PAGERANK PARAMETERS

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The most important page on the web

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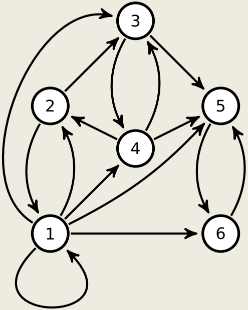
PageRank details



$$\rightarrow \underbrace{\begin{bmatrix} 1/6 & 1/2 & 0 & 0 & 0 & 0 \\ 1/6 & 0 & 0 & 1/3 & 0 & 0 \\ 1/6 & 1/2 & 0 & 1/3 & 0 & 0 \\ 1/6 & 0 & 1/2 & 0 & 0 & 0 \\ 1/6 & 0 & 1/2 & 1/3 & 0 & 1 \\ 1/6 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}}_{\mathbf{P}}$$

$P_{ij} \geq 0$
$\mathbf{e}^T \mathbf{P} = \mathbf{e}^T$

"jump" $\rightarrow$ $\mathbf{v} = [\, \frac{1}{n} \; \dots \; \frac{1}{n} \,]^T$

$v_i \geq 0$
$\mathbf{e}^T \mathbf{v} = 1$

Markov chain $\quad \left[\alpha \mathbf{P} + (1-\alpha)\mathbf{v}\mathbf{e}^T\right] \mathbf{x} = \mathbf{x}$

unique $\mathbf{x} \Rightarrow x_j \geq 0$, $\mathbf{e}^T \mathbf{x} = 1$.

Linear system $\quad (\mathbf{I} - \alpha \mathbf{P})\mathbf{x} = (1-\alpha)\mathbf{v}$

Ignored $\quad$ *dangling nodes patched back to* $\mathbf{v}$

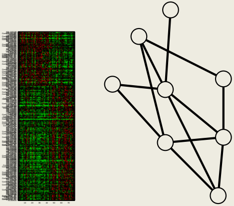
algorithms later

Other uses for PageRank

What else people use PageRank to do

GeneRank

Morrison et al. GeneRank, 2005



Use $(\mathbf{I} - \alpha \mathbf{G}\mathbf{D}^{-1})\mathbf{x} = \mathbf{w}$ to find "nearby" important genes.

Note New paper LabRank with a random scientist?

ProteinRank
ObjectRank
EventRank
IsoRank
Clustering
Sports ranking
Food webs
Centrality
Reverse PageRank
FutureRank
SocialPageRank
BookRank
ArticleRank
ItemRank
SimRank
DiffusionRank
TrustRank
TweetRank

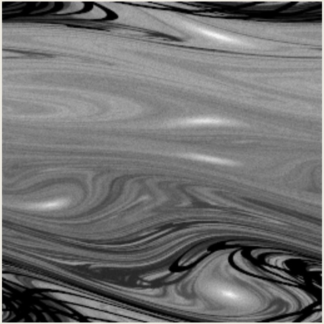
Ulam Networks

Chirikov map

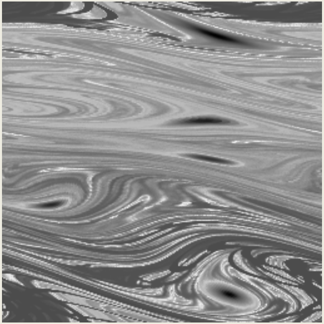
$y_{t+1} = \eta y_t + k \sin(x_t + \theta_t)$
$x_{t+1} = x_t + y_{t+1}$

Ulam network

1. divide phase space into uniform cells
2. form $\mathbf{P}$ based on trajectories.



$\log(\mathrm{E}[\mathbf{x}(A)])$



$\log(\mathrm{Std}[\mathbf{x}(A)])/\log(\mathrm{E}[\mathbf{x}(A)])$

$A \sim \mathrm{Beta}(2, 16)$

Note *White is larger, black is smaller*

Google matrix, dynamical attractors, and Ulam networks, Shepelyansky and Zhirov, arXiv

Choosing alpha

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What is alpha? There's no single answer.

Ask yourself, why am I computing PageRank? Then use the best value for your application.

web-search	$\rightarrow$	tune α for the best feature vector
node centrality	$\rightarrow$	understand what random jumps mean in your graph
find important nodes in a web-graph	$\rightarrow$	use the random surfer interpretation

Author	$\boldsymbol{\alpha}$
Brin and Page (1998)	0.85
Najork et al. (2007)	0.85
Litvak et al. (2006)	0.5
Pan el al. (2004)	0.15
Algorithms (...)	≥ 0.85
Experiment	???

The PageRank limit value

Singular? $(\mathbf{I} - \alpha \mathbf{P})\mathbf{x} = (1-\alpha)\mathbf{v}$

$$\mathbf{P} = \mathbf{X} \begin{bmatrix} \mathbf{I} & 0 \\ 0 & \mathbf{J}_1 \end{bmatrix} \mathbf{X}^{-1}$$

$$\left(\mathbf{I} - \alpha \mathbf{X} \begin{bmatrix} \mathbf{I} & 0 \\ 0 & \mathbf{J}_1 \end{bmatrix} \mathbf{X}^{-1} \right) \mathbf{x} = (1-\alpha)\mathbf{v}$$

$$\mathbf{X} \left(\mathbf{I} - \alpha \begin{bmatrix} \mathbf{I} & 0 \\ 0 & \mathbf{J}_1 \end{bmatrix} \right) \mathbf{X}^{-1}\mathbf{x} = (1-\alpha)\mathbf{v}$$

$$\left(\mathbf{I} - \alpha \begin{bmatrix} \mathbf{I} & 0 \\ 0 & \mathbf{J}_1 \end{bmatrix} \right) \mathbf{y} = (1-\alpha)\mathbf{z}$$

$$(1-\alpha)\mathbf{y}_1 = (1-\alpha)\mathbf{z}_1$$

$$(\mathbf{I} - \alpha \mathbf{J}_2)\mathbf{y}_2 = (1-\alpha)\mathbf{z}_2$$

Boldi et al. 2003: PageRank as a function of the damping parameter

TotalRank

$$\mathbf{t} = \int_0^1 \mathbf{x}(\alpha)\, d\alpha$$

Proposed by Boldi et al. (2005) as a parameter free PageRank.

Generalized PageRank

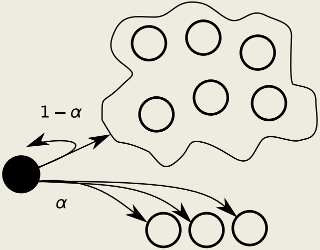
PageRank	$(\mathbf{I} - \alpha \mathbf{P})\mathbf{x} = (1-\alpha)\mathbf{v}$ $\mathbf{x} = \sum_{i=0}^{\infty} (1-\alpha)(\alpha^i)\mathbf{P}^i \mathbf{v}$
Generalized PageRank	$\mathbf{y} = \sum_{i=0}^{\infty} f(i)\mathbf{P}^i \mathbf{v}$ $\sum_i f(i) < \infty$
TotalRank	$f(i) = \frac{1}{i+1} - \frac{1}{i+2}$
LinearRank	$\ldots$
HyperRank	$\ldots$

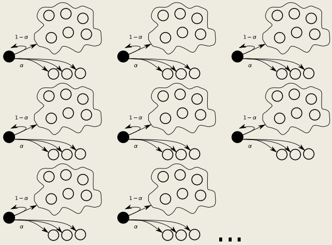
Baeza-Yates et al. 2006

Pick a distribution

Multiple surfers should have an impact!

Each person picks α_i from distribution A





$\downarrow$ $\qquad\qquad\qquad\qquad$ $\downarrow$

$\mathbf{x}(\mathrm{E}[A])$ $\qquad\qquad\qquad\qquad$ $\mathrm{E}[\mathbf{x}(A)]$

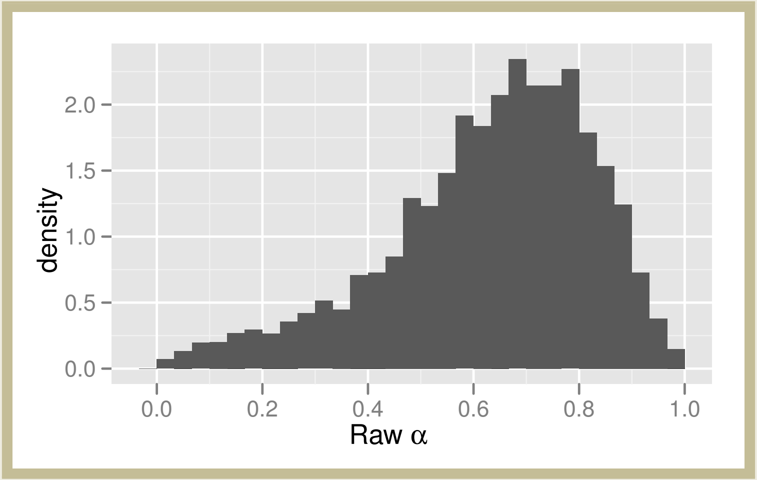
$\searrow$ $\qquad\qquad\qquad\qquad$ $\swarrow$

$$\mathbf{x}(\mathrm{E}[A]) \neq \mathrm{E}[\mathbf{x}(A)]$$

$\text{TotalRank} : \mathrm{E}[\mathbf{x}(A)] : A \sim U[0, 1]$

Constantine & Gleich, Internet Mathematics, in press.

From users



Sample mean $\bar{\mu} = 0.631$.

Gleich et al., WWW2010

Note *257,664 users from Microsoft toolbar data*

Choosing personalization

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Personalization choices

Application specific

- GeneRank : $\mathbf{v}$ = normalized microarray weights
- TopicRank: $\mathbf{v}$ = pages on the same topic
- TrustRank: $\mathbf{v}$ = only pages known to be good
- BadRank: $\mathbf{v}$ = only pages known to be bad (an reverse the graph)

Super-personalized

- Set $\mathbf{v}$ to have only a single non-zero : $\mathbf{v} = \mathbf{e}_i$.

Personalized PageRank

$$\mathbf{B} = (1-\alpha)(\mathbf{I} - \alpha\mathbf{P})^{-1}$$

B_{ij} = "personalized score of page i when jumping to page i"

Related methods

PageRank history

See Vigna 2010: Spectral Ranking and
Franceschet 2010: PageRank: Standing on the shoulder of giants.

Let $\mathbf{A}$ be the adjacency matrix of a graph.

PageRank	$(\mathbf{I} - \alpha\mathbf{P})\mathbf{x} = (1-\alpha)\mathbf{v}$	$(\alpha\mathbf{P} + (1-\alpha)\mathbf{v}\mathbf{e}^T)\mathbf{x} = \mathbf{x}$
Seeley 1949		$\mathbf{P}\mathbf{x} = \mathbf{x}$
Wei 1952		$\mathbf{A}^T\mathbf{x} = \mathbf{x}$
Katz 1953	$(\mathbf{I} - \alpha\mathbf{A})\mathbf{x} = \mathbf{e}$	
Hubbell 1965		$\mathbf{A}^T\mathbf{x} = \mathbf{x} + \mathbf{v}$

Graph centrality

For a graph G, a score assigned to each vertex $v \in V$ is a *centrality score* if larger scores are "more central" vertices and the score is independent of the labeling on the vertices.

Open issues

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The Problem

- We can derive gazillions of small variants
- Which ones are meaningful?
- Justify your existence!
- But nobody does :(
- Note: the same happens for the web

From Vigna, A history of spectral ranking, MMDS2010

Other issues

QUESTIONS?