Physics 129L S5A

1) Density of States

$$H=\frac{P_x^2+P_y^2}{2m}+\frac{1}{2}m\omega^2(x^2+y^2)$$

a) Derive $g(E)$ the number of microstates per unit

$$U_1=\frac{P_x}{\sqrt{2m}},\ U_2=\frac{P_y}{\sqrt{2m}},\ V_1=x\sqrt{\frac{m\omega^2}{2}},\ V_2=y\sqrt{\frac{m\omega^2}{2}}\rightarrow H=U_1^2+U_2^2+V_1^2+V_2^2=E$$

E_{ℓ}^{n} for 4D hypersphere $r=\sqrt{E}$

$$V=\frac{\pi^2}{2}R^4=\frac{(\pi E)^2}{2}$$

$$(x,y,P_x,P_y)\rightarrow(U_1,U_2,V_1,V_2)\rightarrow J=\begin{bmatrix}\partial V_{1x} & \partial V_{1y} & \partial V_{1P_x} & \partial V_{1P_y}\\ \partial V_{2x} & \partial V_{2y} & \partial V_{2P_x} & \partial V_{2P_y}\\ \partial U_{1x} & \partial U_{1y} & \partial U_{1P_x} & \partial U_{1P_y}\\ \partial U_{2x} & \partial U_{2y} & \partial U_{2P_x} & \partial U_{2P_y}\end{bmatrix}$$

$$J=\begin{bmatrix}\sqrt{\frac{m\omega^2}{2}} & & & 0\\ & \sqrt{\frac{m\omega^2}{2}} & & \\ & & (2m)^{-\frac{1}{2}} & \\ 0 & & & (2m)^{-\frac{1}{2}}\end{bmatrix}\rightarrow |J|=\left(\frac{m\omega^2}{2}\right)\left(\frac{1}{2m}\right)=\frac{\omega^2}{4}$$

$$|J^{-1}|=\frac{1}{|J|}=\frac{4}{\omega^2}$$

$$\Omega(E)=|J^{-1}|V=2\left(\frac{\pi E}{\omega}\right)^2$$

$$g(E)=\frac{d\Omega(E)}{dE}=\boxed{\left(\frac{2\pi}{\omega}\right)^2E=g(E)}$$

$$b)\ Z(\beta)=\int_0^\infty g(E)e^{-\beta E}dE=\int_0^\infty\left(\frac{2\pi}{\omega}\right)^2 E e^{-\beta E}dE,\ W=\beta E,\ dW=\beta dE$$

$$Z(\beta)=\left(\frac{2\pi}{\omega}\right)^2\left(\frac{1}{\beta}\right)\int_0^\infty W e^{-W}dW=\left(\frac{2\pi}{\omega}\right)^2\frac{1}{\beta}\left(\frac{-x-1}{e^x}\right)\Big|_0^\infty=\boxed{\left(\frac{2\pi}{\beta\omega}\right)^2=Z(\beta)}$$

$$c)\ H=\frac{p_x^2+p_y^2}{2m}+\underbrace{\frac{1}{2}m\omega^2(x^2+y^2)+\lambda(x^2+y^2)^2}_{V(x,y)}$$

$$r^2=x^2+y^2\to H=\frac{p_x^2+p_y^2}{2m}+\frac{1}{2}m\omega^2r^2+\lambda r^4$$

$$\Omega(E)=\pi\iint_{V(x,y)<E}2m(E-V(x,y))dx\,dy=2\pi m(2\pi)\int_0^{r_{max}}(E-V(r))r\,dr$$

$$=4\pi^2m\int_0^{r_{max}}\underbrace{\left(E-\frac{1}{2}m\omega^2r^2-\lambda r^4\right)}_{I}r\,dr$$

$$?=r_{max}=r(I=0)\to E=\frac{1}{2}m\omega^2r_{max}^2+\lambda r_{max}^4 \qquad U=r_{max}^2$$

$$\lambda U_{max}^2+\frac{1}{2}m\omega^2U_{max}-E=0\to U_{max}=\frac{-\frac{1}{2}m\omega^2+\sqrt{(\frac{1}{2}m\omega^2)^2+4\lambda E}}{2\lambda}=\frac{m\omega^2}{4\lambda}\left(\sqrt{1+\frac{16\lambda E}{m^2\omega^4}}-1\right)$$

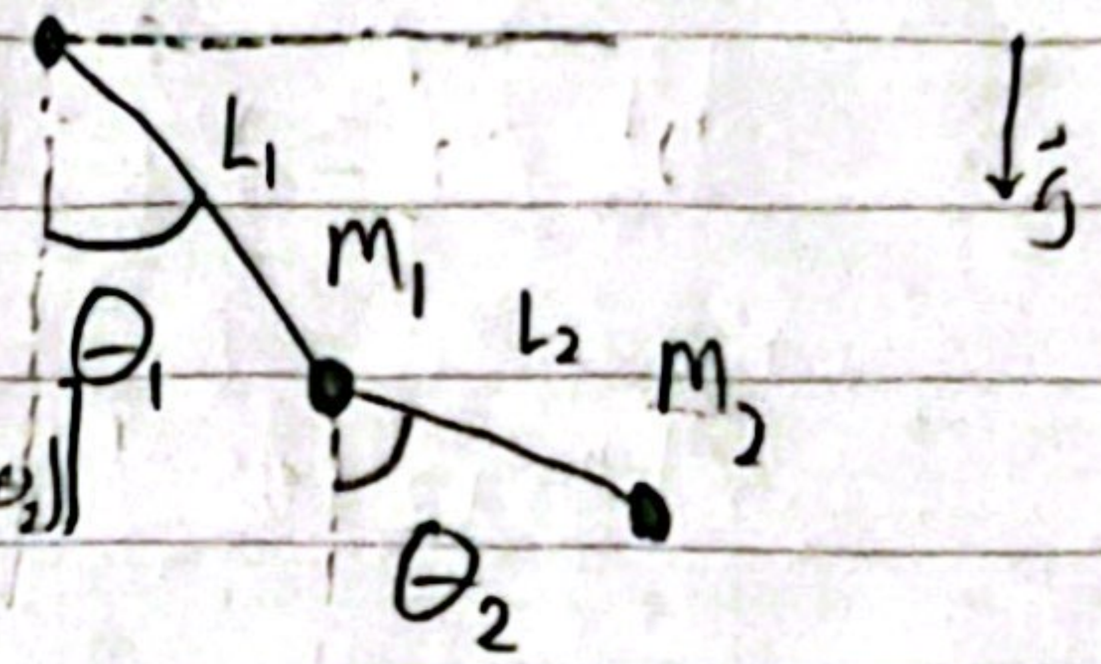
$$U=r^2,\ dU=2r\,dr$$

$$\Omega(E)=\frac{1}{2}4\pi^2m\int_0^{U_{max}}E-\frac{1}{2}m\omega^2U-\lambda U^2dU=2\pi^2m\left[EU_{max}-\frac{1}{4}m\omega^2U_{max}^2-\frac{\lambda U_{max}^3}{3}\right]$$

$$g(E)=\frac{d\Omega(E)}{dE}=2\pi^2mU_{max}=2\pi^2m\left(\frac{m\omega^2}{4\lambda}\left(\sqrt{1+\frac{16\lambda E}{m^2\omega^4}}-1\right)\right)$$

$$=\boxed{\frac{\pi^2m}{2\lambda}\left(\sqrt{m^2\omega^4+16\lambda E}-m\omega^2\right)=g(E)}$$

2) Double pendulum dynamics, Used Taylor



a) $U_1 = m_1 g L_1(1-\cos\theta_1)$, $U_2 = m_2 g\left[L_1(1-\cos\theta_1)+L_2(1-\cos\theta_2)\right]$

$$U=(m_1+m_2)gL_1(1-\cos\theta_1)+m_2 g L_2(1-\cos\theta_2) \xrightarrow[\cos\theta\approx 1]{\text{small }\theta_s} \boxed{U=\frac{1}{2}(m_1+m_2)gL_1\theta_1^2+\frac{1}{2}m_2 g L_2\theta_2^2}$$

$$T_1=\frac{1}{2}m_1L_1^2\dot{\theta}_1^2,\quad T_2=\frac{1}{2}m_2\left[L_1^2\dot{\theta}_1^2+2L_1L_2\dot{\theta}_1\dot{\theta}_2\cos(\theta_1-\theta_2)+L_2^2\dot{\theta}_2^2\right]$$

$$T=\frac{1}{2}(m_1+m_2)L_1^2\dot{\theta}_1^2+m_2L_1L_2\dot{\theta}_1\dot{\theta}_2\cos(\theta_1-\theta_2)+\frac{1}{2}m_2L_2^2\dot{\theta}_2^2$$

$$\boxed{\mathcal{L}=T-U},\quad \overset{\text{E-L}}{\frac{\partial \mathcal{L}}{\partial \theta_i}=\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\theta}_i}}\quad \overset{\downarrow \text{Small } \theta_s\ \cos\theta\approx 1}{\boxed{T=\frac{1}{2}(m_1+m_2)L_1^2\dot{\theta}_1^2+m_2L_1L_2\dot{\theta}_1\dot{\theta}_2+\frac{1}{2}m_2L_2^2\dot{\theta}_2^2}}$$

$$\frac{\partial \mathcal{L}}{\partial \theta_1}=-\frac{1}{2}(m_1+m_2)gL_1(2\theta_1)=-(m_1+m_2)gL_1\theta_1$$

$$\frac{\partial \mathcal{L}}{\partial \dot{\theta}_1}=\frac{1}{2}(m_1+m_2)L_1^2(2\dot{\theta}_1)+m_2L_1L_2\dot{\theta}_2$$ Assumed can small angle approx

$$\text{E-L}\rightarrow -(m_1+m_2)gL_1\theta_1=(m_1+m_2)L_1^2\ddot{\theta}_1+m_2L_1L_2\ddot{\theta}_2$$

$$\frac{\partial \mathcal{L}}{\partial \theta_2}=-m_2gL_2\theta_2,\quad \frac{\partial \mathcal{L}}{\partial \dot{\theta}_2}=m_2L_1L_2\dot{\theta}_1+m_2L_2^2\dot{\theta}_2$$

$$\text{E-L}\rightarrow -m_2gL_2\theta_2=m_2L_1L_2\ddot{\theta}_1+m_2L_2^2\ddot{\theta}_2$$ Eq's of motion

$$\boxed{\begin{bmatrix}(m_1+m_2)L_1^2 & m_2L_1L_2\\ m_2L_1L_2 & m_2L_2^2\end{bmatrix}\begin{bmatrix}\ddot{\theta}_1\\ \ddot{\theta}_2\end{bmatrix}=-\begin{bmatrix}(m_1+m_2)gL_1 & 0\\ 0 & m_2gL_2\end{bmatrix}\begin{bmatrix}\theta_1\\ \theta_2\end{bmatrix}}$$

b) $\mathcal{H} = p\dot{q} - \mathcal{L}$, $P_1 = \frac{\partial \mathcal{L}}{\partial \dot{\theta}_1}$, $P_2 = \frac{\partial \mathcal{L}}{\partial \dot{\theta}_2}$

This is far too much work for part 1 of 2

of Section work for any class...

I'm calling it here and starting 5B