Math 218
Section 81
Summer 2019
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Assignment 3

Problem 1

Let R be the relation on the set Q defined by:

$$(a, b) \in R \text{ if and only if } |a - b| \leq 4$$

R is Reflexive.

Proof. Let $a \in Q$

$$\begin{aligned} |a - a| &\leq 4 \\ |0| &\leq 4 \\ 0 &\leq 4 \\ (a, a) &\in R \end{aligned}$$

□

R is Symmetric.

Proof. Let $a, b \in Q$, Suppose $(a, b) \in R$

$$\begin{aligned} &|a - b| \leq 4 \\ &|b - a| \leq 4 \text{ [Absolute value]} \\ &(b, a) \in Q \end{aligned}$$

□

R is not Transitive.

Proof.

$$\begin{aligned} |10 - 6| \leq 4 \wedge |6 - 2| \leq 4 \\ |10 - 2| > 4 \end{aligned}$$

□

R is not Antisymmetric.

Proof.

$$\begin{aligned} |10 - 6| \leq 4 \\ |6 - 10| \leq 4 \end{aligned}$$

□

Problem 2

Let R be the relation on the set R defined by:

$$(a, b) \in R \text{ if and only if } 5a = b$$

R is not Reflexive.

Proof. Let $a \in R$

$$5a \neq a \text{ [In General]}$$

□

R is not Symmetric.

Proof. Let $a, b \in R$

$$5(5) = 25$$
$$5(25) \neq 5$$

□

R is not Transitive.

Proof. Let $a, b, c \in R$, Suppose $(a, b), (b, c) \in R$

$$5(1) = 5$$
$$5(5) = 25$$
$$5(1) \neq 25$$

□

R is Antisymmetric.

Proof. Let $a, b \in R$, Suppose $(a, b), (b, a) \in R$

$$5(a) = b \wedge 5(b) = a$$
$$5(5(b)) = b$$
$$25(b) = b \text{ iff } b = 0$$
$$5(5(a)) = a$$
$$25(a) = a \text{ iff } a = 0$$

a and b must both the same if $(a, b), (b, a) \in R$, so R is Antisymmetric □

Problem 3

Let R be the relation on the set Z defined by:

$$(a, b) \in \text{ if and only if } ab \geq 1$$

R is not Reflexive.

Proof.

$$0(0) < 1$$

□

R is Symmetric.

Proof. Let $a, b \in Z$, Suppose $(a, b) \in R$

$$a(b) \geq 1$$
$$b(a) \geq 1$$

□

R is Transitive.

Proof. Let $a, b, c \in Z$, Suppose $(a, b), (b, c) \in R$

$$ab \geq 1 \wedge bc \geq 1$$
$$(ab)(bc) \geq 1$$
$$acb^2 \geq 1$$
$$ac \geq b^{-2}$$

□

R is not Antisymmetric.

Proof.

$$1(2) \geq 1$$
$$2(1) \geq 1$$

□

Problem 4

Let R be the relation on the set $Z \times Z$ defined by:

$$((a, b), (c, d)) \in R \text{ if and only if } a + 2b \leq c + 2d$$

R is Reflexive.

Proof. Let $(a, b) \in Z \times Z$

$$a + 2(b) \leq a + 2(b)$$
$$((a, b), (a, b)) \in R$$

□

R is not Symmetric.

Proof. Let $(a, b), (c, d) \in Z \times Z$, Suppose $((a, b), (c, d)) \in R$

$$0 + 2(0) \leq 1 + 2(1)$$
$$0 \leq 2$$
$$1 + 2(1) > 0 + 2(0)$$
$$3 > 0$$

□

R is Transitive.

Proof. Let $(a, b), (c, d), (e, f) \in Z \times Z$, Suppose $(a, b)R(c, d) \wedge (c, d)R(e, f)$

$$a + 2(b) \leq c + 2(d) \wedge c + 2(d) \leq e + 2(f)$$
$$a + 2(b) \leq e + 2(f)$$

□

R is Antisymmetric.

Proof. Let $(a, b), (c, d) \in Z \times Z$, Suppose $(a, b)R(c, d)$

$$a + 2(b) \leq c + 2(d)$$
$$c + 2(d) \leq a + 2(b)$$
$$a + 2(b) = c + 2(b)$$

□

Problem 5

Let $\sim$ be the relation on the set Z defined by:

$$a \sim b \text{ if and only if } a = \pm b.$$

$\sim$ is Reflexive.

Proof. Let $a \in Z$

$$\begin{aligned} a &= \pm a \\ a^2 &= (\pm a)^2 \\ a^2 &= a^2 \end{aligned}$$

$\square$

$\sim$ is Symmetric.

Proof. Let $a, b \in Z$, Suppose $a \sim b$

$$a = \pm b$$
$$a = b \lor a = -b$$
$$a = b \implies b = a$$
$$a = -b \implies -b = a \implies b = -a$$

$\square$

$\sim$ is Transitive.

Proof. Let $a, b, c \in Z$, Suppose $a \sim b, b \sim c$

$$\begin{aligned} a &= \pm b \\ a^2 &= (\pm b)^2 \\ a^2 &= b^2 \\ b &= \pm c \\ b^2 &= (\pm c)^2 \\ b^2 &= c^2 \\ a^2 = b^2 &= c^2 \\ a^2 &= b^2 \end{aligned}$$

$\square$

Find all the elements in the class $\bar{7}$

$$\bar{7} = \{7, -7\}$$

Problem 6

Let $\sim$ be the relation on the set Z defined by:

$$a \sim b \text{ if and only if 8 is a divisor of } 7a + b$$

$\sim$ is Reflexive.

Proof. Let $a \in Z$

$$\begin{aligned} 8 &| 7(a) + a \\ 8(k) &= 7(a) + a \text{ where } K \in \mathbb{Z} \\ 8(k) &= 8(a) \end{aligned}$$

$\square$

$\sim$ is Symmetric.

Proof. Let $a, b \in Z$

$$\begin{aligned} 8 &| 7(a) + b \\ 8(k) &= 7(a) + b, \text{ where } k \in \mathbb{Z} \\ -8(k) &= -7(a) - b \\ -8(k) + 8(a) + 8(b) &= -7(a) + 8(a) - b + 8(b) \\ 8(-k + a + b) &= a + 7(b) \\ 8(-k + a + b) &= 7(b) + a \end{aligned}$$

$\square$

$\sim$ is Transitive.

Proof. Let $a, b, c \in Z$, Suppose $a \sim b, b \sim c$

$$\begin{aligned} 8 &| 7(a) + b \wedge 8|7(b) + c \\ 8(k) &= 7(a) + b \wedge 8(j) = 7(b) + c, \text{ where } k, j \in \mathbb{Z} \\ 8(k) + 8(j) &= 7(a) + b + 7(b) + c \\ 8(k) + 8(j) &= 7(a) + 8(b) + c \\ 8(k) + 8(j) - 8(b) &= 7(a) + c \\ 8(k + j - b) &= 7(a) + c \end{aligned}$$

$\square$

Find four elements in the equivalence class $\bar{5}$

$$\bar{5} = \{5, 13, 21, 29\}$$