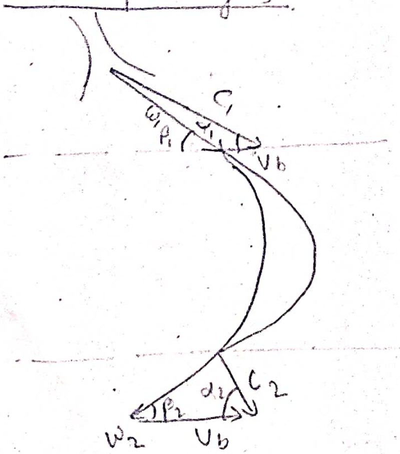
→ $\boxed{\text{Impulse turbine}}$

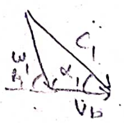




$$D_{mean} = \frac{D_{tip} + D_{root}}{2}$$

Velocity triangles:-





C: absolute velocity
w: Relative velocity
V_b: Blade velocity.
α: flow angle / nozzle angle.
β: Blade angle

Torque will be produced by the difference in swirl / whirl component of velocity

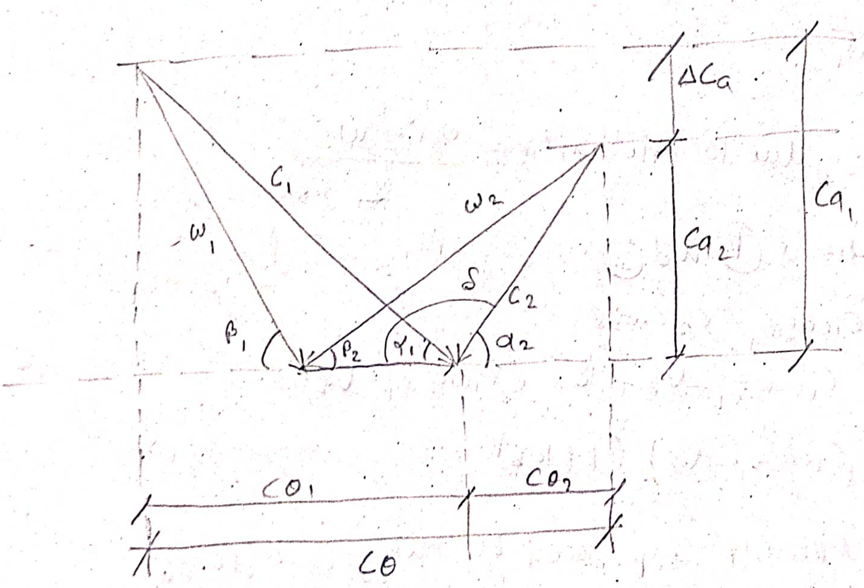
$\Delta C_\theta = C_{\theta_1} - C_{\theta_2}$

Swirl velocity at inlet

Swirl velocity at outlet

tangential component of absolute velocity at the inlet.

Velocity Triangle



C_{a_1} = component of absolute velocity in axial direction at inlet.

\# Blade angle at inlet is equal to the blade angle at outlet
$\Rightarrow \beta_1 = \beta_2$ {symmetrical blade}
for impulse turbine

$\Delta C_\theta = |C_{\theta_1}| - |C_{\theta_2}|$

$= C_1 \cos \alpha_1 - C_2 \cos \alpha_2$

$= C_1 \cos \alpha_1 - C_2 \cos \delta$ —①

From inlet triangle

$$C_1 \cos\alpha_1 - V_b = W_1 \cos\beta_1$$

$$C_1 \sin\alpha_1 = W_1 \sin\beta_1$$

$$\Rightarrow \tan\beta_1 = \frac{C_1 \sin\alpha_1}{C_1 \cos\alpha_1 - V_b}$$

From exit velocity triangle

$$C_2 \cos\delta = V_b + W_2 \cos\beta_2 \quad -(2)$$

K_b = blade friction factor

$$K_b = \frac{W_2}{W_1}$$

$$\text{Loss of energy due to friction} = \frac{W_1^2 - W_2^2}{2}$$

using equations (1) and (2)

$$\Delta C_0 = C_1 \cos\alpha_1 - C_2 \cos\delta$$
$$= C_1 \cos\alpha_1 - V_b + K_b (C_1 \cos\alpha_1 - V_b)$$
$$= (C_1 \cos\alpha_1 - V_b)(1 + K_b)$$

tangential thrust impressed by the jets (steam) at the blades.

Tangential thrust, $P_t = \dot{m}_s \times \Delta C_0$ { $\dot{m}_s$ = mass flow rate of steam }

Work transfer $= P_t \times V_b$

$= \dot{m}_s \Delta C_0 V_b$ } Blading work / Diagram work

Axial thrust, $P_a = \dot{m}_s \Delta C_a$

Blading / Diagram efficiency : $\dfrac{\text{Rate at which work is done on the blades}}{\text{Rate of energy input to blade.}}$

$$\eta_d = \frac{\dot{m}_s \Delta C_0 V_b}{\frac{1}{2} \dot{m}_s C_1^2}$$

$$\eta_d = \frac{m_s \Delta C_\theta V_b}{\frac{1}{2} m_s C_1^2} = \frac{2 \Delta C_\theta V_b}{C_1^2}$$

$$\eta_d = \frac{2 (C_1 \cos\alpha_1 - V_b)(1+K_b) V_b}{C_1^2}$$

$$= 2 V_r^2 \left(\frac{\cos\alpha}{V_r} - 1\right)(1+K_b)$$

$$\boxed{\eta_d = 2 (V_r \cos\alpha - V_r^2)(1+K_b)}$$

where $V_r = \frac{V_b}{C_1}$ } Velocity Ratio.

$$\frac{d\eta_d}{dV_r} = 0 \Rightarrow V_{r,opt} = \frac{\cos\alpha_1}{2}$$

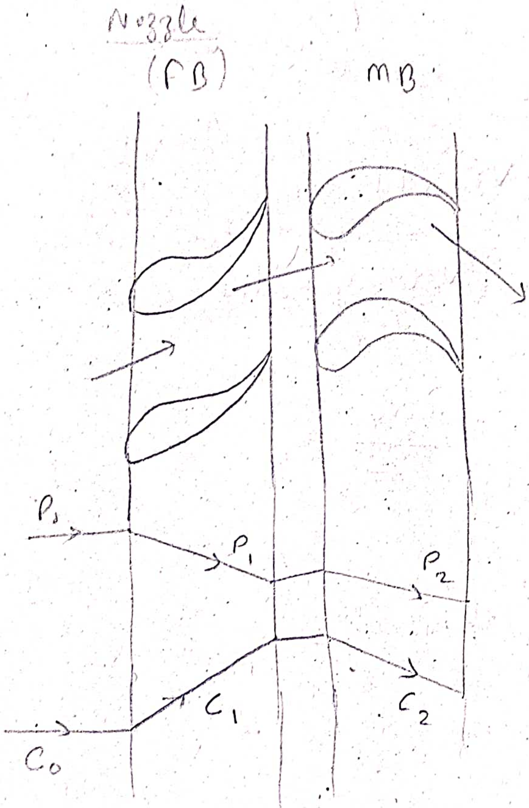
$$\eta_{d\,max} = \frac{1}{2} \cos^2\alpha (1+K_b)$$

$\Rightarrow$ when $\alpha_1 \downarrow \Rightarrow \eta_d \uparrow$

Reaction steam turbine

→ Pressure drop takes place both in nozzles (fixed blades) and in the moving blades

→ moving blades are also in the shape of nozzle.



① Impulse effect of jets (Due to change in momentum)

+

② Reaction force of the exiting jets impressed on the blades in the opposite direction.

the above two effects together allow the runner/wheel to rotate.

also called Impulse-Reaction turbine.

Degree of Reaction (R)

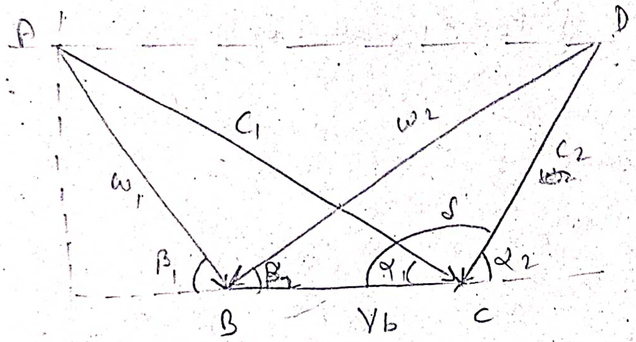
$$R = \frac{\Delta h_{mb}}{\underbrace{\Delta h_{fb} + \Delta h_{mb}}_{\text{total enthalpy drop of steam in a steam}}}$$

Case I If $\Delta h_{mb} = 0 \Rightarrow R = 0$ {Impulse turbine}

Case II $\Delta h_{mb} = \Delta h_{fb} = \frac{\Delta h_{stage}}{2} \Rightarrow R = \frac{1}{2}$ {Parson's Turbine}

Case III :- $\Delta h_{fb} = 0 \Rightarrow R = 1$ {Pure Reaction turbine} → {Hero's turbine}.

Velocity triangles for $R = \frac{1}{2}$ {50% Reaction turbine}



ΔABC and ΔBCD are similar.

for 50% turbine

$$\Delta h_{mb} = \Delta h_{fb} \rightarrow C_1 = W_2$$
$$\beta_2 = \alpha_1$$

Fixed blades and moving blades are similar in shape.

ΔABC and ΔBDC are similar

$\Rightarrow C_2 = w_1$ and $\beta_1 = (180 - \delta)$

$\beta_1 \neq \beta_2$ (as blades are not symmetrical)

work transfer, $W_D = \underbrace{\Delta C_0}_{\text{Change in Swirl velocity}} V_b$

$$\Delta C_0 = |C_{0_1}| - |C_{0_2}|$$

C_{θ_1} : component of absolute velocity in the tangential direction

$C_{\theta_1} = C_1 \cos\alpha_1$

$C_{\theta_2} = C_2 \cos\alpha_2$
$= C_2 \cos(180 - \delta)$

$\Rightarrow \Delta C_\theta = |C_{\theta_1}| - |C_{\theta_2}|$
$= C_1 \cos\alpha_1 - C_2 \cos\delta$
$= C_1 \cos\alpha_1 + \omega_2 \cos\beta_2 - V_b$
$= 2 C_1 \cos\alpha_1 - V_b \quad \{\beta_2 = \alpha_1,\ \omega_2 = C_1\}$

OR

$\Delta C_\theta = \omega_{\theta_1} + \omega_{\theta_2}$
$= \omega_1 \cos\beta_1 + \omega_2 \cos\beta_2$

$\{\omega_2 = c_1,\ \beta_2 = \alpha_1\}$

$\Rightarrow \Delta C_\theta = C_1 \cos\alpha_1 - V_b + C_1 \cos\alpha_1$

$\boxed{\Delta C_\theta = 2 C_1 \cos\alpha_1 - V_b}$

work transfer / Diagram work $= \Delta C_\theta V_b$
$= (2 C_1 \cos\alpha_1 - V_b) V_b$

Diagram efficiency / Blading efficiency

$$\eta_d = \frac{(2 C_1 \cos\alpha_1 - V_b) V_b}{\text{energy input}}$$

energy input per unit mass flow of steam

$$= \frac{C_1^2}{2} + \underbrace{\frac{1}{2}(\omega_2^2 - \omega_1^2)}_{\text{due to chang in relative velocity}}$$

energy input $= \frac{C_1^2}{2} + \frac{C_1^2}{2} - \frac{w_1^2}{2}$

$= C_1^2 - \frac{w_1^2}{2}$

applying cosine formula

$$w_1^2 = C_1^2 + Vb^2 - 2C_1Vb\cos\alpha_1$$

$$\Rightarrow \eta_d = \left[\frac{2(2C_1\cos\alpha_1 - Vb)Vb}{C_1^2 - Vb^2 + 2C_1Vb\cos\alpha_1}\right]$$

$$= \frac{2Vb^2\left(\frac{2C_1\cos\alpha_1}{Vb} - 1\right)}{C_1^2\left(1 - \frac{Vb^2}{C_1^2} + 2\frac{Vb}{C_1}\cos\alpha_1\right)}$$

$$V_r = \frac{Vb}{C_1}$$

$$\eta_d = \frac{2\left(\frac{2\cos\alpha_1}{V_r} - 1\right)V_r^2}{\left(1 - V_r^2 + 2V_r\cos\alpha_1\right)}$$

Optimum, V_r, $\frac{d\eta_d}{dV_r} = 0$

$\Rightarrow V_{r,\,opt} = C_1\cos\alpha_1$

$$\eta_{d_{max}} = \frac{2\cos^2\alpha_1}{1+\cos^2\alpha_1}$$

$$\underbrace{W_{D_{max}}}_{\text{blading work or diagram work corresponding to maximum blading efficiency.}} = (2Vb - Vb)\,Vb = Vb^2.$$

$$(\underbrace{2C_1\cos\alpha_1}_{Vb_{opt}} - Vb)\,Vb$$

50%. Reaction turbine with maximum η_d