AEM 5452/ EE 5251 **Fall 2015**
University of Minnesota **September 8, 2015**

PROBLEM SET #1

Due September 22, 2015 at 4:30

Problems:

1. Do problem 1.4 in your text.
2. Do problem 1.13 in your text
3. Do problem 1.18 in your text.
4. Table 1 is a summary of loans made by three mortgage bankers at The Bank of Minnesota.[1] Loans that have at least one missed payment are considered a *moderate default risk*. Loans that have at least two missed payments are considered a *high default risk*. Once three payments are missed, the loans are considered default.

		Number of Loans With At Least ...		
	Total Number of Loans Issued (A)	**One Missed Payment (B)**	**Two Missed Payments (C)**	**Three Missed Payments (D)**
Banker X	2406	456	622	6
Banker Y	2446	57	1407	76
Banker Z	1455	7	9	2

Table 1: Loan Statistics for Three Mortgage Bankers X, Y and Z.

(a) What is the probability that a loan issued from the Bank of Minnesota will go into default or $P(D)$?

(b) What is the probability that a loan issued from the Bank of Minnesota was made by Banker X or $P(X)$?

(c) Given that a loan was made by Banker X, what is the probability it will go into default or $P(D|X)$?

(d) Given that a loan has defaulted, what is the probability it was made by Banker X or $P(X|D)$?

(e) What is the probability that a loan issued by the Bank of Minnesota will have been made by Banker X and has defaulted or $P(X, D)$? Calculate this probability three ways: Using the data in Table 1 and using your answers from (a)-(d) above with Bayes' rule in two different ways.

[1] A completely fictitious bank just made up for this example

5. Do problem 2.7 in your text but add the following part c) to the problem: c) What is the variance of X?

6. The vector analog of Equation 2.42 in your text is given as follows: Suppose X and Y are random variables with a known joint density $f_{XY}(x, y)$. If the random variables Z and V are related to X and Y as follows:

$$X = g(Z, V) \quad \text{and} \quad Y = h(Z, V)$$

where

$$J(x, y) = \begin{vmatrix} \frac{\partial g}{\partial x} & \frac{\partial g}{\partial x} \\ \frac{\partial g}{\partial x} & \frac{\partial g}{\partial x} \end{vmatrix}$$

then the joint density $f_{ZV}(z, v)$ is given by:

$$f_{ZV}(z, v) = \frac{1}{|J(x, y)|} f_{XY}(x, y)$$

Now, suppose $X \sim U(0, 1)$ and $Y \sim U(0, 1)$. Find the density for the random variable $f_Z(z)$ given by:

$$Z = \sqrt{-2 \ln X} \cos(2\pi Y)$$

in the following two ways:

(a) Empirically by using MATLAB (or OCTAVE or Python) to generate 10,000 samples of X and Y and plotting the histogram of Z.

(b) Analytically using the method outlined above.