応数Ｉ（フーリエ）　課題５

H30 年度 番号 4J42

複素数の復習

例題 9-1,9-2 を参考に章末問題の[演習 1]～[演習 3]を行う.

[演習 1]

(1)

$$e^{j\frac{2\pi}{3}}$$

(2)

$$e^{j\frac{3\pi}{4}}$$

[演習 2]

(1) $\theta = \frac{3}{4}\pi$

$$e^{j\frac{3\pi}{4}} = \cos\left(\frac{3}{4}\pi\right) + jsin\left(\frac{3}{4}\pi\right) = -\frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}$$

(2)

$$e^{j\frac{5\pi}{3}} = \cos\left(\frac{5}{3}\pi\right) + jsin\left(\frac{5}{3}\pi\right) = \frac{1}{2} - j\frac{\sqrt{3}}{2}$$

[演習 3]

$$\cos\theta + j\sin\theta = \frac{e^{j\theta} + e^{-j\theta}}{2} + j\left(\frac{e^{j\theta} + e^{-j\theta}}{2j}\right) = \frac{1}{2}\left\{2e^{j\theta}\right\} = e^{i\theta}$$

複素フーリエ級数・係数

例題 9-3, 9-4 を参考に章末問題の[演習 4]～[演習 5]を行う.

[演習 4]

(1) $T = 2\,, \omega_0 = \frac{2\pi}{T} = \pi$

$$c_0 = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)\,dt = \frac{1}{2}\int_{-1}^{1} f(t)\,dt = \frac{1}{2}\int_{-\frac{1}{2}}^{\frac{1}{2}} 1\,dt = \frac{1}{2}[t]_{-\frac{1}{2}}^{\frac{1}{2}} = \frac{1}{2}(1) = \frac{1}{2}$$

$$c_n = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)\cdot e_0^{-jn\omega_0 t}\,dt = \frac{1}{2}\int_{-1}^{1} f(t)\cdot e_0^{-jn\omega_0 t}\,dt = \frac{1}{2}\int_{-\frac{1}{2}}^{\frac{1}{2}} 1\cdot e_0^{-jn\omega_0 t}\,dt = \frac{1}{2}\left[-\frac{1}{j\omega_0 n}e^{-j\omega_0 nt}\right]_{-\frac{1}{2}}^{\frac{1}{2}}$$

$$= \frac{-1}{2j\omega_0 n}\left(e^{-j\omega_0 n\cdot\frac{1}{2}} - e^{j\omega_0 n\cdot\frac{1}{2}}\right) = -\frac{1}{2j\pi n}\left(e^{-\frac{j\pi n}{2}} - e^{-\frac{j\pi n}{2}}\right)$$

$$= -\frac{1}{2j\pi n}\left\{\cos\left(\frac{\pi n}{2}\right) - jsin\left(\frac{\pi n}{2}\right) - \left(cos\left(\frac{\pi n}{2}\right) + sin\left(\frac{\pi n}{2}\right)\right)\right\} = -\frac{1}{2j\pi n}\left\{-2jsin\left(\frac{\pi n}{2}\right)\right\}$$

$$= \frac{1}{n\pi}sin\left(\frac{n\pi}{2}\right)$$

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(2) $\mathrm{T}=2\,,\omega_0=\frac{2\pi}{T}=\pi$

$$\mathrm{f(t)}=\sum_{n=-\infty}^{\infty}c_n\cdot e^{j\omega_0 nt}=\sum_{n=-\infty}^{\infty}\left\{\frac{1}{n\pi}\sin\left(\frac{n\pi}{2}\right)\right\}\cdot e^{jn\pi t}$$

$$=\frac{1}{2}+\frac{1}{\pi}\sin\left(\frac{\pi}{2}\right)\cdot e^{j\pi t}+\left(-\frac{1}{\pi}\right)\sin\left(-\frac{\pi}{2}\right)\cdot e^{-j\pi t}+\frac{1}{2\pi}\sin(\pi)\,e^{j2\pi t}+\left(-\frac{1}{2\pi}\right)\sin(-\pi)$$

$$+\frac{1}{3\pi}\sin\left(\frac{3}{2}\pi\right)\cdot e^{-j3\pi t}+\left(-\frac{1}{3\pi}\right)\sin\left(-\frac{3}{2}\pi\right)+\cdots$$

$$=\frac{1}{2}+\frac{1}{\pi}\cdot e^{j\pi t}-\frac{1}{\pi}e^{-j\pi t}+0+0+\left(-\frac{1}{3\pi}\right)\cdot e^{j3\pi t}+\frac{1}{3\pi}\cdot e^{-j3\pi t}+\cdots$$

$$=\frac{1}{2}+\frac{1}{\pi}\left(e^{j\pi t}+e^{-j\pi t}\right)-\frac{1}{3\pi}\left(e^{j3\pi t}+e^{-j3\pi t}\right)+\cdots$$

$$=\frac{1}{2}+\frac{1}{\pi}(2\cos(\pi t)-\frac{1}{3\pi}\{2\cos(3\pi t)\}+\cdots$$

$$=\frac{1}{2}+\sum_{n=1}^{\infty}\frac{1}{(2n-1)\pi}\{2\cos\left((2n-1)\pi t\right)\}$$

(3)

$$\mathrm{c}_0=\frac{1}{2}=\frac{1}{2}+j0\,,|\mathrm{c}_0|=\sqrt{\frac{1}{4}}=\frac{1}{2}\,,\theta_0=0$$

$$\mathrm{c}_1=\frac{1}{\pi}\sin\left(\frac{\pi}{2}\right)=\frac{1}{\pi}=\frac{1}{\pi}+j0\,,|\mathrm{c}_1|=\sqrt{\frac{1}{\pi^2}}=\frac{1}{\pi}\,,\theta_1=0$$

$$\mathrm{c}_2=\frac{1}{2\pi}\sin(\pi)=0=0+j0\,,|\mathrm{c}_2|=\sqrt{0}=0\,,\theta_2=0$$

$$\mathrm{c}_3=\frac{1}{3\pi}\sin\left(\frac{3\pi}{2}\right)=-\frac{1}{3\pi}=-\frac{1}{3\pi}+j0\,,|\mathrm{c}_3|=\sqrt{\frac{1}{9\pi^2}}=\frac{1}{3\pi}\,,\theta_3=0$$

$$\mathrm{c}_4=\frac{1}{4\pi}\sin(2\pi)=0=0+j0\,,|\mathrm{c}_4|=\sqrt{0}=0\,,\theta_4=0$$

$$\mathrm{c}_5=\frac{1}{5\pi}\sin\left(\frac{5\pi}{2}\right)=-\frac{1}{5\pi}=-\frac{1}{5\pi}+j0\,,|\mathrm{c}_5|=\sqrt{\frac{1}{25\pi^2}}=\frac{1}{5\pi}\,,\theta_5=0$$

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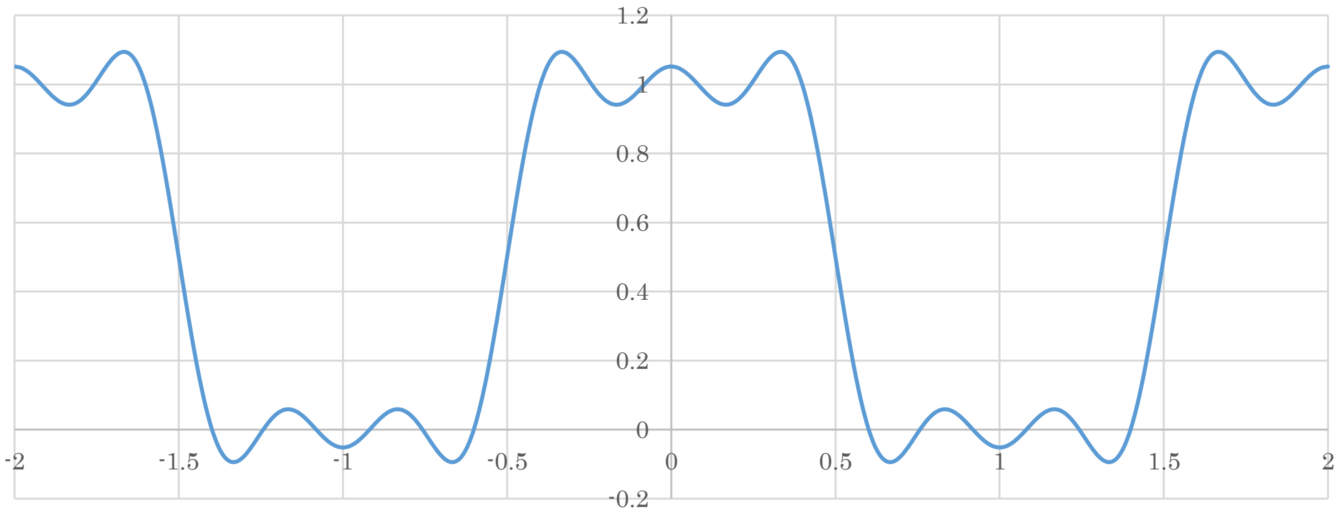
(4) グラフ

$$f(t) = \frac{1}{2} + \sum_{n=1}^{3} \frac{1}{(2n-1)\pi}\{2\cos((2n-1)\pi t)\}$$



[演習 5]

(1) $T = 2\,,\, \omega_0 = \frac{2\pi}{T} = \pi$

$$c_0 = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)\,dt = \frac{1}{2}\int_{-1}^{1} f(t)\,dt = \frac{1}{2}\int_{0}^{1} 1\,dt = \frac{1}{2}[t]_0^1 = \frac{1}{2}(1) = \frac{1}{2}$$

$$c_n = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)\cdot e_0^{-jn\omega_0 t}\,dt = \frac{1}{2}\int_{-1}^{1} f(t)\cdot e_0^{-jn\omega_0 t}\,dt = \frac{1}{2}\int_{0}^{1} 1\cdot e_0^{-jn\omega_0 t}\,dt = \frac{1}{2}\left[-\frac{1}{jn\pi}e^{-jn\pi t}\right]_0^1$$

$$= -\frac{1}{2jn\pi}\{e^{-jn\pi} - 1\} = -\frac{1}{2jn\pi}\left(e^{-j\frac{n\pi}{2}-j\frac{n\pi}{2}} - e^{j\frac{n\pi}{2}-j\frac{n\pi}{2}}\right) = -\frac{1}{2jn\pi}\left(e^{-j\frac{n\pi}{2}} - e^{j\frac{n\pi}{2}}\right)\cdot e^{-j\frac{n\pi}{2}}$$

$$= -\frac{1}{2jn\pi}\left\{2j\sin\left(\frac{\pi n}{2}\right)\left(cos\left(\frac{\pi n}{2}\right) - j\sin\left(\frac{\pi n}{2}\right)\right)\right\} = \frac{1}{n\pi}\sin\left(\frac{\pi n}{2}\right)\left(cos\left(\frac{\pi n}{2}\right) - j sin\left(\frac{\pi n}{2}\right)\right)$$

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(2)

$$f(t) = \sum_{n=-\infty}^{\infty} c_n \cdot e^{j\omega_0 nt} = \sum_{n=-\infty}^{\infty} \left\{\frac{1}{n\pi}\sin\left(\frac{\pi n}{2}\right)\left(cos\left(\frac{\pi n}{2}\right) - jsin\left(\frac{\pi n}{2}\right)\right)\right\} \cdot e^{j\pi t}$$

$$= \frac{1}{2} + \frac{1}{\pi}\sin\left(\frac{\pi n}{2}\right)\left(cos\left(\frac{\pi}{2}\right) - jsin\left(\frac{\pi}{2}\right)\right) \cdot e^{j\pi t}$$

$$-\frac{1}{\pi}\sin\left(-\frac{\pi}{2}\right)\left(cos\left(-\frac{\pi}{2}\right) - jsin\left(-\frac{\pi}{2}\right)\right) \cdot e^{-j\pi t} + \frac{1}{2\pi}\sin(\pi)\big(cos(\pi) - jsin(\pi)\big) \cdot e^{j2\pi t}$$

$$-\frac{1}{2\pi}\sin(-\pi)\big(cos(-\pi) - jsin(-\pi)\big) \cdot e^{-j2\pi t}$$

$$+\frac{1}{3\pi}\sin\left(\frac{3\pi}{2}\right)\left(cos\left(\frac{3\pi}{2}\right) - jsin\left(\frac{3\pi}{2}\right)\right) \cdot e^{j3\pi t}$$

$$-\frac{1}{3\pi}\sin\left(-\frac{3\pi}{2}\right)\left(cos\left(-\frac{3\pi}{2}\right) - jsin\left(-\frac{3\pi}{2}\right)\right) \cdot e^{-j3\pi t} + \cdots$$

$$= \frac{1}{2} - j\frac{1}{\pi}e^{j\pi t} + j\frac{1}{\pi}e^{-j\pi t} - j\frac{1}{3\pi}e^{j3\pi t} + j\frac{1}{3\pi}e^{-3\pi t} + \cdots$$

$$= \frac{1}{2} - j\frac{1}{\pi}\left(e^{j\pi t} - e^{-j\pi t}\right) - j\frac{1}{3\pi}\left(e^{j3\pi t} - e^{-j3\pi t}\right) + \cdots = \frac{1}{2} + \frac{2}{\pi}\sin(\pi t) + \frac{2}{3\pi}\sin(3\pi t) + \cdots$$

$$= \frac{1}{2} + \sum_{n=1}^{\infty} \frac{2}{(2n-1)\pi}\{\sin(2n-1)\}$$

(3)

$$c_0 = \frac{1}{2} = \frac{1}{2} + j0\,, |c_0| = \sqrt{\frac{1}{4}} = \frac{1}{2}\,, \theta_0 = 0$$

$$c_1 = -j\frac{1}{\pi} = 0 - j\frac{1}{\pi}\,, |c_1| = \sqrt{\frac{1}{\pi^2}} = \frac{1}{\pi}\,, \theta_1 = -\frac{\pi}{2}$$

$$c_2 = 0 = 0 + j0\,, |c_2| = \sqrt{0} = 0\,, \theta_2 = 0$$

$$c_3 = -j\frac{1}{3\pi} = 0 - j\frac{1}{3\pi}\,, |c_3| = \sqrt{\frac{1}{9\pi^2}} = \frac{1}{3\pi}\,, \theta_3 = -\frac{\pi}{2}$$

$$c_4 = 0 = 0 + j0\,, |c_4| = \sqrt{0} = 0\,, \theta_4 = 0$$

$$c_5 = -j\frac{1}{5\pi} = 0 - j\frac{1}{5\pi}\,, |c_5| = \sqrt{\frac{1}{25\pi^2}} = \frac{1}{5\pi}\,, \theta_5 = -\frac{\pi}{2}$$

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(4) グラフ

$$f(t)=\frac{1}{2}+\sum_{n=1}^{3}\frac{2}{(2n-1)\pi}\{\sin(2n-1)\}$$