Discrete Structures

Faisal Bukhari, PhD
Associate Professor
Department of Data Science, Faculty of Computing and Information Technology, University of the Punjab

Text book

Discrete Mathematics and Its Application, 7^{th} Edition

Kenneth H. Rosen

References

Chapter 1

1. Discrete Mathematics and Its Application, 6^{th} Edidition

by

Kenneth H. Rose

2. Discrete Mathematics with Applications

by

Thomas Koshy

3. Discrete Mathematical Structures, CS 173

by

Cinda Heeren, Siebel Center

These slides contain material from the above resources.

Logical Equivalences

- **Compound propositions** that have the **same truth values** in all possible cases are called **logically equivalent**.

- The compound propositions p and q are called **logically equivalent** if **p ↔ q is a tautology**. The notation **p ≡ q** denotes that p and q are **logically equivalent**.

- **Remark:** The symbol **≡** is not a **logical connective** and p ≡ q is not a compound proposition but rather is the statement that **p ↔ q** is a **tautology**. The symbol ⇔ is sometimes used instead of **≡** to denote logical equivalence

TABLE 6 Logical Equivalences.

Equivalence	*Name*
$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity laws
$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws

Keywords

"**Neither** P **nor** Q" can be **rephrased** as "**It is not the case** that P, **and it is not the case** that Q"

"It is not the case that P, and it is not the case that Q" formulation, which can be written with symbols as "**$\neg$P and $\neg$Q**"

Logical Equivalences

In these equivalences, **T** denotes the compound proposition that is **always true** and **F** denotes the **compound proposition** that is **always false.**

TABLE 7 Logical Equivalences Involving Conditional Statements.

$p \to q \equiv \neg p \vee q$
$p \to q \equiv \neg q \to \neg p$
$p \vee q \equiv \neg p \to q$
$p \wedge q \equiv \neg(p \to \neg q)$
$\neg(p \to q) \equiv p \wedge \neg q$
$(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$
$(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$
$(p \to q) \vee (p \to r) \equiv p \to (q \vee r)$
$(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$

TABLE 8 Logical Equivalences Involving Biconditional Statements.

$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$
$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$
$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$
$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

In class quiz

$(p \to q) \land (p \to r) \equiv p \to (q \land r)$

$(p \to r) \lor (q \to r) \equiv (p \land q) \to r$

Logical Equivalences Involving Biconditionals

TABLE 8 Logical Equivalences Involving Biconditional Statements.
$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$
$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$
$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$
$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

Logical Equivalences

Example Use **De Morgan's laws** to express the negations of **"Miguel has a cellphone and he has a laptop computer"**

and **"Heather will go to the concert or Steve will go to the concert ".**

Example cont.

Let p: **"Miguel has a cellphone"**

and q: **"He has a laptop computer"**

$\Rightarrow$ $p \land q$

$\neg(p \land q) \equiv \neg p \lor \neg q$

$\neg p$ = Miguel does not have a cell phone

$\neg q$ = He does not have a laptop computer

$\Rightarrow$ $\neg p \lor \neg q$ = Miguel does **not** have a cell phone **or** he does **not** have a laptop computer

Example cont.

"Heather will go to the concert **or** Steve will go to the concert"

Let **p: "Heather will go to the concert"**

and **q: "Steve will go to the concert"**

$\Rightarrow$ $p \vee q$

$\neg(p \vee q) \equiv \neg p \wedge \neg q$

$\neg p$ = Heather will not go to the concert

$\neg q$ = Steve will not go to the concert

$\neg p \wedge \neg q$ = Heather will **not** go to the concert **and** Steve will **not** go to the concert

Logical Equivalences

Example: Show that $\neg(p \wedge \neg q) \vee q \equiv \neg p \vee q$ are logically equivalent. You are not allowed to use the Truth Table.

Logical Equivalences

$\neg(p \wedge \neg q) \vee q \equiv (\neg p \vee \neg\neg q) \vee q$	**using De Morgan Law** $\because \neg(p \wedge q) \equiv \neg p \vee \neg q$
$\neg(p \wedge \neg q) \vee q \equiv (\neg p \vee q) \vee q$	**using double negation** $\because \neg\neg \mathbf{q} \equiv \mathbf{q}$
$\neg(p \wedge \neg q) \vee q \equiv \neg p \vee (q \vee q)$	**using associative Law** $\because (\mathbf{p} \vee \mathbf{q}) \vee \mathbf{r} \equiv \mathbf{p} \vee (\mathbf{q} \vee \mathbf{r})$
$\neg(p \wedge \neg q) \vee q \equiv \neg p \vee q$	**using idempotent Law** $\because q \vee q \equiv q$

Example Show that $[p \wedge (p \to q)] \to q$ is a tautology. **You are not allowed to use the Truth Table.**

We use $\equiv$ to show that $[p \wedge (p \rightarrow q)] \rightarrow q \equiv \mathrm{T}$.

$[p \wedge (p \rightarrow q)] \rightarrow q$		
→	$\equiv [p \wedge (\neg p \vee q)] \rightarrow q$	substitution for $\rightarrow$
→	$\equiv [(p \wedge \neg p) \vee (p \wedge q)] \rightarrow q$	distributive
→	$\equiv [\ \mathrm{F} \vee (p \wedge q)] \rightarrow q$	negation
→	$\equiv (p \wedge q) \rightarrow q$	identity
→	$\equiv \neg(p \wedge q) \vee q$	substitution for $\rightarrow$
→	$\equiv (\neg p \vee \neg q) \vee q$	DeMorgan's
→	$\equiv \neg p \vee (\neg q \vee q\ )$	associative
→	$\equiv \neg p \vee \mathrm{T}$	negation
→	$\equiv \mathrm{T}$	identity

We use ≡ to show that $[p \wedge (p \rightarrow q)] \rightarrow q \equiv T$

$[p \wedge (p \rightarrow q)] \rightarrow q$	Reasons
$\equiv [p \wedge (\neg p \vee q)] \rightarrow q$	$\because p \rightarrow q \equiv \neg p \vee q$
$\equiv [(p \wedge \neg p) \vee (p \wedge q)] \rightarrow q$	$\because p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
$\equiv [\ F \vee (p \wedge q)] \rightarrow q$	$\because p \wedge \neg p \equiv F$
$\equiv (p \wedge q) \rightarrow q$	$\because q \vee F \equiv q$
$\equiv \neg(p \wedge q) \vee q$	$\because p \rightarrow q \equiv \neg p \vee q$
$\equiv (\neg p \vee \neg q) \vee q$	$\because \neg(p \wedge q) \equiv \neg p \vee \neg q$
$\equiv \neg p \vee (\neg q \vee q)$	$\because p \vee (q \vee r) \equiv (p \vee q) \vee r$
$\equiv \neg p \vee T$	$\because \neg q \vee \boldsymbol{q} \equiv \mathbf{T}$
$\equiv T$	$\because \neg q \vee T \equiv T$

Logical Equivalences

Example: Show $\neg(p \to q) \equiv p \land \neg q$ are logically equivalent

Logical Equivalences

$\neg(p \rightarrow q) \equiv \neg(\neg p \vee q)$	using $p \rightarrow q \equiv \neg p \vee q$
$\equiv \neg(\neg p) \wedge \neg q$	De Morgan Law
$\equiv p \wedge \neg q$	Double negation

Suggested Readings

1.3 Propositional Equivalences