応数Ⅰ（フーリエ）　課題７

H30 年度　番号　4J42

フーリエ変換
例題 10-6～10-8 の計算を確認し，章末問題の[演習 3]～[演習 5]を行う．
※例題の図に関しては，実部と虚部のスペクトル，又は振幅と位相スペクトルのどちらかのペアのみでも可

[例題 10-6]の図

$$f(t) = \begin{cases} e^{-2t} & (t \geq 0) \\ 0 & (t < 0) \end{cases}$$

$$F(\omega) = \int_{-\infty}^{\infty} f(t) \cdot e^{-j\omega t}\,dt$$

$$= \int_{0}^{\infty} e^{-2t} \cdot e^{-j\omega t}\,dt = \int_{0}^{\infty} e^{-(2+j\omega)t}\,dt$$

$$= -\frac{1}{2+j\omega}\left[e^{-(2+j\omega)t}\right]_0^{\infty} = -\frac{1}{2+j\omega}(0-1)$$

$$= \frac{1}{2+j\omega} = \frac{(2-j\omega)}{(2+j\omega)(2-j\omega)} = \frac{2-j\omega}{4+\omega^2}$$

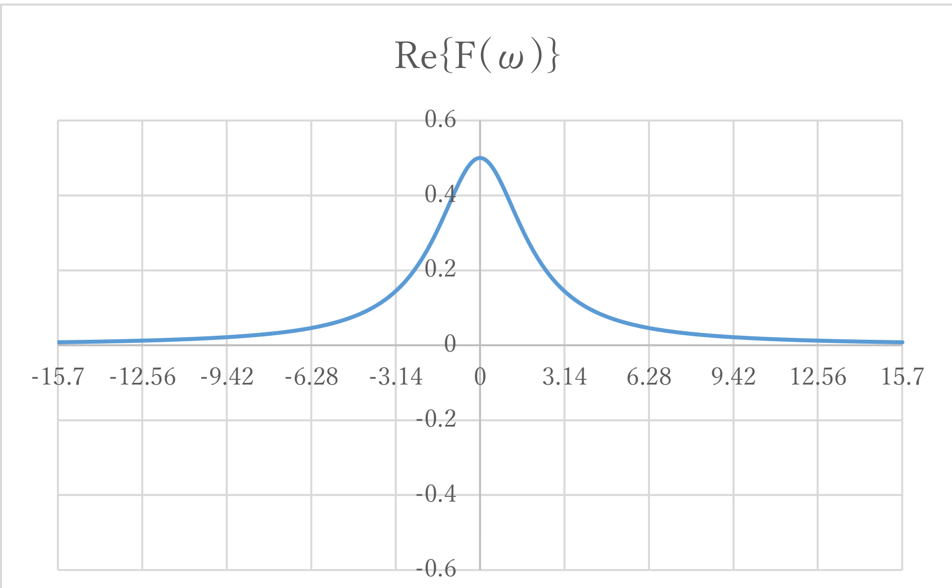
$$\mathrm{Re}\{F(\omega)\} = \frac{2}{4+\omega^2}$$

$$\mathrm{Im}\{F(\omega)\} = -\frac{\omega}{4+\omega^2}$$

$$|F(\omega)| = \sqrt{\left(\frac{2}{4+\omega^2}\right)^2 + \left(-\frac{\omega}{4+\omega^2}\right)^2} = \sqrt{\frac{4+\omega^2}{(4+\omega^2)^2}} = \frac{\sqrt{4+\omega}}{4+\omega^2}$$

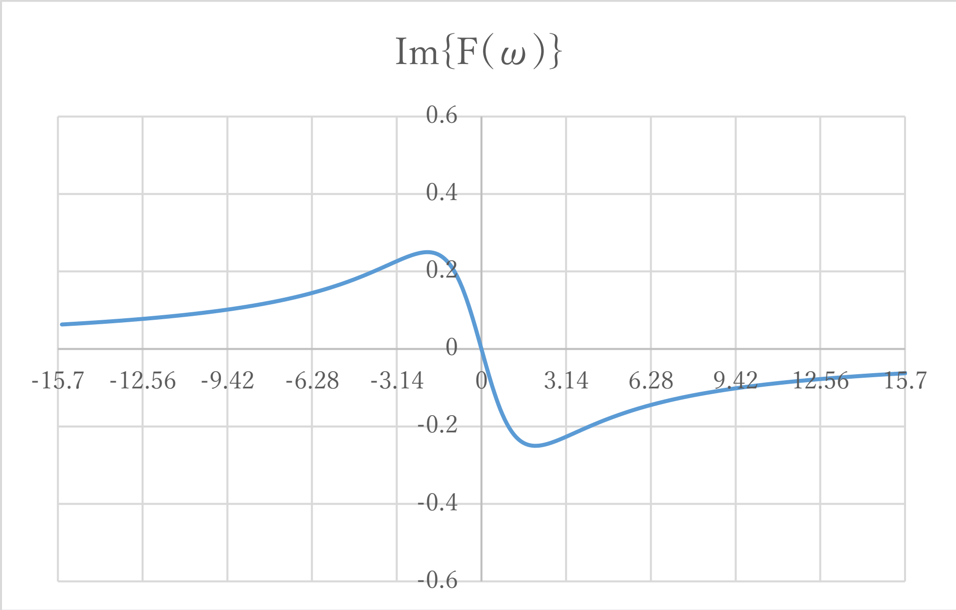
$$\theta(\omega) = \tan^{-1}\frac{Im\{F(\omega)\}}{Re\{F(\omega)\}} = \tan^{-1}\frac{-\omega}{2} = -\tan^{-1}\frac{\omega}{2}$$

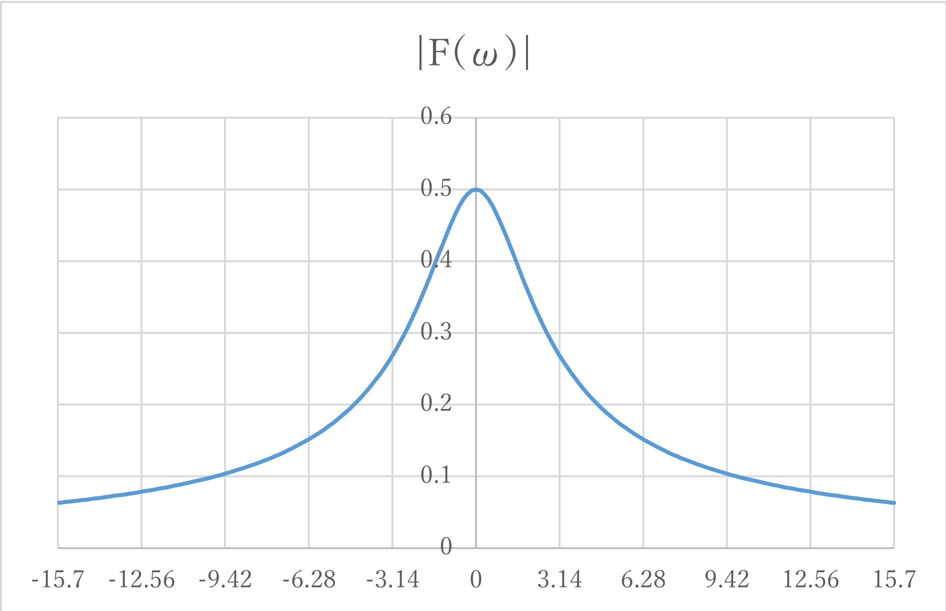
グラフ：

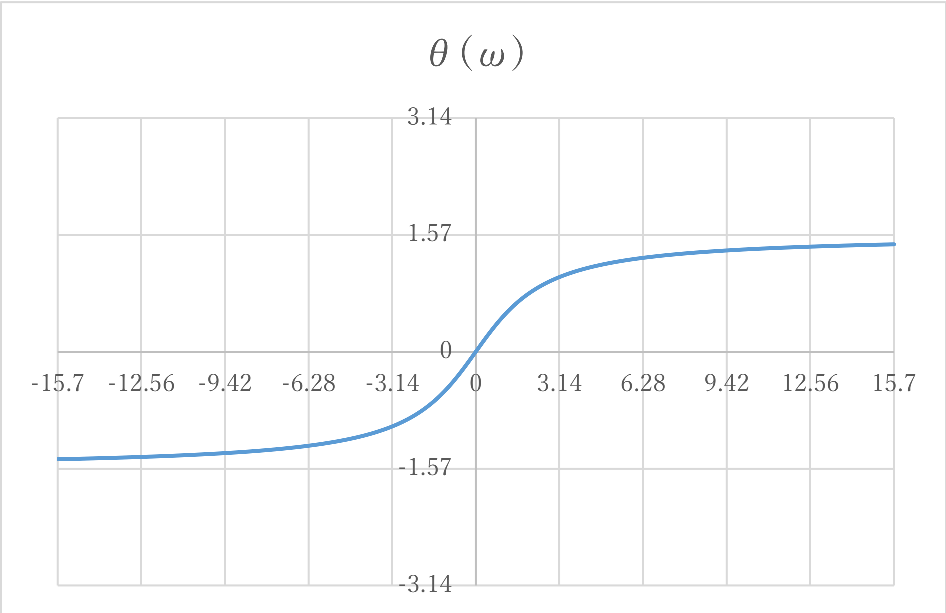


応数Ｉ（フーリエ）　課題７

H30 年度 番号　4J42







応数 I（フーリエ）　課題 7

H30 年度 番号　4J42

[例題 10-7]の図

$$f(t) = e^{-2|t|}$$

$$F(\omega) = \int_{-\infty}^{\infty} f(t) \cdot e^{-j\omega t}\,dt$$

$$= \int_{-\infty}^{\infty} e^{-2|t|} \cdot e^{-j\omega t}\,dt = \int_{-\infty}^{0} e^{2t} \cdot e^{-j\omega t}\,dt + \int_{0}^{\infty} e^{-2t} \cdot e^{-j\omega t}\,dt$$

$$= \frac{1}{2-j\omega}\left[e^{(2-j\omega)t}\right]_{-\infty}^{0} + \frac{1}{-2-j\omega}\left[e^{-(2+j\omega)t}\right]_{0}^{\infty}$$

$$= \frac{1}{2-j\omega}(1-0) + \frac{-1}{2+j\omega}(0-1)$$

$$= \frac{1}{2-j\omega} + \frac{1}{2+j\omega} = \frac{4}{4+\omega^2}$$

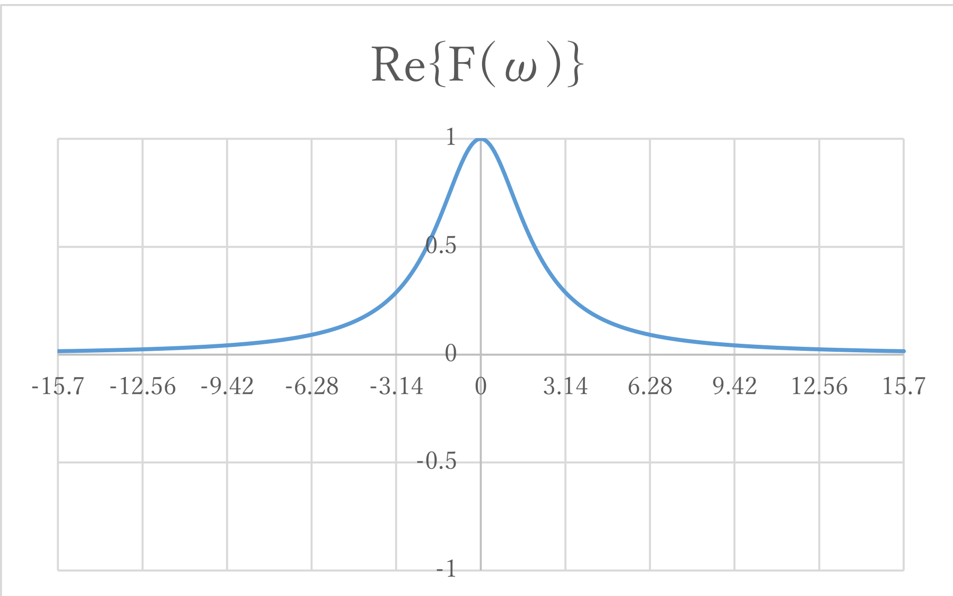
$$\mathrm{Re}\{F(\omega)\} = \frac{4}{4+\omega^2}$$

$$\mathrm{Im}\{F(\omega)\} = 0$$

$$|F(\omega)| = \sqrt{\left(\frac{4}{4+\omega^2}\right)^2 + (0)^2} = \sqrt{\left(\frac{4}{4+\omega^2}\right)^2} = \frac{4}{4+\omega^2}$$

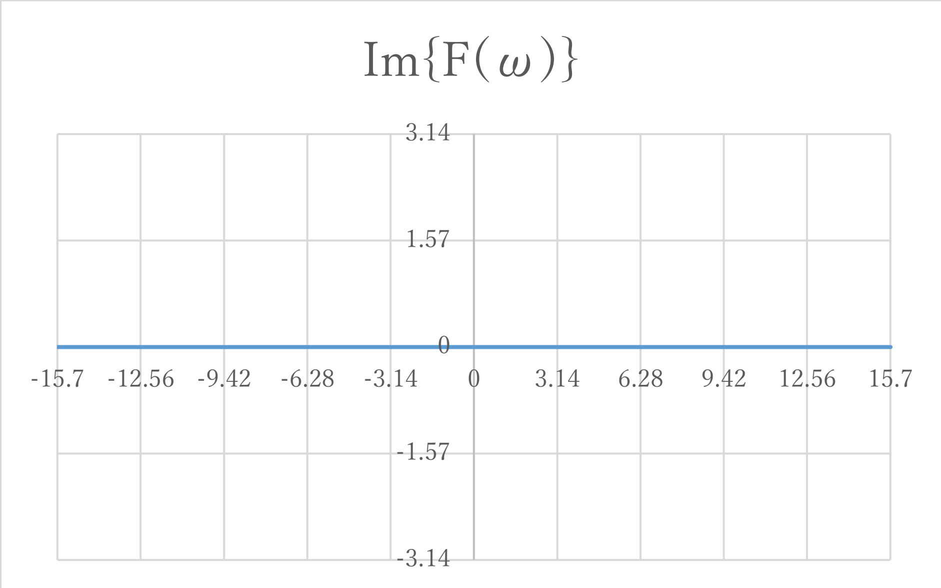
$$\theta(\omega) = \tan^{-1}\frac{Im\{F(\omega)\}}{Re\{F(\omega)\}} = \tan^{-1}\frac{0}{\frac{4}{4+\omega^2}} = 0$$

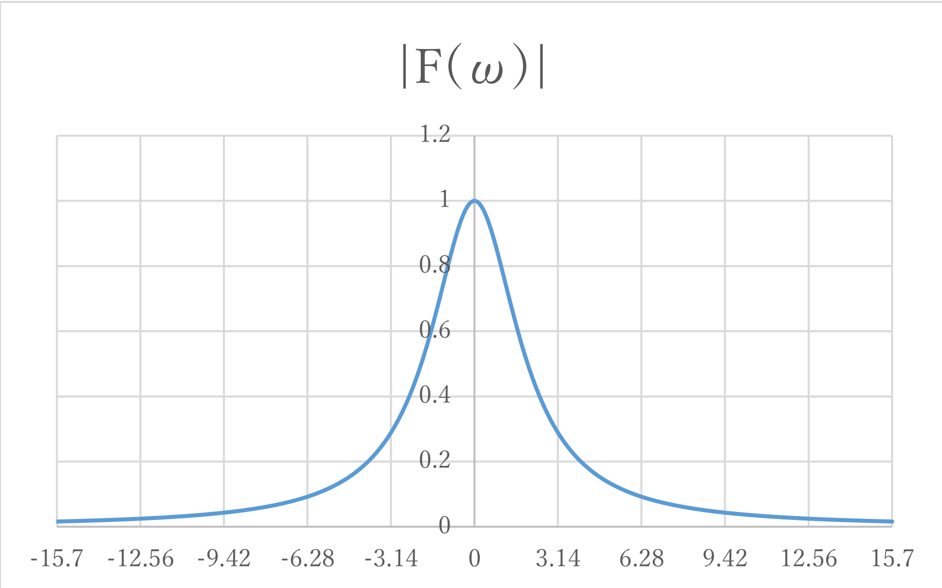
グラフ：

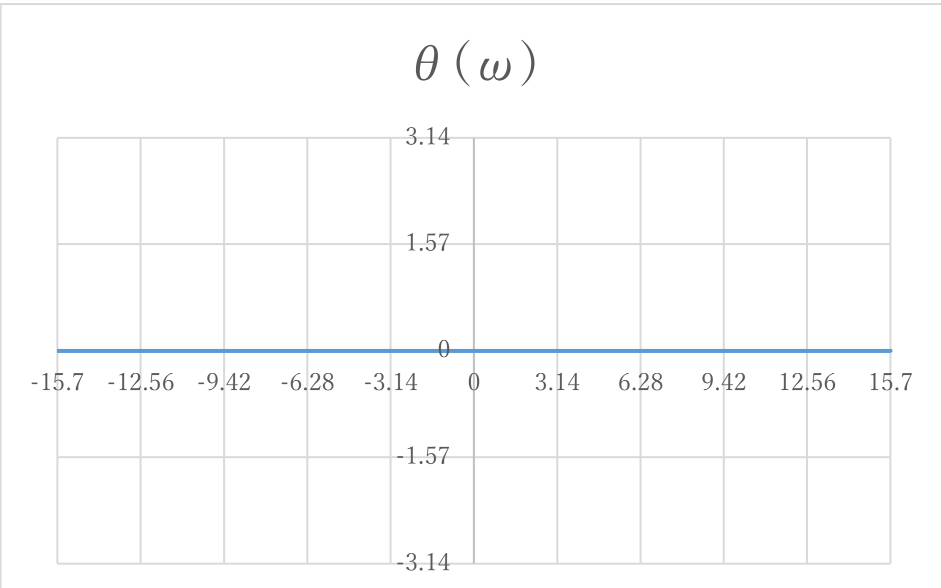


応数Ⅰ（フーリエ）　課題７

H30 年度　番号　4J42







応数Ⅰ（フーリエ）　課題７

H30 年度 番号　4J42

[例題 10-8]の図

$$f(t)=\begin{cases}\cos 2t\cdot e^{-2t} & (t\geq 0)\\ 0 & (t<0)\end{cases}$$

$$F(\omega)=\int_{-\infty}^{\infty}f(t)\cdot e^{-j\omega t}\,dt$$

$$=\int_{0}^{\infty}\cos 2t\cdot e^{-2t}\cdot e^{-j\omega t}\,dt$$

$$=\int_{0}^{\infty}\cos 2t\cdot e^{-(2+j\omega)t}\,dt$$

$$=-\frac{1}{2+j\omega}\left[\cos 2t\cdot e^{-(2+j\omega)t}\right]_0^{\infty}-\int_0^{\infty}-2\sin 2t\cdot-\frac{1}{2+j\omega}e^{-(2+j\omega)t}\,dt=$$

$$=-\frac{1}{2+j\omega}(0-1)-\frac{1}{2+j\omega}\int_0^{\infty}2\sin 2\cdot e^{-(2+j\omega)t}\,dt$$

$$=\frac{1}{2+j\omega}-\frac{2}{2+j\omega}\left\{-\frac{1}{2+j\omega}\left[\sin 2t\cdot e^{-(2+j\omega)t}\right]_0^{\infty}-\frac{1}{2+j\omega}\int_0^{\infty}\cos 2t\cdot e^{-(2+j\omega)t}\,dt\right\}$$

$$=\frac{1}{2+j\omega}-\frac{2}{2+j\omega}\left\{0+\frac{2}{2+j\omega}\int_0^{\infty}\cos 2t\cdot e^{-(2+j\omega)t}\,dt\right\}$$

$$=\frac{1}{2+j\omega}-\frac{4}{(2+j\omega)^2}\int_0^{\infty}\cos 2t\cdot e^{-(2+j\omega)t}\,dt$$

$$=\frac{1}{2+j\omega}-\frac{4}{(2+j\omega)^2}\cdot F(\omega)$$

$$=\frac{1}{2+j\omega}\left(\frac{1}{1+\frac{4}{(2+j\omega)^2}}\right)=\frac{1}{2+j\omega}\left(\frac{(2+j\omega)^2}{(2+j\omega)^2+4}\right)=\frac{2+j\omega}{8+4j\omega-\omega^2}$$

$$=\frac{2+j\omega}{8+4j\omega-\omega^2}\cdot\frac{8-4j\omega-\omega^2}{8-4j\omega-\omega^2}=\frac{16-2\omega^2-j\omega^3+4\omega^2}{(8-\omega^2)+16\omega^2}=\frac{2\omega^2+16-j\omega^3}{\omega^4+6}$$

$$\mathrm{Re}\{F(\omega)\}=\frac{2\omega^2+16}{64+\omega^4}$$

$$\mathrm{Im}\{F(\omega)\}=\frac{-\omega^3}{64+\omega^4}$$

$$|F(\omega)|=\sqrt{\left(\frac{2\omega^2+16}{64+\omega^4}\right)^2+\left(\frac{-\omega^3}{64+\omega^4}\right)^2}=\sqrt{\left(\frac{4+16-\omega^3}{64+\omega^4}\right)^2}=\sqrt{\left(\frac{2\omega^2+16+\omega^3}{8+\omega^2}\right)^2}$$

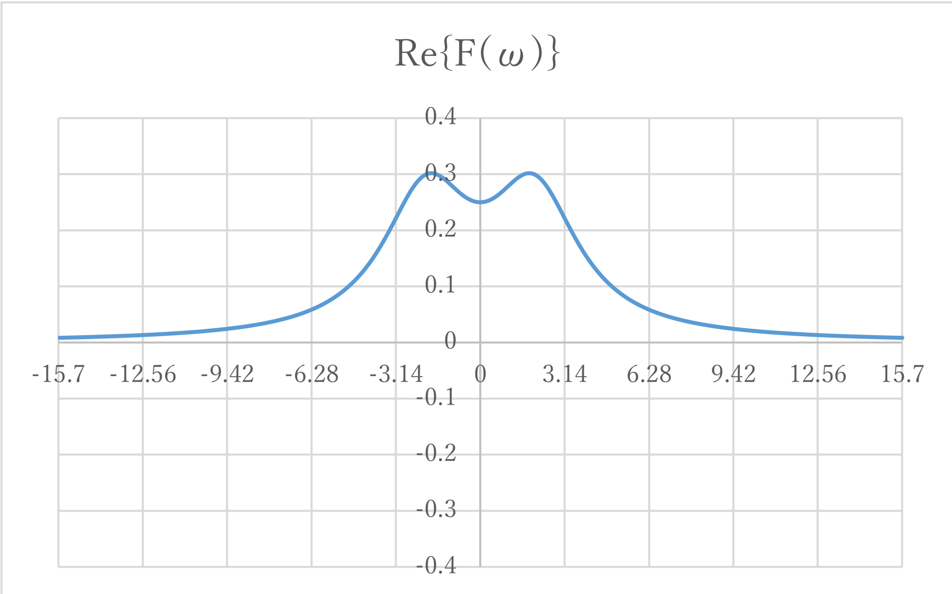
$$=\frac{(64+\omega^4)^2+(\omega^2+4\ \ )}{(64+\omega^4)^2}=\frac{\sqrt{\omega^2+4}}{\sqrt{\omega^4+64}}$$

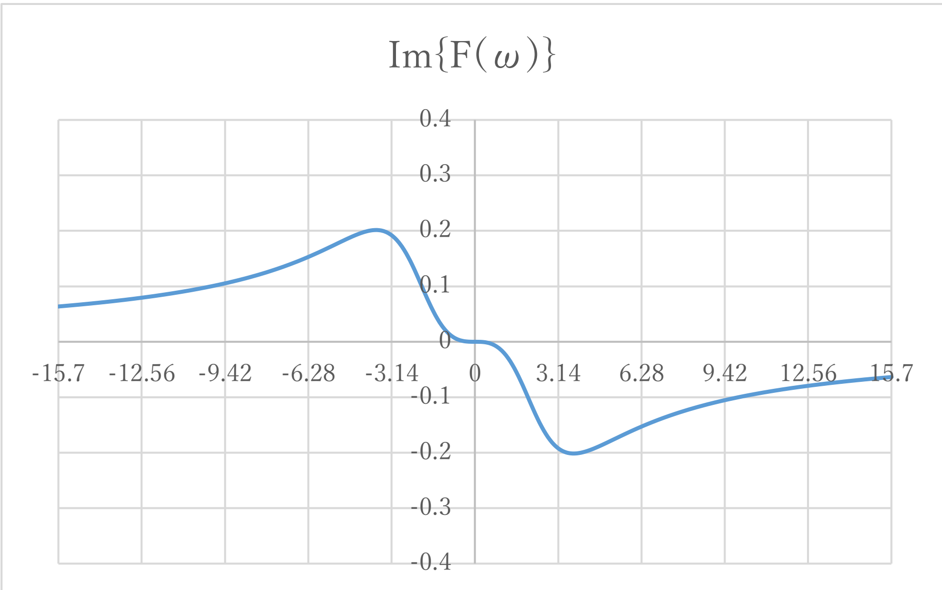
応数Ⅰ（フーリエ）　課題７

H30 年度　番号　4J42

$$\theta(\omega) = \tan^{-1}\frac{Im\{F(\omega)\}}{Re\{F(\omega)\}} = \tan^{-1}\frac{\frac{-\omega^3}{64+\omega^4}}{\frac{2\omega^2+16}{64+\omega^4}} = \tan^{-1} - \frac{\omega^3}{2\omega^2+16} = -\tan^{-1}\frac{\omega^3}{2\omega^2+16}$$

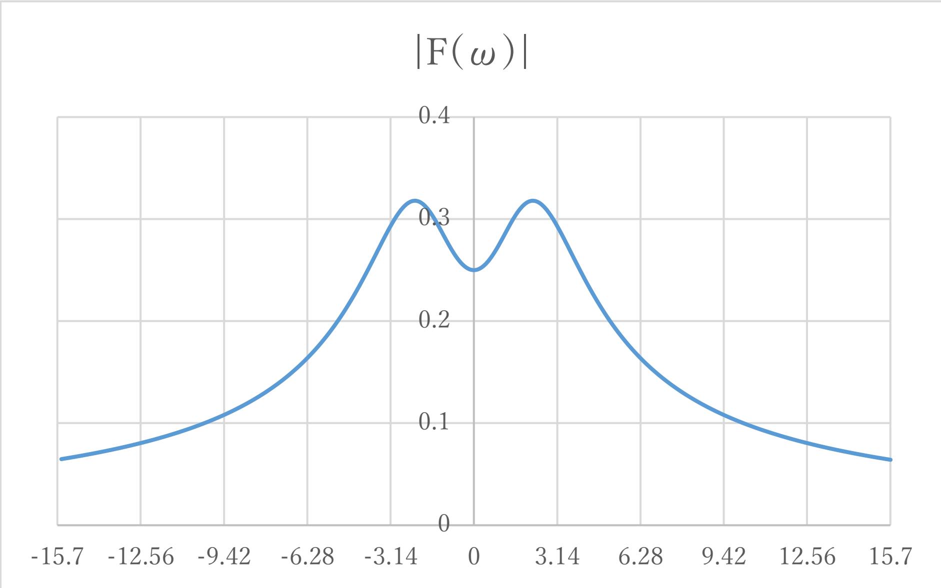
グラフ：

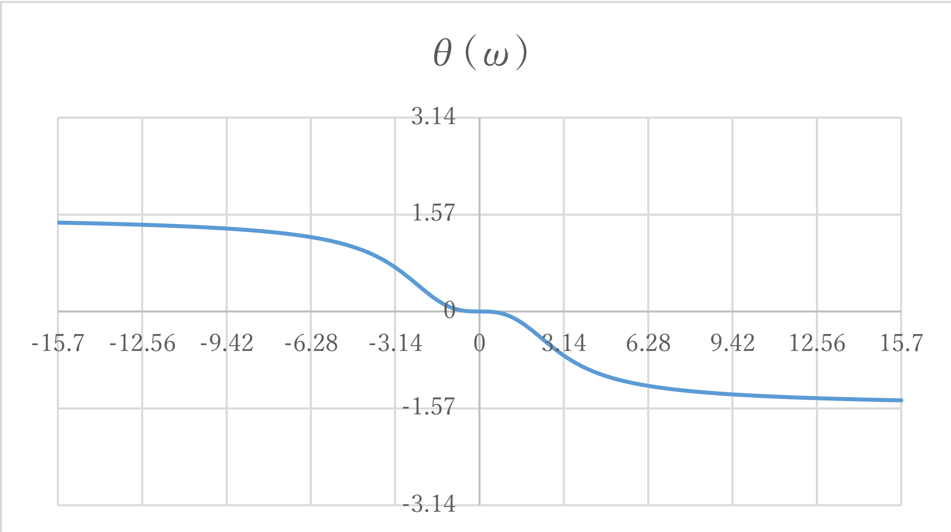




応数Ⅰ（フーリエ）　課題７

H30 年度　番号　4J42





応数Ⅰ（フーリエ）　課題７

H30 年度 番号　4J42

[演習 3]

$$f(t)=\begin{cases}1-|t| & (|t|\le 1)\\ 0 & (|t|>1)\end{cases}$$

$$F(\omega)=\int_{-\infty}^{\infty} f(t)\cdot e^{-j\omega t}\,dt$$

$$=\int_{-1}^{0}(1+t)\cdot e^{-j\omega t}\,dt+\int_{0}^{1}(1+t)\cdot e^{-j\omega t}\,dt$$

$$=-\frac{1}{j\omega}\left[(1+t)\cdot e^{-j\omega t}\right]_{-1}^{0}-\int_{-1}^{0}-\frac{1}{j\omega}e^{-j\omega t}\,dt$$

$$-\frac{1}{j\omega}\left[(1-t)\cdot e^{-j\omega t}\right]_{0}^{1}-\int_{0}^{1}-1\cdot-\frac{1}{j\omega}e^{-j\omega t}\,dt$$

$$=\ -\frac{1}{j\omega}(1-0)+\frac{1}{j\omega}\left[-\frac{1}{j\omega}e^{-j\omega t}\right]_{-1}^{0}-\frac{1}{j\omega}(0-1)-\frac{1}{j\omega}\left[-\frac{1}{j\omega}e^{-j\omega t}\right]_{0}^{1}$$

$$=\frac{1}{j\omega}\left\{-\frac{1}{j\omega}-\left(-\frac{1}{j\omega}e^{j\omega}\right)\right\}-\frac{1}{j\omega}\left\{-\frac{1}{j\omega}e^{-j\omega}-\left(-\frac{1}{j\omega}\right)\right\}$$

$$=-\frac{1}{(j\omega)^2}+\frac{1}{(j\omega)^2}e^{j\omega}+\frac{1}{(j\omega)^2}e^{-j\omega}-\frac{1}{(j\omega)^2}$$

$$=\frac{1}{\omega^2}-\frac{1}{\omega^2}e^{j\omega}-\frac{1}{\omega^2}e^{-j\omega}+\frac{1}{\omega^2}$$

$$=\frac{1}{\omega^2}\left(2-e^{j\omega}-e^{-j\omega}\right)=\frac{2}{\omega^2}(1-\cos\omega)$$

$$=\frac{4}{\omega^2}\sin^2\left(\frac{\omega}{2}\right)=sinc^2\left(\frac{4}{\omega}\right)$$

$$\mathrm{Re}\{F(\omega)\}=sinc^2\left(\frac{4}{\omega}\right)$$

$$\mathrm{Im}\{F(\omega)\}=0$$

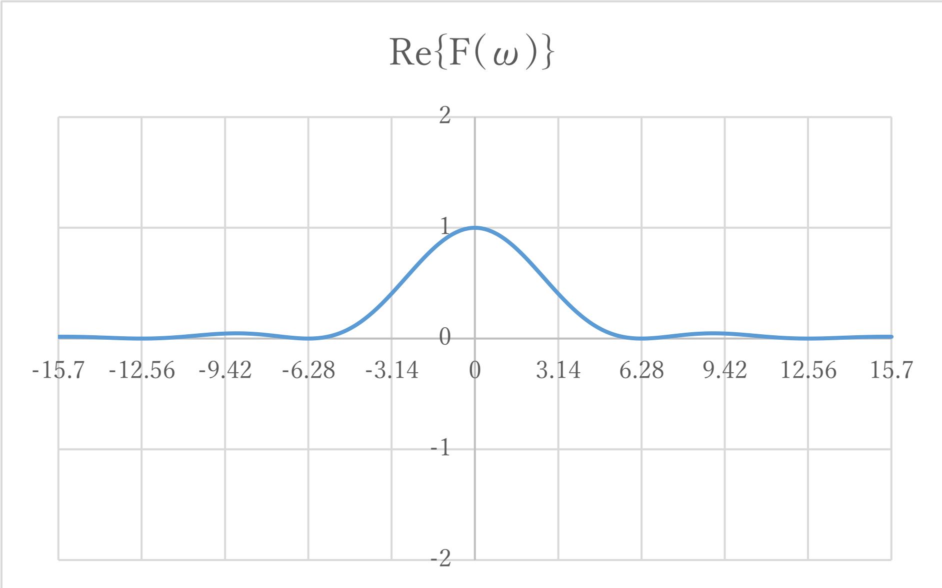
$$|F(\omega)|=\sqrt{\left(sinc^2\left(\frac{4}{\omega}\right)\right)^2+(0)^2}=\sqrt{\left(sinc^2\left(\frac{4}{\omega}\right)\right)^2}=sinc^2\left(\frac{4}{\omega}\right)$$

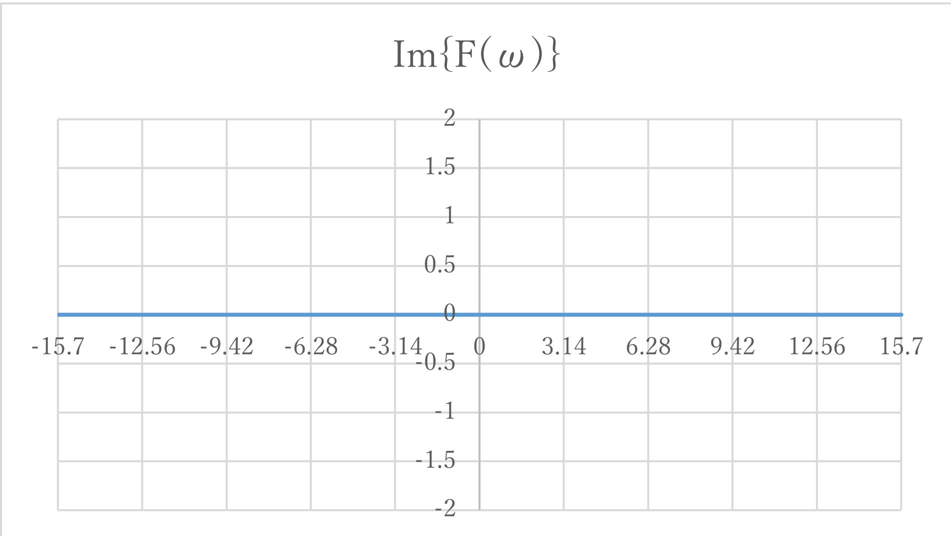
$$\theta(\omega)=\tan^{-1}\frac{Im\{F(\omega)\}}{Re\{F(\omega)\}}=\tan^{-1}\frac{0}{sinc^2\left(\frac{4}{\omega}\right)}=\tan^{-1}0=0$$

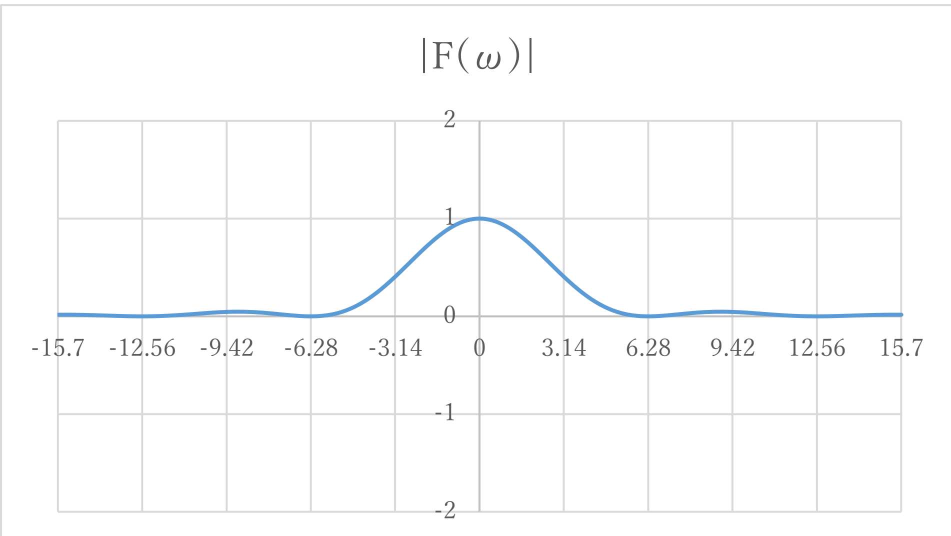
応数Ⅰ（フーリエ）　課題７

H30 年度 番号　4J42

グラフ：

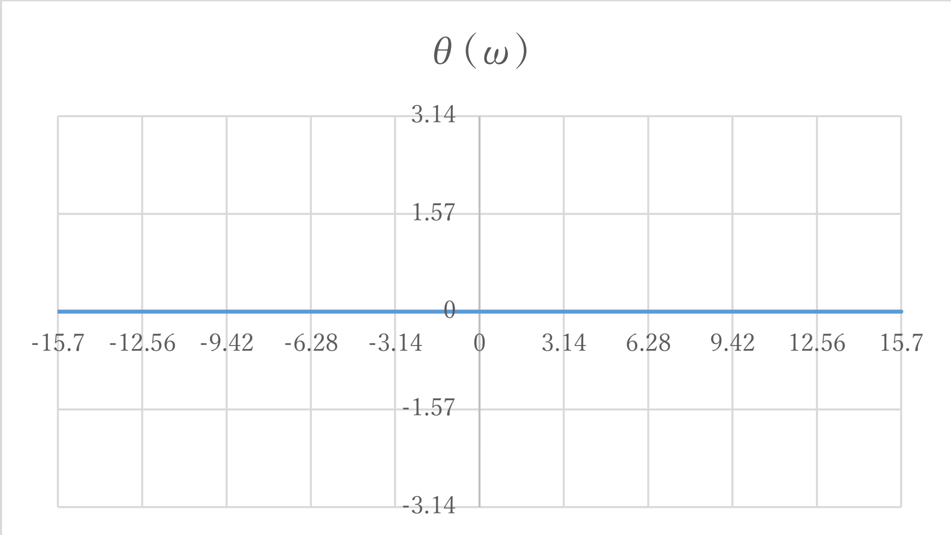






応数Ⅰ（フーリエ）　課題７

H30 年度 番号　4J42



[演習 4]

$$f(t)=\begin{cases}5e^{-3t} & (t\geq 0)\\ 0 & (t<0)\end{cases}$$

$$F(\omega)=\int_{-\infty}^{\infty} f(t)\cdot e^{-j\omega t}\,dt$$

$$=\int_{0}^{\infty} 5e^{-3t}\cdot e^{-j\omega t}\,dt$$

$$=\int_{0}^{\infty} 5e^{-(3+j\omega)t}\,dt$$

$$=-\frac{5}{3+j\omega}\left[e^{-(3+j\omega)t}\right]_0^{\infty}$$

$$=-\frac{5}{3+j\omega}(0-1)=\frac{5}{3+j\omega}=\frac{5(3-j\omega)}{(3+j\omega)(3-j\omega)}$$

$$=\frac{15-5j\omega}{9+\omega^2}=\frac{15}{9+\omega^2}+j\frac{-5\omega}{9+\omega^2}$$

$$\mathrm{Re}\{F(\omega)\}=\frac{15}{9+\omega^2}$$

$$\mathrm{Im}\{F(\omega)\}=\frac{-5\omega}{9+\omega^2}$$

$$|F(\omega)|=\sqrt{\left(\frac{15}{9+\omega^2}\right)^2+\left(\frac{-5\omega}{9+\omega^2}\right)^2}=\sqrt{\left(\frac{15-5\omega}{9+\omega^2}\right)^2}=\sqrt{\frac{25(9+\omega^2)}{(9+\omega^2)^2}}=\frac{5\sqrt{9+\omega^2}}{9+\omega^2}$$

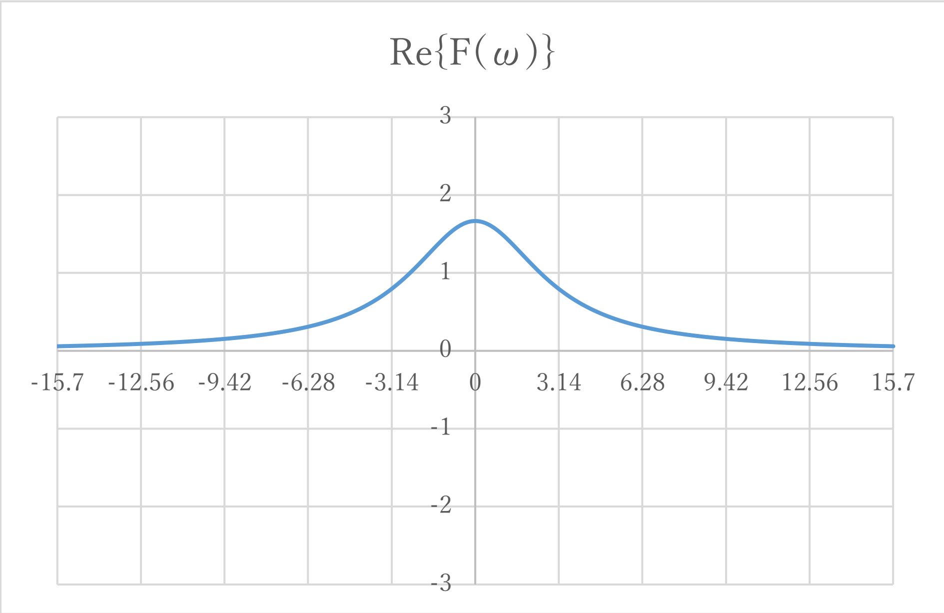
$$=\frac{5}{\sqrt{9+\omega^2}}$$

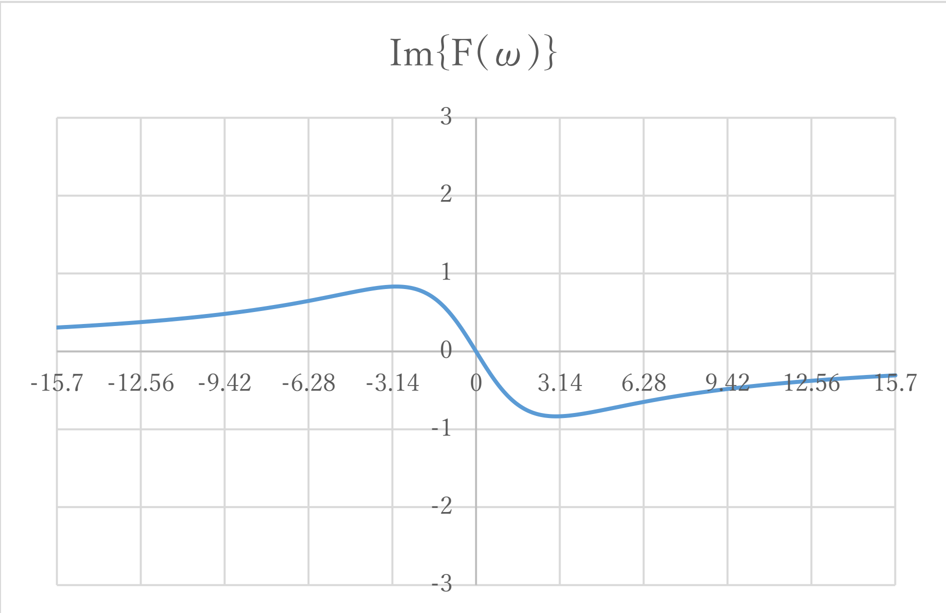
$$\theta(\omega)=\tan^{-1}\frac{Im\{F(\omega)\}}{Re\{F(\omega)\}}=\tan^{-1}\frac{\frac{-5\omega}{9+\omega^2}}{\frac{15}{9+\omega^2}}=\tan^{-1}-\frac{\omega}{3}=-\tan^{-1}\frac{\omega}{3}$$

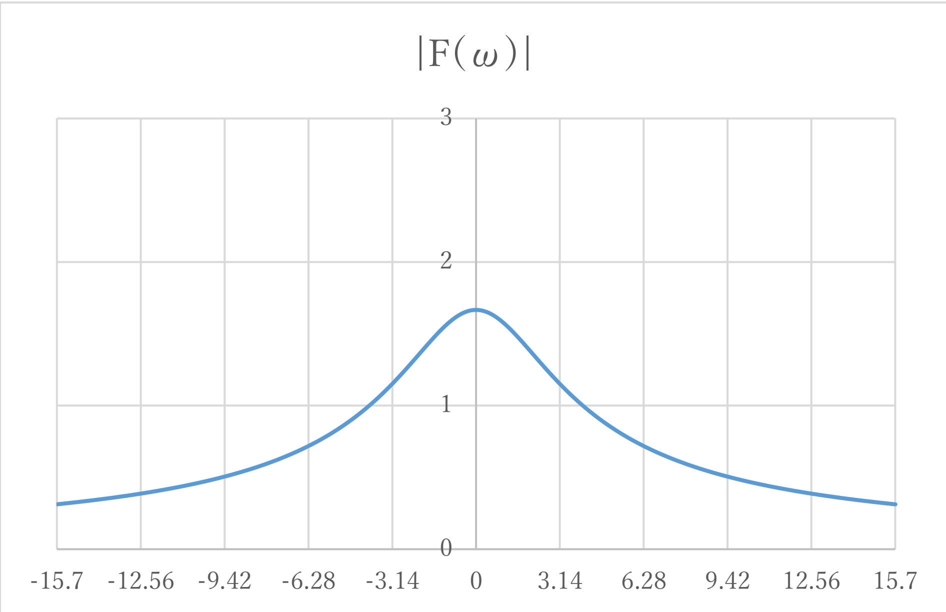
応数 I（フーリエ）　課題７

H30 年度　番号　4J42

グラフ：

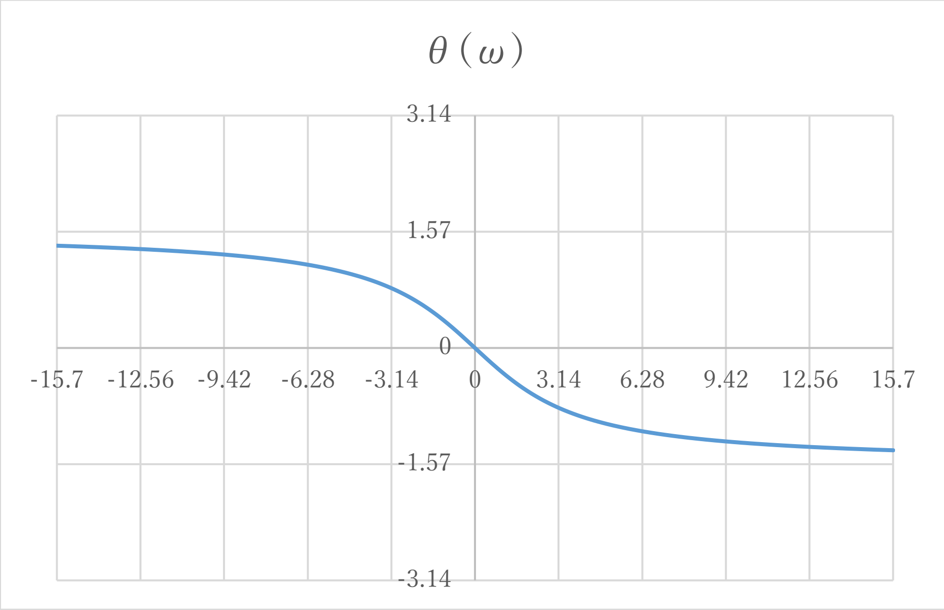






応数Ⅰ（フーリエ）　課題７

H30 年度 番号　4J42



[演習 5]

$$f(t)=\begin{cases}\sin 2t\cdot e^{-2t} & (t\geq 0)\\ 0 & (t<0)\end{cases}$$

$$F(\omega)=\int_{-\infty}^{\infty} f(t)\cdot e^{-j\omega t}\,dt=\int_{0}^{\infty} \sin 2t\cdot e^{-2t}\,dt=\int_{0}^{\infty}\frac{1}{2j}\left(e^{j2t}-e^{-j2t}\right)\cdot e^{-2t}\cdot e^{-j\omega t}$$

$$=\frac{1}{2j}\int_{0}^{\infty}\left\{e^{j2t-2t}\cdot e^{-j\omega t}-e^{-j2t-2t}\cdot e^{-j\omega t}\right\}dt$$

$$=\frac{1}{2j}\int_{0}^{\infty}\{e^{(-2+j(2-\omega))t}-e^{-(2+j(2+\omega))t}\,dt$$

$$=\frac{1}{2j}\left\{\frac{1}{-2+j(2-\omega)}\left[e^{(-2+j(2-\omega))t}\right]_{0}^{\infty}+\frac{1}{2+j(2+\omega)}\left[e^{-(2+j(2+\omega)t)}\right]_{0}^{\infty}\right\}$$

$$=\frac{1}{2j}\left\{\frac{-1}{-2+j(2-\omega)}+\frac{-1}{2+j(2+\omega)}\right\}=\frac{1}{2j}\left\{\frac{1}{2-j(2-\omega)}-\frac{1}{2+j(2+\omega)}\right\}$$

$$=\frac{1}{2j}\left\{\frac{2+j\omega+2j-2-j\omega+2j}{(2+j\omega)^2-(2j)^2}\right\}=\frac{1}{2j}\left\{\frac{4j}{(2+j\omega)^2-(2j)^2}\right\}$$

$$=\frac{2}{4+4j\omega-\omega^2+4}=\frac{2}{4j\omega-\omega^2+8}=\frac{2(8-\omega^2-4j\omega)}{(8-\omega^2+4j\omega)(8-\omega^2-4j\omega)}$$

$$=\frac{16-2\omega^2-8j\omega}{(8-\omega^2)^2+16\omega^2}=\frac{16-2\omega^2-8j\omega}{64-16\omega^2+16\omega^2+\omega^4}=\frac{16-2\omega^2}{\omega^4+64}+j\frac{-8\omega}{\omega^4+64}$$

$$\mathrm{Re}\{F(\omega)\}=\frac{16-2\omega^2}{\omega^4+64}$$

$$\mathrm{Im}\{F(\omega)\}=\frac{-8\omega}{\omega^4+64}$$

応数Ⅰ（フーリエ）　課題７

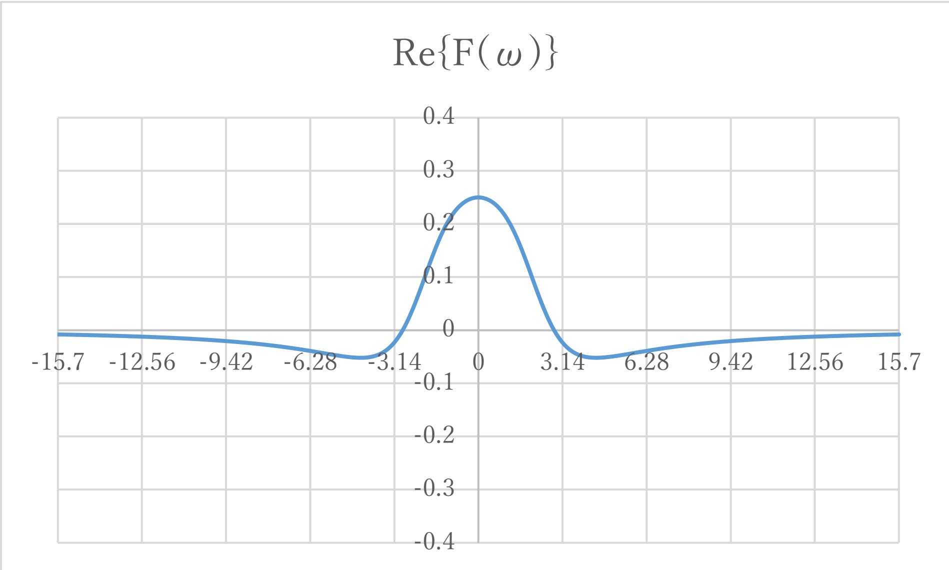
H30 年度 番号　4J42

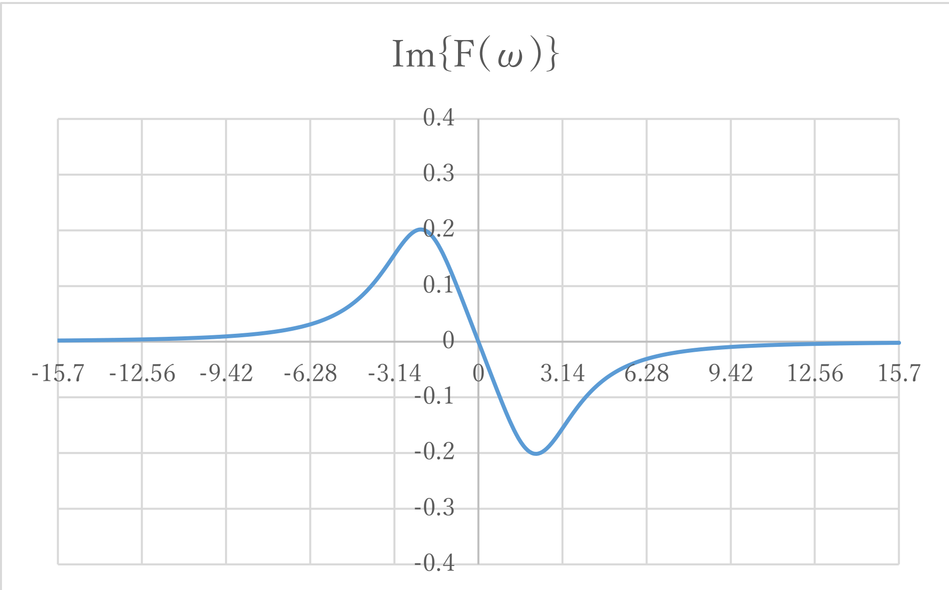
$$|F(\omega)| = \sqrt{\left(\frac{16-2\omega^2}{\omega^4+64}\right)^2 + \left(\frac{-8\omega}{\omega^4+64}\right)^2} = \sqrt{\frac{16-64\omega^2+4\omega^4}{(\omega^4+64)^2}} = \sqrt{\frac{16^2+4\omega^4}{(\omega^4+64)^2}}$$

$$= \sqrt{\frac{4(64+\omega^4)}{(\omega^4+64)^2}} = \frac{2}{\sqrt{\omega^2+64}}$$

$$\theta(\omega) = \tan^{-1}\frac{Im\{F(\omega)\}}{Re\{F(\omega)\}} = \tan^{-1}\frac{\frac{-8\omega}{\omega^4+64}}{\frac{16-2\omega^2}{\omega^4+64}} = \tan^{-1} - \frac{8\omega}{16-2\omega^2} = -\tan^{-1}\frac{4\omega}{8-\omega^2}$$

グラフ：





応数Ⅰ（フーリエ）　課題７

H30 年度　番号　4J42