

ME324: IEOR

Assignment 03: Special Cases of LP and Sensitivity Analysis

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1. Write the following LP model into the equation form for simplex method

$$\begin{aligned} & \text{Maximize } z = 3x_1 + 2x_2 + x_3 \\ & \text{subject to the constraints} \\ & (i) 2x_1 + 5x_2 + x_3 = 12, (ii) 3x_1 + 4x_2 = 11, \\ & x_2, x_3 \geq 0, x_1 \text{ unrestricted.} \end{aligned}$$

HINT: one variable is unrestricted and use proper type of variables for '=' constraints.

2. Solve the following LP problem using the simplex method, and comment on the special cases like degeneracy, alternate optima, unbounded solution, infeasible solution etc.

a.

$$\begin{aligned} & \text{Maximize } z = 5x_1 + 3x_2 \\ & \text{subject to} \\ & (i) x_1 + x_2 \leq 2, (ii) 5x_1 + 2x_2 \leq 10, (iii) 3x_1 + 8x_2 \leq 12, \\ & x_1, x_2 \geq 0. \end{aligned}$$

b.

$$\begin{aligned} & \text{Maximize } z = 6x_1 + 3x_2 \\ & \text{subject to} \\ & (i) 2x_1 + x_2 \leq 8, (ii) 3x_1 + 3x_2 \leq 18, (iii) x_2 \leq 3, \\ & x_1, x_2 \geq 0. \end{aligned}$$

c.

$$\begin{aligned} & \text{Maximize } z = -2x_1 + 3x_2 \\ & \text{subject to} \\ & (i) x_1 \leq 5, (ii) 2x_1 - 3x_2 \leq 6, \\ & x_1, x_2 \geq 0. \end{aligned}$$

d.

$$\begin{aligned} & \text{Maximize } z = 2x_1 + 3x_2 \\ & \text{subject to} \\ & (i) x_1 - x_2 \geq 4, (ii) x_1 + x_2 \leq 6, (iii) x_1 \leq 2, \\ & x_1, x_2 \geq 0. \end{aligned}$$

3. A company wants to produce three products: A, B and C. The per unit profit on these products is INR 4, INR 6 and INR 2, respectively. These products requires two types of

resources, manpower and raw material. The LP model formulate for determining the optimal product mix is as follows:

$$\text{Maximize } z = 4x_1 + 6x_2 + 2x_3$$

subject to the constraints

$$(i) x_1 + x_2 + x_3 \leq 3, (ii) x_1 + 4x_2 + 7x_3 \leq 9,$$

$$x_1, x_2, x_3 \geq 0.$$

where x_1, x_2, x_3 = number of units of products A, B, and C, respectively, to be produced.

- a. Find the optimal product mix and the corresponding profit of the company.
- b. Which constraint should be given the priority to increase z ?
- c. Find the range of right hand side of the preferred constraint found in (b) such that the dual price of the preferred constraint remains same. Assume no change in the other constraint.
- d. Find the range of the profit contribution of product C in the objective function, such that the current optimal product mix remains unchanged.
- e. Find the range of the profit contribution of product A in the objective function such that the current optimal product mix remains unchanged.