Discrete Strutures

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Text book

Discrete Mathematics and Its Application, 7^{th} Edition

Kenneth H. Rosen

References

Chapter 1

1. Discrete Mathematics and Its Application, 7th Edition
by Kenneth H. Rose

2. Discrete Mathematics with Applications
By Thomas Koshy

These slides contain material from the above two books.

BICONDITIONALS (Bi-implication)

- Let p be the statement **"You can take the flight"** and let q be the statement **"You buy a ticket".** Then $p \leftrightarrow q$ is the statement "You can take the flight **if and only if** you buy a ticket."

The truth table for the bi-implications of two propositions

p	Q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Converse, Contrapositive, and Inverse

- **CONVERSE:** The converse of $p \to q$ is the proposition $q \to p$

- **CONTRAPOSITIVE:** The contrapositive of $p \to q$ is the proposition $\neg q \to \neg p$

- **INVERSE:** The contrapositive of $p \to q$ is the proposition $\neg p \to \neg q$

Converse, Contrapositive, and Inverse

- **Equivalent:** When two compound **propositions always** have the **same truth value** we call them **equivalent**

- The **contrapositive, $\neg q \rightarrow \neg p$**, of a conditional statement **$p \rightarrow q$** always has the same truth value as $p \rightarrow q$. A **conditional statement** and its **contrapositive are equivalent**.

- The **converse** and the **inverse** of a conditional statement are also equivalent

Different ways to express conditional statement (implication)

1. ''if p, then q''

2. ''if p, q ''

3. ''p is sufficient for q''

4. ''q if p''

5. ''q when p''

6. ''a necessary condition for p is q''

Different ways to express conditional statement (implication)

7. "q unless $\neg$p"

8. "p implies q"

9. "p only if q"

10"a sufficient condition for q is p"

11"q whenever p"

12"q is necessary for p"

13"q follows from p"

- **Example:** What are the **contrapositive, the converse, and the inverse of the conditional statement**.

  "The home team wins whenever it is raining."?

  "q whenever p" is one of the ways to express the conditional statement p $\rightarrow$ q , the original statement can be rewritten as **"If it is raining, then the home team wins.“**

  Let

  p: “it is raining”

  q: “ the home team wins”

"If it is raining, then the home team wins."

p: "it is raining"

q: " the home team wins"

Contrapositive: The contrapositive of $p \rightarrow q$ is the proposition $\neg q \rightarrow \neg p$

"If the home team does not win then it is not raining"

Converse: The converse of $p \rightarrow q$ is the proposition $q \rightarrow p$

"If the home team wins then it is raining"

Inverse: The contrapositive of $p \rightarrow q$ is the proposition $\neg p \rightarrow \neg q$

"If it is not raining then the home team does not win"

Precedence of Logical Operators

Precedence of Logical Operators	
Operator	**Precedence**
¬	1
^	2
v	3
→	4
↔	5

Truth Tables o f Compound Propositions

Construct the truth table of the compound proposition (p v ¬q) → (p ∧ q)

Truth Table of (p v ¬q) → (p ∧ q)					
p	q	¬q	p v ¬q	p ∧ q	(p v ¬q) → (p ∧ q)
T	T	F	T	T	T
T	F	T	T	F	F
F	T	F	F	F	T
F	F	T	T	F	F

Translating English Sentences

How can this English sentence be translated into a logical expression?

"You can access the Internet from campus only if you are a computer science major or you are not a freshman"

"You can access the Internet from campus only if you are a computer science major or you are not a freshman"

Solution: $p \to q$ can be written **as "p only if q"**

Let

a: "**You can access the Internet from campus**"

c: "**You are a computer science major**"

f: "**You are a freshman**"

$a \to (c \lor \neg f)$

OR

"You can access the Internet from campus only if you are a computer science major or you are not a freshman"

Solution: $p \rightarrow q$ can be written as "p only if q"

Let

p: "**You can access the Internet from campus**"

q: "**You are a computer science major**"

r: "**You are a freshman**"

$p \rightarrow (q \vee \neg r)$

System Specifications

Translating sentences in **natural language (such as English)** into **logical expressions** is an essential part of specifying both **hardware and software systems**.

Express the specification **"The automated reply cannot be sent when the file system is full"** using **logical connectives**.

"The automated reply **cannot** be sent **when** the file system is full"

Solution:

$p \rightarrow q$ or **"q when p"**

Let **p: "The file system is full"**

and q: "The automated reply be sent "

and

$\Rightarrow$ $\neg q$ **when** p

$\Rightarrow$ $p \rightarrow \neg q$ $\quad$ $\because$ q when $p \equiv p \rightarrow q$

Or

$\Rightarrow$ $q \rightarrow \neg p$ $\quad$ $\because p \rightarrow q \equiv \neg q \rightarrow \neg p$

Consistent system specifications [1]

- System specifications should be consistent, that is, they should not contain **conflicting requirements** that could be used to **derive a contradiction**.

- When specifications are **not consistent**, there would be no way to develop a system that **satisfies all specifications**.

Consistent system specifications [2]

Example: Determine whether these system specifications are consistent:

"The diagnostic message is stored in the buffer **or** it is retransmitted"

"The diagnostic message is **not** stored in the buffer"

"**If** the diagnostic message is stored in the buffer, **then** it is retransmitted"

Solution:

Let **p: "The diagnostic message is stored in the buffer"**

q: "It is retransmitted"

"The diagnostic message is stored in the buffer or it is retransmitted“

1. $p \vee q$ (Either p or q or both are true)

"The diagnostic message is not stored in the buffer"

2. $\neg p$

"If the diagnostic message is stored in the buffer, then it is retransmitted“

3. $p \rightarrow q$

Analyzing the system specfication

- ✓ p must be false using **proposition 2**.
- ✓ It means q must be true using **proposition 1**.
- ✓ **p is false** it means **q is true** using **proposition 3**. The system is **consistent**.

Consistent system specifications [3]

Determine whether these system specifications are consistent:

“The diagnostic message is stored in the buffer or it is retransmitted”

"The diagnostic message is not stored in the buffer"

"**If the diagnostic message is stored in the buffer**, **then it is retransmitted"**

"The diagnostic message is not retransmitted"

Solution:

Let **p: "The diagnostic message is stored in the buffer"**

q: "It is retransmitted"

"The diagnostic message is stored in the buffer or it is retransmitted"

1. p v q (Either p or q or both are true)

"The diagnostic message is not stored in the buffer"

2. ¬p

"If the diagnostic message is stored in the buffer, then it is retransmitted"

3. p → q

"The diagnostic message is not retransmitted"

4. ¬q

Let **p: "The diagnostic message is stored in the buffer"**

q: "It is retransmitted"

1. $p \vee q$
2. $\neg p$
3. $p \to q$
4. $\neg q$

q must be false that makes the system inconsistent. Because we get problem in proposition 1.

Boolean Searches

- **Logical connectives** are used extensively in **searches of large collections of information**, such as indexes of Web pages. Because these searches employ techniques from **propositional logic**, they are called **Boolean searches**.

- In **Boolean searches**, the **connective AND** is used to match records that contain **both of two search terms**.

- The connective **OR** is used to match one or both of two search terms

Boolean Searches

- The connective **NOT** (sometimes written as AND NOT) is used **to exclude a particular search term**.

- Careful planning of how logical connectives are used is often required when Boolean searches are used to locate information of potential interest.

Logic Puzzles

- **Puzzles** that can be solved using **logical reasoning** are known as **logic puzzles**. Solving logic puzzles is an excellent way to practice working with the rules of logic.

- Also, **computer programs** designed to carry out **logical reasoning** often use well-known logic puzzles to illustrate their capabilities.

Suggested Readings

1.1 Propositional Logic