Math 218
Section 81
Summer 2019
Christopher Schmitt Assignment 6

Problem 1

Use the binary exponentiation algorithm to compute the remainder when 3^{85} is divided by 53

$$3^2 \equiv 9(mod 53)$$
$$3^4 \equiv 28(mod 53)$$
$$.$$
$$.$$
$$7$$

Problem 2

(a) Find the inverse (reciprocal) of 21, modulo 919.

$$4 \times 919 + 21(744) \equiv 1(mod 919)$$
$$744$$

(b) Use your answer to part (a) to solve the congruence equation.

$$21x \equiv 13(mod 919)$$
$$21(744) \equiv 1(mod 919)$$
$$21(9674) \equiv 13(mod 919)$$
$$21(484) \equiv 13(mod 919)$$

Problem 3

Use the Principle of Mathematical Induction to prove the following formula:

Proof. Basis Step: $n = 1$
LHS $= 1 \times 10 = 10$
RHS $= \frac{1(1+1)(1+14)}{3} = 10$
Inductive Step: $n = k$
$1(10) + 2(11) + 3(12) + \cdots + k(k+9)$
Consider $n = k+1$
LHS $= 1(10) + 2(11) + 3(12) + \cdots + k(k+9) + (k+1)(k+10)$
LHS $= \frac{k(k+1)(k+14)}{3} + (k+1)(k+10)$
LHS $= \frac{k(k+1)(k+14)+3(k+1)(k+10)}{3}$
LHS $= \frac{k^3+18k^2+47k+30}{3}$
RHS $= \frac{(k+1)(k+2)(k+15)}{3}$
RHS $= \frac{k^3+18k^2+47k+30}{3}$ $\square$

Problem 4

Use the Principle of Mathematical Induction to prove that:

$$7 \text{ is a divisor of } 6 * 4^n + 11^n, \text{ for all integers } n \geq 0.$$

Proof. Basis Step: n = 1
$7(u) = 6 * 4^n + 11^n$, where $u \in \mathbf{Z}$
$7(u) = 6 * 4 + 11 = 35 = 7(5)$
Inductive Step: n = k
$7(u) = 6 * 4^k + 11^k$
$7(u) = 6 * 44^k$
Consider n = k + 1
$7(u) = 6 * 4^{k+1} + 11^{k+1}$
$7(u) = 6 * 44^{k+1}$ □