応数 I（フーリエ）　課題６

H30 年度 番号　4J42

フーリエ変換

例題 10-4，10-5 を参考にし，章末問題の[演習 1]～[演習 2]を行う.

[例題 10-4]

$$\mathrm{F}(\omega) = \int_{-\infty}^{\infty} f(t) \cdot e^{-j\omega t}\, dt$$

$$= \int_{-2}^{2} 2 \cdot e^{-j\omega t}\, dt = -\frac{2}{j\omega}\left[e^{-j\omega t}\right]_{-2}^{2} = -\frac{2}{j\omega}\left(e^{-2j\omega} - e^{-2j\omega}\right)$$

$$= -\frac{2}{j\omega}\left(-2\mathrm{j}\sin(2\omega)\right) = \frac{4}{\omega}\left(\sin(2\omega)\right) = \frac{8}{2\omega}\left(\sin(2\omega)\right) = 8\mathrm{sinc}(2\omega)$$

[例題 10-5]

$$\mathrm{F}(\omega) = \int_{-\infty}^{\infty} f(t) \cdot e^{-j\omega t}\, dt$$

$$= \int_{-2}^{2} (2 - |t|) \cdot e^{-j\omega t} = \int_{-2}^{0} (2 + t) \cdot e^{-j\omega t}\, dt + \int_{0}^{2} (2 - t) \cdot e^{-j\omega t}\, dt$$

$$= \int_{0}^{2} (2 - t)\left(\frac{1}{j\omega} e^{-j\omega t}\right)' dt + \int_{0}^{2} (2 - t)\left(\frac{1}{j\omega} e^{-j\omega t}\right)' dt$$

$$= \left(-\frac{1}{j\omega}\right)\left[(2 + t) \cdot e^{-j\omega t}\right]_{-2}^{0} + \int_{-2}^{0} e^{-j\omega t}\, dt$$

$$+ \left(-\frac{1}{j\omega}\right)\left[(2 - t) \cdot e^{-j\omega t}\right]_{0}^{2} - \frac{1}{j\omega}\int_{0}^{2} e^{-j\omega t}\, dt$$

$$= \left(-\frac{1}{j\omega}\right)(2 - 0) + \left(\frac{1}{\omega^2} - \frac{1}{\omega^2} e^{2j\omega}\right)$$

$$+ \left(-\frac{1}{j\omega}\right)\left(0 - (2)\right) + \left(-\frac{1}{\omega^2} e^{-2j\omega} + \frac{1}{\omega^2}\right)$$

$$= -\frac{2}{j\omega} + \frac{2}{j\omega} + \left(\frac{1}{\omega^2} - \frac{1}{\omega^2} e^{2j\omega}\right) + \left(-\frac{1}{\omega^2} e^{-2j\omega} + \frac{1}{\omega^2}\right)$$

$$= \frac{1}{\omega^2}\left(1 - e^{2j\omega}\right) + \frac{1}{\omega^2}\left(-e^{-2j\omega} + 1\right) = \frac{1}{\omega^2}\left(2 - e^{-2j\omega} - e^{2j\omega}\right)$$

$$= \frac{1}{\omega^2}\left(2 - 2\cos(2\omega)\right) = \frac{2}{\omega^2}\ (1 - cos2\omega) = \frac{2}{\omega^2}\ \sin^2(\omega) =\ 2sinc(\omega)$$

[演習 1]

$$\mathrm{e}^{\mathrm{i}\theta} = cos\theta + jsin\theta$$

$$\mathrm{e}^{-\mathrm{j}\theta} = cos\theta - jsin\theta$$

より、

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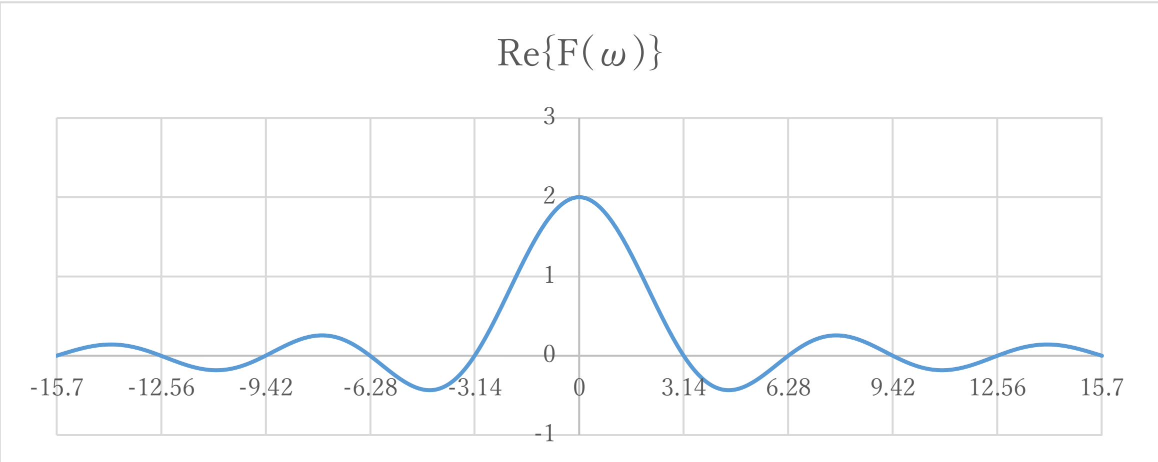
$$2\cos\theta = e^{j\theta} - e^{-j\theta}$$
$$j2\sin\theta = e^{j\theta} - e^{-j\theta}$$

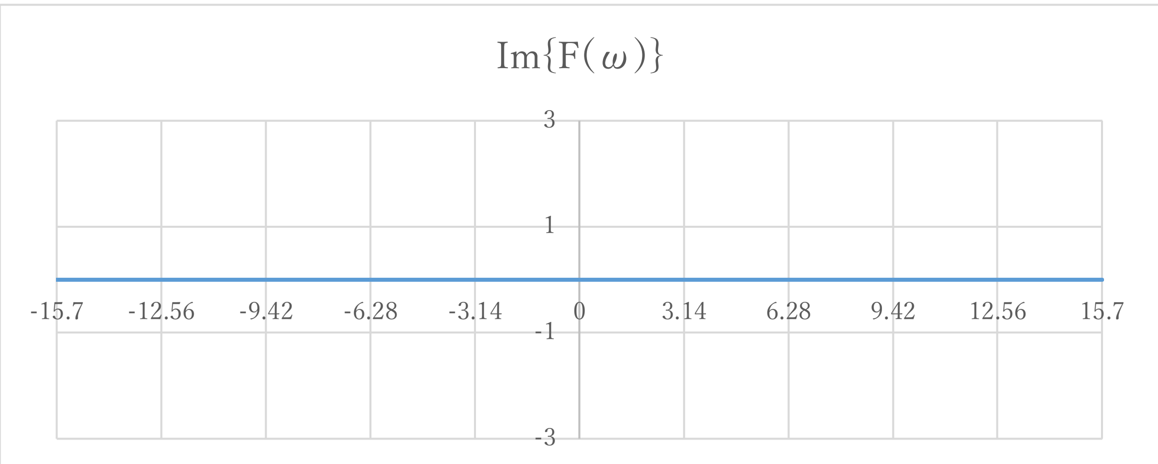
［演習 2］

$$\mathrm{F}(\omega) = \int_{-\infty}^{\infty} f(t)\cdot e^{-j\omega t}\,dt = \int_{-1}^{1} e^{-j\omega t}\,dt = -\frac{1}{j\omega}\left[e^{-j\omega t}\right]_{-1}^{1} = -\frac{1}{j\omega}\left(e^{-j\omega} - e^{j\omega}\right)$$
$$= -\frac{1}{j\omega}(-2j sin\omega) = \frac{2}{\omega}\sin(\omega) = 2sinc(\omega)$$
$$\mathrm{Re}\{\mathrm{F}(\omega)\} = 2sinc(\omega)\,,\mathrm{Im}\{\mathrm{F}(\omega)\} = 0$$
$$|\mathrm{F}(\omega)| = \sqrt{(\mathrm{Re}\{\mathrm{F}(\omega)\}^2 + \mathrm{Im}\{\mathrm{F}(\omega)\}^2)} = \sqrt{|2sinc(\omega)|^2} = 2sinc(\omega)$$

グラフ：

グラフの題名は縦軸　横軸は全て ω $(-5\pi\sim5\pi)$





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