Math 218
Section 81
Summer 2019
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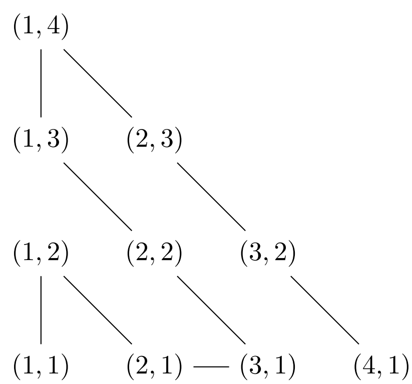
Assignment 5

Problem 1

(a) Prove that the set $12\mathbf{Z}$ of all multiples of 12 is countably infinite.

$$\begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & \cdots \\ | & | & | & | & | & | & \\ 0 & 12 & -12 & 24 & -24 & 36 & \cdots \end{array}$$

(b) Prove that the set $\mathbf{Z}^+ \times \mathbf{Z}^+$ is countably infinite.



Problem 2

(a) Prove that the set $x \in \mathbf{R} | 0.53 < x < 0.54$ (in other words, the interval (0.53, 0.54) is uncountable.

$$f : \mathbf{R} \to (0.53, 0.54)$$
$$f(x) = tan(50\pi(x - 0.535))$$

Since a bijection exists between $\mathbf{R}$ and $(0.53, 0.54)$, $(0.53, 0.54)$ is uncountable.

(b) Let A be the set of all infinite sequences of positive integers. For example, one of the elements of A is the sequence $1, 2, 3, 4, 5, 6, \ldots$ Another element of A is the sequence $1, 2, 4, 8, 16, 32, \ldots$ Prove that A is uncountable.

Proof. Suppose that the set A is countably infinite.
$\Longrightarrow$ A bijection exists between A and $\mathbf{Z}^+$
$\Longrightarrow$ Some function, $f : \mathbf{Z}^+ \to A$ is onto
$\Longrightarrow$ A list can be created matching every member of A to $\mathbf{Z}^+$
However, an element can be constructed that is not in A: $x_1, x_2, x_3, ...$, where x_n is any random integer such that x_n is not equal to k, where k is the n'th element of n'th element of the list. So f cannot be onto. $\square$

Problem 3

Find the quotient and the remainder when a is divided by b, for the following values of a and b:

(a) $a = 148$, $b = 9$, $q = 16$, $r = 4$

(b) $a = -148$, $b = 9$, $q = -17$, $r = 5$

(c) $a = 148$, $b = -9$, $q = -17$, $r = 5$

(d) $a = -148$, $b = -9$, $q = 16$, $r = 4$

Problem 4

Find the binary and hexadecimal representations of the decimal number 2775.

$$101011010111_2$$
$$AD7_{16}$$

Problem 5

Use the Extended Euclidean Algorithm to compute the greatest common divisor of the integers 768 and 46, and to express that greatest common divisor in the form $768x + 46y$, where $x, y \in Z$.

$$2 = 10(768) + 167(46)$$

Problem 6

Factor the integer 155,540 into a product of primes.

$$2^2 \times 5 \times 7 \times 11 \times 101$$