Math 218
Section 81
Summer 2019
Assignment 4

Christopher Schmitt

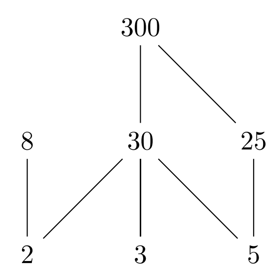
Problem 1

Let $\preceq$ be the relation on the set $A = \{2, 3, 5, 8, 25, 30, 300\}$ defined by:

$$a \preceq b \text{ iff } a \text{ is a divisor of } b$$

$\preceq$ is a partial order on A (you do not need to prove this). Answer each of the following.

(a) Draw the hasse diagram for this partially ordered set.



(b) What are the maximal elements for this poset? $\{8, 300\}$

(c) What are the maximum elements for this poset? None

(d) What are the minimal elements for this poset? $\{2, 3, 5\}$

(e) What are the minimum elements for this poset? None

(f) Find $2 \vee 3$. 30

(g) Find $8 \vee 25$. Does not exist

(h) Find $8 \wedge 300$. 2

(i) Find $8 \wedge 25$. Does not exist

Problem 2

Prove that the function $f : Q \to Q$ defined by $f(x) = 3x + 7$ is injective.

Proof. Let $a, b \in Q$, Suppose $f(a) = f(b)$

$$3(a) + 7 = 3(b) + 7$$
$$3(a) = 3(b)$$
$$a = b$$

□

Problem 3

Prove that the function $f : R \to R$ defined by $f(x) = x^2 - 3x + 5$ is not injective.

Proof.

$$f(0) = f(3)$$

$\square$

Problem 4

Prove that the function $f : Z \times Z \to Z$ defined by $f(n, m) = 3n - 2m - 1$ is onto.

Proof. Let $k \in Z$

$$\begin{aligned} f(n, m) &= 3(n) - 2(m) - 1 \\ f(2, 2) &= 3(2) - 2(2) - 1 = 1 \\ f(2k, 2k) &= 3(2k) - 2(2k) - 1 \\ f(2k, 2k) &= k(1) = k \end{aligned}$$

$\square$

Problem 5

Prove that the function $f : Z \to Z$ defined by $f(n) = 3n + 2$ is not onto.

Proof.

$$\begin{aligned} 1 &\in Z \\ 1 &= 3n + 2 \\ -1 &= 3n \\ \frac{-1}{3} &= n \\ \frac{-1}{3} &\notin Z \end{aligned}$$

So $f : Z \to Z$ cannot produce the value 1, and therefore cannot be onto. $\square$

Problem 6

Let $f : R \setminus \{4\} \to R$ be the function defined by $f(x) = \frac{2x+7}{x-4}$

(a) Prove that this function is injective.

Proof. Let $a, b \in R \setminus \{4\}$ Suppose $f(a) = f(b)$

$$\begin{aligned}
\frac{2(a)+7}{a-4} &= \frac{2(b)+7}{b-4} \\
(2(a)+7)(b-4) &= (2(b)+7)(a-4) \\
2ab - 8a + 7b - 28 &= 2ab - 8b + 7a - 28 \\
-8a + 7b &= -8b + 7a \\
15b &= 15a \\
b &= a
\end{aligned}$$

□

(b) This function is not onto. Determine which element should be removed from the codomain to make it onto. Prove that f is onto when this element is removed from the codomain, and find the inverse f^{-1}.

$$f : R \setminus \{4\} \to R \setminus \{2\}$$

Proof. Let $r \in R \setminus \{2\}$

$$\begin{aligned}
r &= \frac{2x+7}{x-4} \\
r(x-4) &= 2x + 7 \\
rx - 4r &= 2x + 7 \\
rx - 4r - 2x + 8 &= 15 \\
(r-2)(x-4) &= 15 \\
x - 4 &= \frac{15}{r-2} \\
x &= \frac{15}{r-2} + 4 \\
x &= \frac{15 + 4(r-2)}{r-2} \\
x &= \frac{7+4r}{r-2} \\
f(\frac{7+4r}{r-2}) &= r, \text{ where } r \neq 2
\end{aligned}$$

□

$$f^{-1} : R \setminus \{2\} \to R \setminus \{4\}$$

$$f^{-1}(x) = \tfrac{7+4r}{r-2}$$