Wealth Distribution under Heterogenous Preferences[a]

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Abstract

Although the assumptions of heterogeneity of either impatience rate or risk aversion are common in the literature that studies wealth distribution, they do not reflect the fact (and a common belief) that *both parameters vary* across population. This paper takes the micro findings of impatience and risk aversion seriously and studies how the heterogeneity of the two factors affects income and wealth distribution. It enriches the standard framework with more plausible assumptions of agents' preferences and shows that a model where agents have both different discount factors and risk aversion rates can generate a large mass of the poor and a high concentration of wealth at the top.

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1 Introduction

Multiple sources document excessive wealth inequality in the world, which has continuously increased in the past four decades.[3] Recent estimates reveal that the richest 1% of the world population own more than the bottom 99%. Such extremes prompt many authors to study the underlying factors that cause income and wealth inequality, and its implications on politics and public policy.[4]

The expansive scientific literature addressing the issue identifies a variety of reasons why wealth is not equally distributed. As summarised in Leitner (2016), the three main drivers of disproportional wealth accumulation include a) structural differences such as skills, fortune, consumption preferences, etc., b) saving and investment, and c) intergenerational transfers. For instance, Venti and Wise (2001) find that wealth dispersion amongst households with similar incomes households is attributed, to a large extent, to their saving decisions. Kotlikoff and Summers (1981, 1988), Modigliani (1988), Gale and Scholtz (1994) and others underline the importance of inheritance and intergenerational transfer of bequests or gifts. This paper studies the first channel, the structural differences, and focuses on the effect of heterogeneity of consumer preferences on wealth distribution.

In the macroeconomics literature, distribution of wealth is usually modelled using an incomplete markets framework with uninsurable idiosyncratic shock to earnings. In this framework, agents face fluctuations in their income, and self-insure by saving in good times so that they can consume their savings in bad times. Consequently, the agents that experience bad income shocks become poor, and the agents that enjoy a long sequence of good income shocks become relatively rich. On the one hand, this precautionary savings mechanism allows the agents to smooth consumption effectively[5] which results in only a small number of them being borrowing constrained. On the other hand, the agents may only save up to a certain limit. When the benefits of the accumulated wealth outweigh the labor income, the agents start to dissave, i.e. their saving rates become negative. This setup is used in so-called Bewley-Huggett-

[3]See, e.g., Piketty (2014), Oxfam (2016), Cowell, Karagiannaki and McKnight (2012), Facundo et al. (2017).

[4]Stiglitz (2012) describes the origins and the dimensions of economic inequality, and discusses its adverse effects. Many authors demonstrate that the consequences of economic inequality can lead to redistribution in democracies, social unrest, and evolution of political regimes. It is commonly argued that poor majority populations in democracies can outvote the rich and achieve redistribution, see e.g. Meltzer and Richard (1981). The greater the inequality, the more incentives the poor have to seize the wealth of the rich through illegal means such as revolutions, coups, etc. as in Alesina and Perotti (1994). The fear of future high redistribution under a democratic regime causes an autocracy's ruling elite to embark on the transition path to democratization, see e.g. Acemoglu and Robinson (2000, 2006). However, recent studies, see e.g. Haggard and Kaufman (2012, 2016), find evidence that the transition is also determined by other political and institutional factors.

[5]This important observation is reviewed in many papers. See, e.g., Lucas (1987), Cochrane (1989), Marcet and Singleton (1998), Shao and Silos (2014).

Aiyagari[6] models to study the income and wealth distributions. Under plausible assumptions and calibrations, the findings of this class of models do not match the real data where: a) the rich have constantly high saving rates, and b) there is a large mass of the poor hitting borrowing constraints. In order to achieve higher wealth concentration among the rich, and generate a large number of agents with very little wealth, several authors introduce additional features to the basic framework.

The features that help better match the stylised facts of wealth distribution and primarily affect the savings mechanisms include: a) preference heterogeneity as in Krusell and Smith (1998), Hendricks (2007), Carroll et al. (2017), Coen-Pirani (2004) and Cozzi (2012); b) the intergenerational bequest motive, proposed in De Nardi (2004) and De Nardi and Yang (2016), that requires the rich to save more to bequeath; c) a stochastic rate of return as studied in Benhabib et al. (2011, 2015, 2016); d) an extraordinary productivity process making agents self-insure against much higher earnings risk assumed in Castaneda et al. (2003); e) entrepreneurial or different types of skills yielding higher stochastic rates of return as adopted in Castaneda et al. (1997), Quadrini (1999, 2000) and Cagetti and De Nardi (2006); and f) health and marital risk modelled in Hubbard, Skinner, and Zeldes (1995) and Cubeddu and Rios-Rull (1996) that reflects the fact that both long-term health deterioration and marital status have a dramatic impact on people's rates of saving.[7]

Preference heterogeneity has been added to existing models in different flavours. While Krusell and Smith (1998) allow for three types of agents that are characterised by their *different time discount factors*, Hendricks (2007) and Carroll et al. (2017) consider models with *a distribution of time discount factors*. In Coen-Pirani (2004), there are two types of agents that have different rates of risk aversion. Cozzi (2012) assumes heterogenous agents whose risk aversion rates are drawn from log-normal distribution. Although the above-mentioned assumptions of heterogeneity of either impatience rate or risk aversion are common in the literature, they do not reflect the fact (and a common belief) that *both parameters are dispersed* across population. In fact, Cagetti (2003) estimates discount factors and risk aversion of three groups (individuals without schooling, high school graduates, and college graduates), and finds that higher education groups exhibit a greater degree of patience and risk aversion. Likewise, Booji and Van Praag (2009) find that "both parameters [impatience and risk aversion] strongly vary over individuals, while they are moderately negatively correlated". In their study of the relation between impatience, risk aversion and cognitive ability, Dohmen et al. (2010) provide evidence that there is substantial variation of risk aversion and time discount factors among individuals. Becker et al. (2018) analyse, among others, variations of risk aversion and patience across coun-

[6] Due to Bewley(1977), Hugget (1993) and Aiyagari (1994).

[7] De Nardi (2016) provides a recent survey of the corresponding models.

tries.

This paper takes the micro findings on impatience and risk aversion seriously and studies how the heterogeneity of the two factors affects income and wealth distributions in a macroeconomic framework. It also enriches the standard framework by using more plausible assumptions of agents' preferences. My model is related to Krusell and Smith (1998), but the heterogenous preferences of agents are reflected not only in different time discount factors but also in different risk aversion rates. I consider three types of agents which are characterised by a tuple comprising a discount factor and a risk aversion rate. In a production economy with incomplete markets, idiosyncratic income shocks, and aggregate uncertainty, the average lifespan of the three types of individuals are matched to that of a generation. I simulate the model using different sets of preference parameters to analyse their effects on income and wealth distribution. I find that a setting where agents differ both in terms of discount factors and risk aversion rates generates a wealth distribution able to match features in the data: a larger mass of poor agents than previous models, and and a realistic concentration of wealth at the top.

Section 2 lays out the model. Section 3 compares outcomes with respect to different parameter calibrations. Section 4 concludes. All parameter values as well as the computational strategy of the model simulations are summarised in Appendix A and Appendix B, respectively.

2 Model

This section introduces the model and characterises the equilibrium. The model is a general equilibrium framework with incomplete markets and aggregate uncertainty. In a production economy, agents face different employment states (either employed or unemployed) depending on the idiosyncratic income shock. The model is based on Krusell and Smith (1998) and extended with unemployment benefits as in Den Haan et al. (2010), and heterogenous preferences in time discount factors and risk aversion.

Households
There is a continuum (measure one) of agents who live in discrete time. The agent maximises his expected lifetime utility subject to the sequence of budget and borrowing constraints. At each point in time the agent receives an individual income shock that determines his employment state: if employed, he earns a wage, if unemployed, he receives an unemployment benefit. To insure against future unemployment shocks, the agent invests in capital, the only type of savings available in the

economy. Each agent i solves the following problem when choosing the optimal levels of saving and consumption from time $t=0$ onwards:

$$\max_{\{c_{i,t},k_{i,t+1}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta_i^t \frac{c_{i,t}^{1-\gamma_i}-1}{1-\gamma_i} \tag{1}$$

$$\text{s.t.} \quad \begin{aligned} & c_{i,t}+k_{i,t+1} = r_t k_{i,t} + (1-\tau_t)\bar{l}e_{i,t}w_t + \mu(1-e_{i,t})w_t + (1-\delta)k_{i,t}, \\ & c_{i,t} \geq 0 \quad \text{and} \quad k_{i,t+1} \geq b. \end{aligned} \tag{2}$$

In the equations above, the time-varying variables $c_{i,t}$, $k_{i,t}$, w_t, r_t, τ_t and $e_{i,t}$ denote an agent's consumption, capital investment, wage rate, interest rate, tax rate and employment shock at time t, respectively. In each period, the agent supplies $\bar{l}$ units of labor, and receives an individual employment shock $e_{i,t}$. When employed, $e_{i,t}=1$, and he earns an after-tax income $(1-\tau_t)\bar{l}w_t$. When unemployed, $e_t=0$, and he collects an unemployment benefit μw_t. The benefit rate μ, capital depreciation rate δ and borrowing limit b are constant over time. The agent's preferences are represented by the CRRA utility function where β_i is a time discount factor and γ_i is a risk aversion rate of agent i.

Production

Given that the economy's production function is Cobb-Douglas, identical competitive firms combine aggregate capital K_t and labor $\bar{l}N_t$ of employed agents N_t to produce final output Y_t:

$$Y_t = a_t K_t^{\alpha} \left(\bar{l}N_t\right)^{1-\alpha}, \tag{3}$$

where α is the capital share and a_t is an aggregate technology shock. The standard first order conditions for profit maximisation imply that the factor prices, w_t and r_t are given by:

$$w_t = (1-\alpha)a_t \left(\frac{K_t}{\bar{l}N_t}\right)^{\alpha} \quad \text{and} \quad r_t = \alpha a_t \left(\frac{K_t}{\bar{l}N_t}\right)^{\alpha-1}. \tag{4}$$

Government

The government is not allowed running deficits. It collects taxes from employed agents N_t, and spends the receipts on benefits to those receiving negative income shock, i.e. unemployed agents $1-N_t$. The balanced government budget at time t is described by the following equation:

$$\bar{l}N_t\tau_t w_t = \mu(1-N_t)w_t. \tag{5}$$

Shocks

The uncertainty in the economy is due to aggregate productivity shock and individual shocks. All the shocks are first-order Markov processes.

The aggregate shock a_t is exogenous and governs the transition of the economy between good and bad states, a_g and a_b. The transition probabilities are chosen to match the average duration of both states to eight quarters. When the economy is in a good state, $a_t = a_g$, the unemployment rate is $u_g = 1 - N_t$. When the economy is in a bad state, $a_t = a_b$, the unemployment rate is $u_b = 1 - N_t$.

There are also two kinds of individual shocks: income shocks and preference type shocks. There is no correlation between them by assumption. In each period, an agent i observes his own income shock $e_{i,t}$ that determines his employment status. The income shocks are uncorrelated across the agents.

The preference type shock defines the level of patience, β_i, and the attitudes toward risk, γ_i, of agent i. There are three types of agents, *low*, *medium*, and *high* with the following preferences: $\{(\beta_L, \gamma_L), (\beta_M, \gamma_M)$ and $(\beta_H, \gamma_H)\}$. The corresponding values of impatience, risk aversion rates as well as transition probabilities among the types are part of the experiments and described in the next section.

Aggregate Law of Motion

The aggregate state of the economy is described by aggregate productivity a_t and agents' distributions of wealth and employment status Γ_t. It is updated according to an aggregate laws of motion which is based on the first moment of Γ_t and takes the (standard in the related literature) form:[8]

$$\begin{aligned} \ln K_{t+1} &= b_{0,g} + b_{1,g} \ln K_t \quad \text{if} \quad a = a_g, \\ \ln K_{t+1} &= b_{0,b} + b_{1,b} \ln K_t \quad \text{if} \quad a = a_b. \end{aligned} \tag{6}$$

Equilibrium

The agent's problem can be expressed recursively with the following Bellman equation:

$$V(k, e, \beta, \gamma, a, K) = \max_{c,k'} \left\{ \frac{c^{1-\gamma} - 1}{1-\gamma} + \beta E_{\beta', \gamma', e', a' | \beta, \gamma, e, a} V(k', e', \beta', \gamma', a', K') \right\} \tag{7}$$

[8]Krusell and Smith (1996, 1998) show that the suggested law of motion has excellent fit and inclusion of further moments of the distribution does not improve forecasting power. In the same fashion, e.g. Cozzi (2015), Shin (2012) and many others, assume law of motion with only the first moment of the distribution.

$$
\begin{aligned}
\text{s.t.} \quad & c + k' = rk + (1-\tau)\bar{l}ew + (1-\delta)k \quad && \text{if} \quad e = 1, \\
& c + k' = rk + \mu w + (1-\delta)k && \text{if} \quad e = 0, \\
& c \geq 0 \quad \text{and} \quad k' \geq b,
\end{aligned} \tag{8}
$$

$$
\begin{aligned}
\ln K' &= b_{0,g} + b_{1,g} \ln K \quad && \text{if} \quad a = a_g, \\
\ln K' &= b_{0,b} + b_{1,b} \ln K && \text{if} \quad a = a_b.
\end{aligned} \tag{9}
$$

The definition of a recursive equilibrium for this economy requires a) agents' value functions V, b) agents' decision rules for consumption c and savings k', c) the government's decision rules τ, d) the factor prices r and w, e) macroeconomic aggregates K and L, and f) the law of motion (9) such that 1) c and k' are solutions to the agents' problem, 2) r and w are factor marginal productivities, 3) the markets for goods, labor and capital clear, and the government budget is balanced, and 4) the aggregate capital stock in the economy evolves according to the law of motion.

3 Simulation Results

To solve the model, I use a stochastic version of the Krusell and Smith (1998) algorithm outlined in Maliar et al. (2010). A precise solution algorithm with necessary adjustments of this study is detailed in Appendix B. The preferences of the agents are allowed to differ in three experiments. First, the benchmark model allows for preference heterogeneity only in the time discount factors. Second, I consider a model where the agents that have the highest level of patience are also more risk averse. Third, I assume a model where agents' attitudes towards risk and impatience are correlated. In each experiment, an economy is populated by $N = 10,000$ agents and simulated for $T = 1,100$ periods where the first 100 periods are skipped to ensure the independency of the initial state. Below, I describe the estimated laws of motion as well as agents' individual policy functions for each case considered, and subsequently compare generated wealth distributions.

Parametrisation

The model is calibrated at the quarterly frequency. Most of the parameter values are conventional and aligned with the previous studies of Krusell and Smith (1998) and Den Haan et al. (2010). All the parameter values together with their sources are reported in Appendix A.

The aggregate productivity shock a_t is a symmetric two-state Markov process. Given that the aggregate productivity is 2% lower in recessions than in expansions and normalising $E(a_t) = 1$, the corresponding values in good and bad states are $a_b = 0.99$ and $a_g = 1.01$. Following business cycles, the expected duration of switching between the two states is 8 quarters on average. The process is governed by a transition matrix that is shown in Table 5, Appendix A. The capital share of output is traditionally calibrated to $\alpha = 0.36$, while capital depreciates at rate $\delta = 0.025$.

The individual employment shock is also a two-state Markov process that defines an employment status of an agent. The transition probabilities, given in Table 6, Appendix A, ensure that the unemployment rate is a function of the aggregate productivity, and the average duration of unemployment is 1.5 quarters in good states and 2.5 quarters in bad states. When the economy is in a good state, $a_t = a_g$ and the unemployment rate is 4%, i.e. $u_g = 1 - N_t = 0.04$. When the economy is in a bad state, $a_t = a_b$ and the unemployment rate is 10%, i.e. $u_b = 1 - N_t = 0.1$. Assuming that the unemployment benefits are 15% of the wage of the employed, $\mu = 0.15$. Normalisation of labor supply in a bad state $\bar{l}N_t = \bar{l}(1 - u_b) = 1$ pins down $\bar{l} = 1/0.9$.

Experiments

Agents' time preference rates are parameterised according to the estimates in Carroll et al. (2017) who calibrate time discount factors to match wealth distribution in a life cycle model, in a model with the Krusell and Smith (1998) aggregate process and in a model with the Friedman/buffer stock aggregate process.[9] In the baseline, I use the estimates obtained in the Carroll et al. (2017) version with the Krusell and Smith (1998) aggregate process. Time discount factors β are distributed uniformly in the population between 0.984 - 0.0102 and 0.984 + 0.0102, and the rate of risk aversion γ is 1. In practice, I discretise this distribution and choose transition probabilities so that (i) the invariant distribution of the three types of agents has equal population shares at each of the three tuples $(\beta_L, \gamma_L) = (0.984 - \frac{2}{3}0.0102, 1)$, $(\beta_M, \gamma_M) = (0.984, 1)$, and $(\beta_H, \gamma_H) = (0.984 + \frac{2}{3}0.0102, 1)$[10] (ii) immediate transitions between values of β_L and β_H occur with a probability of zero, and (iii) the average duration of β_L and β_H is 50 years. A corresponding transition matrix is in Table 7, Appendix A.

To introduce heterogeneity in risk aversion, I assign different values to the three types. Although the rates of risk aversion vary anywhere from close to 0 up till 5 in related studies, the extreme values are usually considered in robustness checks. Both Krusell and Smith (1998) and Carroll et al. (2017) rely on a rate of risk aversion equal to 1 in their main models and investigate the sensitivity of their findings with higher (up to 5) and lower (up to 0.5) values. Cozzi (2015) uses a risk aversion rate of 2 in his model. Cagetti and De Nardi (2006) and Hendricks (2007) assume a risk aversion rates of 1.5. Based on the previous studies, I consider four reasonable values, $\gamma \in \{0.5, 1.0, 2.0, 5.0\}$, in the experiments. Table 1 reports the experimen-

[9] These parameter values are adopted in Krueger et al. (2016).

[10] Since β is uniformly distributed with support $[0.984 - 0.0102, 0.984 + 0.0102]$, $P(\beta \leq 0.984 - \frac{2}{3}0.0102) = \frac{1}{3}$, $P(0.984 - \frac{2}{3}0.0102 < \beta \leq 0.984 + \frac{2}{3}0.0102) = \frac{1}{3}$ and $P(\beta > 0.984 + \frac{2}{3}0.0102) = \frac{1}{3}$.

tal settings of the preferences together with the distribution of wealth for selected percentiles.

Table 1: Results and data: wealth distribution for selected percentiles.

(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Risk aversion parameter					Data
γ_L	1.0	0.5	1.0	1.0	0.5	
γ_M	1.0	1.0	1.0	1.0	1.0	$SCF16^*$
γ_H	1.0	1.0	2.0	5.0	5.0	
Population	Percentage of net wealth held by population					
top 1%	18.9	18.8	21.0	24.7	25.0	38.6
top 5%	45.7	46.4	47.1	49.3	50.4	65.1
top 10%	63.0	64.3	63.3	62.9	64.4	77.1
top 20%	82.6	84.3	82.0	79.5	81.4	88.3
top 30%	92.5	94.3	91.7	89.2	91.3	93.7
top 40%	96.8	98.5	96.2	94.5	96.6	96.9
top 60%	99.0	100.6	98.8	98.2	100.2	99.9
bottom 40%	1.0	-0.6	1.2	1.8	-0.2	0.1
bottom 20%	0.1	-0.6	0.2	0.3	-0.5	-0.5
bottom 10%	-0.2	-0.4	-0.1	-0.1	-0.4	-0.5
	Gini coefficient					
	79.0	81.4	78.7	77.3	80.2	86.0
Measure	Model performance					
R^2	0.99	0.99	0.99	0.99	0.99	–
$\hat{\sigma}$	0.03%	0.02%	0.03%	0.04%	0.03%	–

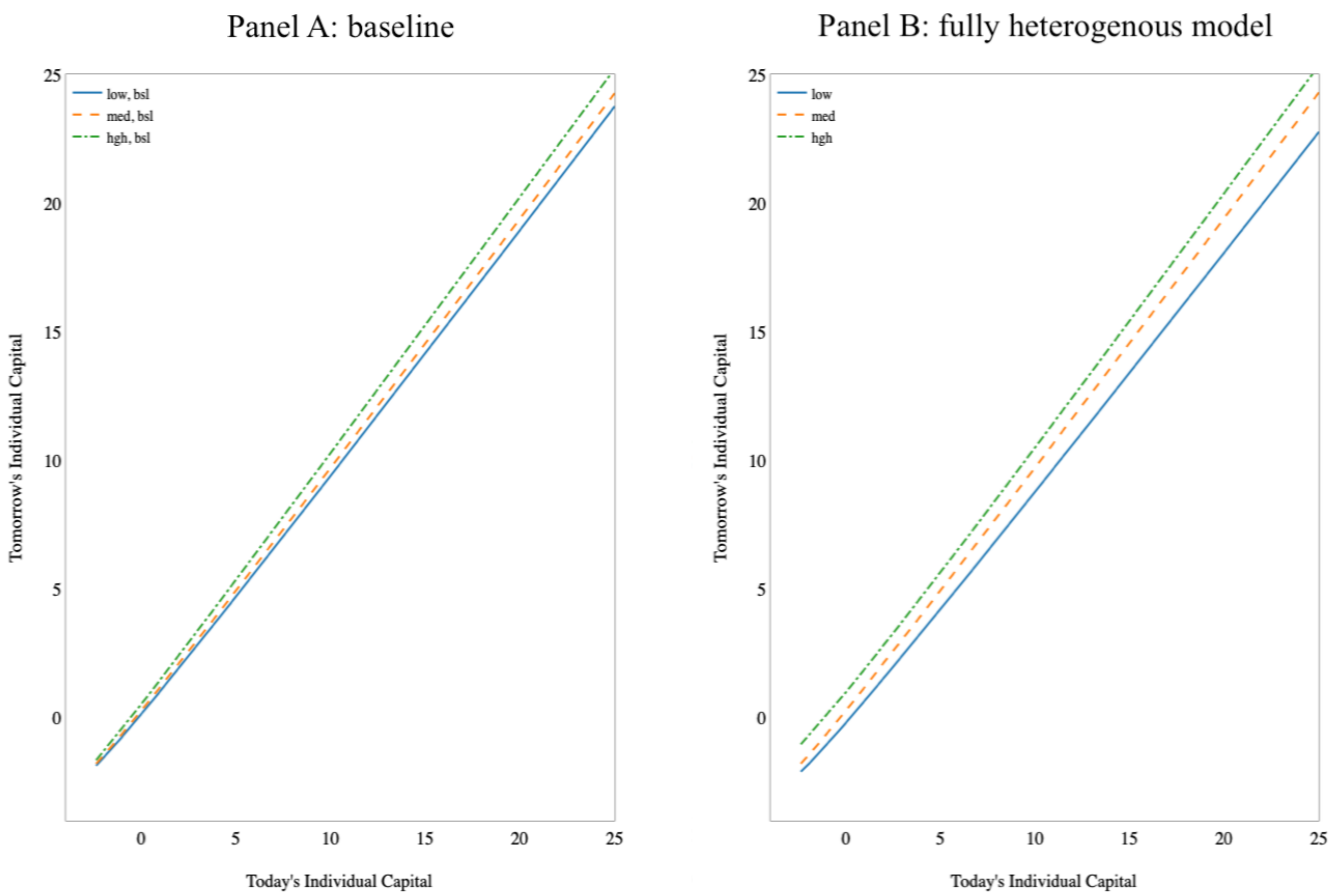
Note: *own calculations using data from the 2016 Survey of Consumer Finances.

In the first experiment, I only allow for the low type agents to have a different (lower) rate of risk aversion. Although the generated wealth distribution listed in Column 3 shows a larger mass of poor agents than that of the baseline in Column 2, there are no improvements in matching the upper tail of the distribution as implied by actual data in Column 7. Higher risk aversion rates are introduced in the models, where only the high-type agents are allowed to differ from the rest. The corresponding wealth distributions displayed in Column 4 and 5 reveal that higher risk aversion rates among the high-type agents succeeded in generating a larger concentration at the top. To further understand how preference heterogeneity affects wealth distribution, I assign the low-type agents low risk aversion rates and the high-type agents high risk aversion rates. The estimated wealth distribution in this fully heterogenous model (FHM), shown in Column 6, inherits both features: a larger number of poor agents and fewer extremely rich agents. Interestingly, the excessive mass of agents with low wealth obtained in the version with low-type heterogeneity in Column 3, is now less pronounced and better fits the data. The differences in the estimated Gini coefficient are in the magnitude of a few percentage points. The experiment with risk aversion heterogeneity records a slightly higher value of 80.2, yet is far below the value of 86.0 found in the data.

The model fit measured by R^2 and the regression error, $\hat{\sigma}$, of the estimated aggregate law of motion are reported in the lower panel of Table 1. Both R^2 and $\hat{\sigma}$ indicate an excellent performance of the aggregate law of motion.

The findings presented above suggest that the larger concentration of wealth at the top is induced by the high-type agents, while the poor mainly are low-type agents. By assumption, the high type agents are more patient and risk averse implying that they save more to insure against future risks and accumulate greater wealth than the impatient and more risk-seeking agents. Indeed, Figure 1 demonstrates that the high-type agents are saving more than the medium-type agents, and the low-type agents save the least, both in the baseline and in the FHM.

Figure 1: Agents' decision rules: baseline vs. fully heterogenous model.



Also, the observed gaps between the decision rules of the low-, medium- and high-types are wider in the FHM than in the baseline. These discrepancies between the decision rules in the baseline and the FHM are driven by two forces. A higher (lower) rate of risk aversion prompts the high (low)- type agents to save more (less). More saving and therefore aggregate capital accumulation in the economy leads to a decrease in the interest rate. The saving motive stemming from risk aversion is thus partially offset by weaker incentives to save at a lower interest rate.

Following the logic regarding agents' saving habits of the agents, we should expect the high types to be richer and the low types to be poorer. Table 2 breaks down the shares of the agent types in Column 6, and confirms that the wealthiest are predominantly of the high type and the poorest are mostly of the low type.

Table 2: Percentage of the types of agents in the selected percentiles of the FHM.*

	top							bottom		
agent type	1%	5%	10%	20%	30%	40%	60%	40%	20%	10%
high	72.00	79.20	81.9	82.10	81.07	77.2	55.30	0.20	0.10	0.38
medium	27.00	19.60	17.20	16.85	17.80	21.0	42.13	8.50	5.4	20.12
low	1.00	1.20	0.90	1.05	1.13	1.80	2.57	91.3	94.50	79.50

Note: * the agents' preferences are fully heterogenous as in Table 1, Column 6, $(\beta_L, \gamma_L) = (0.984 - \frac{2}{3}0.0102, 0.5)$, $(\beta_M, \gamma_M) = (0.984, 1.0)$, and $(\beta_H, \gamma_H) = (0.984 + \frac{2}{3}0.0102, 5.0)$.

Robustness

Since there is no one commonly accepted set of discount factors, and the estimates vary across the studies, I conduct robustness checks with different sets of discount factors for agents' types and different shares of the types of agents in the population.

Following Krusell and Smith (1998), I try setting the discount factors of the low-, medium- and high-type agents to $\beta_L = 0.9858$, $\beta_M = 0.9894$, and $\beta_H = 0.9930$. Also, the type transition matrix displayed in Table 8, Appendix A is calibrated so that low- and high-type agents each comprise 10% of the population, and the remaining 80% are of medium type. Other assumptions are maintained. The outcomes of these experiments, shown in Table 3, resemble the previous results. When low-type agents are less risk averse, as in Column 2, there are more poor agents in the economy; when high-type agents are more risk averse, as in Column 4 and 5, there is a larger concentration of wealth at the top. If the two features are combined, as in Column 6, the model generates both a large mass of poor agents and a few that are extremely rich. These properties are generated by virtue of the saving behaviour of the low- and high- type agents. In comparison to the previous results, the effect is less pronounced for the shares of the low type and high-type agents only comprise 10% of the entire population in these experiments.

Table 3: Robustness 1: wealth distribution for selected percentiles.

(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Risk aversion parameter					Data
γ_L	1.0	0.5	1.0	1.0	0.5	
γ_M	1.0	1.0	1.0	1.0	1.0	$SCF16^*$
γ_H	1.0	1.0	2.0	5.0	5.0	
Population	Percentage of wealth held by population					
top 1%	21.0	21.1	22.7	23.6	23.7	38.6
top 5%	56.3	56.8	57.5	57.3	57.9	65.1
top 10%	74.6	75.2	71.4	65.9	66.5	77.1
top 20%	86.2	86.9	81.7	74.5	75.2	88.3
top 30%	89.0	89.7	85.4	79.8	80.5	93.7
top 40%	91.4	92.1	88.6	84.2	85.0	96.9
top 60%	95.5	96.2	94.0	91.7	92.5	99.9
bottom 40%	4.5	3.8	6.0	8.3	7.5	0.1
bottom 20%	1.4	0.8	2.0	2.7	1.9	-0.5
bottom 10%	0.4	0.1	0.6	0.9	0.3	-0.5
	Gini coefficient					
	78.3	79.4	74.8	69.0	70.4	86.0
Measure	Model performance					
R^2	0.99	0.99	0.99	0.99	0.99	
$\hat{\sigma}$	0.03%	0.03%	0.04%	0.03%	0.03%	

Note: *own calculations using data from the 2016 Survey of Consumer Finances.

In the second robustness test, I specify the distributions of the discount factors as in the Carroll et al. (2017) version with a) life cycle model where the time discount factors are uniformly distributed between 0.9814 - 0.0182 and 0.9814 + 0.0182,[11] and b) with the Friedman/buffer stock aggregate process where the time discount factors are uniformly distributed between 0.9867 - 0.0067 and 0.9867 + 0.0067.[12] The transition probabilities governing the individual type shocks are as in the baseline and displayed in Table 7, Appendix A.

Throughout, the simulation outcomes of these robustness checks, shown in Table 4, maintain the intended properties of larger concentration of the wealth at the top and poor at the bottom. The results of the FHM in Columns 3 and 5 are closer to the data in Column 6 then the version with no heterogeneity in risk aversion in Columns 2 and 4 for most of the reported percentiles. However, the FHM generates too many poor agents in Column 5 due to a large dispersion of the time discount factor that prompts overly low value $\beta_L = 0.9692(6)$ for the low type in comparison to the previous experiments (in Krusell and Smith (1998), $\beta_L = 0.9858$).

[11] Since β is uniformly distributed with support $[0.9814 - 0.0182, 0.9814 + 0.0182]$, and using the same logic as before, $\beta_L = 0.9692(6)$, $\beta_M = 0.9814$, and $\beta_H = 0.9935(3)$.

[12] Since β is uniformly distributed with support $[0.9867 - 0.0067, 0.9867 + 0.0067]$, and using the same logic as before, $\beta_L = 0.9822(3)$, $\beta_M = 0.9867$, and $\beta_H = 0.9911(6)$.

Table 4: Robustness 2: wealth distribution for selected percentiles.

(1)	(2)	(3)	(4)	(5)	(6)
	Patience and risk aversion parameters				Data
(β_L, γ_L)	(0.9822(3), 1.0)	(0.9822(3), 0.5)	(0.9692(6), 1.0)	(0.9692(6), 0.5)	
(β_M, γ_M)	(0.9867, 1.0)	(0.9867, 1.0)	(0.9814, 1.0)	(0.9814, 1.0)	$SCF16^{*}$
(β_H, γ_H)	(0.9911(6), 1.0)	(0.9911(6), 5.0)	(0.9935(3), 1.0)	(0.9935(3), 5.0)	
Population	Percentage of wealth held by population				
top 1%	21.1	24.5	16.2	19.2	38.6
top 5%	47.2	52.5	44.3	46.1	65.1
top 10%	62.5	64.4	63.7	64.1	77.1
top 20%	80.2	79.5	85.5	84.6	88.3
top 30%	89.8	88.7	95.8	95.1	93.7
top 40%	94.6	94.3	99.3	99.5	96.9
top 60%	97.8	99.0	100.2	101.3	99.9
bottom 40%	2.2	1.0	-0.2	-1.3	0.1
bottom 20%	0.5	-0.1	-0.3	-0.9	-0.5
bottom 10%	0.0	-0.2	-0.3	-0.5	-0.5
	Gini coefficient				
	76.8	78.2	81.4	82.2	86.0
Measure	Model performance				
R^2	0.99	0.99	0.99	0.99	
$\hat{\sigma}$	0.03%	0.03%	0.03%	0.03%	

Note: *own calculations using data from the 2016 Survey of Consumer Finances.

4 Conclusions

This study extends the standard framework with more plausible assumptions of preference heterogeneity. It investigates how discount factor and risk aversion heterogeneity affect wealth distribution. Based on the assumption that risk aversion and patience are correlated, the model could generate a larger mass of poor agents and few extremely rich agents. The effect is more pronounced when the two rates are more evenly distributed among agents and there are no predominant types.

There are several interesting ways to extend the paper in future work. First, it is possible to use Bayesian techniques to estimate a joint distribution of discount factor and risk aversion rate. Such distribution should provide a more realistic approximation of agents' preferences. Second, the current model can be extended to allow more types of agents to study the importance of heterogeneity of the two rates.

Appendix A

Table 5: Parameter Values

variable	value	source	variable	value	source
a_g	1.01	Krusell and Smith (1998)	a_b	0.99	Krusell and Smith (1998)
u_g	0.04	Krusell and Smith (1998)	u_b	0.1	Krusell and Smith (1998)
$\bar{l}$	1/0.9	Krusell and Smith (1998)	α	0.36	Krusell and Smith (1998)
δ	0.025	Krusell and Smith (1998)	μ	0.15	Den Haan et al. (2010)
b	-2.4	Krusell and Smith (1998)			

Table 6: Transition Probabilities, Aggregate and Individual Income Shocks

	$a = a_b, e = 0$	$a = a_b, e = 1$	$a = a_g, e = 0$	$a = a_g, e = 1$
$a = a_b, e = 0$	0.525	0.35	0.03125	0.09375
$a = a_b, e = 1$	0.038889	0.836111	0.002083	0.122917
$a = a_g, e = 0$	0.09375	0.03125	0.291667	0.583333
$a = a_g, e = 1$	0.009115	0.115885	0.024306	0.850694

Source: Den Haan et al. (2010).

Table 7: Transition Probabilities, Individual Type Shock

	(β_L, γ_L)	(β_M, γ_M)	(β_H, γ_H)
(β_L, γ_L)	0.995	0.005	0.0
(β_M, γ_M)	0.005	0.99	0.005
(β_H, γ_H)	0.0	0.005	0.995

Source: own calculations.

Table 8: Transition Probabilities, Individual Type Shock

	(β_L, γ_L)	(β_M, γ_M)	(β_H, γ_H)
(β_L, γ_L)	0.995	0.005	0.0
(β_M, γ_M)	0.000625	0.99875	0.000625
(β_H, γ_H)	0.0	0.005	0.995

Source: Krusell and Smith (1998).

Appendix B

The solution algorithm is the stochastic-simulation Krusell-Smith algorithm as in Maliar et al. (2010). I adjust it in order to accommodate the heterogeneity of the three types of agents.

- *Step 1.*
  Generate and fix:
  a) grid (k,K,e,a) with polynomial distribution of 200 points on interval $[k_{min},k_{max}]$ in k dimension, linear distribution of 10 points in K dimension, 2 points a_b and a_g in a dimension, and 2 points $e=0$ and $e=1$ in e dimension,
  b) a time series of length T for the aggregate shocks,
  c) a time series of length T for the idiosyncratic employment shocks for each agent of N,
  d) a time series of length T for the type shocks for each agent of N.

  Initialise:
  a) a vector of coefficients b of the aggregate law of motion (9),
  b) capital distribution across N agents where all values are steady state values of capital,
  c) set $k'=0.9k$ for points on grid (k,K,e,a).

- *Step 2.* Given b and the aggregate law of motion (9), compute a solution to the individual problem according to *the agent's problem* in Section 2 for the three types.
  For each type:
  a) For each point on grid (k,K,e,a):
     - calculate next period period consumption:
     $$c' = \left(1-\frac{\mu u'}{\bar{l}L'}\right) w'\bar{l}e' + \mu w'(1-e') + (1-\delta+r')k' - k'(k') \tag{10}$$
     - set $c' = \max[c', 10^{-10}]$
     - find $\tilde{k}'$:
     $$\tilde{k}' = \left(1-\frac{\mu u}{\bar{l}L}\right) w\bar{l}e + \mu w(1-e) + (1-\delta+r)k - \beta E\left[\frac{1-\delta+r'}{c'^{\gamma}}\right]^{-\frac{1}{\gamma}} \tag{11}$$
     - set $\tilde{k}' = \min[\max[\tilde{k}', k_{min}], k_{max}]$
     - update $k' = \eta\tilde{k}' + (1-\eta)k'$, where η is an updating parameter.
  b) iterate on a) until $||\tilde{k}' - k'|| < k_{conv}$, where $k_{conv} = 10^{-8}$ is a precision parameter.

- *Step 3.* Use the individual policy rules computed in *Step 2* to simulate the economy over T periods. In each period:
  a) given the individual policy rules, find optimal savings for each agent of N,
  b) calculate the first moment (mean), m_{t+1}, of cross sectional distribution of capital.

- *Step 4.* Update the vector of coefficients b:
  a) regress the new time series of the first moments of cross sectional distribution of capital, m_{t+1}, from *Step 3* on m_t and a_t to obtain coefficients $\tilde{b}$.
  b) $b = \lambda\tilde{b} + (1-\lambda)b$, where λ is an updating parameter.

- *Step 5.* Iterate on *Steps 2 - 4* until convergence $||\tilde{b} - b|| < b_{conv}$, where $b_{conv} = 10^{-8}$ is a precision parameter.

References

Acemoglu Daron, and Robinson James (2006). ”Why did the West extend the franchise? Democracy, inequality, and growth in historical perspective.” *Quarterly Journal of Economics*, 115: 1167-1199.

Aiyagari S. Rao (1994). ”Uninsured idiosyncratic risk and aggregate saving.” *The Quarterly Journal of Economics 109(3)*, 659-684.

Benhabib Jess, Alberto Bisin, and Shenghao Zhu (2011). ”The Distribution of Wealth and Fiscal Policy in Economies with Finitely-Lived Agents.” *Econometrica (79)*, 123-157.

Benhabib Jess, Alberto Bisin, and Shenghao Zhu (2015). ”The Wealth Distribution in Bewley Models with Capital Income.” *Journal of Economic Theory*, 489 - 515.

Benhabib Jess, Alberto Bisin, and Shenghao Zhu (2016). ”The Distribution of Wealth in the Blanchard-Yaari Model.” *Macroeconomic Dynamics, 20*, 466-481.

Adam S.Booij, Bernard M.S.van Praag (2009). ”A simultaneous approach to the estimation of risk aversion and the subjective time discount rate.” *Journal of Economic Behavior and Organization, 70*, Pages 374-388.

Armin Falk, Anke Becker Thomas Dohmen Benjamin Enke David Huffman Uwe Sunde (2018). ”Global Evidence on Economic Preferences.” *The Quarterly Journal of Economics, 133/4:* 1645 - 1692.

Bewley Truman F. (1977). ”The Permanent Income Hypothesis: A Theoretical Formulation.” *Journal of Economic Theory 16*, 252-259.

Cagetti, Marco (2003). ”Wealth Accumulation Over the Life Cycle and Precautionary Savings.” *Journal of Business and Economic Statistics, Vol. 21,* 339-353.

Cagetti Marco and De Nardi Mariacristina (2006). ”Entrepreneurship, frictions, and wealth.” *Journal of Political Economy, 114(5)*, 835 - 870.

Christopher Carroll, Jiri Slacalek, and Kiichi Tokuoka (2017). ”The distribution of wealth and the marginal propensity to consume.” *Journal of Quantitative Economics, (8)*, 977 - 1020.

Castaneda Ana, Diaz-Gimenez Javier and Rios-Rull Jose-Victor (1997). ”Unemployment spells, cyclically moving factor shares and income distribution dynamics.” *Manuscript. Federal Reserve Bank of Minneapolis.*

Castaneda Ana, Diaz-Gimenez Javier and Rios-Rull Jose-Victor (2003). ”Accounting for the us earnings and wealth inequality”. *Journal of political economy 111(4)*, 818 - 857.

Cochrane John H. (1989). ”The Sensitivity of Tests of the Intertemporal Allocation of Consumption to Near-Rational Alternatives.” *American Economic Review, vol. 79(3)*, 319 - 337.

Coen-Pirani, Daniele (2004). ”Effects of Differences in Risk Aversion on the Distribution of Wealth.” *Macroeconomic Dynamics, Vol. 8,* 617-632.

Cowell, Frank, Eleni Karagiannaki and Abigail McKnight (2012). ”Mapping and Measuring the Distribution of Household Wealth: A Cross-Country Analysis.” *CASE working paper 165, London School of Economics.*

Cozzi, Marco (2012). ”Risk Aversion Heterogeneity, Risky Jobs and Wealth Inequality.” *Queen’s University, Department of Economics, Working Papers, 1286*.

Cozzi, Marco (2015). ”The Krusell-Smith Algorithm: Are Self-Fulfilling Equilibria Likely?” *Computational Economics, Vol. 46, 4,* 653 - 670.

Cubeddu, Luis and Rios-Rull, Jose-Victor (1997). "Marital risk and capital accumulation." *Staff Report 235, Federal Reserve Bank of Minneapolis.*

Davies J.B. Shorrocks A.F. (2000). "The distribution of wealth." in A.B. Atkinson and F. Bourguignon (eds), *Handbook of Income Distribution,* 605 - 675.

De Nardi Mariacristina (2004). "Wealth inequality and intergenerational links". *Review of Economic Studies, 71(3)*:743 - 768.

De Nardi Mariacristina and Yang Fang (2016). "Wealth inequality, family background, and estate taxation." *Journal of Monetary Economics 77*, 130 - 145.

De Nardi Mariacristina (2016). "Quantitative models of wealth inequality: A survey." *Working Paper 21106, National Bureau of Economic Research.*

Den Haan, Wouter J. (2010). "Assessing the accuracy of the aggregate law of motion in models with heterogeneous agents." *Journal of Economic Dynamics and Control, 34*, 79 - 99.

Den Haan, Wouter J., Judd, Kenneth L., and Juillard, Michel (2010). "Computational Suite of Models with Heterogeneous Agents: Incomplete Markets and Aggregate Uncertainty." *Journal of Economic Dynamics and Control, 34*, 1 - 3.

Dohmen, Thomas, Armin Falk, David Huffman, and Uwe Sunde (2010). "Are Risk Aversion and Impatience Related to Cognitive Ability?" *American Economic Review, 100 (3)*, 1238 - 60.

Facundo Alvaredo, Lucas Chancel, Thomas Piketty, Emmanuel Saez and Gabriel Zucman (2017). "Global Inequality Dynamics: New Findings from WID.world." *American Economic Review, 107(5)*, 404 - 409.

Shane Frederick, George Loewenstein and Ted O'Donoghue (2002). "Time Discounting and Time Preference: A Critical Review." *Journal of Economic Literature, XL*, 351 - 401.

Gale W.G. Scholz J.K. (1994). "Intergenerational transfers and the accumulation of wealth." *Journal of Economic Perspectives,* 145 - 160.

Christian Gollier and Richard Zeckhauser (2005). "Aggregation of Heterogeneous Time Preferences." *Journal of Political Economy 113*, 878-896.

Haggard, S., and Kaufman, R. (2012). "Inequality and regime change: Democratic transitions and the stability of democratic rule." *American Political Science Review, 106(3)*, 495 - 516.

Haggard, S., and Kaufman, R. (2016). "Dictators and Democrats. Masses, Elites, and Regime Change." *Princeton University Press.*

Haliassos, Michael, and Carol C. Bertaut (1995). "Why Do So Few Hold Stocks?" *Economic Journal, 105(432),* 1110-29.

Hendricks, Lutz (2007). "How Important is Discount Rate Heterogeneity for Wealth Inequality?" *Journal of Economic Dynamics and Control, Vol. 31*, 3042-3068.

Hubbard, R. Glenn, Skinner, Jonathan and Zeldes, Stephen P. (1995). "Precautionary saving and social insurance". *Journal of Political Economy 103*, 360 - 399.

Huggett, M. (1993). "The Risk-Free Rate in Heterogeneous-Agents, Incomplete Markets Economies." *University Rochester, Center for Economic Research, Working Paper 399*, 953-969.

Kotlikoff L.J. Summers L.H. (1981). "The role of intergenerational transfers in aggregate capital accumulation." *Journal of Political Economy,* 372 - 391.

Kotlikoff L.J. Summers L.H. (1988). ”The contribution of intergenerational transfers to total wealth: a reply.” *in D. Kessler and A. Masson (eds), Modelling the Accumulation and Distribution of Wealth ,Clarendon Press, Oxford,* 53 - 67.

Krueger, D., Mitman, K., and Perri, F. (2016). ”Macroeconomics and Household Heterogeneity.” *in Taylor, JB and Uhlig, H., editors. Handbook of Macroeconomics, Amsterdam, North Holland,* 843-921.

Krusell Per and Smith Anthony A. (1996): ”Income and Wealth Heterogeneity in the Macroeconomy.” *Journal of Political Economy, 106*, 867 - 896.

Krusell Per and Smith Anthony A. (1998): ”Income and Wealth Heterogeneity in the Macroeconomy.” *Journal of Political Economy, 106*, 867 - 896.

Leitner Sebastian (2016): ”Drivers of wealth inequality in euro area countries: the effect of inheritance and gifts on household gross and net wealth distribution analysed by applying the Shapley value approach to decomposition”. *European Journal of Economics and Economic Policies: Intervention, 13/1*, 114 - 136.

Lucas Robert E., Jr. (1987). ”Models of Business Cycles.” *New York: Basil Blackwell,* pp. 115.

Lilia Maliar, Serguei Maliar and Fernando Valli (1987). ”Solving the Incomplete Markets Model with Aggregate Uncertainty Using the Krusell-Smith Algorithm.” *Journal of Economic Dynamics and Control 34,* 42 - 49.

Mankiw, N. Gregory, and Stephen Zeldes (1991). ”The Consumption of Stockholders and Nonstockholders.” *Journal of Financial Economics, 29(1),* 97-112.

Modigliani F. (1988). ”The role of intergenerational transfers and life cycle saving in the accumulation of wealth.” *Journal of Economic Perspectives,* 15 - 40.

Marcet Albert and Singleton Kenneth J. (1998). ”Equilibrium Asset Prices and Savings of Heterogeneous Agents in the Presence of Incomplete Markets and Portfolio Constraints”. *Universitat Pompeu Fabra Economics WP No. 319.*

Oxfam (2016). ”An economy for 1%.” *Oxfam International,* 210 Oxfam Briefing Paper.

Quadrini Vincenzo (1999): ”The Importance of Entrepreneurship for Wealth Concentration and Mobility”. *Review of Income and Wealth, 45*, 1 - 19.

Quadrini Vincenzo (2000). ”Entrepreneurship, Savings and Social Mobility.” *Review of Economic Dynamics, 3*, 1 - 40.

Shao Enchuan, and Silos Pedro (2014). ”Wealth Inequality and Employment Fluctuations” *Economic Inquiry, Vol.52*, 778-795.

Shin Inyong (2012). ”Income inequality and economic growth.” *Economic Modelling, Vol.29, 5*, 2049 - 2057.

Stiglitz Joseph Eugene (2012). ”The price of inequality: How today’s divided society endangers our future.” *WW Norton & Company.*

Piketty Tomas (2014). ”Capital in the Twenty First Century.” *Harvard University Press*.

Vissing-Jorgensen, Annette (2003). ”Perspectives on Behavioral Finance: Does ’Irrationality’ Disappear with Wealth? Evidence from Expectations and Actions”. *NBER Macroeconomics Annual*.

Wise, David A., and Steven F. Venti. (2001): ”Choice, Chance, and Wealth Dispersion at Retirement.” *Issues in Aging in the United States and Japan. Ed. David A. Wise. University of Chicago Press*.