Name: Shane Marie M. Ignacio

Section: UCOS 3-1

FINALS – ACTIVITY #2

1. **Solutions of Equations** - Involves finding the values of variables that satisfy a given equation.
   1) **Algebraic Equations -** For simple polynomial or rational functions, use algebraic techniques:
      a. **Linear equations:** Solve directly (e.g., $2x + 3 = 7 \rightarrow x = 2$)
      b. Quadratic equations: Solve using factoring, completing the square, or the quadratic formula:
      $$\boldsymbol{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{29}}$$
      c. Higher-degree polynomials: Use factoring, synthetic division, or numerical methods.

   2) **Transcendental Equations -** Equations involving non-algebraic functions (e.g., $e^x, \sin x\,, \ln x$)
      a. **Graphical methods:** Plot the functions and find intersection points.
      b. **Iterative methods**: Such as the Newton-Raphson method, which uses derivatives:
      $$x_n + 1 = x_n - \frac{f(x_n)}{f'(x_n)}$$
   3) **Equations from Calculus Applications**
      a. Finding roots in derivative-based problems:
      b. To find where a function has a horizontal tangent line, solve $f'(x) = 0$
      c. Critical points and optimization:
      d. Use $f''(x)$ (second derivative test) to classify these points
   4) **Numerical Approximations –** When an analytical solution is impractical:
      a. **Bisection Method:** Divide the interval [a,b] and find where the sign changes.
      b. **Secant Method:** Use a line connecting two points near the root:
      $$x_n + 1 = x_n - f(x_\eta) \cdot \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})}$$
   5) **Systems of Equations**
      a. Linear systems: Solve using substitution, elimination, or matrix methods (e.g., Gaussian elimination).

2. Transcendental Curve Tracing - Transcendental curve tracing involves analyzing and sketching the graphs of transcendental functions, which are functions that cannot be expressed as finite polynomials or rational functions.

1) **Domain and Range**
   a. Permissible x-values
   b. Identify the set of all permissible values for x (domain).
   c. Determine the set of possible output values (range).
   d. Range: $(-e^{-x}, e^{-x})$
2) **Symmetry**
   a. About x-axis (y -> -y)
   b. **Even function**: About y-axis (x -> -x)
   c. **Odd function**: About Origin (x,y -> -x,-y)
3) **Intercepts**
   a. **X-intercept**: Solve f(x) = 0.
   b. **Y-intercept**: Evaluate f(0).
4) **Behavior**
   a. $\lim_{x \to \pm\infty}$
5) **Asymptotes**
   a. **Vertical Asymptotes**: Occur where the function approaches infinity as x approaches specific values.
   b. **Horizontal Asymptotes**: The end behavior as $x \to \pm\infty$
   c. **Oblique Asymptotes**: Slant lines the curve approaches (if applicable).
6) **Regions**
   a. VA and x-intercep**t**
7) **Critical Points:**
   a. Find the derivative $f'(x)$ and solve $f'(x) = 0$ to identify critical points.
8) **Use** $f'(x)$:
   a. Positive $(f'(x) > 0)$ indicates the function is increasing.
   b. Negative $(f'(x) < 0)$ indicates the function is decreasing.
9) **Concavity and Inflection Points:**
   a. Determine $(f''(x)$ (second derivative)
   b. Positive $(f''(x) > 0)$ implies the function is concave up.
   c. Negative $(f''(x) < 0)$ implies the function is concave down.
   d. Inflection points occur where $(f''(x) = 0)$ or changes sign.
10) **Behavior at Infinity:**
   a. Study the limits as $x \to \infty$ or $x \to -\infty$ for insights into the behavior of the function.

9) **Sketch the Curve**

a. Combine all findings to create a rough but accurate sketch of the curve.

Example 1 :

Function: f(x) = $e^{-x}\sin(x)$

1. **Domain:** All real numbers (-∞, ∞)
2. **Range:** $(-e^{-x}, e^{-x})$
3. **Symmetry:** None (neither even nor odd).
4. **Intercepts:**
   - x-intercept: $x = n\pi$ where n is an integer
   - y-intercept: $f(0) = 0$
5. **Asymptotes:** No vertical or horizontal asymptotes (but decays to zero as $x \to \infty$)
6. **Critical Points:**
   - Solve $f'(x) = e^{-x}(-\sin(x)) + \cos(x)) = 0$
   - Critical points occur when tan(x) = 1
7. **Concavity:**
   - Analyze $f''(x)$ to determine concavity changes and inflection points.
8. **Behavior at Infinity:**
   - As (x→∞), $f(x) \to 0$