Homework-key

S.1. Translate the following scenario into FOPL.

Fred is a collie and a trained dog. Sam is Fred's master. It's Saturday and it's cold outside. Spaniels are good dogs and so are trained collies. If a dog is a good dog and has a master, then he will be with his master. Sam is at the park on Saturdays when it's not cold outside. Otherwise, he is at the museum. Where is Fred?

$$collie(fred) \wedge trained(fred)$$

$$master(fred, sam)$$

$$day(saturday) \wedge cold(saturday)$$

$$\forall X\; spaniel(X) \vee (collie(X) \wedge trained(X)) \rightarrow gooddog(X)$$

$$\forall X \forall Y \forall Z\; gooddog(X) \wedge master(X, Y) \wedge location(Y, Z) \rightarrow location(X, Z)$$

$$day(saturday) \wedge\; \neg cold(saturday) \rightarrow location(sam, park)$$

$$day(saturday) \wedge\; cold(saturday) \rightarrow location(sam, museum)$$

$$\exists X\; location(fred, X)$$

S.2. Use Modus Ponens to show the location of Fred.

S.3. Convert the FOPL expressions obtained in S.1. above into clause form.

S.4. Use resolution refutation to prove that Fred is at the museum.

S.5. Convert the following sentences into clause form:

i) $\forall X\; p(X) \rightarrow \big(a(X) \wedge b(X)\big) \vee \neg c(X, d)$

$\forall$X (¬ p(X) $\vee$ (a(X) ∧ b(X)) $\vee$ ¬ c(X, d)) Removing Implication

¬ p(X) $\vee$ (a(X) ∧ b(X)) $\vee$ ¬ c(X, d) dropping universal quantifier

(¬ p(X) $\vee$ ¬ c(X, d)) $\vee$ (a(X) ∧ b(X))) by commutative and associative property of $\vee$

(¬ p(X) $\vee$ ¬ c(X, d) $\vee$ a(X)) ∧ (¬ p(x) $\vee$ ¬ c(X, d) $\vee$ b(X)) distributing $\vee$ over ∧

Clause 1: ¬ p(X) $\vee$ ¬ c(X, d) $\vee$ a(X)

Clause 2: ¬ (p(Y) $\vee$ ¬ c(Y, a) $\vee$ b(Y) Standardizing variables across clauses

ii) $\exists Y\; q(Y, c) \wedge \big(\forall Z\; a(Z) \rightarrow \neg b(Y)\big)$

∃Y (q(Y, c) ∧ (∀Z ¬ a(Z) v ¬ b(Y))) Removing Implication

∃Y ∀Z (q(Y, c) ∧ (¬ a(Z) v ¬ b(Y))) Prenex normal form

∀Z (q(a, c) ∧ (¬ a(Z) v ¬ b(a))) Skolemizing to remove ∃y

q(a, c) ∧ (¬ a(Z) v ¬ b(a)) Dropping universal quantifier

Clause 1: q(a, c)

Clause 2: ¬ a(Z) v ¬ b(a)