ME322: Machine Design

Hydrodynamic bearings

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Reynolds Equation

- The generalized Reynolds equation as derived by Osborne Reynolds in 1885 is given as

$$\frac{\partial}{\partial x}\left[\rho h^3 \frac{\partial p}{\partial x}\right]+\frac{\partial}{\partial z}\left[\rho h^3 \frac{\partial p}{\partial z}\right]=6\mu \mathrm{U}\left(\frac{\partial \rho h}{\partial x}\right) \qquad \text{…………..… (1)}$$

where p is the fluid pressure, h is the film thickness, ρ is the fluid density, μ is the viscosity of the fluid and U is the sliding speed in x- direction. x and z are the coordinates in the direction of sliding and the length of the bearing.

- The Reynolds equation for incompressible fluid (ρ = constant) is given as

$$\frac{\partial}{\partial x}\left[h^3\frac{\partial p}{\partial x}\right]+\frac{\partial}{\partial z}\left[h^3\frac{\partial p}{\partial z}\right]=6\mu\mathrm{U}\left(\frac{\partial h}{\partial x}\right) \qquad \text{..................} \text{ (2)}$$

- The Reynolds equation was first solved by Sommerfeld (1905) for infinitely long journal bearing.

- Later Ockvirk (1953) solved the equation for infinitely short journal bearing.

- Though there are approximate analytical solutions of this two-dimensional partial differential equation, the solutions are too clumsy.

- For bearings with finite length there is no exact analytical solution of Reynolds equation. However, the numerical solution provided by Raimondi and Boyd is considered to be close to experimental results.

Note: Refer to books like " Introduction to Tribology of Bearings" by B C Majumdar for detail derivation of Reynolds equation

Raimondi and Boyd method

- There is no exact solution to Reynolds equation for a journal bearing or a slider bearing having a finite length.
- However, AA Raimondi and John Boyd of Westinghouse Research Laboratory solved this equation on computer using the iteration technique.
- The results of this work are available in the form of charts and tables. In the Raimondi and Boyd method, the performance of the bearing is expressed in terms of dimensionless parameters.
- The values of these parameters for a full journal bearing (360^o arc) with side flow are given in Table 1.

Dimensionless parameters

1. Eccentricity Ratio

- The radial clearance 'c' is given by $c = R - r$ …………… (3)

where c=radial clearance (mm), R=radius of bearing (mm) and r=radius of journal (mm)

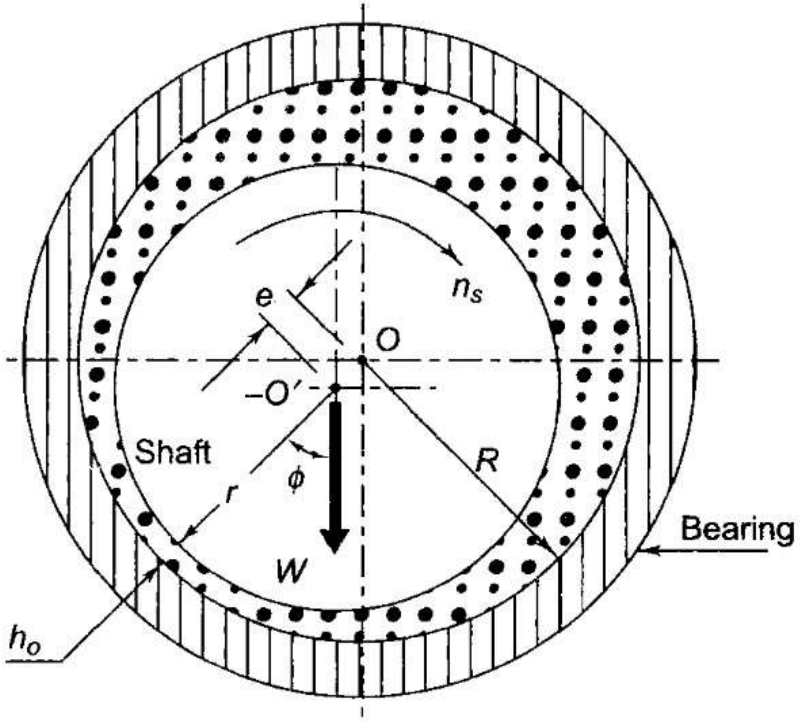

Fig 1: Journal bearing

- The eccentricity ratio is defined as the ratio of eccentricity to radial clearance.

  Therefore,

$$\varepsilon = \frac{e}{c} \qquad \text{..................} (4)$$

2. Minimum film thickness

From Fig.1,

$$R = e + r + h_0 \qquad \text{................} (a)$$

where, h_0 = minimum film thickness (mm)

Using eq. 3, eq. 4 and eq. (a), we get

$$c = R - r = e + h_0 = c\varepsilon + h_0$$

or

$$c(1-\varepsilon) = h_0$$

$$\therefore \quad \varepsilon = 1 - \left(\frac{h_0}{c}\right) \qquad \text{.................} (5)$$

The quantity $\left(\frac{h_0}{c}\right)$ is called the minimum film thickness variable.

3. Sommerfeld Number

Sommerfeld number is given as

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu n_s}{P} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(6)}$$

where, *S* = Sommerfeld number

μ = viscosity of the lubricant (N-s/mm^2) or (Mpa-s)

n_s = journal speed (rev./s)

P = unit bearing pressure, i.e., load per unit of the projected area (N/mm^2) or (MPa)

Note: The Sommerfeld number contains all the variables that are controlled by the designer

4. Attitude Angle

The angle ϕ shown in Fig. 1 is called the angle of eccentricity or eccentricity ratio. It locates the position of minimum film thickness with respect to the direction of load. The values of ϕ given in Table 1 are in degrees.

Table 1: Dimensionless performance parameters for full journal bearing with side flow

$\left(\frac{l}{d}\right)$	ε	$\left(\frac{h_0}{c}\right)$	S	ϕ	$\left(\frac{r}{c}\right)f$	$\left(\frac{Q}{rcn_s l}\right)$	$\left(\frac{Q_s}{Q}\right)$	$\left(\frac{p}{p_{\max}}\right)$
∞	0	1.0	∞	(70.92)	∞	π	0	-
	0.1	0.9	0.240	69.10	4.80	3.03	0	0.826
	0.2	0.8	0.123	67.26	2.57	2.83	0	0.814
	0.4	0.6	0.0626	61.94	1.52	2.26	0	0.764
	0.6	0.4	0.0389	54.31	1.20	1.56	0	0.667
	0.8	0.2	0.021	42.22	0.961	0.760	0	0.495

Table 1: *Continued…*

$\left(\frac{l}{d}\right)$	ε	$\left(\frac{h_0}{c}\right)$	S	ϕ	$\left(\frac{r}{c}\right) f$	$\left(\frac{Q}{rcn_s l}\right)$	$\left(\frac{Q_s}{Q}\right)$	$\left(\frac{p}{p_{\max}}\right)$
∞	0.9	0.1	0.0115	31.62	0.756	0.411	0	0.358
	0.97	0.03	-	-	-	-	0	-
	1.0	0	0	0	0	0	0	0
1	0	1.0	∞	(85)	∞	π	0	-
	0.1	0.9	1.33	79.5	26.4	3.37	0.150	0.540
	0.2	0.8	0.631	74.02	12.8	3.59	0.280	0.529
	0.4	0.6	0.264	63.10	5.79	3.99	0.497	0.484
	0.6	0.4	0.121	50.58	3.22	4.33	0.680	0.415
	0.8	0.2	0.0446	36.24	1.70	4.62	0.842	0.313
	0.9	0.1	0.0188	26.45	1.05	4.74	0.919	0.247
	0.97	0.03	0.00474	15.47	0.514	4.82	0.973	0.152
	1.0	0	0	0	0	0	1.0	0

Table 1: *Continued…*

$\left(\frac{l}{d}\right)$	ε	$\left(\frac{h_0}{c}\right)$	S	ϕ	$\left(\frac{r}{c}\right)f$	$\left(\frac{Q}{rcn_s l}\right)$	$\left(\frac{Q_s}{Q}\right)$	$\left(\frac{p}{p_{max}}\right)$
$\left(\frac{1}{2}\right)$	0	1.0	∞	(88.5)	∞	π	0	-
	0.1	0.9	4.31	81.62	85.6	3.43	0.173	0.523
	0.2	0.8	2.03	74.94	40.9	3.72	0.318	0.506
	0.4	0.6	0.779	61.45	17.0	4.29	0.552	0.441
	0.6	0.4	0.319	48.14	8.10	4.85	0.730	0.365
	0.8	0.2	0.0923	33.31	3.26	5.41	0.874	0.267
	0.9	0.1	0.0313	23.66	1.60	5.69	0.939	0:206
	0.97	0.03	0.00609	13.75	0.610	5.88	0.980	0.126
	1.0	0	0	0	0	-	1.0	0
$\left(\frac{1}{4}\right)$	0	1.0	∞	(89.5)	∞	π	0	-
	0.1	0.9	16.2	82.31	322.0	3.45	0.180	0.515
	0.2	0.8	7.57	75.18	153.0	3.76	0.330	0.489
	0.4	0.6	2.83	60.86	61.1	4.37	0.567	0.415

Table 1: *Continued…*

$\left(\frac{l}{d}\right)$	ε	$\left(\frac{h_0}{c}\right)$	S	ϕ	$\left(\frac{r}{c}\right)f$	$\left(\frac{Q}{rcn_s l}\right)$	$\left(\frac{Q_s}{Q}\right)$	$\left(\frac{p}{p_{\max}}\right)$
$\left(\frac{1}{4}\right)$	0.6	0.4	1.07	46.72	26.7	4.99	0.746	0.334
	0.8	0.2	0.261	31.04	8.8	5.60	0.884	0.240
	0.9	0.1	0.0736	21.85	3.50	5.91	0.945	0.180
	0.97	0.03	0.0101	12.22	0.922	6.12	0.984	0.108
	1.0	0	0	0	0	-	1.0	0

5. Coefficient of friction variable

The coefficient of friction variable (CVC) is given by

$$(CFV) = \left(\frac{r}{c}\right) f \qquad \text{.....................} \; (7)$$

where *f* is the coefficient of friction.

6. Coefficient of flow variable

The flow variable (FV) is given by

$$(FV) = \frac{Q}{rcn_s l} \qquad \text{.....................} \; (8)$$

where,

l=length of the bearing (mm)

Q=flow of the lubricant (mm^3/s)

- Q represents the total flow of the lubricating oil, a part of which is circulated around the journal, while the remaining oil flows as side leakage (Q_s).
- Q_s represents the side leakage, which can be calculated from the values of parameter $\left(\frac{Q_s}{Q}\right)$ given in the table 1.
- The maximum pressure ($P_{\max}$) developed in the film is calculated from the ratio given in last column of the table 1.
- The values are based on the assumption that the oil is supplied at the atmospheric pressure. If the oil is supplied at the higher pressure, the maximum pressure ($P_{\max}$) will also increase by the corresponding value.

Frictional Torque

The frictional torque is given by,

$$\left(M_t\right)_f = fW_r \text{ N-mm}$$

$$\text{Frictional Power} = (2\pi n_s)(fWr)\text{N-mm/s}$$

$$= (2\pi n_s)(fWr)\left(10^{-3}\right)\text{W}$$

$$=(2\pi n_s)(fWr)\left(10^{-6}\right)\text{kW}$$

Therefore,

$$\left(kW\right)_f = \frac{2\pi n_s fWr}{10^6} \qquad \text{………………} \; (9)$$

Temperature Rise

- Viscous friction results in heat generation.
- Rise in temperature of the lubricant takes place due to heat generation

- Assuming that the total heat generated in the bearing is carried away by the total oil flow in the bearing, the expression for temperature rise can be determined.
- The heat generated (H_g) is given by,

$$H_g = (kW)_f = (2\pi n_s)(fWr)\left(10^{-6}\right)\text{kW} \quad \text{or kJ/s}$$

Substituting,

$$f = \left(\frac{c}{r}\right)(CFV) \quad \text{and} \quad W = 2plr$$

$$H_g = (kW)_f = (4\pi)\left(10^{-6}\right) rcn_s\, lp \quad (CFV) \quad \text{..................} \text{ (a)}$$

The heat carried away by oil flow is (H_g) is given by

$$H_c = mC_p\, \Delta t \qquad \text{.................} \text{ (b)}$$

Where, m=mass of the lubricating oil passing through the bearing (kg/s)

C_p = specific heat of lubricating oil (kJ/kg°C)

Δt = temperature rise (°C)

The mass of the lubricating oil is given by $m = pQ(10^{-6})$ kg/s

Substituting $Q = rcn_s l(FV)$

The mass is given by

$$m = p(rcn_s l)(FV)(10^{-6}) \text{ kg/s} \quad \text{................ (c)}$$

Substituting Eq. (c) in Eq. (b),

$$H_c = C_p \, \Delta t p(rcn_s l)(FV)(10^{-6}) \quad \text{................ (d)}$$

Equating the expressions for H_g and H_c,

$$\Delta t = \left(\frac{4\pi p}{\rho C_p}\right)\frac{(CFV)}{(FV)} \quad \text{................ (10)}$$

For most lubricating oils,

$\rho = 0.86$ and $C_p = 1.76 \text{ kJ/kg}\,^0C$

Substituting these values in Eq. (10)

$$\Delta t = \frac{8.3p(CFV)}{(FV)} \qquad \text{……………….. (11)}$$

The average temperature of the lubricant is given by

$$T_{av} = T_i + \left(\frac{\Delta t}{2}\right) \qquad \text{……………….. (12)}$$

where T_i is the inlet pressure

Bearing Design-Selection of parameters

Very often, in the preliminary stages of journal bearing design, it is required to select suitable values for the following parameters:

- Length to Diameter ratio
- Unit bearing pressure
- Start-up load
- Radial clearance
- Minimum oil film thickness
- Maximum oil film temperature

Length to Diameter Ratio

- Shaft diameter is generally known/ given/ determined considering bending, torsion and other considerations.
- The designer has to decide on the length of the bearing to obtain a given bearing capacity.
- The performance of the bearing depends on the length to diameter ratio.
- A long bearing has more load carrying capacity compared with a short bearing.
- A short bearing, on the other hand, has greater side flow, which improves heat dissipation.
- The long bearings are more susceptible to metal to metal contact at the two edges, when the shaft is deflected under load. The longer the bearing, the more difficult it is to get sufficient oil flow through the passage between the journal and the bearing.

- The design trend is to use (*l*/*d*) ratio as 1 or less than 1 when side flow is the criterion for cooling the bearing.
- When the shaft and the bearing are precisely aligned, the shaft deflection is within the limit and cooling of lubricant and bearing does not pose a serious problem, the (*l*/*d*) ratio can be taken as more than 1.
- In practice, the (*l*/*d*) ratio varies from 0.5 to 2.0, but in the majority of applications, it is taken as 1 or less than 1.

Unit Bearing Pressure

- The unit bearing pressure is the load per unit of projected area of the bearing in running condition.
- It depends upon a number of factors, such as bearing material, operating temperature, the nature and frequency of load and service conditions

- The values of unit bearing pressure, based on past experience, are given in Table 2.

Table 2: Permissible bearing pressures

Application		Unit bearing pressure (p) (N/mm^2)
Diesel engines:	Main bearing	5-10
	Crank pin	7-14
	Gudgeon pin	13-14
Automotive engines:	Main bearing	3-4
	Crank pin	10-14
Air compressors	Main bearing	1-1.5
	Crank pin	1.5-3.0
Centrifugal pumps	Main bearing	0.5-0.7
Electric motors	Main bearing	0.7-1.5
Transmission shafting	Light duty	0.15
	Heavy duty	1.00
Machine tools	Main bearing	2

Start-up Load

- The unit bearing pressure for starting conditions should not exceed 2 N/mm^2.
- The start-up load is the static load when the shaft is stationary. It mainly consists of the dead weight of the shaft and its attachments.
- The start-up load can be used to determine the minimum length of the bearing on the basis of starting conditions.

Radial Clearance

- The radial clearance should be small to provide the necessary velocity gradient.
- However, this requires costly finishing operations, rigid mountings of the bearing assembly and clean lubricating oil without any foreign particles. This increases the initial and maintenance costs.

- The practical value of radial clearance is 0.001 mm per mm of the journal radius or $c=(0.001)\ r$.
- The practical values of radial clearances for commonly used bearing materials are given in Table 3.

Table 3: Radial Clearance

Material	Radial clearance
Babbitts	(0.001) r to (0.00167) r
Copper-lead	(0.001) r to (0.01) r
Alluminium-Alloy	(0.002) r to (0.0025) r

Minimum oil film thickness

- The surface finish of the journal and the bearing is governed by the value of the minimum oil film thickness selected by the designer and vice versa.
- There is a lower limit for the minimum oil film thickness, below which metal to metal contact occurs and the hydrodynamic film breaks.
- This lower limit is given by, $h_0 = (0.0002)r$

Maximum oil film temperature

- The lubricating oil tends to oxidise when the operating temperature exceeds 120°.
- Also, the surface of babbitt bearing tends to soften at 125°C (for bearing pressure of 7 N/mm^2) and at 190°C (for bearing pressure of 1.4 N/mm2). Therefore, the operating temperature should be kept within these limits.
- In general, the limiting temperature is 90°C for bearings made of babbitts.

Design Criteria

- Bearings can be designed for two different conditions

1. Maximum load carrying capacity
2. Minimum frictional loss

- The optimum values of $\left(\frac{h_0}{c}\right)$ for full journal bearing for these conditions are as follows:

$\left(\frac{l}{d}\right)$ ratio	$\left(\frac{h_0}{c}\right)$ For maximum load	$\left(\frac{h_0}{c}\right)$ For minimum friction
∞	0.66	0.60
1	0.53	0.30
0.5	0.43	0.12
0.25	0.27	0.03

The designer can use the above values for design of bearings under optimum conditions.

- Designers also need to know the viscosity-temperature relationship. This can be gathered from the charts (Fig.2 is an example)

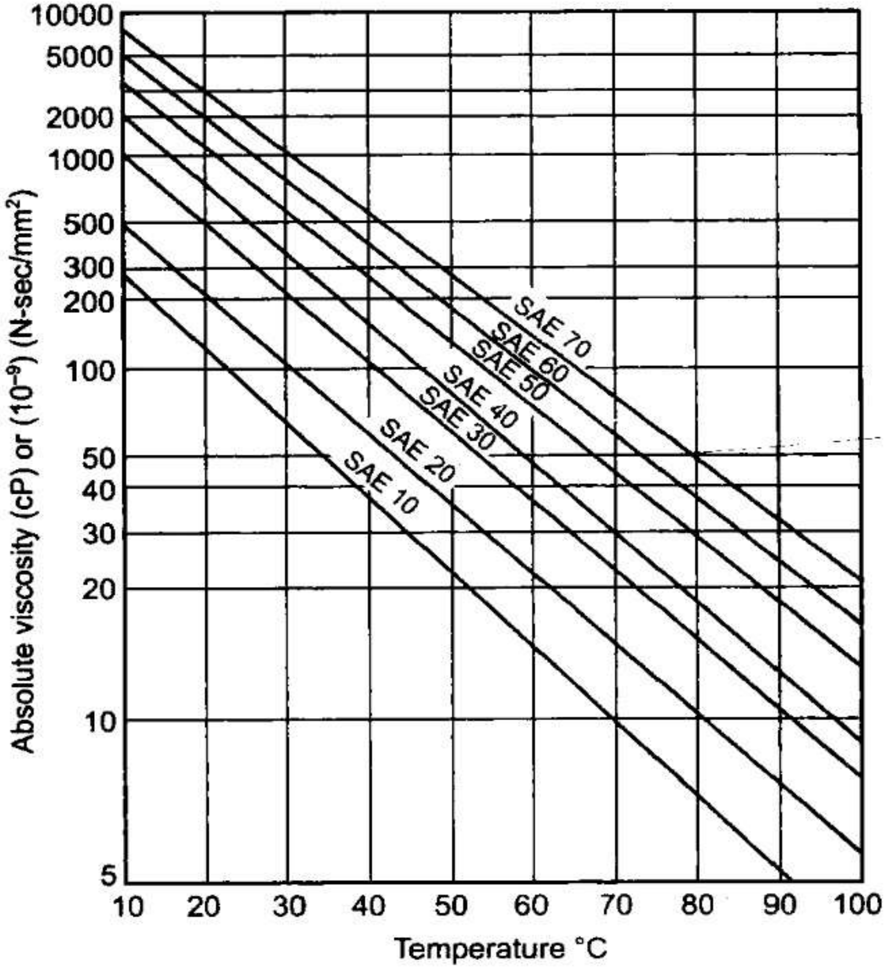

Fig. 2: Viscosity-Temperature relationship for different lubricants