Physics 129L S5B

1) Fermi-Dirac Statistics

$\varepsilon_m = (\varepsilon, 2\varepsilon, 3\varepsilon, \ldots M\varepsilon)$, Each energy level contributes factor $1+e^{-\beta(m\varepsilon-\mu)}$

$$Z=\left(\sum_{j=0}^{1}\exp(-\beta j(\varepsilon-\mu))\right)\left(\sum_{j=0}^{1}\exp(-\beta j(2\varepsilon-\mu))\right)\cdots\left(\sum_{j=0}^{1}\exp(-\beta j(M\varepsilon-\mu))\right)$$

$$\boxed{Z=\prod_{m=1}^{M}\left(1+e^{-\beta(m\varepsilon-\mu)}\right)}$$

2) Bose-Einstein Condesate

a) $E_0 = 0$, $E_1 = \varepsilon$, N particles, $N-K$ particles at E_1, K at E_0, $K = 0, 1, 2, \ldots N$

$\boxed{\text{There are } N+1 \text{ microstates}}$, e.g. $N=3 \to (3,0), (2,1), (1,2), (0,3) \to 4$ microstates

b) $Z_c = \sum_{K=0}^{N} \binom{N}{K} e^{-\beta K\varepsilon} = \sum_{K=0}^{N} \frac{N!}{K!(N-K)!} e^{-\beta K\varepsilon} = \boxed{(1+e^{-\beta\varepsilon})^N = Z_c}$

$$\boxed{P(E_s) = \binom{N}{\frac{E}{\varepsilon}} \frac{e^{-\beta E}}{Z}}$$

c) ? $= \langle n_0\rangle_c, \langle n_\varepsilon\rangle_c$,

$$\boxed{\begin{aligned}\langle n_0\rangle_c &= \frac{1}{Z_c}\sum_{K=0}^{N}(N-K)\binom{N}{K}e^{-\beta K\varepsilon} = \frac{N}{1+e^{-\beta\varepsilon}} \\ \langle n_\varepsilon\rangle_c &= \frac{1}{Z_c}\sum_{K=0}^{N}K\binom{N}{K}e^{-\beta K\varepsilon} = \frac{N}{1+e^{\beta\varepsilon}}\end{aligned}}$$

$\langle n_0\rangle_c + \langle n_\varepsilon\rangle_c = N\left(\frac{1}{1+e^{-\beta\varepsilon}} + \frac{1}{1+e^{\beta\varepsilon}}\right) = 1 \ \forall\ \beta, \varepsilon$ ✓

See plot in P2.Py

d) $$\boxed{Z = \sum_{K=0}^{N} e^{-\beta K\varepsilon} = \frac{e^{\beta\varepsilon} - e^{-N\beta\varepsilon}}{e^{\beta\varepsilon}-1} \to P(\varepsilon) = \frac{e^{-\beta\varepsilon}}{Z} = \frac{e^{-\beta\varepsilon}(e^{\beta\varepsilon}-1)}{e^{\beta\varepsilon}-e^{-N\beta\varepsilon}}}$$

e) $\langle n_0\rangle, \langle n_\varepsilon\rangle = ?$

I'm calling it here. Having this class require, week-to-week doing different disciplines at an upper division/graduate level, is not doing it for me!!.