Math 218
Section 81
Summer 2019
Christopher Schmitt Assignment 2

Problem 1

Let x be a fixed positive real number. Prove that there exists a unique positive real number y such that $xy = 3$.

Proof. Let x be a fixed positive real.

$$\begin{aligned} xy &= 3 \\ \frac{xy}{x} &= \frac{3}{x} \\ y &= \frac{3}{x}, \text{ which is a positive real.} \end{aligned}$$

Suppose $x(y_1) = 3$
Suppose $x(y_2) = 3$

$$\begin{aligned} x(y_1) &= x(y_2) \\ \frac{x(y_1)}{x} &= \frac{x(y_2)}{x} \\ y_1 &= y_2, \text{ thus any two solution are the same.} \end{aligned}$$

□

Problem 2

Write the negation of the following statement in English. Do not simply add the words "not" or "it is not the case that" or similar before the existing statement.

"Every dog likes some flavor of Brand XYZ dog food."

Solution

"Some dog does not like some flavor of Brand XYZ dog food."

Problem 3

Consider the following compound statements:

$$(p \implies q) \wedge (\neg r \implies q) \text{ and } (p \vee \neg r) \implies q$$

are logically equivalent by constructing truth tables.

p	q	r	(p	→	q)	&	(~	r	→	q)
T	T	T	T	T	T	T	F	T	T	T
T	T	F	T	T	T	T	T	F	T	T
T	F	T	T	F	F	F	F	T	T	F
T	F	F	T	F	F	F	T	F	F	F
F	T	T	F	T	T	T	F	T	T	T
F	T	F	F	T	T	T	T	F	T	T
F	F	T	F	T	F	T	F	T	T	F
F	F	F	F	T	F	F	T	F	F	F

p	q	r	(p	∨	~	r)	→	q
T	T	T	T	T	F	T	T	T
T	T	F	T	T	T	F	T	T
T	F	T	T	T	F	T	F	F
T	F	F	T	T	T	F	F	F
F	T	T	F	F	F	T	T	T
F	T	F	F	T	T	F	T	T
F	F	T	F	F	F	T	T	F
F	F	F	F	T	T	F	F	F

Problem 4

Prove that the compound statements in Problem 3 are logically equivalent by using the basic logical equivalences. At each step, state which basic logical equivalence you are using.

1. $(p \implies q) \wedge (\neg r \implies q)$

Since $(p \implies q) \wedge (r \implies q) \equiv (p \vee r) \implies q$, we can skip right to:

2. $(p \vee \neg r) \implies q$

Problem 5

Write the compound statements in Problem 3 in disjunctive normal form. (Since they are logically equivalent, they have the same disjunctive normal form, so you only need to give one answer.

$$(p \wedge q \wedge r) \vee (p \wedge q \wedge \neg r) \vee (\neg p \wedge q \wedge r) \vee (\neg p \wedge q \wedge \neg r) \vee (\neg p \wedge \neg q \wedge r)$$

Problem 6

Consider the premises:

1. It is not snowing today and it is windy;

2. School will be canceled only if it is snowing today;
3. If school is not canceled today, then our study group will meet;
4. If I did not get A on the exam, then our study group did not meet.

Do these premises imply the conclusion "I will get A or B on the exam"? If so, explain why by using the rules of inference. If not, explain why not.

1. $\neg S \wedge W$
2. $C \iff S$
3. $\neg C \implies G$
4. $\neg A \implies \neg G$

Steps

1. Contrapositive of 4: $G \implies A$
2. School is not canceled because $C \iff S$ and $\neg S \wedge W$
3. School is not canceled, so the group meets because $\neg C \implies G$
4. And $G \implies A$ as seen earlier. So $A \vee B$ is true because G is true.