CS 483
Section 01
Fall 2020
Christopher K. Schmitt Assignment 2

Problem 1

1. L_1 is not regular
2. L_2 is regular, $(0|1)*1(0|1)*1(0|1)*1(0|1)*$
3. L_3 is regular, $(0|1)(00|01|10|11)*$
4. L_4 is regular, $(0)|(1)|(0(0|1)*0)|(1(0|1)*1)$
5. L_5 is not regular
6. L_6 is not regular

Problem 2.1

Theorem: L_1 is not regular.

Proof. Suppose L_1 is regular. Let p be the pumping length given by the pumping lemma.
Because any valid arithemtic expression requires balanced parenthesis, there must be an equal number of opening and closing parenthesis. We use this to show that L_1 is not regular.
Choose s to be $o^p c^p$, where o corresponds to the opening parenthesis and c corresponds to the closing parenthesis.
By the pumping lemma, $s = xyz$ and $xy^i z \in L_1$, for any $i \geq 0$.
Because $|xy| \leq p$, x and y must all be opening parenthesis.
This leads to a contradiction, Because $y \neq \epsilon$ and $xy^0 z \notin L_1$. Thus the lemma is broken. $\square$

Problem 2.2

Theorem: L_5 is not regular.

Proof. Suppose L_5 is regular. Let p be the pumping length given by the pumping lemma.
Choose s to be $0^p 110^p$.
By the pumping lemma, $s = xyz$ and $xy^i z \in L_1$, for any $i \geq 0$.
$|xy| \leq p$, so xy must contain only zeros.
By the pumping lemma, $xz \in L_5$, But xz cannot be a palindrome if $i \geq 1$ given that $y \neq \epsilon$ (x must contain at least one more zero than z). $\square$

Problem 2.3

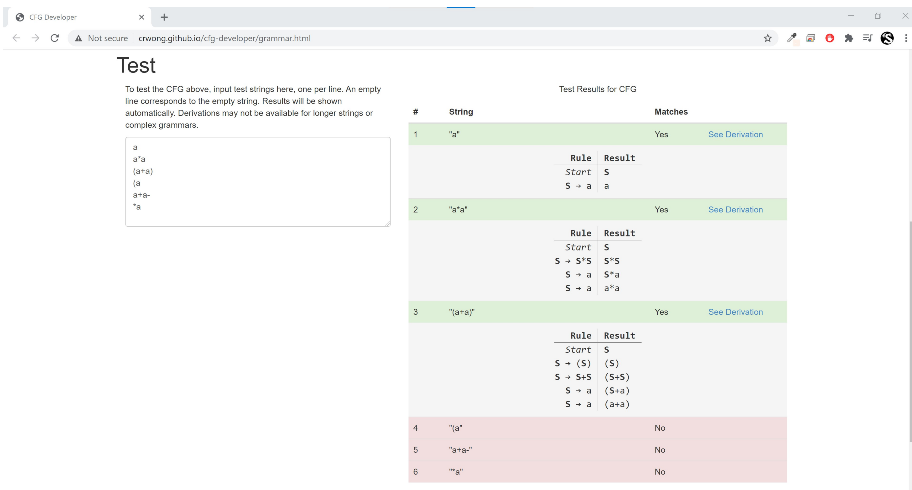
Theorem: L_6 is not regular.

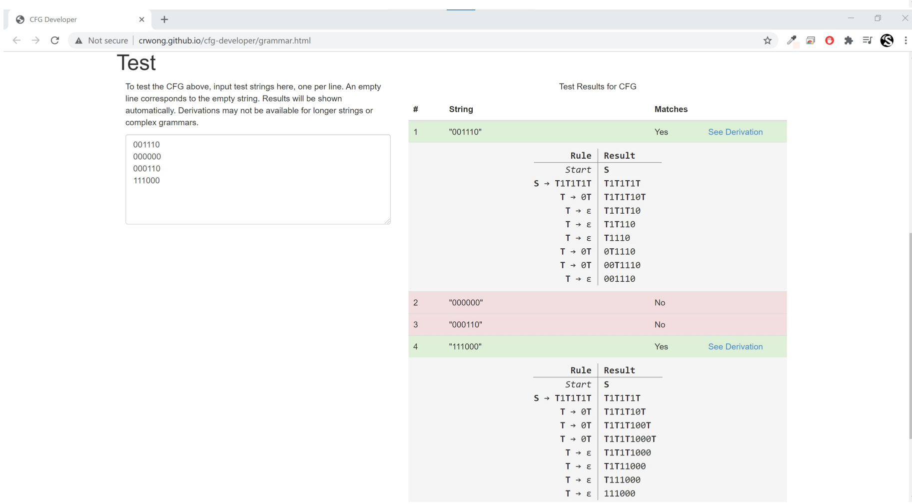
Proof. Suppose L_6 is regular. Let p be the pumping length given by the pumping lemma
Choose s to be 0^p1^p.
By the pumping lemma, $s = xyz$ and $xy^iz \in L_6$, for any $i \geq 0$.
y cannot be ϵ and $|xy| \leq p$, so y only contains zeros.
The pumping lemma says that $xy^2z \in L_6$, but xy^2z contains more zeros than ones, and therefore cannot be in L_6. $\square$

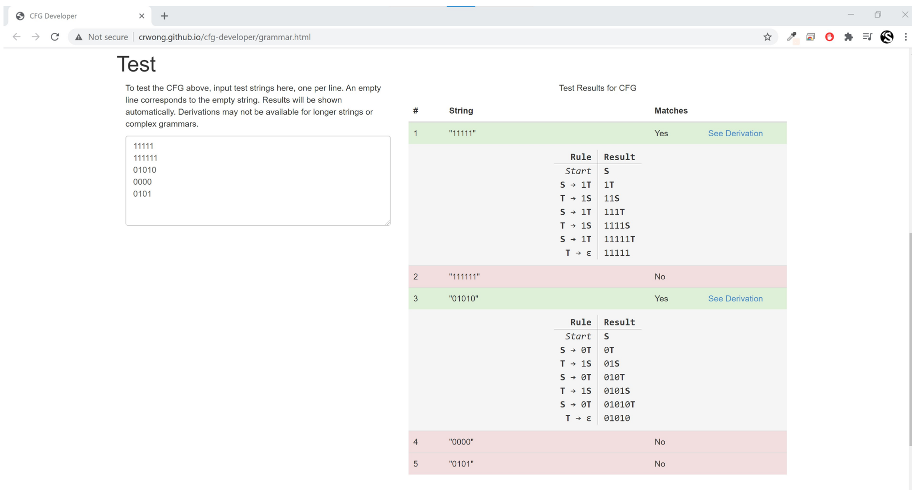
Problem 3

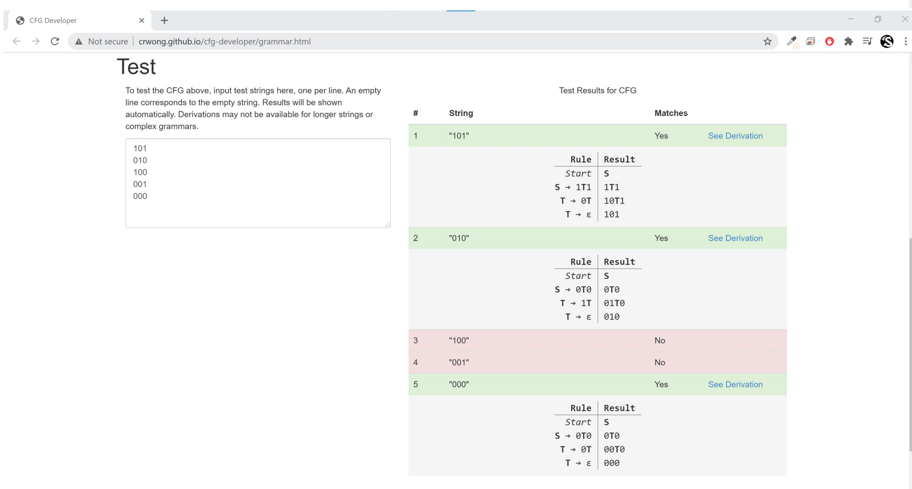
L_1	$S \to a \mid (S) \mid S + S \mid S - S \mid S * S \mid S \div S$
L_2	$S \to T1T1T1T$ $T \to \epsilon \mid 0T \mid 1T$
L_3	$S \to 0T \mid 1T$ $T \to \epsilon \mid 0S \mid 1S$
L_4	$S \to 0T0 \mid 1T1 \mid 0 \mid 1$ $T \to \epsilon \mid 0T \mid 1T$
L_5	$S \to \epsilon \mid 0S0 \mid 1S1 \mid 0 \mid 1$
L_6	$S \to \epsilon \mid SS \mid 0S1 \mid 1S0$

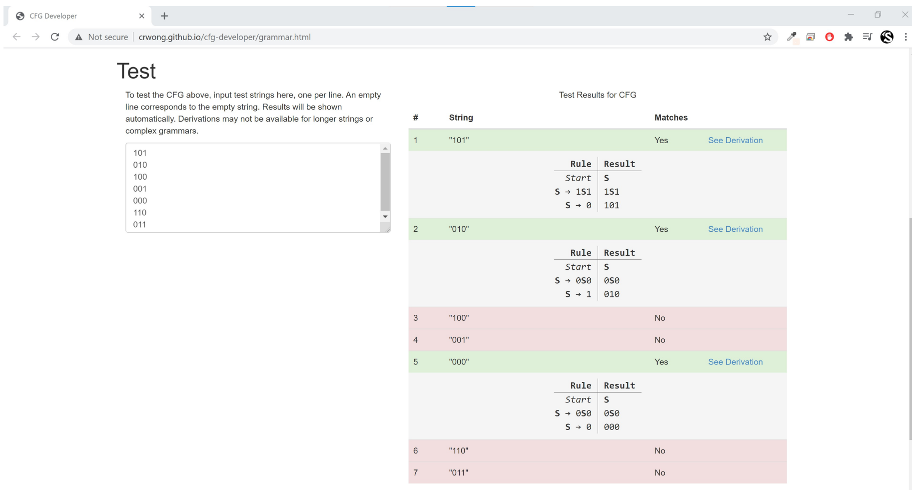
Problem 4

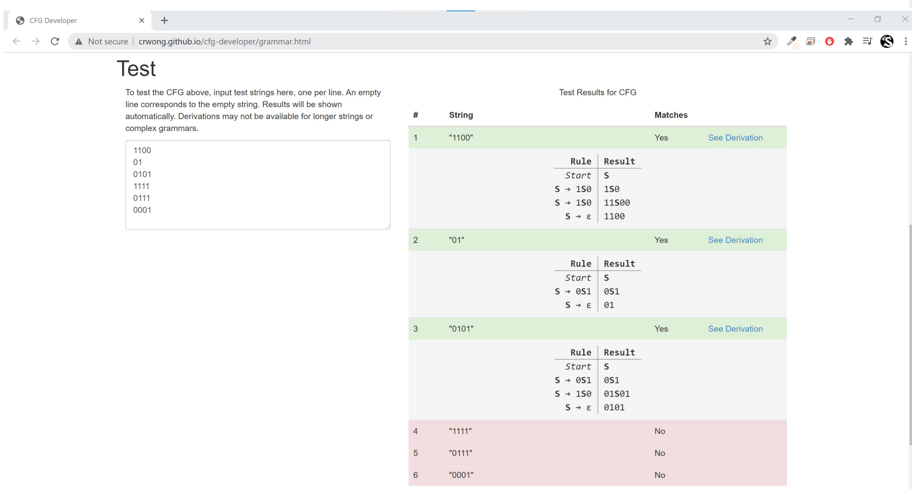

CFG Developer
Not secure | crwong.github.io/cfg-developer/grammar.html
Test
To test the CFG above, input test strings here, one per line. An empty line corresponds to the empty string. Results will be shown automatically. Derivations may not be available for longer strings or complex grammars.
a
a*a
(a+a)
(a
a+a-
*a
Test Results for CFG
String Matches
1 "a" Yes See Derivation
Rule Result
Start S
S → a a
2 "a*a" Yes See Derivation
Rule Result
Start S
S → S*S S*S
S → a S*a
S → a a*a
3 "(a+a)" Yes See Derivation
Rule Result
Start S
S → (S) (S)
S → S+S (S+S)
S → a (S+a)
S → a (a+a)
4 "(a" No
5 "a+a-" No
6 "*a" No


CFG Developer
Not secure | crwong.github.io/cfg-developer/grammar.html
Test
To test the CFG above, input test strings here, one per line. An empty line corresponds to the empty string. Results will be shown automatically. Derivations may not be available for longer strings or complex grammars.
001110
000000
000110
111000
Test Results for CFG
String Matches
1 "001110" Yes See Derivation
Rule Result
Start S
S → T1T1T1T T1T1T1T
T → 0T T1T1T10T
T → ε T1T1T10
T → ε T1T110
T → ε T1110
T → 0T 0T1110
T → 0T 00T1110
T → ε 001110
2 "000000" No
3 "000110" No
4 "111000" Yes See Derivation
Rule Result
Start S
S → T1T1T1T T1T1T1T
T → 0T T1T1T10T
T → 0T T1T1T100T
T → 0T T1T1T1000T
T → ε T1T1T1000
T → ε T1T11000
T → ε T111000
T → ε 111000

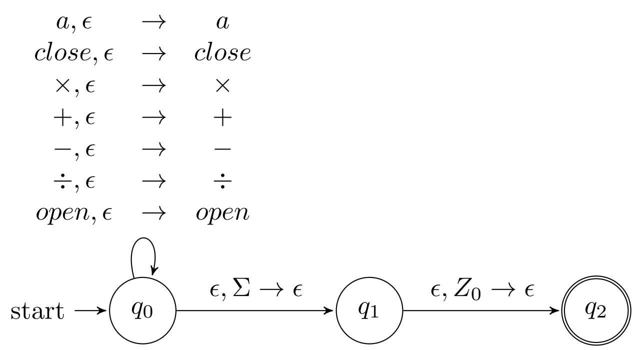

CFG Developer
Not secure | crwong.github.io/cfg-developer/grammar.html
Test
To test the CFG above, input test strings here, one per line. An empty line corresponds to the empty string. Results will be shown automatically. Derivations may not be available for longer strings or complex grammars.
11111
111111
01010
0000
0101
Test Results for CFG
String Matches
1 "11111" Yes See Derivation
Rule Result
Start S
S → 1T 1T
T → 1S 11S
S → 1T 111T
T → 1S 1111S
S → 1T 11111T
T → ε 11111
2 "111111" No
3 "01010" Yes See Derivation
Rule Result
Start S
S → 0T 0T
T → 1S 01S
S → 0T 010T
T → 1S 0101S
S → 0T 01010T
T → ε 01010
4 "0000" No
5 "0101" No


CFG Developer
Not secure | crwong.github.io/cfg-developer/grammar.html
Test
To test the CFG above, input test strings here, one per line. An empty line corresponds to the empty string. Results will be shown automatically. Derivations may not be available for longer strings or complex grammars.
101
010
100
001
000
Test Results for CFG
String Matches
1 "101" Yes See Derivation
Rule Result
Start S
S → 1T1 1T1
T → 0T 10T1
T → ε 101
2 "010" Yes See Derivation
Rule Result
Start S
S → 0T0 0T0
T → 1T 01T0
T → ε 010
3 "100" No
4 "001" No
5 "000" Yes See Derivation
Rule Result
Start S
S → 0T0 0T0
T → 0T 00T0
T → ε 000

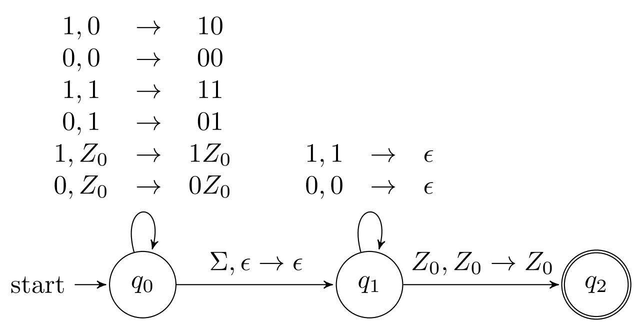

CFG Developer
Not secure | crwong.github.io/cfg-developer/grammar.html
Test
To test the CFG above, input test strings here, one per line. An empty line corresponds to the empty string. Results will be shown automatically. Derivations may not be available for longer strings or complex grammars.
101
010
100
001
000
110
011
Test Results for CFG
String Matches
1 "101" Yes See Derivation
Rule Result
Start S
S → 1S1 1S1
S → 0 101
2 "010" Yes See Derivation
Rule Result
Start S
S → 0S0 0S0
S → 1 010
3 "100" No
4 "001" No
5 "000" Yes See Derivation
Rule Result
Start S
S → 0S0 0S0
S → 0 000
6 "110" No
7 "011" No


CFG Developer
Not secure | crwong.github.io/cfg-developer/grammar.html
Test
To test the CFG above, input test strings here, one per line. An empty line corresponds to the empty string. Results will be shown automatically. Derivations may not be available for longer strings or complex grammars.
1100
01
0101
1111
0111
0001
Test Results for CFG
String Matches
1 "1100" Yes See Derivation
Rule Result
Start S
S → 1S0 1S0
S → 1S0 11S00
S → ε 1100
2 "01" Yes See Derivation
Rule Result
Start S
S → 0S1 0S1
S → ε 01
3 "0101" Yes See Derivation
Rule Result
Start S
S → 0S1 0S1
S → 1S0 01S01
S → ε 0101
4 "1111" No
5 "0111" No
6 "0001" No

Problem 5.1



Problem 5.2



Problem 5.3