Problem 1

a. f is not one-to-one, both 1 and 3 map to 6

b. f is not onto, f does not cover 10

c. f is a correspondence

d. g is one-to-one

e. g is onto

f. g is a correspondence

Problem 2

Proof.
Suppose $\mathcal{B}$ is countable. Then there is a correspondence between the naturals, $\mathbb{N}$, and $\mathcal{B}$. We define a function, $f(n)$, which maps each natural to an element in $\mathcal{B}$. It may look something like this:

n	$f(n)$
1	01000101 . . .
2	10011101 . . .
3	01111101 . . .
4	01010000 . . .
$\vdots$	$\vdots$

now suppose we take the sequence $s \in \mathcal{B}$, which is constructed like so: $s_i = 0$ when $f(i)_i = 1$, and $s_i = 1$ when $f(i)_i = 0$. Essentially, the i'th position of s is always the opposite of a single bit in every $f(n)$. Because $s \in \mathcal{B}$, but f cannot produce s, there is no bijection between $\mathcal{B}$ and $\mathbb{N}$, so $\mathcal{B}$ is not countable.

$\square$

Problem 3

Proof.
Suppose T is decided by $\mathcal{R}$. We can construct TM $\mathcal{Z}$ as follows:

$\mathcal{Z}$, on input $< M, w >$:

1. Create a TM $\mathcal{Z}_0$ as follows. On input x:

   (a) Reject if x is not 10, or 01

(b) Accept if x is 01

(c) Run M on w, and accept if M accepts

2. Run $\mathcal{R}$ on $< \mathcal{Z}_0 >$

3. Accept if $\mathcal{R}$ accepts, otherwise reject

We know that T in undecidable because $\mathcal{Z}$ decides A_{TM} (which is not decidable).

$\square$