Discrete Structures

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Text book

Discrete Mathematics and Its Application, 7th Edition

Kenneth H. Rosen

References

Chapter 1

1. Discrete Mathematics and Its Application, 7^{th} Edition

by

Kenneth H. Rose

2. Discrete Mathematics with Applications

by

Thomas Koshy

These slides contain material from the above two books.

ASSESSMENT OF STUDENT PERFORMANCE

- The course teacher will be responsible for assessing a students' performance. The following assessment methods will be used:
- Tests, quizzes, assignments, presentations, attendance, and class participation.
- The midterm examinations, which shall be held eight weeks after the start of a semester.
- The final examinations, which shall be held at the end of the semester.
- Grade distribution in a course will be as follows:
  - **Midterm Examination 35%**
  - **Final Examination 40%**
  - **Sessional (assignment, test etc) 25%**
- Final project/research thesis shall be examined by the concerned project supervisor and the College Final Project Committee constituted for this purpose.
- Minimum 50% marks (grade 'D') are required to pass a course.

Grading System of FCIT

PUCIT GRADING SYSTEM

Percent Marks	Letter Grade	Grade points
85-100	A	4.00
80-84	A-	3.70
75-79	B+	3.30
70-74	B	3.00
65-69	B-	2.70
61-64	C+	2.30
58-60	C	2.00
55-57	C-	1.70
50-54	D	1.00
Below 50	F	0.00
Withdrawl	W	
Incomplete	I	

- The grade point average (GPA) is computed by multiplying the number of credit hours of each course by the grade point. The sum total is then divided by total number of credit hours.

Grading breakup

I. Midterm = 25 points
II. Final term = 35 points
III. Quizzes = $2.5 \times 8 = 20$ points (A total of 8 quizzes)
IV. Assignments = 1.25 × 4 = 5 points (A total of 4 assignments)
V. Presentations = 5 points.

Logic

Logic is the study of **methods and principles** of reasoning in all its possible form.

OR

It is the basis of **mathematical reasoning**.

OR

It is the study of the principles and methods that distinguishes between a **valid** and **invalid arguments**.

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Logic

Logic has numerous applications in computer science. These rules are used in the design of **computer circuits**, the **construction of computer programs**, the verification of the **correctness of programs**, and in many other ways.

Proposition [1]

- A **proposition** is a **declarative sentence** (that is, a sentence that declares a fact) that is either true or false, but not both.

OR

- It is a **statement or sentence** which is either **true or false** but **not both**.

OR

- A **declarative sentence** that is either true or false, but not both, is a **proposition (or a statement),** which we will denote by **lowercase letter p, q, r, s or t** are boolean variables (or logic variables).

Proposition [2]

Example: Are the following declarative sentences are propositions:

1. Allama Iqbal was a great poet.

It is a **proposition** and its truth value is **true**.

2. Islamabad is a capital city of Pakistan.

It is a **proposition** and its truth value is **true**.

3. $1 + 1 = 22$

It is a **proposition** and its truth value is **false**.

4. $6 > 600$

It is a **proposition** and its truth value is **false**.

Proposition [3]

5. 2 + 2 = 8

It is a **proposition** and its truth value is **false**.

6. Muree is a hill station

It is a **proposition** and its truth value is **true**.

7. Islam is a religion of tolerance.

It is a **proposition** and its truth value is **true.**

Is proposition or not?

Example 2 Consider the following sentences.

1 . What time is it?

Not a **proposition**.

2 . Read this carefully.

Not a **proposition**.

3 . $x + 1 = 2$.

It is not a **proposition**. It is **neither true nor false**.

4 . $x + y = Z$.

It is not a **proposition**. It is **neither true nor false**.

Is proposition or not?

5. Let me go!

It is not a **proposition** because it is **not declarative sentence.**

6. x + 3 = 5

It is not a **proposition** because it is **not declarative sentence**.

7. Close the door!

It is not a **proposition** because it is **not declarative sentence**.
It is a command

8. What is my line?

It is not a **proposition** because it is **not declarative sentence**.

Proposition [4]

If a **proposition is true**, we say that it has a truth value of **"true".**
If it is **false**, it has a truth value is **"false".**

Rule: If the **sentence** is preceded by other sentences that make the pronoun or variable reference clear then the sentence is a statement.

Example:

$x = 1, x > 2$

$x > 2$ is a statement with truth value of false.

Proposition [5]

Example: George Boole was a famous mathematician. He is renowned.

He is renowned is a statement with **truth value true**.

Example: Bill Gates is an American. He is very rich.

He is very rich is a statement with **truth value true**.

Understanding statements (Quiz)

1. x + 4 is negative
2. May I come in?
3. Logic is interesting
4. It is sunny today
5. -10 > 0
6. x + y = 12
7. Islam is a religion of peace
8. PUCIT is the largest computer science institution in Pakistan
9. 7+7 = 150
10. How are you studying discrete mathematics?

Types of Proposition [1]

There are two types of proposition:

a) Simple proposition

b) Compound proposition

Simple proposition could be used to **build a compound statement.**

New propositions are formed from **existing propositions** using **logical operators**.

Types of Proposition [2]

Example:

1. "Discrete mathematics is a prerequisite of analysis of algorithms’’ **and** "Analysis of algorithms is very important subject in computer science’’.

2. "1 + 1 = 2’’ **and** "2 + 2 = 4’’

3. "Lahore is a provincial capital of Pakistan" **or** ‘’Islamabad is a capital of Pakistan’’

Logical connectives

- There are four important logical connectives--**conjunctions**, **disjunctions**, **conditional statements**, and **bi-conditional statements** as well as negations.

- We can use these connectives to build up complicated **compound propositions** involving any number of propositional variables.

- **AND, OR, NOT**, **if…then** (implication or conditionals), **if and only if** (**bi-implications or bi-conditional**) are called logical connectives.

Connectives

Connective	Meanings	Symbol	Called
Negation	not	~	Tilde
Conjunction	and	^	Hat
Disjunction	or	v	Vel
Conditional (implication)	if…then…	->	Arrow
Bi-conditional (Bi-implications)	if and only if	<->	Double arrow

Propositional Logic – negation [1]

- Let p be a proposition. The **negation of p**, denoted by **$\neg$p (also denoted by $\overline{p}$)**, is the statement **"It is not the case that p. "**

OR

- The proposition $\neg$p is read **"not p."** The truth value of the negation of p, **$\neg$ p, is the opposite of the truth value of p**.

OR

- Suppose p is a proposition. The negation of p is written $\neg$p and has meaning:

"It is not the case that p."

Truth table for negation

p	$\neg$ p
T	F
F	T

Propositional Logic – negation[2]

Example 3: Find the negation of the proposition

"Today is Friday."

Solution: The negation is

"**It is not the case** that today is Friday "

OR

"Today is not Friday "

Propositional Logic – negation[3]

Example 4: Find the **negation of the proposition** "**At least 10 inches of rain fell today in Miami.**" and express this in simple English.

Solution: The negation is

"**It is not the case that** at least 10 inches of rain fell today in Miami."

This negation can be more simply expressed by
"Less than 1 0 inches of rain fell today in Miami."

Propositional Logic – negation[3]

- **Remark:** Strictly speaking, sentences involving variable times such as those in Examples 3 and 4 are **not propositions unless a fixed time is assumed**.
- The same holds for variable places unless a **fixed place is assumed** and for pronouns unless a particular person is assumed.
- We will always assume **fixed times, fixed places, and particular people** in such sentences unless otherwise noted.

Propositional Logic – conjunction [1]

Let p and q be propositions. The conjunction of p and q, denoted by **$p \wedge q$**, is the proposition "p and q.'' The **conjunction $p \wedge q$** is true when both p and q are true and is false otherwise

OR

Let p and q be the two simple propositions. The compound propositions "p and q" is denoted by **$p \wedge q$**. It will be true when both are true.

OR

The conjunction of **two arbitrary propositions** p and q, denoted by **$p \wedge q$**, is the proposition p and q. It is formed by **combining the propositions** using the word **and**, called connective.

Propositional Logic – conjunction [2]

- **Example:** Find the conjunction of the propositions p and q where p is the proposition **"Today is Friday"** and q is the proposition "It is raining today."

Solution: The conjunction of these propositions, $p \wedge q$, is the proposition "Today is Friday and it is raining today "

The Truth Table for the Conjunction of Two Propositions.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Propositional Logic – disjunction [1]

- Let p and q be propositions. The **disjunction** of p and q, denoted by **p v q**, is the proposition "p or q ." The disjunction p v q is false when both **p and q are false** and is true otherwise.

OR

Let p and q be the two simple propositions. The compound proposition "p or q" is denoted by p v q will be true if at least one is true and is false when both p and q are false.

OR

Way of combining two propositions p and q is by using the connective or. The resulting proposition p or q is the disjunction of p and q and is devoted by p v q.

Propositional Logic – disjunction [2]

Example: "Students who have taken calculus **or** computer science can take this class.’’

The Truth Table for the Disjunction of Two Propositions.

p	q	p v q
T	T	T
T	F	T
F	T	T
F	F	F

Rule

- Note: No of rows in the truth table depend on the number of propositions.

- **No of rows = 2^n**

  where n = Number of propositions

Suggested Readings

1.1 Propositional Logic