Math 218
Section 81
Summer 2019
Christopher Schmitt Assignment 1

Problem 1

Are the following statements TRUE or FALSE? Explain why.

(a) If π is rational, then so is 2

(b) If π is irrational, then so is 2

Solution

(a) True, since $False \rightarrow True$

(b) False, since $True \nrightarrow False$

Problem 2

Consider the statement: "If an animal is an rhinoceros, then it has a horn."

(a) Write down the CONVERSE of this statement.

(b) Write down the CONTRAPOSITIVE of this statement.

Solution

(a) "If an animal has a horn, then it is a rhinoceros"

(b) "If an animal does not have a horn, then it is not a rhinoceros"

Problem 3

Let x be a real number. Using the definition of rational number, write a direct proof of the following: If x is rational, then $x^2 + 5$ is also rational.

Proof. Let x be a rational number.

$$\begin{aligned}
x &= \frac{a}{b}, \text{ where } a, b \text{ are integers and } b \neq 0 \\
x^2 &= \frac{a^2}{b^2} \\
x^2 + 5 &= \frac{a^2}{b^2} + 5 \\
x^2 + 5 &= \frac{a^2 + 5b^2}{b^2}, \text{ which is rational.}
\end{aligned}$$

□

Problem 4

Let x be a positive real number. Using the definition of a rational number, write a proof by contraposition of the following: If x irrational, then $\sqrt{x+6}$ is also irrational.

Proof. (By contraposition) Let $\sqrt{x+6}$ be rational.

$$\begin{aligned}
\sqrt{x+6} &= \frac{a}{b}, \text{ where } a, b \text{ are integers and } b \neq 0 \\
x+6 &= \frac{a^2}{b^2} \\
x &= \frac{a^2}{b^2} - 6 \\
x &= \frac{a^2 - 6b^2}{b^2}, \text{ which is rational}
\end{aligned}$$

□

Problem 5

let n be an integer. Using the definition of odd/even, write a proof of the following: n is even if and only if $2n^2 + 5n + 7$ is odd.

Proof. $(p \Rightarrow q)$ Suppose that n is even.

$$\begin{aligned}
n &= 2(k), \text{ where } k \text{ is an integer} \\
2n^2 + 5n + 7 &= 2(2k)^2 + 5(2k) + 7 \\
2n^2 + 5n + 7 &= 8k^2 + 10k + 7 \\
2n^2 + 5n + 7 &= 8k^2 + 10k + 6 + 1 \\
2n^2 + 5n + 7 &= 2(4k^2 + 5k + 3) + 1, \text{ which is odd.}
\end{aligned}$$

$(\neg p \Rightarrow \neg q)$ Suppose that n is odd.

$$\begin{aligned}
n &= 2(k) + 1, \text{ where } k \text{ is an integer} \\
2n^2 + 5n + 7 &= 2(2k+1)^2 + 5(2k+1) + 7 \\
2n^2 + 5n + 7 &= 8k^2 + 18k + 14 \\
2n^2 + 5n + 7 &= 2(4k^2 + 9k + 7), \text{ which is even.}
\end{aligned}$$

□

Problem 6

Using the definition of odd and even, write a proof of the following: n^2+3n+7 is odd.

Proof. (By cases) Suppose that n is even.

$$\begin{aligned}
n &= 2(k), \text{ where k is an integer} \\
n^2+3n+7 &= (2k)^2+3(2k)+7 \\
n^2+3n+7 &= 4k^2+6k+7 \\
n^2+3n+7 &= 4k^2+6k+6+1 \\
n^2+3n+7 &= 2(2k^2+3k+3)+1, \text{ which is odd.}
\end{aligned}$$

Suppose that n is odd.

$$\begin{aligned}
n &= 2(k)+1, \text{ where k is an integer} \\
n^2+3n+7 &= (2k+1)^2+3(2k+1)+7 \\
n^2+3n+7 &= 4k^2+10k+11 \\
n^2+3n+7 &= 4k^2+10k+10+1 \\
n^2+3n+7 &= 2(2k^2+5k+5)+1, \text{ which is odd.}
\end{aligned}$$

□

Problem 7

Prove that there exists positive integers a, b such that $a^2+b^2=100$

Proof. (By example)

$$6^2+8^2=100$$

□