

Assignment 1 - Part V

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1 Part V.I) CNN

1.1 Task 1

We know,

The formula for calculating the output dimensions of a convolution layer,

$$Output = \frac{(I - K + 2P)}{S} + 1$$

where:

$$\begin{aligned} \text{Input size, } I &= 32 \\ \text{Kernel size, } K &= 5 \\ \text{Padding, } P &= 0 \\ \text{Stride, } S &= 1 \end{aligned}$$

Therefore,

$$\begin{aligned} O &= \frac{(32 - 5 + 2(0))}{1} + 1 \\ &= \frac{27}{1} + 1 \\ &= 28 \end{aligned}$$

Thus,

The output size after first convolution layer is $10 \times 28 \times 28$

1.2 Task 2

The number of learnable parameters in a convolution layer is calculated by:

$$\text{Total parameters} = (K \times K \times C) \times F + B$$

where:

Kernel Size, K	=	5
Number of channels, C	=	3
Number of filters, F	=	10
Bias, B	=	10

Thus,

$$(5 \times 5 \times 3) \times 10 + 10$$

$$= (75 \times 10) + 10$$

$$= 750 + 10$$

$$= 760$$

Therefore, the total number of learnable parameters is 760.

1.3 Task 3

We know,

The formula for calculating the output dimensions of a convolution layer,

$$\text{Output} = \frac{(I - K + 2P)}{S} + 1$$

where:

Input size, I	=	32
Kernel size, K	=	5
Padding, P	=	1
Stride, S	=	1

Therefore,

$$O = \frac{(32 - 5 + 2(1))}{1} + 1$$

$$= \frac{29}{1} + 1$$

$$= 30$$

Thus,

The output size after first convolution layer is $10 \times 30 \times 30$

1.4 Task 4

If the input was a greyscale image, the image channel would reduce from 3 (RGB) to 1 (Grey).

The number of learnable parameters in a convolution layer would be calculated as:

$$\text{Total parameters} = (K \times K \times C) \times F + B$$

where:

Kernel Size, K	=	5
Number of channels, C	=	1
Number of filters, F	=	10
Bias, B	=	10

Thus,

$$(5 \times 5 \times 1) \times 10 + 10$$

$$= (25 \times 10) + 10$$

$$= 250 + 10$$

$$= 260$$

Therefore, the total number of learnable parameters is 260.

1.5 Task 5

Given the tasks involving multi-class classification with 5 output classes, the most appropriate activation function is **softmax**.

Softmax:

- This function transforms raw logits into probabilities.
- It allows each output to be interpretable as a probability for a class

Why other activation functions would not work:

- **ReLU** is used in hidden layers but does not normalize outputs to probabilities.
- **Sigmoid** is better suited for binary classification
- **Tanh** does not provide valid probability distributions.

Thus, **softmax** is the best choice for multi-class classification.

1.6 Task 6

The softmax function is given by:

$$\sigma(z_i) = \frac{e^{z_i}}{\sum_j e^{z_j}}$$

if we add a constant c to all inputs,

$$\sigma(z_i + c) = \frac{e^{z_i + c}}{\sum_j e^{z_j + c}}$$

and simplify the expression as follows,

$$= \frac{e^c e^{z_i}}{e^c \sum_j e^{z_j}}$$

Since e^c is common in both numerator and denominator, it can be canceled out,

$$= \frac{e^{z_i}}{\sum_j e^{z_j}} = \sigma(z_i)$$

Therefore, the soft max function remains unchanged under constant shifts.

The significance of shift invariance is as follows:

- It helps in preventing numerical problems like overflow caused by excessively high values.
- It helps to ensure that models remain adaptable to changes or transformations in the input data.

2 References:-

2.1 Lecture Notes, CNN Architecture I, Alina Vereshchaka