KARLSTADS UNIVERSITET - INSTITUTIONEN FÖR INGENJÖRSVETENSKAP OCH FYSIK

FYGA 25 EXPERIMENTELL PROBLEMLÖSNING

Rolling Cylinders

Handledare
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Abstract/Summary:

This experiment is about how the time for a cylinder that rolls over an inclined plane changes by different factors. The independent factors in this experiment were: the slope of the plane , length of the distance that the cylinder rolled over, and cylinder's inner and outer diameter.
The purpose of the laboratory was to find how the above invoices affect the time it takes for the cylinder to roll down the length and find a formula for the time that it takes for the cylinder. We varied the above invoices separately with the purpose of to see how those variables affect the time. The data from each measurement were logarithmized to get a linear fit curve. This process was done in Matlab.
To sum up, the final formula is as follow:

$$t = 0.25 * L^{1/2} * sin(v)^{-1/2} * g^{-1/2} + 0.28 * L^{1/2} * g^{-1/2} * sin(v)^{-1/2} * d^{2} * D^{-2}$$

In this formula it is the time that it takes for a cylinder to roll over an inclined plane.

1. Introduction:

This report discusses an experiment to study the relationship of how time for a cylinder that rolls over an inclined plane changes by a number of factors. These factors divide into separate measurement series where one factor is varied and the other factors are constant. In this way, the analysis of the factors can generate a final formula.

2. Aim

The purpose of the lab is to create an understanding of how different factors can affect the time on a rolling cylinder such as the inside radius, outside radius, angle and the length of the plane and find a general formula that would be valid independent of the environment:

3. Method

Labs began by making an estimate of how long a rolling cylinder needs to roll down an inclined plane. Then factors that can affect the cylinder were determined which were the inner radius and the outer radius, the length of the plane and angle in the plane. Then two tests were made one for hollow cylinders and one for solid cylinders and through the two tests it was discovered that the inner and outer radius only affects hollow cylinders while solid cylinders rolled just as fast regardless of how big the outer radius is, therefore two different series of measurements were made one for hollow and one for solid cylinders. Since two different measurement series are to be made for solid cylinders, k_1 was chosen as constant, while for non-solid cylinders, k_2 was chosen as constant.
Then the experiment was started with solid cylinders that had invoices length and angle one factor investigated at a time and keep the other constant that affected rolling time and it was done by reading a batch at the beginning and at the end to calculate the time using the Pasco Smartgate program in addition that angle varied by raising and lowering the plant at the same time a caliper and a tape measure are used to measure the length and the inner and outer radius. Then the experiment was done on hollow cylinders that also had inner and outer contours - the radius, length and angle. For each experiment, 5 different cylinders were used and five series of measurements were made. Finally everything was recorded and calculated.

4. Material

- Wooden blocks
- Wooden board
- Cylinders with different dimensions, inner and outer radius. Solid and non-solid.
- Lasers
- Stand
- Ruler, tape measure, caliper
- Pasco Capstone
- Pasco Smartgate program

5. Theory

How long does it take for a cylinder to roll down an inclined plane?
The theory in this lab will take place on two different occasions. Then four different independent variables will be tested and they are ,diameter of the cylinders i.e. inner diameter (d) and outer diameter (D), the angle between the plane and the floor (v), the length where the cylinder starts to roll to the last point where the time is measured (L).
At the first opportunity, only the normal cylinders that have no hole, which means that there is no inner diameter, should be used. Then only one variable should be varied and keep the other independent variables constant. Then the cylinder will roll 5 times and the average value must be calculated and put on the table.

To calculate the time the cylinder needs to roll down an inclined plane, the *Timing Pasco Capstone* program will be used. The program consists of two lasers that are connected to the computer, one laser will be placed where the cylinder starts to roll and the other will be placed at the last point the cylinder rolls (see the figure below). When a cylinder rolls and hits the first laser the program starts counting the time and when it hits the second laser the program gives the time taken by the cylinder.

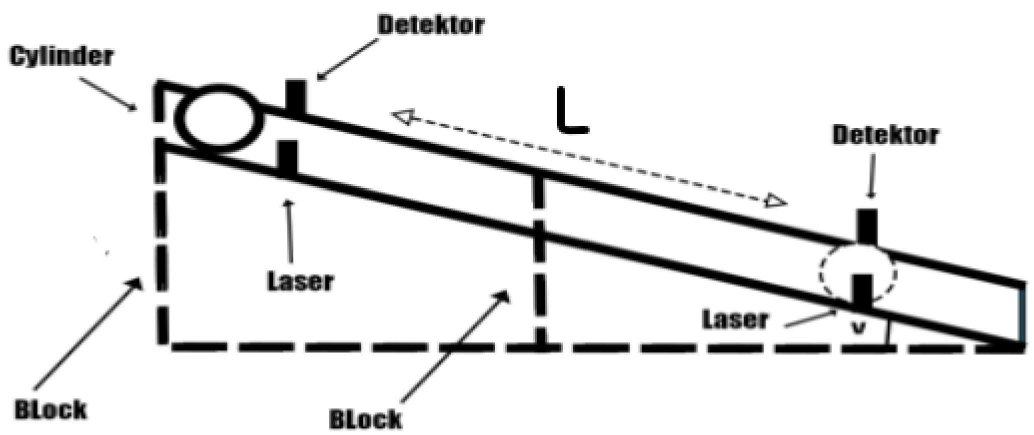

Figure 1 :The figure shows what the path should look like when the experiment would be carried out, there is a cylinder on top of the inclined plan that is to be rolled down, and there are two lasers and detectors that will count the time for the cylinder to roll down

6. Result

Variable definition:

Table 1: the definition of all variables that have been used.

Definition	Variable
Outer Diameter	D
Inner Dimater	d
Time for solid cylinder	ts
Time	t
Time for non-solid cylinder	t-ts
Angle of the plane	v
Length of the plane	L
Gravitational acceleration	g = 9.82
constants	k_1, k_2

Solid Cylinder

Table 2: The table below reports the measurement values for how long (ts) it took for three solid cylinders with different diameters (D) to roll down the plane. Constant values are the length of the plane (l) = 1.40 m and the plane's angle (v) = 0.09 rad.

D [m]	Ts [s]
0.04	1.89
0.06	1.86
0.08	1.80

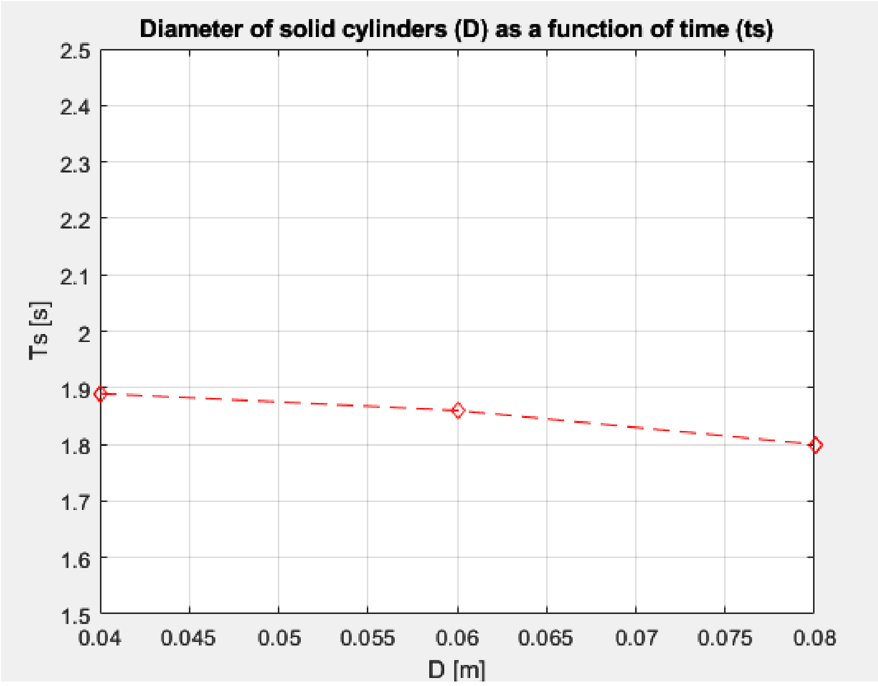

Figure 2: Figure 2 shows the data from Table 2 and a fit plotted on a graph.

Based on the measured values from Table 1 and the graph in Figure 1, it can be assumed that how fast a solid cylinder rolling down a plane solely depends on the angle and length of the plane, not on the diameter of the cylinder.

The length of the plane, (L)

Table 3: The table below shows the measured values for how the length of the plane (L) affects the time (ts) it takes for a solid cylinder to roll down the plane. Constant values are the diameter of the cylinders (D) = 0.08 m, and the angle of the plane (v) = 0.091 rad.

L [m]	ts [s]
0.60	1.03
0.80	1.28
1.00	1.48
1.20	1.64
1.40	1.80

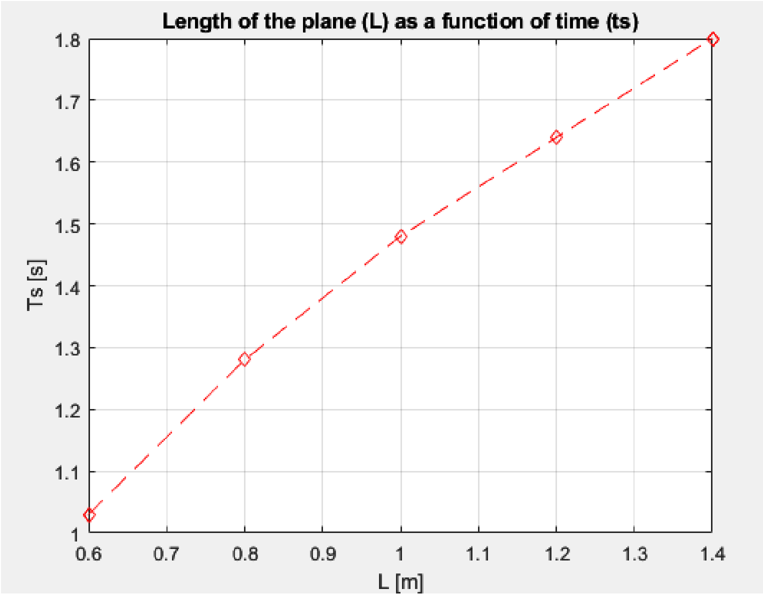

Figure 3: The figure shows the data and a fit for the data from Table 3

To obtain the exponent x to the length of the plane in the approach, the values in Table 6 are logarithmized and are plotted into a new graph.

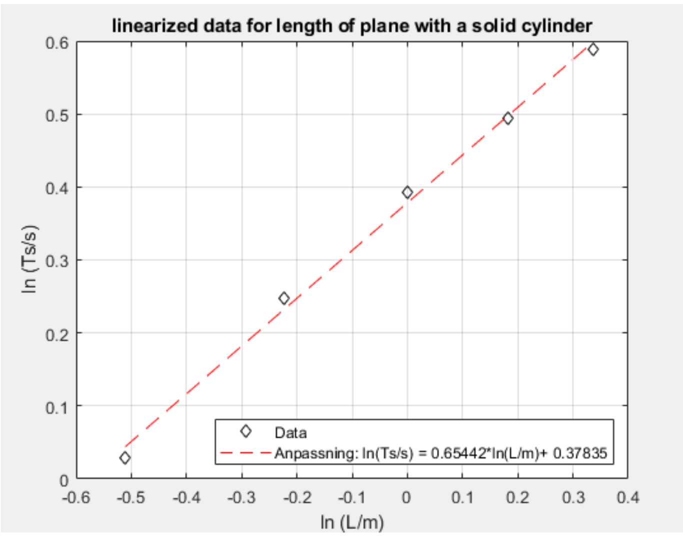

Figure 4: The figure shows a linearized version of Figure 3

According to Figure 4, the exponent x for the length of the plane (L) can be derived. The index is given by calculating the slope of the graph in Figure 4. The slope is calculated by selecting two points on the graph and calculating the difference between these points. where the two differences are separated from each other.

$$x = \Delta ln(ts/s) \, / \, \Delta ln(L/m) = (0.38 - 0.26) \, / \, 0 - (-0.2) = 0.6 \approx 0.5 = 1/2$$

Angle, v

Table 4: The table below shows the measured values for how the angle of the plane (v) affects the time (ts) it takes for a solid cylinder to roll down the plane. Constant values are the length of the track (L) = 1.4 m, as well as diameter of the cylinders (D) = 0.08 m.

Sin v [rad]	ts [s]
0.0575	2.55
0.0775	2.06
0.0900	1.78
0.1150	1.59
0.1275	1.46

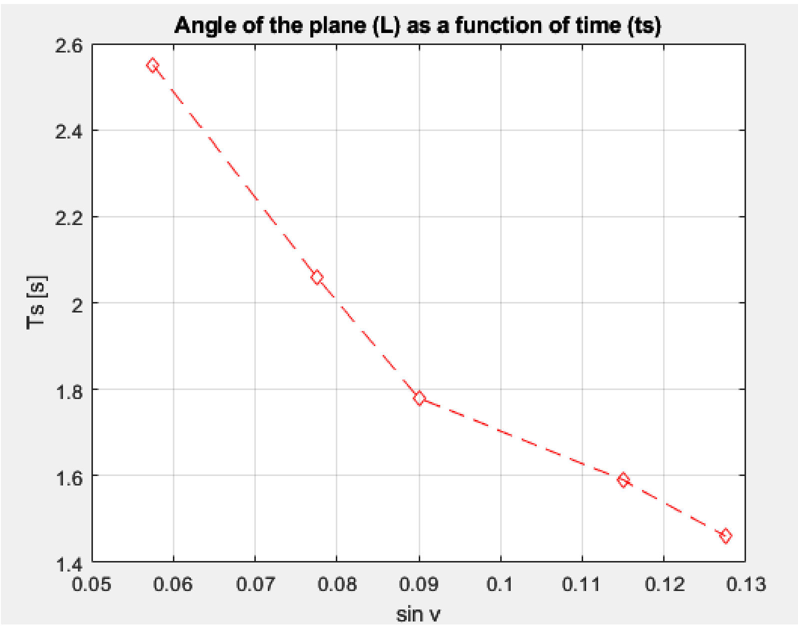

Figure 5: The figure shows the data and a fit for the data from Table 3.

To obtain the exponent *w* to sin(v) in the approach, the values in Table 8 are logarithmized and plotted to a new grap.

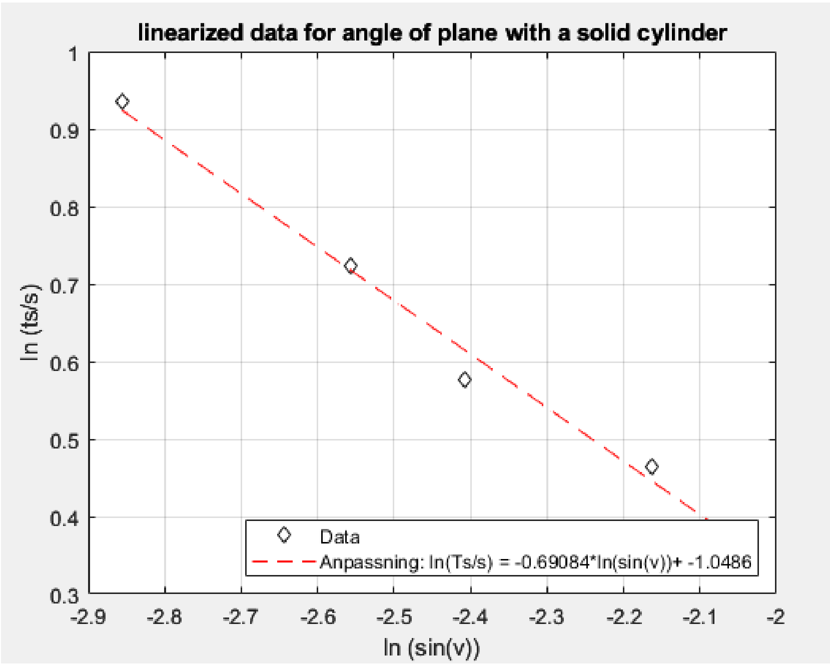

Figure 5: This figure shows a linearized version of Figure 4.

From Figure 5, the exponent w of sin(v) can be derived. The index is calculated by
The slope of the graph in Figure 5. The slope is calculated by selecting two points on the graph and calculating the difference between these points. where the two differences are separated from each other.

$$w = \Delta ln(ts/s) / \Delta ln(sin(v)) = (0.89 - 0.61) / (- 2.8) - (- 2.4) = - 0.7 \approx - 0.5 = - 1/2$$

non-Solid Cylinder:

Innerdiameter, I

Table 4: The table below shows the measured values for how a varying inner diameter (d) of a cylinder affects the time (t) it takes for the cylinder to roll down the plane. Constant values are those : the length of plane (L) = 1.4 m, the angle of the plane (v) = 0.09 rad, and the outer diameter of the cylinders (D) = 0.08 m. The time (ts) = 1.96 s for a solid cylinder on the plane.

d [m]	t [s]	t-ts [s]
0.01	1.963	0.003
0.025	1.97	0.01
0.04	2.01	0.02
0.055	2.05	0.05
0.07	2.12	0.28

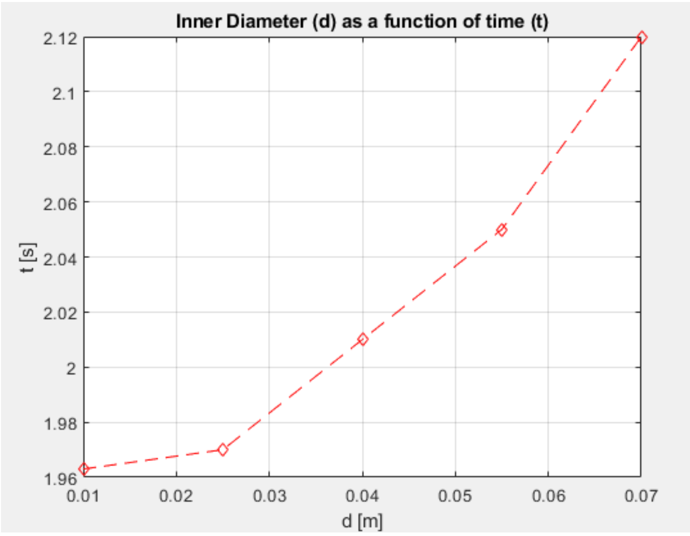

Figure 6 : The figure shows the data and a fit for the data from Table 4.

To obtain the exponent y for the inner diameter in the approach, the values in Table 4 are logarithmized and plotted to a new graph.

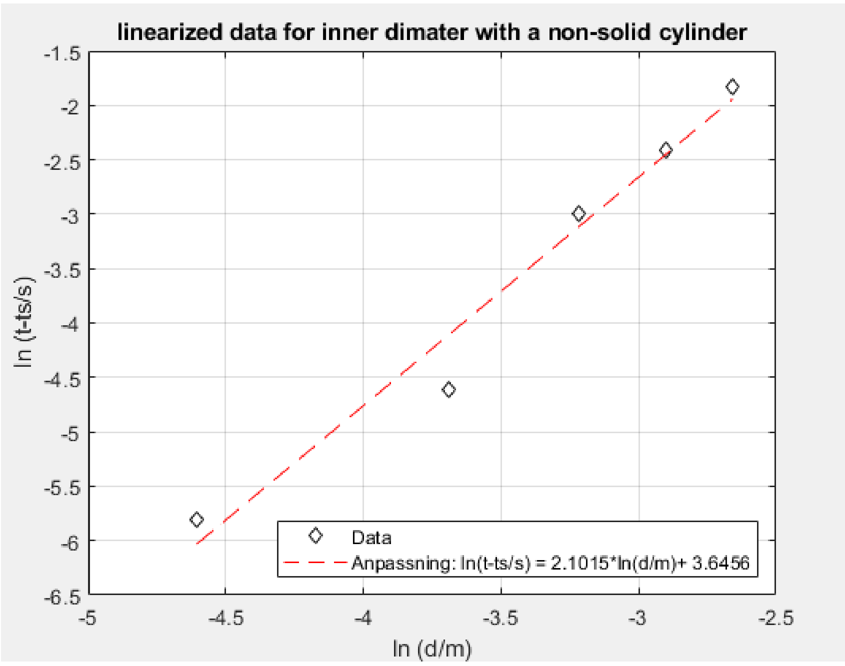

Figure 7: The figure shows a logarithmic version of Figure 6.

Based on Figure 7, the exponent y for the inner diameter (d) can be derived. The exponent is derived by calculating the slope of the graph in Figure 7. The slope is calculated by selecting two points in the graph and the difference between these points is calculated. The two differences are divided by each other. We used MatLab to know the exponent.

$y \ = \ \Delta ln(t \ - \ ts/s) \ / \ \Delta ln(d/m) \ \ = \ \ 2.1015 \approx 2$

Outer diameter, D

Table 5: The table below shows the measured values for how a varying outer diameter (D) of a non-solid cylinder affects the time (t) it takes for the cylinder to roll down the plane. Constant values are those : length of the plane (L) = 1.4 m, the angle of the plane (v) = 0.09 rad, and the inner diameter of the cylinders (d) = 0.04 m. The time (ts) = 1.96 s for a solid steel cylinder on the plane.

D (m)	t [s]	t-ts [s]
0.05	2.21	0.25
0.06	2.12	0.16
0.07	2.07	0.11
0.08	2.06	0.10
0.09	2.04	0.08

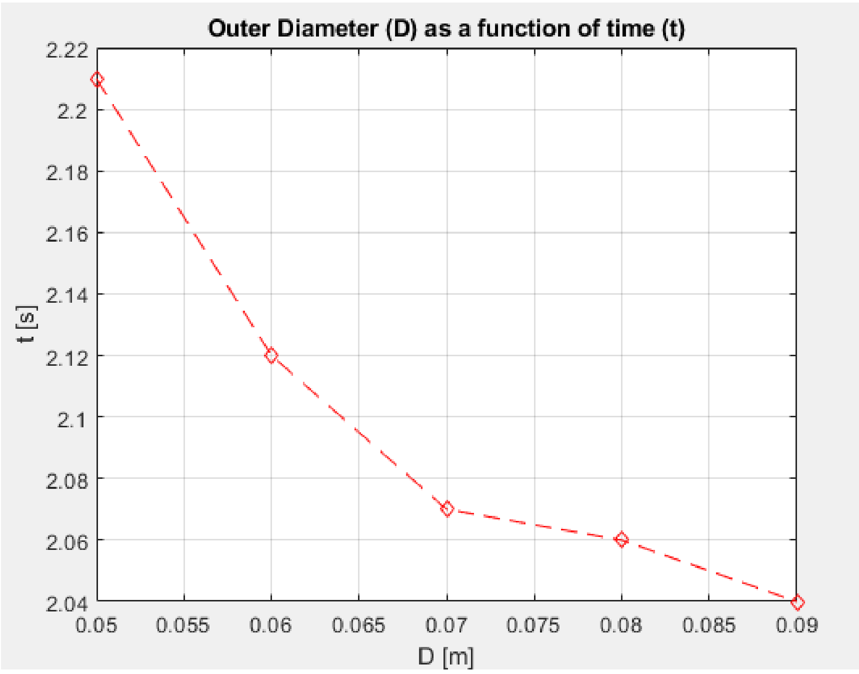

Figure 8 : The figure shows the data and a fit for the data from Table 5.

To obtain the exponent z to the outer diameter in the approach, the values in Table 5 are logarithmized and are plotted into a new graph.

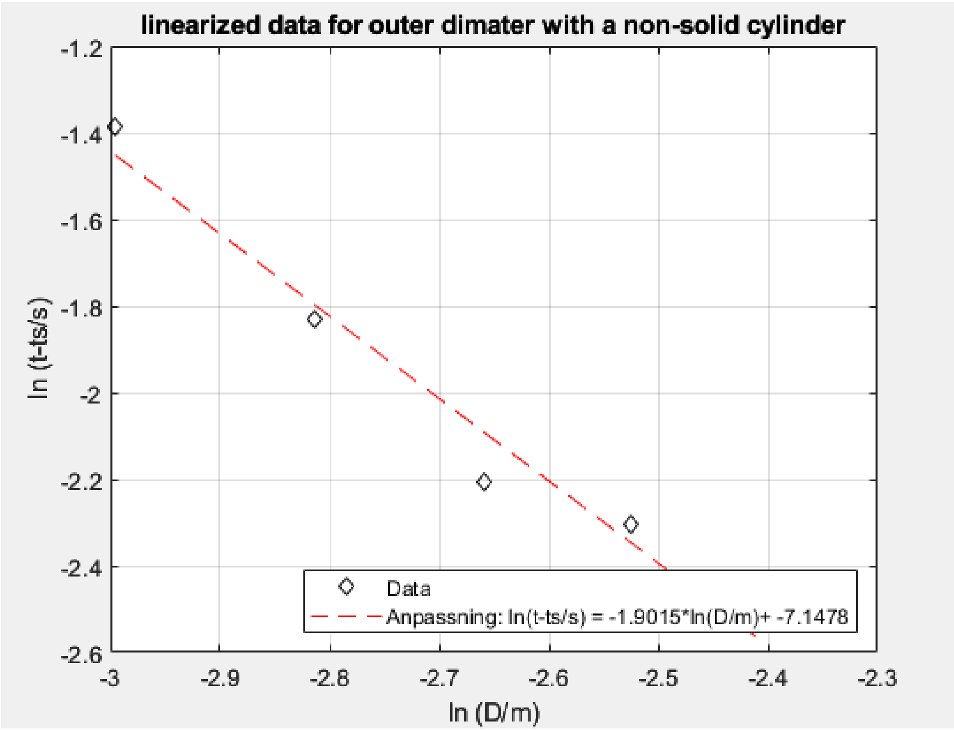

Figure 9: The figure shows a linearized version of Figure 8.

Based on Figure 9, the exponent z for the outer diameter (D) can be derived. The exponent is derived by calculating the slope of the graph in Figure 9. The slope is calculated by selecting two points in the graph and the difference between these points is calculated. The two differences are divided by each other. We used MatLab to know the exponent.

$$z = \Delta ln(t - ts/s) \,/\, \Delta ln(D/m) = -1.9015 = -2$$

Dimensionsanalys:

Now all the exponents are given to all factors, which means that one can perform a dimensional analysis to calculate the dimension of the constants k_1 and k_2.

The approaches: $t_s = k * L^x * sin(v)^w$ and $t = K * d^y * D^z$

The approach with given exponents: $t_s = k * L^{1/2} * sin(v)^{-1/2}$

and $t = K * d^2 * D^{-2}$

Dimensional analysis for the approach:

$t_s = [k] * L^{1/2} * [sin(v)]^{-1/2}$ and $t = [K] * L^{1/2} * [sin(v)]^{-1/2} * L^2 * L^{-2}$

Here we have sin(v) dimensionless, which will be : $t_s = [k] * L^{1/2}$ $and\ t = [K] * L^{1/2}$

We also know that the force of gravity affects the result, therefore we choose to add the force of gravity to the approach.

The approach : $t_s = k * L^x * sin(v)^w * g^e and\ t = K * L^x * g^e * sin(v)^w * d^y * D^z$

Dimensionsanalyse for the approach:

$t_s = [k] * (LT^{-2})^e * L^{1/2} and\ t = [K] * (LT^{-2})^e * L^{1/2}$

One can summarize that k_1 and k_2 have the same exponent and dimension.

$T = (LT^{-2})^e * L^{1/2} = (LT^{-2})^e * L^{1/2}$ that showed e = -½ .

The value of K we get by :

$t = K * (d/D)^2$

$K = k * g^{-1/2} * L^{1/2} * sin(v)^{-1/2}$

And the value of k can we get by calculating :

$k = K * g^{1/2} * L^{-1/2} * sin(v)^{1/2}$

Calculate the constants K and k

Table 6: K describes the constant for the approach to non-solid cylinders, which means that if we want to calculate the constant K, we need to calculate the average value of K for the approach to non-solid cylinders:

Constant values	$t = K * (d/D)^2$	K
t-ts(s) = 0.003 , d(m) =0.01 , D(m) = 0.08	0.003 = $K * (0.01/0.08)^2$	0.192
t-ts(s) = 0.01 , d(m) =0.025 , D(m) = 0.08	0.01 = $K * (0.025/0.08)^2$	0.1024
t-ts(s) = 0.05 , d(m) =0.04 ,	0.05 = $K * (0.04/0.08)^2$	0.2

D(m) = 0.08		
t-ts(s) = 0.09, d(m) =0.055 , D(m) = 0.08	$0.09 = K * (0.055/0.08)^2$	0.1904
t-ts(s) = 0.16 , d(m) =0.07 , D(m) = 0.08	$0.16 = K * (0.07/0.08)^2$	0.2089
t-ts(s) = 0.25 , d(m) =0.04 , D(m) = 0.05	$0.25 = K * (0.04/0.05)^2$	0.3906
t-ts(s) = 0.16 , d(m) =0.04 , D(m) = 0.06	$0.16 = K * (0.04/0.06)^2$	0.3600
t-ts(s) = 0.11 , d(m) =0.04 , D(m) = 0.07	$0.11 = K * (0.04/0.07)^2$	0.3369
t-ts(s) = 0.10 , d(m) =0.04 , D(m) = 0.08	$0.10 = K * (0.04/0.08)^2$	0.4000
t-ts(s) = 0.08 , d(m) =0.04 , D(m) = 0.09	$0.08 = K * (0.04/0.09)^2$	0.405
		<K> = 0.2786

Table 7: k is the constant for soiled cylinders, which means that if we want to calculate the constant k, we need to calculate the average value of k for the approach to solid cylinders:

Constant values	$k = K * g^{1/2} * L^{-1/2} * sin(v)^{1/2}$	k
L(m) = 0.6 , v(rad) = 0.09 g = 9.82 m/s^2, K = 0.2786	$k = (0.2786 * \sqrt{g * sin(0.09)})/(\sqrt{0.6})$	0.3381
L(m) = 0.8 , v(rad) = 0.09 g = 9.82 m/s^2 , K = 0.2786	$k = (0.2786 * \sqrt{g * sin(0.09)})/(\sqrt{0.8})$	0.2928
S(m) = 1.0 , v(rad) = 0.09 g = 9.82 m/s^2, K = 0.2786	$k = (0.2786 * \sqrt{g * sin(0.09)})/(\sqrt{1})$	0.2619
S(m) = 1.2 , v(rad) = 0.09 g = 9.82 m/s^2, K = 0.2786	$k = (0.2786 * \sqrt{g * sin(0.09)})/(\sqrt{1.4})$	0.2391
S(m) = 1.4 , v(rad) = 0.09 g = 9.82 m/s^2, K = 0.2786	$k = (0.2786 * \sqrt{g * sin(0.09)})/(\sqrt{1.4})$	0.2214
S(m) = 1.4 , v(rad) = 0.0575 g = 9.82 m/s^2, K = 0.2786	$k = (0.2786 * \sqrt{g * sin(0.0575)})/(\sqrt{1.4})$	0.1769
S(m) = 1.4 , v(rad) = 0.0775 , g = 9.82 m/s^2, K = 0.2786	$k = (0.2786 * \sqrt{g * sin(0.0775)})/(\sqrt{1.4})$	0.2054

S(m) = 1.4 , v(rad) = 0.09 g = 9.82 m/s^2, K = 0.2786	$k = (0.2786 * \sqrt{g * sin(0.09)})/(\sqrt{1.4})$	0.2214
S(m) = 1.4 , v(rad) = 0.1150 g = 9.82 m/s^2, K = 0.2786	$k = (0.2786 * \sqrt{g * sin(0.1150)})/(\sqrt{1.4})$	0.2502
S(m) = 1.4 , v(rad) = 0.1275 , g = 9.82 m/s^2, K = 0.2786	$k = (0.2786 * \sqrt{g * sin(0.1275)})/(\sqrt{1.4})$	0.2634
		<k> = 0.2471

Results :
The final approach :

$$t = 0.25 * L^{1/2} * sin(v)^{-1/2} * g^{-1/2} + 0.28 * L^{1/2} * g^{-1/2} * sin(v)^{-1/2} * d^2 * D^{-2}$$

Discussion:

The materials that were used in this experiment have not been consistent which caused more friction to the surface of the plane. More friction could have affected the result.
The start position of the cylinder can also affect the result. If the cylinder starts too far up the plane will have a speed before the timing starts which results in affected measurement values.
If the experiment had been repeated we would have made more measurements to ensure the result. It leads to less deviation.

Error analysis

Since the laboratory consists of human measurements and that this was done with non-specific materials and during a limited period of time with few measurements, there are measurement errors in laboratories. Since the linearization was done by linearization in matlab, with control calculations from a lab group of 4 people with little training in the subject, this is also an error that can be seen in the final formula. The errors that were added are considered to be errors of approximately 0.5 of the measurement values used. The final numbers displayed in the formula will be displayed as whole or half numbers.

Results

The result looks like it should be able to work, nothing that should be used in further research as it probably does not follow the standard needed in research dealing with rolling cylinders. The result can be seen as an investigation into how a laboratory can be carried out or see how a report should be written.

Conclusion:

With the formula given, these work on measured values and will facilitate calculations on rolling cylinders. Since it appears that the measurements made had to be redone, it shows that it is not just doing this lab.

It has slowly but surely taken the laboratory forward and finally produced a formula that should work. As I said before, this experiment has been computerized to a certain level, but things like the length measurements and how the cylinder has been released have not been done by a computer but by a human.

The experiment has then been as good as possible with the tools that were used but which in other experiments could have been done with more precision with the help of other tools.
The formula appearance was created early in the procedure and has changed a bit after that:

$$t = a * L^{x} * sin(v)^{w} + K * L^{x} * sin(v)^{w} * d^{y} * D^{z}$$

After calculations, it looks like this:

$$t = 0.25 * L^{1/2} * sin(v)^{-1/2} * g^{-1/2} + 0.28 * L^{1/2} * g^{-1/2} * sin(v)^{-1/2} * d^{2} * D^{-2}$$