Discrete Structures

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Text book

Discrete Mathematics and Its Application, 7th Edition

Kenneth H. Rosen

References

Discrete Mathematics and Its Application, 7h Edition

By Kenneth H. Rose

Probability & Statistics for Engineers & Scientists, Ninth edition, Ronald E. Walpole, Raymond H. Myer

Probability Demystified, Allan G. Bluman

https://en.wikipedia.org/wiki/Law_of_large_numbers

These slides contain material from the above resources.

Intersection [1]

Intersection: The **intersection** of two events A and B, denoted by the symbol **A∩B**, is the event containing all elements that are **common to A and B**.

Example: Let **E** be the event that a person selected at random in a classroom is majoring in **engineering**, and let **F** be the event that the person is **female**. Then **E∩F** is the event of all female engineering students in the classroom.

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Intersection [2]

Example: Let **$V = \{a, e, i, o, u\}$** and **$C = \{l, r, s, t\}$;** then it follows that **$V \cap C = \emptyset$**. That is, V and C have no elements in common and, therefore, **cannot both simultaneously occur**.

Union

Union: The **union** of the two events A and B, denoted by the symbol **AUB**, is the event containing all the elements that belong to A or B or both.

Notation for Addition Rule

P(A or B) = **P**(in a single trial, event *A* occurs or event *B* occurs or they both occur)

Mutually Exclusive or Disjoint

Two events A and B are **mutually exclusive, or disjoint,**

if **A∩B = { }** or **Ø**

OR

Two events A and B are **mutually exclusive**, **or disjoint,**

if **A∩B = Ø**, that is, if A and B have no elements in common.

OR

Events ***A*** and ***B*** are **disjoint** (or **mutually exclusive**) if they **cannot occur at the same time**. (That is, disjoint events do not overlap.)

Addition Rule I

Addition Rule I: When two events are **mutually exclusive** or **disjoint events**

P(A or B) = P(A) + P(B)

OR

$$P(A \cup B) = P(A) + P(B)$$

Example: When a die is rolled, find the probability of getting a **2** or a **3**.

Solution:

S = { 1, 2, 3, 4, 5, 6}; n(S) = 6

Let **A** be the event of getting a **"2"**

A = {2}; n(A) = 1

P(A) = n(A)/n(S) = $\frac{1}{6}$ = **0.1667 (or 16.67%)**

Let **B** be the event of getting a **"3"**

B = {3}; n(B) = 1

P(B) = n(B)/n(S) = $\frac{1}{6}$ = **0.1667 (or 16.67%)**

Since events **A** and **B** are **mutually exclusive**, so

P(A U B) = $\frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$ = 0.3333 **(or 33.33%)**

Addition Rule I [2]

A cable television company offers programs on **eight** different channels, **three** of which are affiliated with **ABC**, **two** with NBC, and **one** with **CBS**. The other **two** are an **educational channel** and the **ESPN sports** channel. Suppose that a person subscribing to this service turns on a television set without first selecting the channel. Let ***A*** be the event that the program belongs to the **NBC network** and ***B*** the event that it belongs to the **CBS network**. Since a television program cannot belong to more than one network, the events *A* and *B* have no programs in common.

Therefore, the intersection **A ∩ B** contains no programs, and consequently the events *A* and *B* are

Addition Rule I [3]

Example: In a committee meeting, there were **5** freshmen, **6** sophomores, **3** juniors, and **2** seniors. If a student is selected at random to be the chairperson, find the probability that the chairperson is a **sophomore** or a **junior**.

Addition Rule I [4]

Solution:

Let A be the event of selecting a chairperson as a **"sophomore"**

$P(A) = \frac{6}{16} = \frac{3}{8} =$ **0.3750**

Let B be the event of selecting a chairperson as a **"junior"**

$P(B) = \frac{3}{16} =$ **0.1875**

Since **A** and **B** are **mutually exclusive or disjoint events**

$P(A \cup B) = P(A) + P(B)$

$= \frac{6}{16} + \frac{3}{16} = \frac{9}{16} =$ **0.5625 (or 56.25%)**

Addition Rule I [5]

Example: A card is selected at random from a deck. Find the probability that the card is an **ace or** a **king**.

Heart	A ♥	2 ♥	3 ♥	4 ♥	5 ♥	6 ♥	7 ♥	8 ♥	9 ♥	10 ♥	J ♥	Q ♥	K ♥
Diamond	A ♦	2 ♦	3 ♦	4 ♦	5 ♦	6 ♦	7 ♦	8 ♦	9 ♦	10 ♦	J ♦	Q ♦	K ♦
Spade	A ♠	2 ♠	3 ♠	4 ♠	5 ♠	6 ♠	7 ♠	8 ♠	9 ♠	10 ♠	J ♠	Q ♠	K ♠
Club	A ♣	2 ♣	3 ♣	4 ♣	5 ♣	6 ♣	7 ♣	8 ♣	9 ♣	10 ♣	J ♣	Q ♣	K ♣

Solution:

Let **A** be the event of selecting an **ace**

P(A) = $\frac{4}{52} = \frac{1}{13}$ = **0.0769 (or 7.69%)**

Let **B** be the even of selecting a **king**

P(B) = $\frac{4}{52} = \frac{1}{13}$ = **0.0769 (or 7.69%)**

Since **A** and **B** are **mutually exclusive or disjoint events**

P(AUB) = P(A) + P(B)

$= \frac{4}{52} + \frac{4}{52}$ = 0.1538 **(or 15.38%)**

Addition Rule II [1]

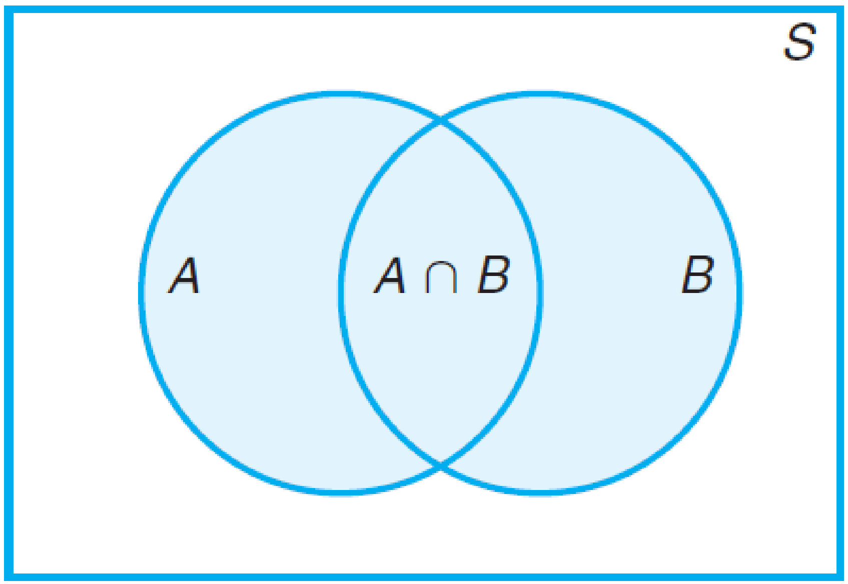
- When two events are **not mutually exclusive**, you need to add the probabilities of each of the two events and **subtract the probability of the outcomes that are common** to both events. In this case, addition **rule II** can be used.

- **Addition Rule II:** If A and B are two events that are not mutually exclusive, then

$$P(A \text{ or } B) = P(A) + P(B) - P(A \cap B)$$

When A and B are two events that are not mutually exclusive

If *A* and *B* are two events, then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.



Additive rule of probability

Example: A card is selected at random from a deck of 52 cards. Find the probability that it is a **6** or a **diamond**.

Addition Rule II [2]

Solution:

Let A be the event of getting a **"6".**

P(A) = $\frac{4}{52} = \frac{1}{13} =$ **0.0769 (or 7.69%)**

Let B be the event of getting a **"diamond".**

P(B) = $\frac{13}{52} = \frac{1}{4} =$ **0.2500 (or 25%)**

Addition Rule II [3]

Let **A ∩ B** be the event of getting a **"6"** and a **"diamond"**

P(A∩B) = $\frac{1}{52}$ = **0.0192 (or 1.9231%)**

Since A and B are **not mutually exclusive**, so

P(A∪B) = P(A) + P(B) – P(A ∩ B)

$$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52}$$

$$= \frac{4}{13} = 0.3077 \text{ (or } 30.77\%)$$

Addition Rule II [4]

Example: A die is rolled. Find the probability of getting an even number or a number less than 4.

Addition Rule II [5]

Solution:

S = {1, 2, 3, 4, 5, 6}

n(S) = 6

Let A be the event of getting an even number

A = { 2, 4, 6}, n(A) = 3

P(A) = $\frac{3}{6}$ = $\frac{1}{2}$ = 0.50 (or 50%)

Let B be the event of getting a number less than 4

B= {1, 2, 3}, n(B) = 3

P(B) = $\frac{3}{6}$ = $\frac{1}{2}$ = 0.50 (or 50%)

Addition Rule II [6]

Let A ∩ B be the event of getting an **"even number"** and a **"number less than 4"**

$A \cap B = \{2\}$

P(A∩B) $= \frac{1}{6} = 0.1667$ or (16.67%)

Since A and B are **not mutually exclusive**, so

$$\mathbf{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

$$= \frac{3}{6} + \frac{3}{6} - \frac{1}{6} \;\; \mathbf{= \frac{5}{6} = 0.8333\ (or\ 83.3333\ \%)}$$

Table

A table can be used for the sample space when two dice are rolled.

	Die 2					
Die 1	1	2	3	4	5	6
1	(1, 1)	(2, 1)	(3, 1)	(4, 1)	(5, 1)	(6, 1)
2	(1, 2)	(2, 2)	(3, 2)	(4, 2)	(5, 2)	(6, 2)
3	(1, 3)	(2, 3)	(3, 3)	(4, 3)	(5, 3)	(6, 3)
4	(1, 4)	(2, 4)	(3, 4)	(4, 4)	(5, 4)	(6, 4)
5	(1, 5)	(2, 5)	(3, 5)	(4, 5)	(5, 5)	(6, 5)
6	(1, 6)	(2, 6)	(3, 6)	(4, 6)	(5, 6)	(6, 6)

Addition Rule II [7]

Example: Two dice are rolled; find the probability of getting **doubles** or a **sum of 8**.

Addition Rule II [8]

Solution:

Let A be the event of getting doubles

A = {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)}, n(A) = 6

P(A) = $\frac{6}{36} = \frac{1}{6}$ = **0.1667 (or 16.67%)**

Let B be the event of getting a sum of 8

A = {(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)}, n(A) = 5

P(B) = $\frac{5}{36}$ = **0.1389 (or 13.89%)**

Addition Rule II [9]

Let **A∩B** be the event of getting a **'doubles'** and a **'sum of 8'**

A∩B = {(4, 4)}

P(A∩B) = $\frac{1}{36}$ = 0.0277 (or 2.7777 %)

Since A and B are **not mutually exclusive**, so

P(AUB) = P(A) + P(B) – P(A ∩ B)

$$= \frac{6}{36} + \frac{5}{36} - \frac{1}{36} = \frac{10}{36}$$

$= \frac{5}{18}$ = **0.2777or (27.7777%)**

Addition Rule II [10]

Let **P** be the event that an employee selected at random from an oil drilling company **smokes cigarettes**.

Let **Q** be the event that the employee selected drinks **alcoholic beverages**.

Then the event **P ∪ Q** is the set of all employees who either **drink** or **smoke** or do **both**.

Example : A coin is tossed twice. What is the probability that at **least 1 head** occurs?

Solution : The sample space for this experiment is

S = {HH,HT, TH, TT}

n(S) = 4

Let **A** be the event of getting at **least 1 head**

A = {HH,HT, TH}

n(A) = 3

$$\therefore P(A) = \frac{n(A)}{n(s)}$$

$$= \frac{3}{4} = (0.75 \text{ or } 75\%)$$

Example : A die is loaded in such a way that **an even number** is **twice** as likely to occur as an **odd number**. If ***E*** is the event that a **number less than 4** occurs on a single toss of the die, find ***P*(*E*).**

Solution

P(Even number) = $2p$

P(Odd number) = p

S = { 1, 2, 3, 4, 5 ,6}

$\because Sum\ of\ probability = 1$

$\therefore$ **p + 2p + p + 2p + p +2p** = 1$\Rightarrow$ $9\text{p} = 1 \Rightarrow$ **p =** $\mathbf{\frac{1}{9}}$

$\Rightarrow$ P(Even number) = $\mathbf{\frac{2}{9}}$

P(Odd number) = $\mathbf{\frac{1}{9}}$

E = {1, 2, 3}

P(E) = P(1) + P(2) + P(3)

$= \mathbf{\frac{1}{9}} + \mathbf{\frac{2}{9}} + \mathbf{\frac{1}{9}} = \mathbf{\frac{4}{9}}$ (or 0.4444 or 44%)

Example A die is loaded in such a way that **an even number** is **twice** as likely to occur as an odd number.

Let ***A*** be the event that an **even number** turns up and let ***B*** be the event that a **number divisible** by **3** occurs. Find $\mathbf{P(A \cup B)}$ and $\boldsymbol{P(A \cap B)}$.

Solution

S = {1, 2, 3, 4, 5, 6}

P(Even number) = 2p

P(Odd number) = p

S = { 1, 2, 3, 4, 5 ,6}

$\because Sum\ of\ probability = 1$

$\therefore$ **p + 2p + p + 2p + p +2p** = 1$\Rightarrow$ $9\mathrm{p} = 1 \Rightarrow$ **p =** $\mathbf{\frac{1}{9}}$

$\Rightarrow$ P(Even number) = $\mathbf{\frac{2}{9}}$

P(Odd number) = $\mathbf{\frac{1}{9}}$

A = { 2, 4, 6}

P(A) = $\mathbf{\frac{2}{9} + \frac{2}{9} + \frac{2}{9} = \frac{6}{9} = \frac{2}{3}}$

B = {3, 6}

P(B) = $\mathbf{\frac{1}{9} + \frac{2}{9} = \frac{3}{9} = \frac{1}{3}}$

$A \cap B = \{6\}$

$P(A \cap B) = \frac{2}{9}$

Since A and B are **not mutually exclusive**, so

$P(A \cup B) = P(A) + P(B) – P(A \cap B)$

$= \frac{6}{9} + \frac{3}{9} - \frac{2}{9} = \frac{7}{9}$ (or 0.7778 or 77.7778%)

- To find ***P*(*A* or *B*),** begin by associating use of the word **"or"** with addition.

- Consider whether **events *A*** and ***B*** are **disjoint**; that is, can they happen at the same time?

- If they are **not disjoint** (that is, they can happen at the same time), be sure to avoid (or at least compensate for) **double-counting** when adding the relevant probabilities.

- If you understand the importance of not double counting when you find *P*(*A* or *B*), you don't necessarily have to calculate the value of $\mathbf{P(A) + P(B) - P(A \cap B)}$

Errors made when applying the addition rule

- Errors made when applying the addition rule often involve **double-counting**; that is, events that **are not disjoint** are treated as if they were. One indication of such an error is a total probability that **exceeds 1.**

- However, errors involving the addition rule do not **always cause** the total probability to **exceed 1**.

Suggested Readings

Probability & Statistics for Engineers & Scientists, Ninth Edition, Ronald E. Walpole, Raymond H. Myer

2.1 Sample space

2.2 Events

2.3 Counting Sample Points

Discrete Mathematics and Its Application, 7th Edition Kenneth H. Rosen

7.1 An Introduction to Discrete Probability

7.2 Probability Theory