Discrete Structures

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Text book

Discrete Mathematics and Its Application, 7th Edition

Kenneth H. Rosen

References

Chapter 1

1. Discrete Mathematics and Its Application, 6^{th} Edititition
by Kenneth H. Rose

2. Discrete Mathematics with Applications
by Thomas Koshy

3. Discrete Mathematical Structures, CS 173
by Cinda Heeren, Siebel Center

These slides contain material from the above resources.

Negating Quantified Expressions [1]

The rules for negations for quantifiers are called De Morgan's laws for quantifiers.

De Morgan's Laws for Quantifiers.			
Negation	Equivalent Statement	When Is Negation True?	When False?
$\neg \exists x P(x)$	$\forall x \neg P(x)$	For every x , P(x) is false	There is an x for which P (x) is true.
$\neg \forall x P(x)$	$\exists x \neg P(x)$	There is an x for which P(x) is false	P (x) is true for every x .

First approach

Example Express the statement **“Some student in this class has visited Mexico”** using predicates and quantifiers

“Some student in this class has visited Mexico”

or

“There is a student in this class with the property that the student has visited Mexico.”

UD = All students in this class

We can introduce a variable *x*, so that our statement becomes

“There is a student *x* in this class having the property that *x* has visited Mexico.”

S(x): “*x* is a student in this class.” (Redundant as it is already covered in the domain or UD)

M(x): “x has visited Mexico”

$\Rightarrow \exists x M(x)$

Second approach

"Some student in this class has visited Mexico"

or

"There is a person *x* having the properties that *x* is a student in this class and *x* has visited Mexico."

UD = All people

S(x): "*x* is a student in this class."

M(x): "x has visited Mexico"

$\Rightarrow$ **$\exists x(S(x) \wedge M(x))$**

Note: The statement in the previous slide cannot be expressed as ***∃x(S(x) → M(x)),*** which is **true** when **there is someone not in the class** because, in that case, for such a person *x*, ***S(x) → M(x)*** becomes either **F→T** or **F → F**, both of which are true.

First approach

Example Express the statement **"Every student in this class has visited either Canada or Mexico"** using predicates and quantifiers

“Every student in this class has visited either Canada or Mexico”

or

“For every *x* in this class, *x* has the property that *x* has visited Mexico or *x* has visited Canada.”

UD = The students in this class

S(x): “*x* is a student in this class” (Redundant as it is already covered in the domain or UD)

M(x): “x has visited Mexico”

C(x): “x has visited Canada”

$\Rightarrow \forall x\ (C(x) \vee M(x))$

Precedence of Quantifiers

- The quantifiers **∀** and **∃** have **higher precedence then all logical operators** from propositional calculus

- ∀xP(x) **v** Q(x) mean **(∀xP (x)) v Q(x)** ≠ **∀x(P(x) v Q(x))**

Second approach

“Every student in this class has visited either Canada or Mexico”

or

“For every person *x*, if *x* is a student in this class, then *x* has visited Mexico or *x* has visited Canada.”

UD = All people

S(x): “*x* is a student in this class”

C(x): “x has visited Canada”

M(x): “x has visited Mexico”

$\Rightarrow \forall x\,(S(x) \rightarrow (C(x) \vee M(x)))$

Suggested Readings

1.4 Predicates and Quantifiers