

# ASSIGNMENT:1

## DISCRETE MATHEMATICS

**NOTE: Cheating and plagiarism cases will be marked 0. Do your own work.**

1. Let  $p$ ,  $q$ , and  $r$  be the propositions

$p$  : Grizzly bears have been seen in the area.

$q$  : Hiking is safe on the trail.

$r$  : Berries are ripe along the trail.

Write these propositions using  $p$ ,  $q$ , and  $r$  and logical connectives (including negations).

a) Berries are ripe along the trail, but grizzly bears have not been seen in the area.

b) Grizzly bears have not been seen in the area and hiking on the trail is safe, but berries are ripe along the trail.

c) If berries are ripe along the trail, hiking is safe if and only if grizzly bears have not been seen in the area.

d) It is not safe to hike on the trail, but grizzly bears have not been seen in the area and the berries along the trail are ripe.

e) For hiking on the trail to be safe, it is necessary but not sufficient that berries not be ripe along the trail and for grizzly bears not to have been seen in the area.

f) Hiking is not safe on the trail whenever grizzly bears have been seen in the area and berries are ripe along the trail

2. Write each of these propositions in the form " $p$  if and only if  $q$ " in English.

a) If it is hot outside you buy an ice cream cone, and if you buy an ice cream cone it is hot outside.

b) For you to win the contest it is necessary and sufficient that you have the only winning ticket.

c) You get promoted only if you have connections, and you have connections only if you get promoted.

d) If you watch television your mind will decay, and conversely.

e) The trains run late on exactly those days when I take it.

3. For each of these sentences, state what the sentence means if the logical connective or is an inclusive or (that is, a disjunction) versus an exclusive or. Which of these meanings of or do you think is intended?

a) To take discrete mathematics, you must have taken calculus or a course in computer science.

b) When you buy a new car from Acme Motor Company, you get \$2000 back in cash or

a 2% car loan.

c) Dinner for two includes two items from column A or three items from column B.

d) School is closed if more than 2 feet of snow falls or if the wind chill is below  $-100$

4. Let  $p$  and  $q$  be the propositions “The election is decided” and “The votes have been counted,” respectively. Express each of these compound propositions as an English sentence.

a)  $\neg p$       b)  $p \vee q$

c)  $\neg p \wedge q$       d)  $q \rightarrow p$

e)  $\neg q \rightarrow \neg p$       f)  $\neg p \rightarrow \neg q$

g)  $p \leftrightarrow q$       h)  $\neg q \vee (\neg p \wedge q)$

5. Suppose that during the most recent fiscal year, the annual revenue of Acme Computer was 138 billion dollars and its net profit was 8 billion dollars, the annual revenue of Nadir Software was 87 billion dollars and its net profit was 5 billion dollars, and the annual revenue of Quixote Media was 111 billion dollars and its net profit was 13 billion dollars. Determine the truth value of each of these propositions for the most recent fiscal year.

a) Quixote Media had the largest annual revenue.

b) Nadir Software had the lowest net profit and Acme Computer had the largest annual revenue.

c) Acme Computer had the largest net profit or Quixote Media had the largest net profit.

d) If Quixote Media had the smallest net profit, then Acme Computer had the largest annual revenue.

e) Nadir Software had the smallest net profit if and only if Acme Computer had the largest annual revenue

6. Let  $p$ ,  $q$ , and  $r$  be the propositions

$p$  : You have the flu.

$q$  : You miss the final examination.

$r$  : You pass the course.

Express each of these propositions as an English sentence.

a)  $p \rightarrow q$       b)  $\neg q \leftrightarrow r$

c)  $q \rightarrow \neg r$       d)  $p \vee q \vee r$

e)  $(p \rightarrow \neg r) \vee (q \rightarrow \neg r)$

f)  $(p \wedge q) \vee (\neg q \wedge r)$

7. For each of these sentences, determine whether an inclusive or, or an exclusive or, is intended. Explain your answer.

- a) Experience with C++ or Java is required.
- b) Lunch includes soup or salad.
- c) To enter the country you need a passport or a voter registration card.
- d) Publish or perish.

8. Construct a truth table for each of these compound propositions.

- a)  $p \wedge \neg p$
- b)  $p \vee \neg p$
- c)  $(p \vee \neg q) \rightarrow q$
- d)  $(p \vee q) \rightarrow (p \wedge q)$
- e)  $(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$
- f)  $(p \rightarrow q) \rightarrow (q \rightarrow p)$

9. Steve would like to determine the relative salaries of three coworkers using two facts. First, he knows that if Fred is not the highest paid of the three, then Janice is. Second, he knows that if Janice is not the lowest paid, then Maggie is paid the most. Is it possible to determine the relative salaries of Fred, Maggie, and Janice from what Steve knows? If so, who is paid the most and who the least? Explain your reasoning.

10. Five friends have access to a chat room. Is it possible to determine who is chatting if the following information is known? Either Kevin or Heather, or both, are chatting. Either Randy or Vijay, but not both, are chatting. If Abby is chatting, so is Randy. Vijay and Kevin are either both chatting or neither is. If Heather is chatting, then so are Abby and Kevin. Explain your reasoning.

\*11. Each inhabitant of a remote village always tells the truth or always lies. A villager will give only a “Yes” or a “No” response to a question a tourist asks. Suppose you are a tourist visiting this area and come to a fork in the road. One branch leads to the ruins you want to visit; the other branch leads deep into the jungle. A villager is standing at the fork in the road. What one question can you ask the villager to determine which branch to take?

12. An explorer is captured by a group of cannibals. There are two types of cannibals—those who always tell the truth and those who always lie. The cannibals will barbecue the explorer unless he can determine whether a particular cannibal always lies or always tells the truth. He is allowed to ask the cannibal exactly one question.

- a) Explain why the question “Are you a liar?” does not work.
- b) Find a question that the explorer can use to determine whether the cannibal always lies or always tells the truth.

**13.** Express these system specifications using the propositions  $p$  “The message is scanned for viruses” and  $q$  “The message was sent from an unknown system” together with logical connectives (including negations).

**a)** “The message is scanned for viruses whenever the message was sent from an unknown system.”

**b)** “The message was sent from an unknown system but it was not scanned for viruses.”

**c)** “It is necessary to scan the message for viruses whenever it was sent from an unknown system.”

**d)** “When a message is not sent from an unknown system it is not scanned for viruses.”

**14.** Show that each of these conditional statements is a tautology by using truth tables.

**a)**  $[\neg p \wedge (p \vee q)] \rightarrow q$

**b)**  $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$

**c)**  $[p \wedge (p \rightarrow q)] \rightarrow q$

**d)**  $[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$

**15.** Show that  $(p \vee q) \wedge (\neg p \vee r) \rightarrow (q \vee r)$  is a tautology.

**16.** Show that  $(p \rightarrow q) \rightarrow r$  and  $p \rightarrow (q \rightarrow r)$  are not logically equivalent.

**17.** Use De Morgan’s laws to find the negation of each of the following statements.

**a)** Jan is rich and happy.

**b)** Carlos will bicycle or run tomorrow.

**18.** When three professors are seated in a restaurant, the host asks them: “Does everyone want coffee?” The first professor says: “I do not know.” The second professor then says: “I do not know.” Finally, the third professor says: “No, not everyone wants coffee.” The hostess comes back and gives coffee to the professors who want it. How did she figure out who wanted coffee?

**19.** Express these system specifications using the propositions  $p$  “The user enters a valid password,”  $q$  “Access is granted,” and  $r$  “The user has paid the subscription fee”

and logical connectives (including negations).

- a) "The user has paid the subscription fee, but does not enter a valid password."
- b) "Access is granted whenever the user has paid the subscription fee and enters a valid password."
- c) "Access is denied if the user has not paid the subscription fee."

**20.** Are these system specifications consistent? "If the file system is not locked, then new messages will be queued. If the file system is not locked, then the system is functioning normally, and conversely. If new messages are not queued, then they will be sent to the message buffer. If the file system is not locked, then new messages will be sent to the message buffer. New messages will not be sent to the message buffer."