Discrete Strutures

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Text book

Discrete Mathematics and Its Application, 7th Edition

Kenneth H. Rosen

References

Chapter 1

1. Discrete Mathematics and Its Application, 7th Edition
by Kenneth H. Rose

2. Discrete Mathematics with Applications
By Thomas Koshy

These slides contain material from the above two books.

Propositional Logic – exclusive or

Let p and q be propositions. The **exclusive or** of p and q, denoted by **$p \oplus q$** , is the proposition that is **true** when **exactly one of p and q is true** and is false otherwise

The truth table for the exclusive or of two propositions

p	Q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Propositional Logic – exclusive or

- The exclusive or is also equivalent to **the negation of a logical biconditional**

  $p \oplus q \equiv \neg(p \leftrightarrow q)$

Conditional Statements– implication

- Let p and q be propositions. The **conditional statement $p \rightarrow q$** is the proposition "**if p, then q** ."

  - The **conditional statement $p \rightarrow q$** is **false** when **p is true** and **q is false**, and true otherwise.

  - In the **conditional statement $p \rightarrow q$**, **p** is called the hypothesis (or antecedent or premise) and **q** is called the **conclusion (or consequence).**

Conditional Statements– implication

- Note that the **statement $p \to q$** is true when both **p and q are true** and when **p is false** (no matter what truth value q has).

The truth table for the implication of two propositions

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Different ways to express conditional statement (implication)

1. ''if p, then q''
2. ''if p, q ''
3. ''p is sufficient for q''
4. ''q if p''
5. ''q when p''
6. ''a necessary condition for p is q''

Different ways to express conditional statement (implication)

7. "q unless $\neg$p"

8. "p implies q"

9. "p only if q"

10. "a sufficient condition for q is p"

11. "q whenever p"

12. "q is necessary for p"

13. "q follows from p"

Understanding Implication

A useful way to understand the truth value of a conditional statement is to think of an **obligation or a contract**.

For example, the pledge many politicians make when running for office is

"If I am elected, then I will lower taxes."

Understanding Implication

- If the politician is elected, voters would expect this politician to lower taxes.

- Furthermore, if the politician is not elected, then voters will not have any expectation that this person will lower taxes, although the person may have sufficient influence to cause those in power to lower taxes.

- It is only when the **politician is elected but does not lower taxes** that voters can say that the politician has broken the campaign pledge.

- **This last scenario corresponds to the case when p is true but q is false in $p \rightarrow q$.**

Understanding Implication

"If you get 100% on the final, then you will get an A "

- If you manage to get a 100% on the final, then you would expect to receive an A.
- If you do not get 100% you may or may not receive an A depending on other factors.
- However, **if you do get 100%,** but the professor does not give you an A**, you will feel cheated**.

Example1 [1]

- "If I am elected, then I will lower taxes."

Truth table for the conditional statement $p \rightarrow q$

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Example 1 cont

- "If I am elected, then I will lower taxes."

Determine the truth value of each of the following implications:

1. I am elected and I lower taxes.

   The truth value of the proposition is **true**.

2. I am elected and I do not lower taxes.

   The truth value of the proposition is **False**.

3. I am not elected and I do not lower taxes

   The truth value of the proposition is **true**.

4. I am not elected and I lower taxes.

   The truth value of the proposition is **true**.

Example2

- "If you get 100% on the final, then you will get an A."

Truth table for the conditional statement $p \to q$

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Example2 cont.

- "**If** you get 100% on the final, **then** you will get an A." Determine the truth value of each of the following implications:

1. I get 100% marks on the final and I get an A.
   The truth value of the proposition is **true**.

2. I get 100% marks on the final and I do not get an A.
   The truth value of the proposition is **false**.

3. I do not get 100% marks on the final and I do not get an A.
   The truth value of the proposition is **true**.

4. I do not get 100% marks on the final and I get an A.
   The truth value of the proposition is **true**.

Example3

Let p be the statement ''Maria learns discrete mathematics'' and q the statement ''Maria will find a good job.'' Express the **statement $p \rightarrow q$** as a **statement in English**.

Solution:

- Let p: "Maria learns discrete mathematics " and

  q: "Maria will find a good job,"

  $p \rightarrow q$ represents the statement

- "**If** Maria learns discrete mathematics, **then** she will find a good job."

OR

- "For Maria to get a good job, it is **sufficient** for her to learn discrete mathematics." **($\because$p is sufficient for q)**

OR

- "Maria will find a good job **unless** she does **not** learn discrete mathematics ." **($\because$ "q unless $\neg$p")**

Example 3

"If Maria learns discrete mathematics, then she will find a good job."

Determine the truth value of each of the following implications:

1. Maria learns discrete mathematics and she gets a good job.

   TRUE

2. Maria learns discrete mathematics and she does not get a good job.

   FALSE

Example 3 cont.

"If Maria learns discrete mathematics, then she will find a good job."

Determine the **truth value** of each of the **following implications**:

3. Maria does not learn discrete mathematics and she does not get a good job.

TRUE

4. Maria does not learn discrete mathematics and she gets a good job.

TRUE

Example 4

- Let **p: "It is cold"**, and **q: "I wear a jacket".** If p then q. Determine the truth value of each of the following implications:

1. It is cold and I wear a jacket.

   TRUE

2. It is cold and I do not wear a jacket.

   FALSE

Example 4 cont.

- Let p: "It is cold", and q: "I wear a jacket". **If p then q**. Determine the truth value of each of the following implications:

3. It is not cold and I do not wear a jacket.

   TRUE

4. It is not cold and I wear a jacket.

   TRUE

Example 5

- Let **p: " You pass analysis of algorithm with A",** and **q: "You get TA ship in algorithm"**. **If p then q**. Determine he truth value of each of the following **implications**:

1. You pass analysis of algorithm with A and you get a TA ship in algorithm.

   The truth value of the proposition is **true.**

2. You pass analysis of algorithm with A and you do not get a TA ship in algorithm.

   The truth value of the proposition is **false**.

Example 5 cont.

- Let **p: " You pass analysis of algorithm with A",** and **q: "You get TA ship in algorithm"**. **If p then q**. Determine he truth value of each of the following **implications**:

3. You do not pass analysis of algorithm with A and you get a TA ship in algorithm.

The truth value of the **proposition is true**.

4. You do not pass analysis of algorithm with A and you do not get a TA ship in algorithm.

The truth value of the **proposition is true**.

- The **if-then construction** used in many programming languages is different from that used in logic.

- Most programming languages contain statements such as if p then S, where p is a proposition and S is a program segment (one or more statements to be executed).

- When execution of a program encounters such a statement, S is executed if p is true, but S is not executed if p is false

What is the value of the variable x after the statement

if $2 + 2 = 4$ then $x := x + 1$

if $x = 0$ before this statement is encountered? (The symbol := stands for assignment. The statement $x := x + 1$ means the assignment of the value of $x + 1$ to x.)

Solution:

Because $2 + 2 = 4$ is true, the assignment statement $x := x + 1$ is executed.

Hence, x has the value $0 + 1 = 1$ after this statement is encountered

BICONDITIONALS (Bi-implication)

- Let p and q be propositions. The **biconditional statement $p \leftrightarrow q$** is the proposition "p if and only if q". The biconditional statement $p \leftrightarrow q$ is true when **p and q have the same truth values**, and is false otherwise. Biconditional statements are also called bi-implications.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Common ways to express bi-implications

- "p is necessary and sufficient for q "
- "if p then q , and conversely"
- "p iff q"
- The last way of expressing the biconditional statement $p \leftrightarrow q$ uses the abbreviation **"iff"** for "if and only if."
- Note that **$p \leftrightarrow q$** has exactly the same truth value **as $(p \rightarrow q) \wedge (q \rightarrow p)$**.

In class quiz

Prove the following result using truth table:

$$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$$

Suggested Readings

1.1 Propositional Logic