Example: (SunRay Transport)

SunRay Transport Company ships truckloads of grain from the three silos to four mills. The supply (in truckloads) and the demand (also in truckloads) together with the unit transportation costs per truckloads on the different routes are summarized in Table 1. The unit transportation costs are in hundreds of dollars.

Table 1: SunRay Transportation Model

		Mills				
		1	2	3	4	Supply
Silos	1	10	2	20	11	15
	2	12	7	9	20	25
	3	4	14	16	18	10
Demand		5	15	15	15	

The model seeks the minimum-cost shipping schedule between the silos and the mills.

Tranportation tableau

		Mill				
		1	2	3	4	Supply
Silo	1	10	2	20	11	15
	2	12	7	9	20	25
	3	4	14	16	18	10
Demand		5	15	15	15	

→ observation: demand = supply.

Step1: Determination of a starting basic feasible solution

Methods

1. Northwest - corner method
2. Least - cost method
3. Vogel approximation method

North-West Corner Method

Mill

Silos	1	2	3	4	Supply
1	[10] 5	[2] 10	[20]	[11]	~~15~~ ~~10~~ 0
2	[12]	[7] 5	[9] 15	[20] 5	~~25~~ ~~20~~ ~~5~~
3	[4]	[14]	[16]	[18] 10	10
Demand	~~5~~ 0	~~15~~ ~~5~~ 0	~~15~~ 0	~~15~~ 10	

$\begin{cases} x_{11}=5,\ x_{12}=10,\ x_{22}=5,\ x_{23}=15,\ x_{24}=5,\ x_{34}=10 \\ \text{Rest is zero.} \end{cases}$

Transportation cost : $5\times10 + 10\times2 + 5\times7 + 15\times9 + 5\times20 + 10\times18$

$= \$520$ (in hundreds of dollars)

Least-cost Method

Mills

Silos	1	2	3	4	Supply
1	[10]	[2] 15	[20]	[11] 0	~~15~~ 0 x
2	[12]	[7]	[9] 15	[20] 10	~~25~~ ~~10~~ 0 x
3	[4] 5	[14]	[16]	[18] 5	~~10~~ ~~5~~ 0 x
Demand	~~5~~ 0 x	~~15~~ 0 x	~~15~~ 0 x	~~15~~ ~~10~~ 0 x	

$x_{12}=15,\ x_{14}=0,\ x_{23}=15,\ x_{24}=10,\ x_{31}=5,\ x_{34}=5$

Transportation cost: $15\times2 + 0\times11 + 15\times9 + 10\times20 + 5\times4 + 5\times18$

$= \$475$

* Better than North-West corner method solution

Vogel- Approximation method (VAM)

Mill

Silo	1	2	3	4	Supply	Row penalty		
1	[10]	[2] 15	[20]	[11] 0	~~15~~ 0 x	10−2=8	9	9
2	[12]	[7]	[9] 15	[20] 10	~~25~~ 10	9−7=2	2	11
3	[4] 5	[14]	[16]	[18] 5	~~10~~ 0 ~~5~~ 0 x	14−4=10	2	2
Demand	~~5~~ 0 x	~~15~~ 0 x	~~15~~ 0 x	~~15~~ 10				
Column penalty	10−4=6	7−2=5	16−9=7	18−11=7				
	—	5	7	7				
	—	—	7	7				

$x_{12} = 15, \; x_{14} = 0, \; x_{23} = 15, \; x_{24} = 10, \; x_{31} = 5$
$x_{34} = 5$

Transportation cost $= 15\times2 + 0\times11 + 15\times9 + 10\times20 + 5\times4 + 5\times18 = \$475$

* Cost is better than North-West corner method solution
* But, it is same to least-cost method.
* Mostly, VAM produces better solution than least cost method.

Step2 & Step3: Iterative Computations of the Transportation Algorithm

* Determine leaving and entering variables.
* Method of multipliers
  - For all basic variables $u_i + v_j = c_{ij}$ row i & column j
* - Calculate $u_i + v_j - c_{ij}$ for all nonbasic variables

It #1

	$v_1 = 10$	$v_2 = 2$	$v_3 = 4$	$v_4 = 15$	Supply
$u_1 = 0$	[10] 5 ($5-\theta$)	[2] 10 ($10+\theta$)	[20] -16	[11] 4	15
$u_2 = 5$	[12] 3	[7] 5 ($5-\theta$)	[9] 15	[20] 5 ($5+\theta$)	25
$u_3 = 3$	[4] 9 θ	[14] -9	[16] -9	[18] 10 ($10-\theta$)	10
Demand	5	15	15	15	

North-west corner solution

Basic variables	Equation	Solution
x_{11}	$u_1 + v_1 = 10$	$u_1 = 0,\ v_1 = 10$
x_{12}	$u_1 + v_2 = 2$	$u_1 = 0,\ v_2 = 2$
x_{22}	$u_2 + v_2 = 7$	$v_2 = 2,\ u_2 = 5$
x_{23}	$u_2 + v_3 = 9$	$u_2 = 5,\ v_3 = 4$
x_{24}	$u_2 + v_4 = 20$	$u_2 = 5,\ v_4 = 15$
x_{34}	$u_3 + v_4 = 18$	$v_4 = 15,\ u_3 = 3$

* Reduced cost : $\boxed{u_i + v_j - c_{ij}}$ ASK significance?

* Transportation problem is a minimization problem
  - Entering variable; Most positive reduced cost
    - entering variable: x_{31}

* Leaving variable.
  - Maximum amount of shipment θ through x_{31} route such that
    1. Supply limits and demand requirements remain satisfied
    2. Shipments through all routes remain nonnegative
  - Construct a closed-loop which is starting and ending at entering variable through current all basic variables.

$x_{11} = 5-\theta \geq 0$

$x_{22} = 5-\theta \geq 0$

$x_{34} = 10-\theta \geq 0$

leaving variable : x_{11} or x_{22}

— let's take x_{11}

$\theta_{max} = 5$

It#2

	$v_1=1$	$v_2=2$	$v_3=4$	$v_4=15$
$u_1=0$	$\boxed{10}$ $\boxed{-9}$	$\boxed{2}$ $15-\theta$	$\boxed{20}$ $\boxed{-16}$	$\boxed{11}$ θ $\boxed{4}$
$u_2=5$	$\boxed{12}$ $\boxed{-6}$	$\boxed{7}$ $0+\theta$	$\boxed{9}$ 15	$\boxed{20}$ $10-\theta$
$u_3=3$	$\boxed{4}$ 5	$\boxed{14}$ $\boxed{-9}$	$\boxed{16}$ $\boxed{-9}$	$\boxed{18}$ 5

θ_{max} -

Basic variables	equation	solution
x_{12}	$u_1+v_2=2$	$u_1=0,\ v_2=2$
x_{22}	$u_2+v_2=7$	$v_2=2,\ u_2=5$
x_{23}	$u_2+v_3=9$	$u_2=5,\ v_3=2$
x_{24}	$u_2+v_4=20$	$u_2=5,\ v_4=15$
$\cancel{x_{31}}$	$\cancel{u_3+v_1}=4$	v
x_{34}	$u_3+v_4=18$	$v_4=15,\ u_3=3$
x_{31}	$u_3+v_1=4$	$u_3=3,\ v_1=1$

$\theta_{max} = 10$

It #3

	$v_1 = -3$	$v_2 = 2$	$v_3 = 4$	$v_4 = 11$
$u_1 = 0$	$\boxed{10}$ $\boxed{-13}$	$\boxed{2}$ 5	$\boxed{20}$ $\boxed{-16}$	$\boxed{11}$ 10
$u_2 = 5$	$\boxed{12}$ $\boxed{-10}$	$\boxed{7}$ 10	$\boxed{9}$ 15	$\boxed{20}$ $\boxed{-4}$
$u_3 = 7$	$\boxed{4}$ 5	$\boxed{14}$ $\boxed{-5}$	$\boxed{16}$ $\boxed{-5}$	$\boxed{18}$ 5

Basic variables	Equations	Solutions
x_{12}	$u_1 + v_2 = 2$	$u_1 = 0,\ v_2 = 2$
x_{14}	$u_1 + v_4 = 11$	$u_1 = 0,\ v_4 = 11$
x_{22}	$u_2 + v_2 = 7$	$v_2 = 2,\ u_2 = 5$
x_{23}	$u_2 + v_3 = 9$	$u_2 = 5,\ v_3 = 4$
x_{34}	$u_3 + v_4 = 18$	$v_4 = 11,\ u_3 = 7$
x_{31}	$u_3 + v_1 = 4$	$u_3 = 7,\ v_1 = -3$

- No entering variable.
  - Thus, it is optimal transportation tableau.

The optimum solution is

$5 \times 2 + 10 \times 11 + 10 \times 7 + 15 \times 9 + 5 \times 4 + 5 \times 18 = \$435$

* If we start from the least-cost method or VAM solution we need one iteration less to get the optimum solution.