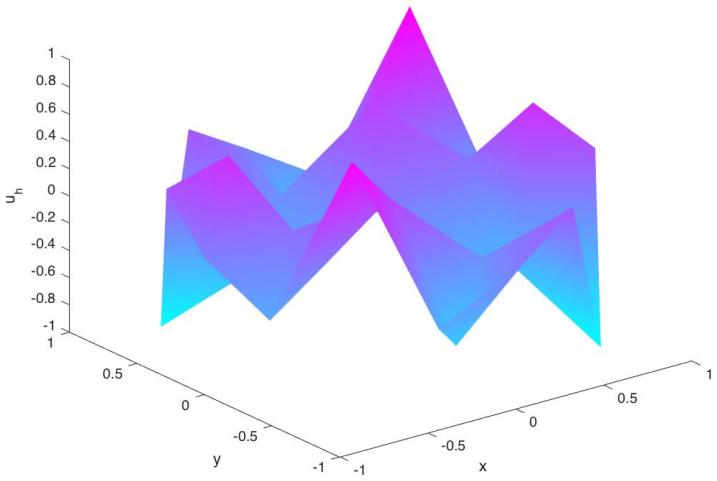
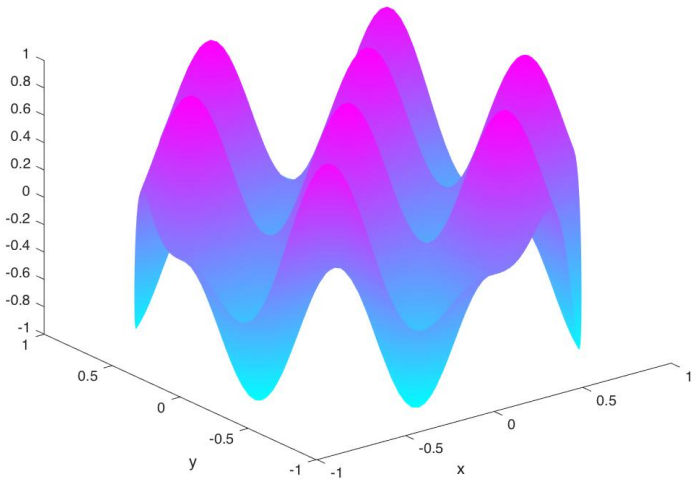
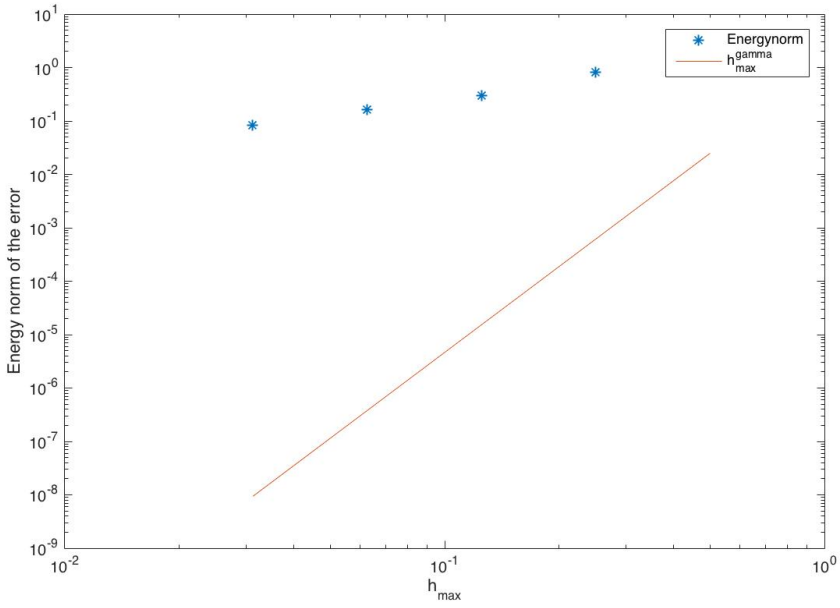
Part A



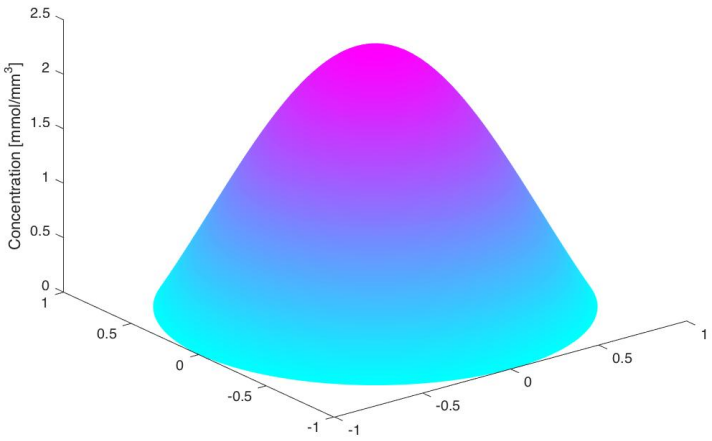


h=1/2

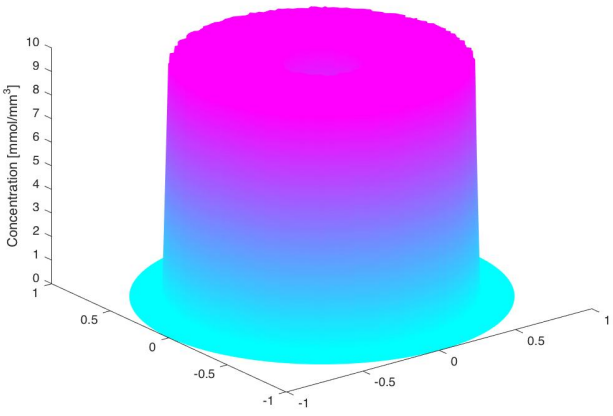
h=1/32



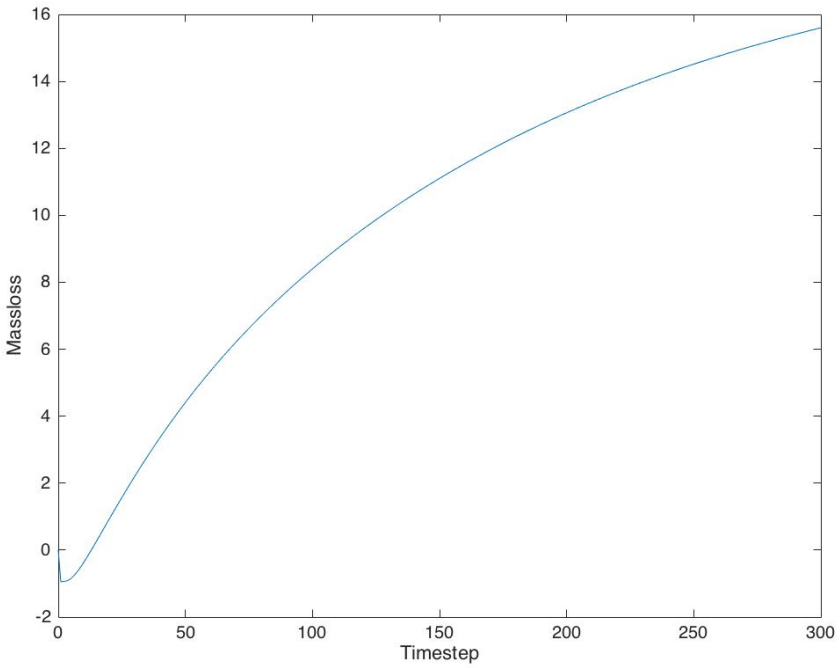
Gamma is around 5


Concentration [mmol/mm^3]
2.5
2
1.5
1
0.5
0
1
0.5
0
-0.5
-1
-1
-0.5
0
0.5
1

T=30


Concentration [mmol/mm^3]
10
9
8
7
6
5
4
3
2
1
0
1
0.5
0
-0.5
-1
-1
-0.5
0
0.5
1

T=0


Massloss
16
14
12
10
8
6
4
2
0
-2
0
50
100
150
200
250
300
Timestep

```
close all
clear all
%comment
geometry =



B2A
@circleg;
error2 = zeros(5,1);
step = zeros(5,1);
for h = 1:5
    step(h) = 1/2^h;
    hmax = 1/2^h;
    [p,e,t] = initmesh(geometry ,'hmax',hmax);
    np = size(p,2);
    A = sparse(np, np);
    bk = zeros(np,1);
    r = zeros(np,1);
    I = eye(length(p));

    for K = 1:size(t,2);                    % loop over the triangles
        nodes = t(1:3,K);                     % find triangle K?s nodes
        % compute the (3 x 3) stiffness matrix AK
        nodeCoordinates = p(:,nodes);
        [area, b ,c] = Gradients(nodeCoordinates(1,:), nodeCoordinates(2,:));
        CoG = sum(nodeCoordinates, 2)/3;
        AK = area*[b*b' + c*c'];
        A(nodes,nodes) = A(nodes,nodes)+AK;   % add AK(i,j), i,j=1,2,3,
        % to A(nodes(i),nodes(j))

        %Compute bik
        bk(nodes,1) = bk(nodes,1)+ [1; 1; 1].*myFun(CoG(1), CoG(2))*area/3;

    end

    %Compute rik by iterating through the edges
%     for E = 1:size(e,2)
%         nodes = e(1:2, E);
%         x = p(1, nodes);
%         y = p(2, nodes);
%         length_E = sqrt((x(1)-x(2))^2+(y(1)-y(2))^2);
%         r(nodes,1) = r(nodes,1) + length_E*myExacFun(x,y)'/2;
%     end

    A(e(1,:),:) = I(e(1,:),:);
    bk(e(1,:)) = 0;

    fixed = unique([e(1,:) e(2,:)]);            % boundary nodes

    g = myExacFun(p(1, fixed),p(2, fixed));
    free = setdiff([1:np],fixed);               % interior nodes
    bk = bk(free)-A(free,fixed)*g';             % modify load vector
    A = A(free,free);                           % modify stiffness matrix
    Z = zeros(np,1);                            % allocate solution vector
    Z(fixed) = g';                              % insert fixed node values
    Z(free) = A\bk;                             % solve for free node values

    %Error
    exactSol = myExacFun(p(1, free),p(2, free));
    error1 = exactSol'-Z(free);
    error2(h) = sqrt(error1'*A*error1);
    if (h == 1 | h == 5)
         pdesurf(p,t,Z)
         xlabel('x','fontsize',16)
         ylabel('y','fontsize',16)
         zlabel('u_h','fontsize',16)
         figure
    end
    %pdemesh(p,e,t)
    %     figure
end

loglog(step,error2,'*')
gamma = abs((error2(end)-error2(1))/(1/2-1/32));
hold on
loglog(step,step.^gamma);
xlabel('h_{max}','fontsize',16)
ylabel('Energy norm of the error','fontsize',16)
legend('Energynorm', 'h_{max}^{gamma}')
```

B2C

```
close all
clear all
tic
L = 300;                % number of time steps
T = 30;                  % final time
t = linspace(0,T,L+1);   % time grid
h = 5;
alph = 0.01;

geometry = @circleg;
hmax = 1/2^h;
[p,e,t2] = initmesh(geometry ,'hmax',hmax);
U = labfun(p(1,:), p(2,:))';          % inital condition

A = StiffMat2D(p,t2,1);
M = MassMat2D(p, t2);
b0 = LoadVec2D(p, t2, @labfun);
b1 = LoadVec2D(p, t2, @labfun2);
np = size(p,2);

fixed = unique([e(1,:) e(2,:)]);            % boundary nodes
free = setdiff([1:np],fixed);               % interior nodes
g = zeros(size(fixed,2),1);

b0 = b0(free);             % modify load vector
b1 = b1(free);
A = A(free,free);
M = M(free,free);
U(fixed) = 0;

Mass0 = 0;
UR = U;
for K = 1:size(t2, 2);
    nodes = t2(1:3,K);
    area = polyarea(p(1,nodes), p(2,nodes));
    Mass0 = Mass0 + 1/3*sum(U(nodes))*area;
end

pdesurf(p,t2,U)
MassT = zeros(1,L);
zlabel('Concentration [mmol/mm^3]', 'fontsize', 16);

for l = 1:L
    k = t(l+1) - t(l);
    U(free) = (M+k/2*A*alph)\((M- k*alph/2*A)*U(free)+k/2*(b1+b0));    %nota that b should be
zero...
    b0 = 0;
    %           pdesurf(p,t2,U)
    %           drawnow
    %           pause(T/100);
    for K = 1:size(t2, 2);
        nodes = t2(1:3,K);
        area = polyarea(p(1,nodes), p(2,nodes));
        MassT(l) = MassT(l) + 1/3*sum(U(nodes))*abs(area);
    end
end
figure
pdesurf(p,t2,U);
zlabel('Concentration [mmol/mm^3]', 'fontsize', 16)

figure
MassLoss = Mass0 - MassT;
plot(0:300, [0 MassLoss]);
ylabel('Massloss', 'fontsize', 16);
xlabel('Timestep', 'fontsize', 16);
toc
```

LOAD2VEC

```
function b = LoadVec2D(p,t,f)
np = size(p,2);
nt = size(t,2);
b = zeros(np,1);
for K = 1:nt
    loc2glb = t(1:3,K);
    x = p(1,loc2glb);
    y = p(2,loc2glb);
    area = polyarea(x,y);
```

```
    bK = [f(x(1),y(1));
        f(x(2),y(2));
        f(x(3),y(3))]/3*area; % element load vector
    b(loc2glb) = b(loc2glb) ...
        + bK; % add element loads to b
end
```

MyFun

```
function z = myFun(x,y)
z =8*pi^2*sin(2*pi*x).*sin(2*pi*y);
```

STIFF2MAT

```
function A = StiffMat2D(p,t,a)
np = size(p,2);
nt = size(t,2);
A = sparse(np,np);
for K = 1:nt
    loc2glb = t(1:3,K); % local-to-global map
    x = p(1,loc2glb); % node x-coordinates
    y = p(2,loc2glb); % node y-
    [area,b,c] = Gradients(x,y);
    xc = mean(x); yc = mean(y); % element centroid
    abar = 1; % value of a(x,y) at centroid
    AK =    (b*b'...
        +c*c')*area; % element stiffness matrix


    A(loc2glb,loc2glb) = A(loc2glb,loc2glb) ...
        + AK; % add element stiffnesses to A
end
size(A)
```

MyExacFun

```
function z = myExacFun(x,y)
z = sin(2*pi*x).*sin(2*pi*y);
```

Gradients

```
function [area,b,c] = Gradients(x,y)
area=polyarea(x,y);
b=[y(2)-y(3); y(3)-y(1); y(1)-y(2)]/2/area;
c=[x(3)-x(2); x(1)-x(3); x(2)-x(1)]/2/area;
```

LabFun2

```
function z = labfun2(x,y)
z = 0;
```