Problem 1

Solve each of the following counting problems, writing your answer as an integer in the standard decimal notation.

In how many ways can a student club that consists of 20 students select 3 students to attend a conference?

$$\binom{20}{3} = 1140$$

How many strings of 0s and 1s have length 11 and have exactly eight ones?

$$\binom{11}{8} = 165$$

How many strings of 0s and 1s have length 11 and have at most three ones?

$$2^{11} - 2^{7} = 1920$$

A donut shop has donuts in six different flavors: chocolate frosted, chocolate creme, glazed, jelly, plain, and Boston creme. The donut shop has an unlimited supply of each flavor. In how many different ways can the donut shop create a box of 18 donuts? Note that the box can include multiple donuts of the same flavor, and that donuts of the same flavor are considered indistinguishable.

$$\binom{n+r-1}{r} = \binom{6+18-1}{18} = \binom{23}{18} = 33649$$

The donut shop now wants to create a box of 18 donuts, including at least four glazed ones. In how many ways can this be done?

$$\binom{n+r-1}{r} = \binom{6+14-1}{14} = \binom{19}{14} = 11628$$

The donut shop now wants to create a box of 18 donuts, including at most four glazed ones. In how many ways can this be done?

$$\binom{6+18-1}{18} - \binom{6+13-1}{13} = \binom{23}{18} - \binom{18}{13} = 33649 - 8568 = 25018$$

The donut shop now wants to create a box of 18 donuts, including exactly four glazed ones. In how many ways can this be done?

$$1 * \binom{5+14-1}{14} = \binom{18}{14} = 3060$$

I want to buy one donut every day from Monday through Friday at the donut shop. In how many ways can this be done?

$$\binom{n+r-1}{r} = \binom{6+5-1}{5} = 252$$

How many "words" can be formed by rearranging the letters of the word MATHEMATICALLY? The "words" do not have to be actual English words.

$$\frac{14!}{2! * 3! * 2! * 2!} = 1816214400$$

Find the coefficient of x^8y^4 in the expansion of the binomial $(x + 2y)^{12}$

$$\binom{12}{4} = 495$$

$$495(x^8)(2y^4) = 990x^8y^4$$

Problem 2

$$(x + y)^{2k} = \sum_{i=0}^{2k} x^i y^{2k-1} \binom{2k}{i}$$

$$(x + y)^{2J+1} = \sum_{i=0}^{2J+1} x^i y^{2J} \binom{2J+1}{i}$$

Theorem:

$$\sum_{0 \le i \le n, n \ odd} \binom{n}{i} = \sum_{0 \le i \le n, n \ even} \binom{n}{i}$$

$$\sum \binom{2k+1}{i} = \sum \binom{2j}{i}$$

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