Discrete Structures

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Text book

Discrete Mathematics and Its Application, 7th Edition

Kenneth H. Rosen

References

Chapter 1

1. Discrete Mathematics and Its Application, 6th Edittition by Kenneth H. Rose

2. Discrete Mathematics with Applications by Thomas Koshy

3. Discrete Mathematical Structures, CS 173 by Cinda Heeren, Siebel Center

4. Discrete Mathematics with applications 5th Edition by Susanna S. EPP

These slides contain material from the above resources.

Conjunction and disjunction

Conjunctions:

English expressions: **and**, yet, **but**, however, moreover, nevertheless, still, also, both, although, additionally.

Disjunction:

English expressions: or

Necessary and Sufficient Conditions

Definition

If *r* and *s* are statements:

r is a **sufficient condition** for *s* means **"if *r* then *s*."**

r is a **necessary condition** for *s* means **"if not *r* then not *s*."**

r is a **necessary condition** for *s* also means **"if *s* then *r*."**

Consequently,

r is a **necessary and sufficient condition** for *s* means **"*r* if, and only if, *s*."**

Uniqueness quantifier

- The uniqueness quantifier is denoted by **∃!** Or **$\exists_1$** The notation **∃!xP(x)** denotes **"There exists a unique x such that P(x) is true."** Other phrases for uniqueness quantification include **"there is exactly one"** and **"there is one and only one."**

- Generally, it is best to stick with **existential and universal quantifiers** so that **rules of inference** for these quantifiers can be used.

Quantifiers with Restricted Domains [1]

- An abbreviated notation is often used to restrict the **domain of a quantifier**. In this notation, a condition a variable must satisfy is included after the quantifier

Quantifiers with Restricted Domains [2]

- **Example** What do the statements **$\forall x < 0\ (x^2 > 0)$** mean, where the domain in each case consists of the real numbers?

- The statement $\forall x < 0\ (x^2 > 0)$ states that for every real number x with $x < 0$, $x^2 > 0$. That is, it states **"The square of a negative real number is positive."**

- This statement is the same as **$\forall x(x < 0 \rightarrow x^2 > 0)$.**

Quantifiers with Restricted Domains [3]

- **Example:** What do the statement $\forall y \neq 0\ (y^3 \neq 0)$ mean, where the domain consists of the real numbers?

- The statement $\forall y \neq 0\ (y^3 \neq 0)$, states that for every real number y with $y \neq 0$ and, we have $y^3 \neq 0$. That is, it states **"The cube of every nonzero real number is nonzero."**

- This statement is equivalent to $\boldsymbol{\forall y(y \neq 0 \rightarrow y^3 \neq 0)}$.

Quantifiers with Restricted Domains [4]

- **Example:** What do the statement $\exists z > 0\ (z^2 = 2)$ mean, where the domain consists of the real numbers?

- The statement $\exists z > 0\ (z^2 = 2)$, that there exists a real number z with $z > 0$ such that $z^2 = 2$. That is, it states **"There is a positive square root of 2"**

- This statement is equivalent to $\exists z(z > 0 \land z^2 = 2)$.

Precedence of Quantifiers

- The quantifiers **∀** and **∃** have **higher precedence then all logical operators** from propositional calculus

- $\forall x P(x)$ **v** $Q(x)$ mean **$(\forall x P(x)) \vee Q(x)$** $\neq$ **$\forall x(P(x) \vee Q(x))$**

Binding Variables

- When a **quantifier** is used on the **variable x**, we say that this occurrence of the **variable is bound**. An occurrence of a variable that **is not bound** by a **quantifier** or set equal to a particular value is said to be **free**.

- The part of a **logical expression** to which a **quantifier is applied** is called the **scope** of this **quantifier**.

Binding Variables

- **Example** In the statement $\exists x(x + y = 1)$, the **variable x is bound** by the **existential quantification $\exists x$** , but the **variable y is free** because it is not bound by a quantifier and no value is assigned to this variable.

- This illustrates that in the statement $\exists x(x + y = 1)$, **x is bound**, but **y is free**.

Logical Equivalences Involving Quantifiers

- Statements involving predicates and quantifiers are logically equivalent if and only **if they have the same truth value** no matter which **predicates** are substituted into these statements and which **domain of discourse** is used for the variables in these **propositional functions**.

- We use the notation **S ≡ T** to indicate that two statements S and T involving predicates and quantifiers are **logically equivalent**.

Negating Quantified Expressions [1]

The rules for negations for quantifiers are called **De Morgan's** laws for quantifiers.

De Morgan's Laws for Quantifiers.			
Negation	Equivalent Statement	When Is Negation True?	When False?
$\neg \exists x P(x)$	$\forall x \neg P(x)$	For every x , P (x) is false	There is an x for which P (x) is true.
$\neg \forall x P(x)$	$\exists x \neg P(x)$	There is an x for which P (x) is false	P (x) is true for every x .

Suggested Readings

- **1.4 Predicates and Quantifiers**