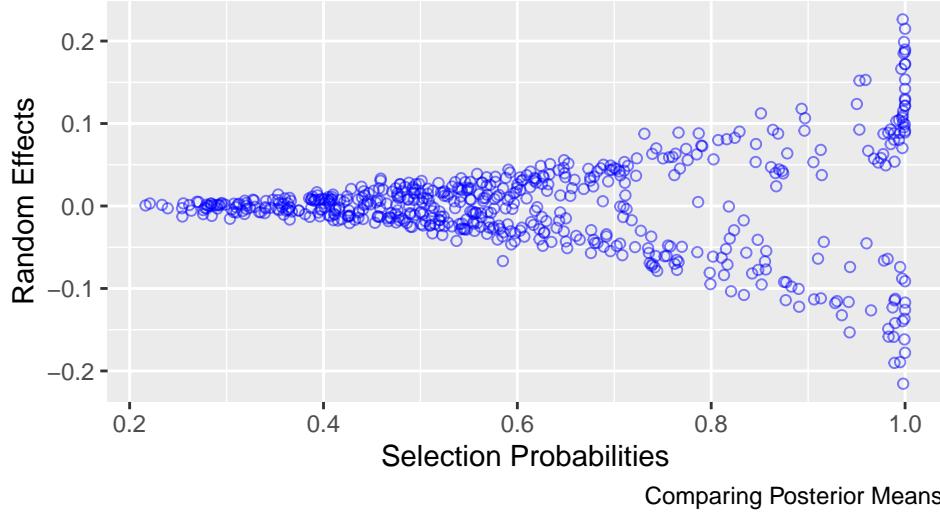


Additional Analysis on Random Effects

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2024-12-26

SSD Model: Selection Probabilities vs Random Effects



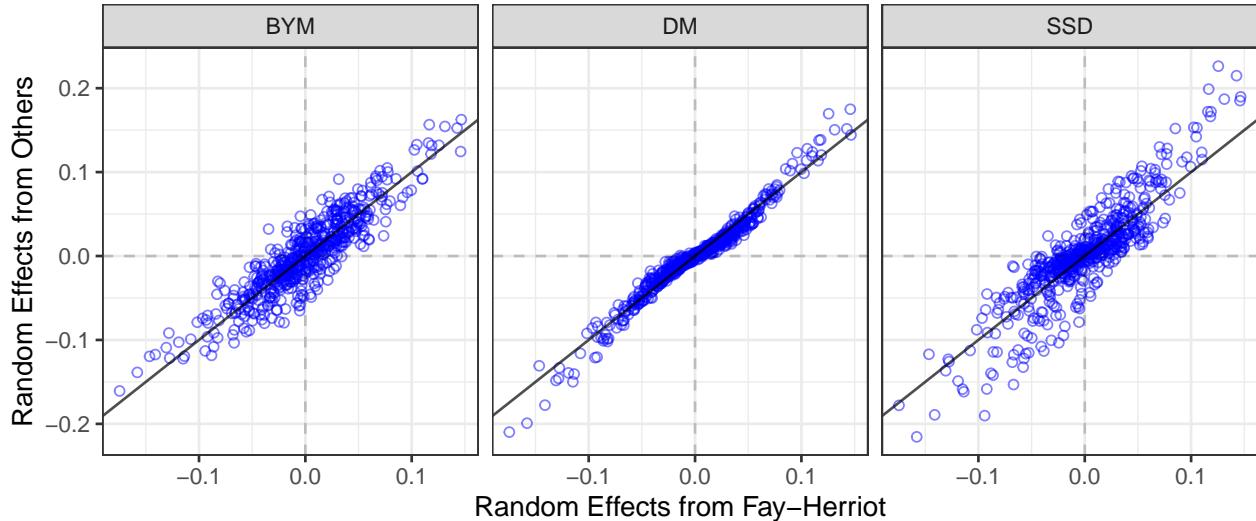
We can see that the areas where the selection probability is high (close to 1) also have larger magnitude random effects (both negative and positive). This makes sense given the posterior selection probability is

$$\hat{p}_i = \frac{p \cdot \phi(y_i | x_i^\top \beta + v_{1i} + v_{2i}, d_i)}{p \cdot \phi(y_i | x_i^\top \beta + v_{1i} + v_{2i}, d_i) + (1-p) \cdot \phi(y_i | x_i^\top \beta, d_i)}$$

for area $i = 1, \dots, n$. Thus, if the random effect is large for area i , the difference between $\phi(y_i | x_i^\top \beta + v_{1i} + v_{2i}, d_i)$ and $\phi(y_i | x_i^\top \beta, d_i)$ will be larger, impacting the selection probability.

Comparing Random Effects from Different Models

Scatterplot Comparisons (Posterior Means)



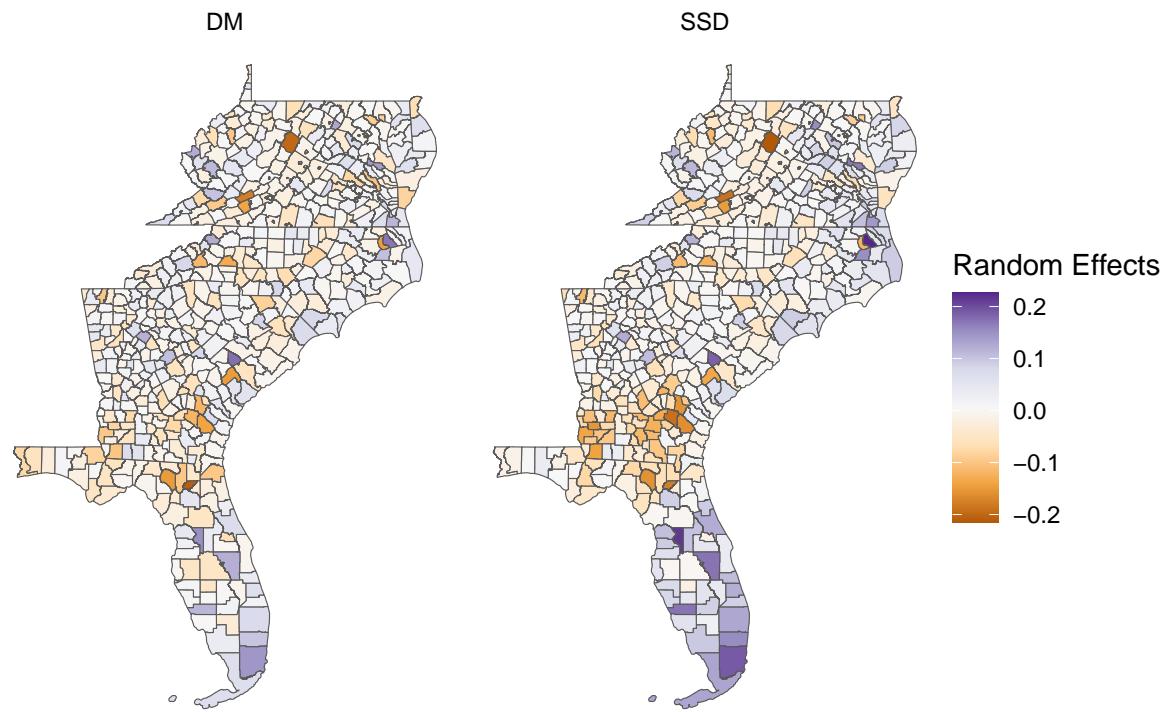
We can see that the DM model most closely resembles the Fay-Herriot random effects, as they both assume independent and identically distributed (IID) random effects. The SSD model is somewhere between the BYM and the DM model, as expected. Of the models, SSD model allows for the biggest range in the random effect values.

Testing for Spatial Dependence

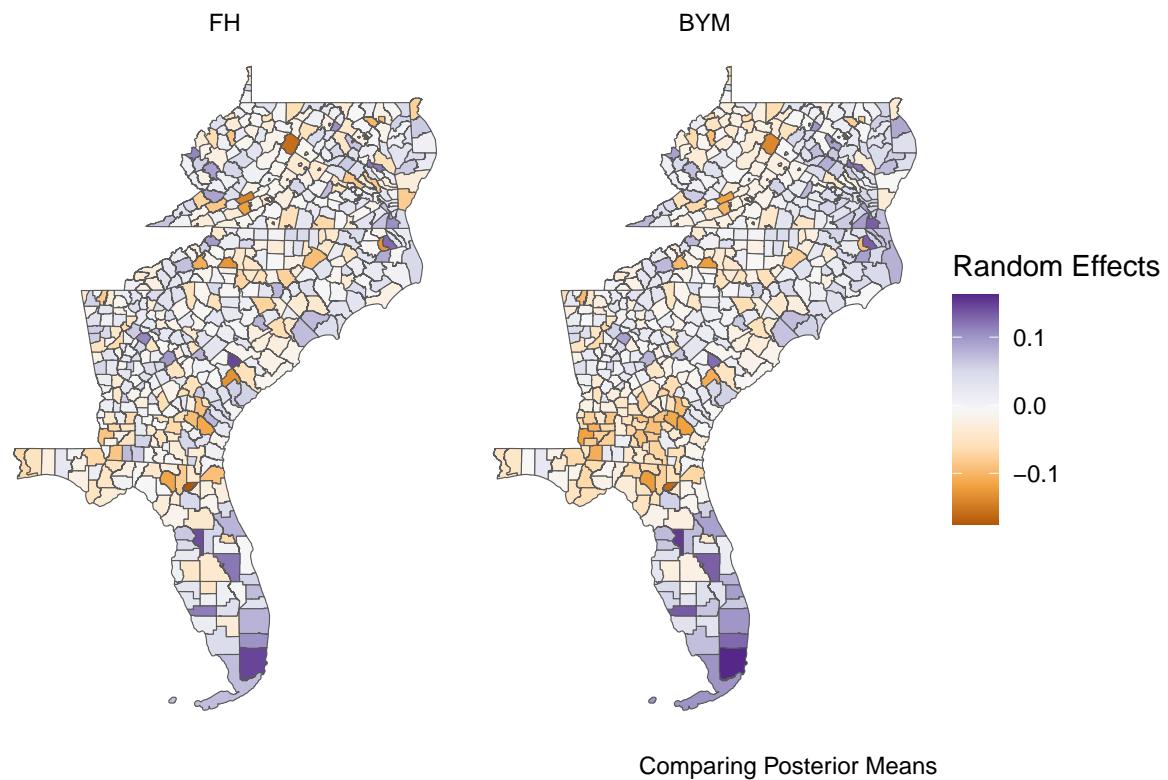
A Monte-Carlo simulation of a Geary's C test was performed on the random effects for the Fay-Herriot and Datta-Mandal. Here are the p-values:

```
## [1] 0.0002499375  
## [1] 0.002249438
```

Map of Random Effects (Posterior Means)

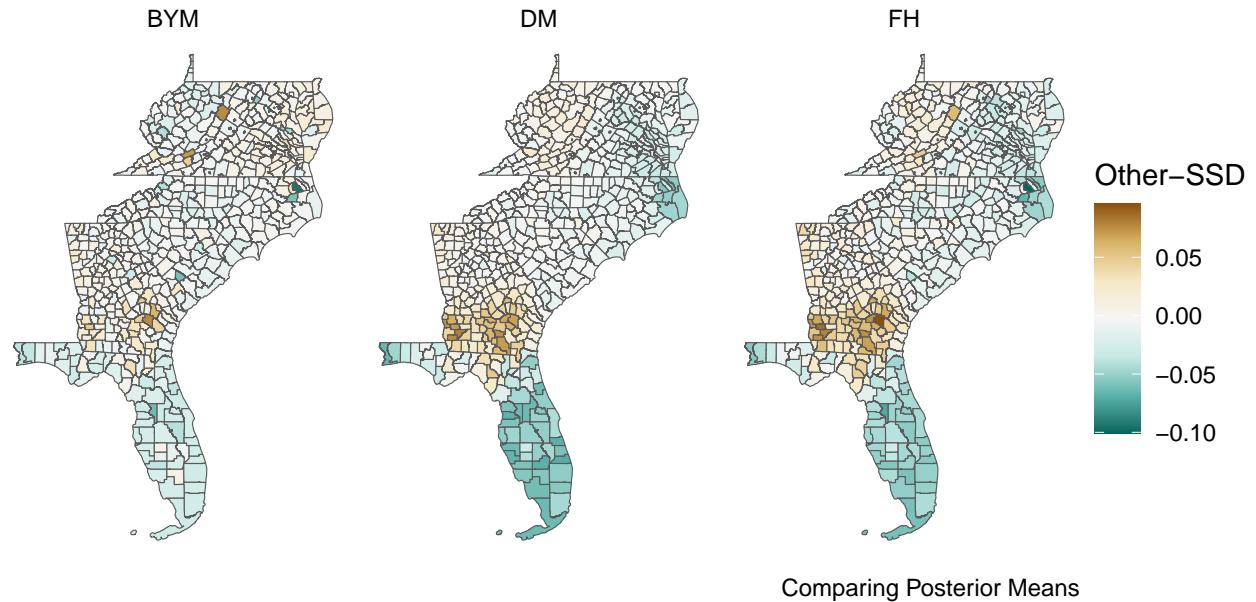


Comparing Posterior Means



We can see that all of the models have fairly similar spatial patterns. They also agree on the areas that have larger magnitude random effects.

Map of Random Effect Differences Between SSD Model and Others



Comparing Posterior Means

Here is a comparison of the differences between the SSD random effects and the other models. As expected, difference compared to the IID models (DM & FH) have a strong spatial pattern. The difference between the BYM and the SSD is marked by a few spots of large differences. It is interesting, however, that it does have a spatial pattern down by Florida, which suggests that incorporating spatial selection can capture spatial patterns that the BYM alone cannot.