

## THEOREMS OF DIFFERENTIATION

### RULE 1:

#### Derivative of a constant

$$\begin{aligned} 1. f(x) &= 7 \\ f'(x) &= 0 \end{aligned}$$

### RULE 2:

#### Derivative of a power

$$\begin{aligned} f(x) &= x^4 & n &= 4 \\ &= 4x^{4-1} \\ f'(x) &= 4x^3 \end{aligned}$$

### RULE 3:

#### Derivative of a constant times a function.

$$\begin{aligned} f(x) &= 5x^3 \\ &= 5 * x^3 \\ &= 5 * 3x^{3-1} \\ &= 5 * 3x^2 \\ f'(x) &= 15x^2 \end{aligned}$$

### RULE 4:

#### Derivative of sum and difference

$$\begin{aligned} f(x) &= x^3 + 4x - 6 \\ &= 3x^{3-1} + 4 - 0 \\ &= 3x^{3-1} + 4 * 1x^{1-1} - 0 \\ &= 3x^2 + 4 * 1x^0 - 0 \\ &= 3x^2 + 4 - 0 \\ f'(x) &= 3x^2 + 4 \end{aligned}$$

### RULE 5:

#### Derivative of a Product

$$\begin{aligned} h(x) &= x^2 * e^x \\ &= 2x * e^x + x^2 * e^x \\ &= 2xe^x + x^2e^x \\ h(x) &= e^x(2x + x^2) \end{aligned}$$

$$\begin{aligned} f(x) &= x^2 & g(x) &= e^x \\ f'(x) &= 2x & g'(x) &= e^x \end{aligned}$$

**RULE 6:**  
**Derivative of a Quotient**

$$h(x) = \frac{x^2 \sin x}{(\sin x)^2}$$

$$h'(x) = \frac{2x \sin x - x^2 \cos x}{(\sin x)^2}$$

$$h'(x) = \frac{2x \sin x - x^2 \cos x}{\sin^2 x}$$

$$\begin{array}{ll} f(x) = & g(x) = \\ f'(x) = & g'(x) = \end{array}$$

**RULE 7:**  
**Derivative of a reciprocal**

$$f(x) = \frac{1}{x^2}$$

$$f'(x) = \frac{-2}{x^3}$$

$$\frac{1}{x^2} = x^{-2}$$

$$\frac{d}{dx} \frac{1}{x^2} = -2x^{-3}$$

**RULE 8:**  
**Derivative of a radical**

$$f'(x) = \frac{1}{2} \sqrt{x^3}$$

$$= x^{\frac{3}{2}-1}$$

$$= \frac{3}{2} x^{\frac{3}{2}-1}$$

$$= \frac{3}{2} x^{\frac{1}{2}}$$

**RULE 9:****Derivative of a function with a radical exponent**

$$h(x) = x^{3/4}$$

$$= \frac{3}{4} x^{3/4 - 1}$$

$$= \frac{3}{4} x^{-1/4}$$

$$h'(x) = \frac{3}{4^4 \sqrt{x}}$$

**RULE 10:****GENERAL POWER RULE**

$$f(x) = (3x+2)^5$$

$$= 5(3x+2)^4 \cdot 3$$

$$f'(x) = 15(3x+2)^4$$

**RULE 11:****CHAIN RULE**

$$h(x) = (\sin(2x))$$

$$= f(g(x)) \cdot g'(x)$$

$$= \cos 2x \cdot 2$$

$$h'(x) = 2\cos 2x$$

$$f'(x) = \cos 2x$$

$$g(x) = 2x$$

$$g'(x) = 2$$

## IMPLICIT DIFFERENTIATION

**1.  $x^2 + y^2 - 2x + 3y = 4$**

$$= 2x^{2-1} + 2y^{2-1}y' - 2(1)x^{1-1} + 3(1)y^{1-1}y' = 0$$

$$= 2x + 2yy' - 2 + 3y' = 0$$

$$= 2yy' + 3y' = -2x + 2$$

$$= y'(2y+3) = \frac{-2x+2}{2y+3}$$

$$y' = \frac{-2x+2}{2y+3}$$

$$y = f(x)$$

$$y' = f'(x)$$

**2.  $x^3 - 3x + y^2 = 16$**

$$= 3x^{3-1} - 3 + 2yy' = 0$$

$$= 2yy' = -3x^2 + 3$$

$$\frac{2y}{2y} = \frac{-3x^2 + 3}{2y}$$

\* cancelled  $2yy'/2y$

$$y' = \frac{-3x^2 + 3}{2y}$$

**3.  $Y + (2x - 5)^3 = 3$  (CHAIN RULE)**

$$= y' + 3(2x-5)^2(2) = 0$$

$$= y' + 6(2x-5)^2 = 0$$

$$y' = -6(2x-5)^2$$

**4.  $x^2 + 3x^4y^2 + y^2 = -4x$  (PRODUCT RULE)**

$$= 2x + 12x^3y^2 + 6x^4yy' + 2yy' = -4$$

$$= 6x^4yy' + 2yy' = -4 - 2x - 12x^3y^2$$

$$= y'(6x^4y + 2y) = \frac{-4 - 2x - 12x^3y^2}{6x^4y + 2y}$$

$$\text{*cancelled } (6x^4y + 2y)$$
$$\frac{-4 - 2x - 12x^3y^2}{6x^4y + 2y}$$

$$y' = \frac{-4 - 2x - 12x^3y^2}{6x^4y + 2y}$$