Problem 1

Let A be the set $\{x, y, z\}$ and B be the set $\{x, y\}$

1. A is not a subset of B
2. B is a proper subset of A
3. $A \cup B = \{x, y, z\}$
4. $A \cap B = \{x, y\}$
5. $A \times B = \{(x, x), (x, y), (y, x), (y, y), (z, x), (z, y)\}$
6. $\mathcal{P}(B) = \{\emptyset, \{x\}, \{y\}, B\}$

Problem 2

If A has a elements, and B has b elements, how many elements are in $A \times B$?

$$|A \times B| = ab$$

This is because, in order to pair every member of A with each memeber of B, $|A| \cdot |B|$ tuples are required.

Problem 3

If C is a set with c elements, how many elements are in the power set of C?

$$|\mathcal{P}(C)| = 2^c$$

Each element of of $\mathcal{P}(C)$ can either contain or exclude every element of C. There will always be exactly 2^c unique ways to do this.

Problem 4

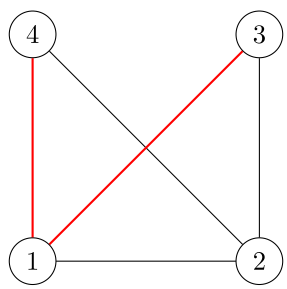
Let X be the set $\{1, 2, 3, 4, 5\}$ and Y be the set $\{6, 7, 8, 9, 10\}$. Let $f : X \to Y$ and $g : X \times Y \to Y$

1. $f(2) = 7$
2. The range of f is Y, the domain of f is X
3. $g(2, 10) = 6$
4. The range of g is Y, the domain of g is $X \times Y$
5. $g(4, f(4)) = 8$

Problem 5

1. $R = \{(a, a), (b, b), (c, c), (a, b), (b, a), (b, c), (c, b)\}$
2. $R = \{(a, a), (b, b), (c, c), (a, b), (a, c), (b, c)\}$
3. $R = \{(a, b), (b, a), (a, c), (c, a), (b, c), (c, b)\}$

Problem 6



Node	Degree
1	3
2	3
3	2
4	2

Problem 7

$$G = (V, E)$$
$$V = \{1, 2, 3, 4, 5, 6\}$$
$$E = \{\{1, 4\}, \{1, 5\}, \{1, 6\}, \{2, 4\}, \{2, 5\}, \{2, 6\}, \{3, 4\}, \{3, 5\}, \{3, 6\}\}$$

Problem 8

If $a = b$, then $a - b = 0$. The error in the proof is the division by $(a - b)$. This operation is undefined when the denominator is 0, therefore the proof is invalid.

Problem 9

Theorem. $S(n) = 1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$

Proof. By induction on n

Base: When $n = 1$, $S(n) = 1 = \frac{1(1+1)}{2}$

Inductive: Suppose $S(k) = \frac{k(k+1)}{2}$
$\implies S(k+1) = 1 + 2 + 3 + \cdots + k + (k+1)$
$\implies S(k+1) = S(k) + (k+1) = \frac{k(k+1)}{2} + (k+1)$
$\implies S(k+1) = \frac{(k+1)((k+1)+1)}{2} = 1 + 2 + \cdots + k + (k+1)$

□

Problem 10

Theorem. For any $n \in \mathbb{Z}$, if $n^3 + 5$ is odd then n is even.

Proof. Suppose that, if $n^3 + 5$ is odd then n is also odd.

$\implies n^3$ is odd, because the product of odd numbers is odd

$\implies n^3 + 5$ is even, because the sum of two odd numbers is even

$\therefore n^3 + 5$ is even, and then supposition is incorrect

□

Problem 11

Theorem. In a set of 51 random integers in $[1, 100]$, there are at least two integers that divide eachother without remainder.

Proof. Partition the set of 51 integers such that each subset conforms to the relation that each element of the subset is a multiple of another element of the subset. If this is down by grouping multiples of odd numbers such that: $\{k, 2k, 4k, 8k, \ldots, 2^i k\}$, where k is any odd number in $[1, 100]$, we will have 50 subsets. By the pigeonhole principle, $\lceil \frac{51}{50} \rceil = 2$, so at least 2 random elements will be part of the same subset. Because subsets are constructed by their multiples, the larger one will divide by the smaller one without remainder. □