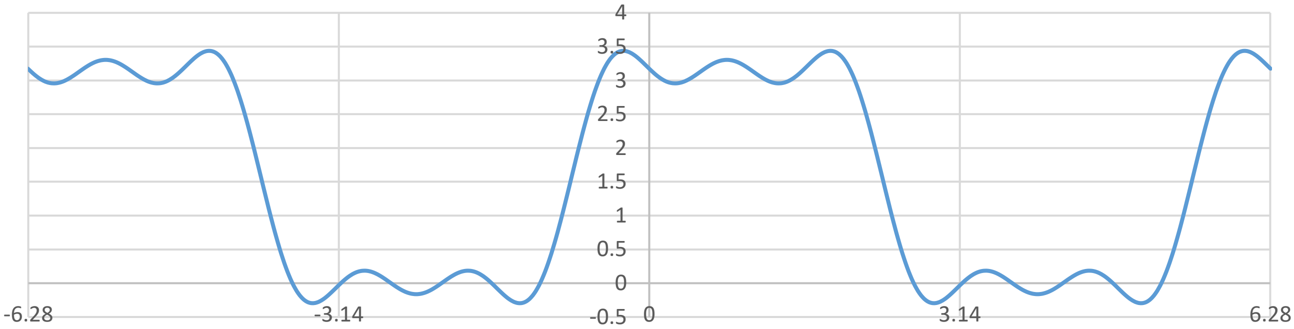
応数Ⅰ（フーリエ）　課題４

H30 年度 番号　　4J42

合成波のグラフ化と実フーリエ級数と係数の計算

[演習 2]　(2)

$$f(t)=\frac{\pi}{2}+\sqrt{2}\cos t+\sqrt{2}\sin t+\frac{\sqrt{2}}{3}\cos 3t-\frac{\sqrt{2}}{3}\sin 3t-\frac{\sqrt{2}}{5}\cos 5t-\frac{\sqrt{2}}{5}\sin 5t$$



例題 8-15 を再度自分で解き直し，章末問題の[演習 3]～[演習 5]を行う．

（例題は分かっているなら計算過程は省略してもかまわない）

[例題 8-15]

$$a_0=\frac{2}{T}\int_{-2}^{2}f(t)\,dt=\frac{2}{T}\left\{\int_{-2}^{0}(2+t)\,dt+\int_{0}^{2}(2-t)\,dt\right\}=\frac{2}{T}\left\{\left[2t+\frac{1}{2}t^2\right]_{-2}^{0}+\left[2t-\frac{1}{2}t^2\right]_{0}^{2}\right\}=2$$

$$a_n=\frac{2}{T}\int_{-2}^{2}f(t)\cdot cosn\omega_0 t\,dt\ ,\left(T=4,\omega_0=\frac{2\pi}{4}=\frac{\pi}{2}\right)$$

$$a_n=\frac{1}{2}\int_{-2}^{2}f(t)\cdot\cos\left(\frac{n\pi}{2}t\right)\mathrm{dt}=\frac{1}{2}\left\{\int_{-2}^{0}(2+t)\cos\left(\frac{n\pi}{2}t\right)dt+\int_{0}^{2}(2-t)\cos\left(\frac{n\pi}{2}t\right)dt\right\}$$

$$=\int_{-2}^{0}\cos\left(\frac{n\pi}{2}t\right)dt+\frac{1}{2}\int_{-2}^{0}t\left(\frac{2}{n\pi}\sin\left(\frac{n\pi}{2}t\right)\right)'dt+\int_{0}^{2}\cos\left(\frac{n\pi}{2}t\right)dt-\frac{1}{2}\int_{0}^{2}t\cdot\left(\frac{2}{n\pi}\sin\left(\frac{n\pi}{2}t\right)\right)'dt$$

$$=-\frac{2}{n\pi}\left[\sin\left(\frac{n\pi}{2}t\right)\right]_{0}^{2}-\frac{1}{2}\left\{\frac{2}{n\pi}\left[t\cdot\sin\left(\frac{n\pi}{2}t\right)\right]_{-2}^{0}+\frac{4}{n^2\pi^2}\left[t\cdot\cos\left(\frac{n\pi}{2}t\right)\right]_{-2}^{0}\right\}+\frac{2}{n\pi}\left[\sin\left(\frac{n\pi}{2}t\right)\right]_{0}^{2}$$

$$-\frac{1}{2}\left\{\frac{2}{n\pi}\left[t\cdot\sin\left(\frac{n\pi}{2}t\right)\right]_{0}^{2}-\frac{4}{n^2\pi^2}\left[\cos\left(\frac{n\pi}{2}t\right)\right]_{0}^{2}\right\}=\frac{4}{n^2\pi^2}(1-\cos(n\pi))$$

$$f(t)=\frac{a_0}{2}+\sum_{n=1}^{\infty}a_n cosn\omega_0 t+\sum_{n=1}^{\infty}b_n sinn\omega_0 t=1+\sum_{n=1}^{\infty}\frac{4}{n^2\pi^2}\{1-\cos(n\pi)\}\cos\left(\frac{n\pi}{2}t\right)$$

[演習 3]

(1)

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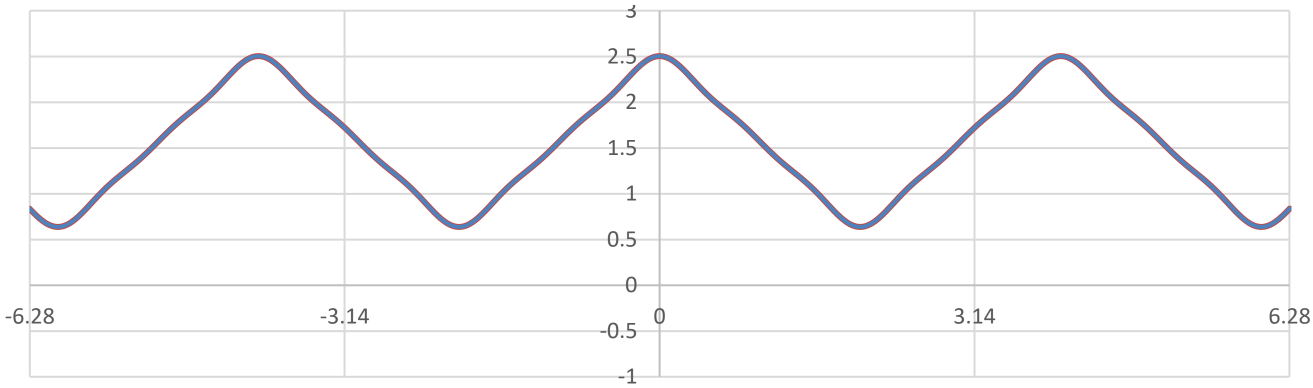
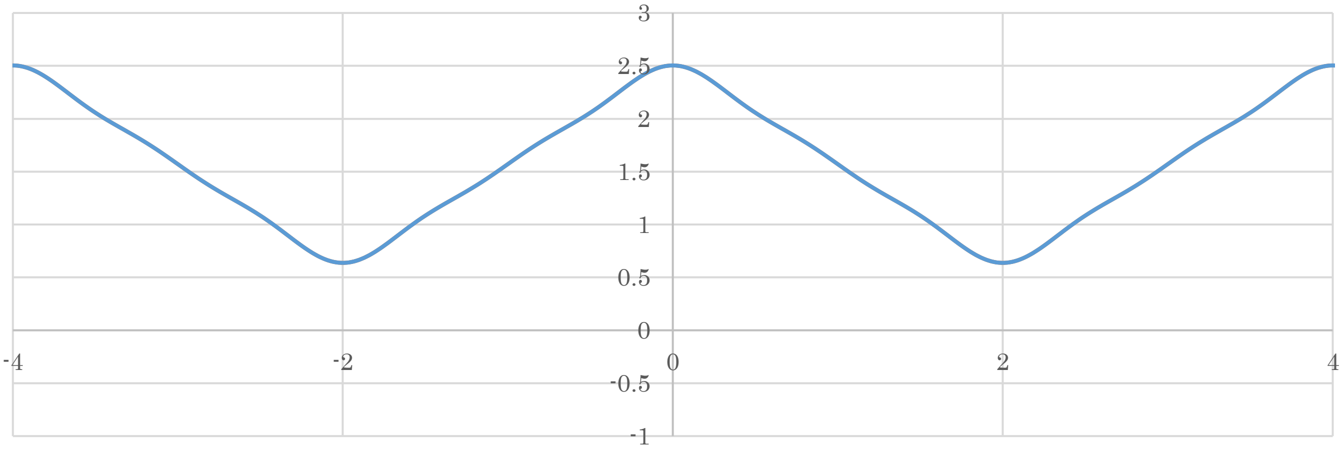
$$f(t) = 1 + \sum_{n=1}^{5} \frac{8}{n^2\pi^2} \sin^2\frac{n\pi}{2} \cdot \cos\left(\frac{n\pi}{2}\right) \quad (T: -2\pi \sim 2\pi)$$



(2)

$$f(t) = 1 + \sum_{n=1}^{5} \frac{8}{n^2\pi^2} \sin^2\frac{n\pi}{2} \cdot \cos\left(\frac{n\pi}{2}\right) \quad (T: -4 \sim 4)$$



[演習 4]

$$a_0 = \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} t \quad dt = \int_{-1}^{1} t\, dt = \left[\frac{1}{2}t^2\right]_{-1}^{1} = \frac{1}{2} - \frac{1}{2} = 0$$

$$a_n = \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} t \cdot \cos(n\omega_0 t) \quad dt = \int_{1}^{1} t \cdot \cos(\pi t)\, dt = \frac{1}{n\pi}[t \cdot \sin(n\pi t)]_{-1}^{1} - \frac{1}{n\pi}\int_{-1}^{1} \sin(n\pi t)\, dt = 0$$

$$b_n = \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} t \cdot \sin(n\omega_0 t)\, dt = \int_{-1}^{1} t \cdot \sin(n\pi t)\, dt = \frac{1}{n\pi}[t \cdot \cos(n\pi t)]_{-1}^{1} - \frac{1}{n\pi}\int_{-1}^{1} \cos(n\pi t)\, dt$$

$$= \frac{1}{n\pi}[-t \cdot \cos(n\pi t)]_{-1}^{1} - \int_{-1}^{1} -\frac{1}{n\pi}\cos(n\pi t)\, dt$$

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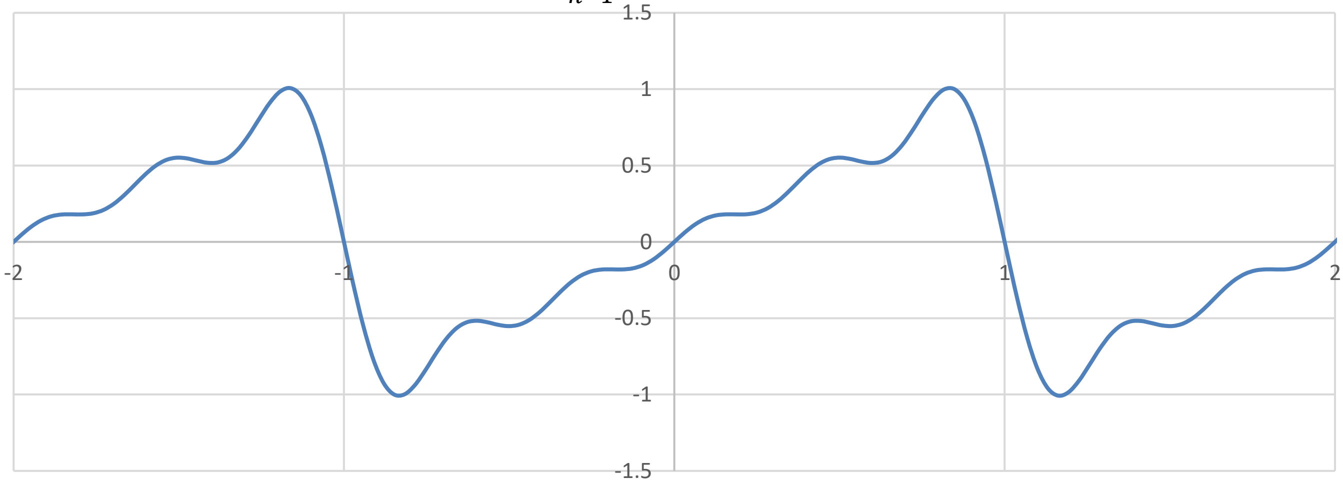
$$= \frac{1}{n\pi}\{-\cos(n\pi) - \cos(n\pi)\} + \frac{1}{n^2\pi^2}[\sin(n\pi t)]_{-1}^{1} = -\frac{2}{n\pi}\cos(n\pi)$$

$$\mathrm{f(t)} = \frac{\mathrm{a}_0}{2} + \sum_{n=1}^{\infty}(a_n\cos(n\omega_0 t) + b_n\sin(n\omega_0 t)) = \sum_{n=1}^{\infty} -\frac{2}{n\pi}\cos(n\pi)\cdot\sin(n\pi t)$$

グラフ

$$f(t) = \sum_{n=1}^{5} -\frac{2}{n\pi}\cos(n\pi)\cdot\sin(n\pi t)$$



[演習 5]

(1)　$\mathrm{T} = 0.02$ [s] ,$\omega_0 = 100\pi rad[rad/s]$

(2)

$$\mathrm{a}_0 = \frac{2}{T}\int_{-\frac{T}{4}}^{\frac{T}{4}} f(t)\,dt = \frac{2}{T}\left[\frac{1}{\omega_0}\sin(\omega_0 t)\right]_{-\frac{T}{4}}^{\frac{T}{4}} = \frac{2}{\pi}\sin\left(\frac{\pi}{2}\right) = \frac{2}{\pi}$$

$$\mathrm{a}_\mathrm{n} = \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)\cdot\cos(n\omega_0 t)\,dt = 100\int_{-\frac{1}{200}}^{\frac{1}{200}}\cos(100\pi t)\cdot\cos(100n\pi t)\,dt$$

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$$= 50\int_{-\frac{1}{200}}^{\frac{1}{200}}\{\cos(100\pi t+100n\pi t)+\cos(100t-100n\pi t)\}\,dt$$

$$=\frac{1}{\mathrm{T}}\ \{\left[\frac{1}{\omega_0(1+n)}\sin(\omega_0(1+n)t)\right]_{-\frac{T}{4}}^{\frac{T}{4}}+\left[\frac{1}{\omega_0(1-n)}\sin(\omega_0(1-n)t)\right]_{-\frac{T}{4}}^{\frac{T}{4}}\}$$

$$=\frac{1}{\mathrm{T}}\{\frac{2}{\omega_0(1+n)}\sin\left(\frac{\omega_0(1+n)T}{4}\right)+\frac{2}{\omega_0(1-n)}\sin\left(\frac{\omega_0(1-n)T}{4}\right)\}$$

$$=\frac{1}{\pi(\mathrm{n}+1)}\sin\left(\frac{(1+n)\pi}{2}\right)+\frac{1}{n(1-n)}\sin\left(\frac{(1-n)\pi}{2}\right)$$

ただし、n = 1 の時

$$a_{n=1}=50\int_{-\frac{1}{200}}^{\frac{1}{200}}\{\cos(100\pi t+100n\pi t)+\cos(100t-100n\pi t)\}\quad dt=\frac{1}{T}\left[\frac{1}{2\omega_0}sin2\omega_0 t+t\right]_{-\frac{T}{4}}^{\frac{T}{4}}$$

$$=\frac{1}{T\omega_0}(\sin\left(\frac{\omega_0T}{2}\right)-(\sin\left(-\frac{\omega_0T}{2}\right)+\frac{1}{T}\left(\frac{T}{4}+\frac{T}{4}\right)=2\sin\left(\frac{\omega_0T}{2}\right)+\frac{1}{2}=0+\frac{1}{2}=\frac{1}{2}$$

$$\mathrm{b_n}=\frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}f(t)\cdot\sin(n\omega_0 t)\,dt=100\int_{-\frac{1}{200}}^{\frac{1}{200}}\cos(100\pi t)\cdot\sin(100n\pi t)\,dt=0$$

以上より,

$$\mathrm{f(t)}=\frac{\mathrm{a_0}}{2}+\sum_{n=1}^{\infty}(a_n\cos(n\omega_0 t)+b_n\sin(n\omega_0 t))=\frac{2}{\pi}\ +\frac{1}{2}$$

$$+\sum_{n=2}^{\infty}\left\{\frac{1}{\pi(\mathrm{n}+1)}\sin\left(\frac{(1+n)\pi}{2}\right)+\frac{1}{\pi(1-n)}\sin\left(\frac{(1-n)\pi}{2}\right)\right\}\cos(100\pi t)$$

(3)

$$f(t)=\sum_{n=1}^{5}\left\{\frac{1}{\pi(n+1)}\sin\left(\frac{(1+n)\pi}{2}\right)+\frac{1}{\pi(1-n)}\sin\left(\frac{(1-n)\pi}{2}\right)\right\}\cos(100n\pi)$$