

Name: _____

Roll Number: _____

Quiz-1

Max. Time: 20 min

Max. Points: 20

Note: Solve all parts. Limit your written responses to the provided space.

Q.1. [8] Choose by putting a check mark on the most appropriate option. Note: No cutting/overwriting is allowed.

i. In some cases, a matrix may be row reduced to more than one matrices in echelon form by using different sequences of row operations.

(A) True (B) False

ii. The row reduction algorithm applies only to augmented matrices for a linear system.

(A) True (B) False

iii. The equation $Ax = b$ is consistent only if the augmented matrix $[A \ b]$ has a pivot position in every row.

(A) True (B) False

iv. The echelon form of a matrix is unique.

(A) True (B) False

v. The pivot positions in a matrix depend on whether row interchanges are used in the row reduction process.

(A) True (B) False

vi. A 3×5 coefficient matrix of a linear system with pivots in every row can be inconsistent.

(A) True (B) False

vii. An underdetermined linear system (fewer equations than the unknowns) is always consistent.

(A) True (B) False

viii. The points corresponding to $\begin{bmatrix} -2 \\ 5 \end{bmatrix}$ and $\begin{bmatrix} 4 \\ -10 \end{bmatrix}$ lie on a line through the origin.

(A) True (B) False

Q.2. [7+5]

a) Row-reduce the following matrix to reduced echelon form.

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 5 & 6 & 7 \\ 6 & 7 & 8 & 9 \end{bmatrix}$$

Solution:

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 5 & 6 & 7 \\ 6 & 7 & 8 & 9 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -3 & -6 & -9 \\ 0 & -5 & -10 & -15 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Name: _____

Roll Number: _____

b) Consider the matrix A below. Check whether vector \mathbf{b} is in the span of the columns of A .

$$A = \begin{bmatrix} 2 & 0 & 6 \\ -1 & 8 & 5 \\ 1 & -2 & 1 \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} 10 \\ 3 \\ 3 \end{bmatrix}$$

Solution:

$$[\mathbf{a}_1 \quad \mathbf{a}_2 \quad \mathbf{a}_3 \quad \mathbf{b}] = \begin{bmatrix} 2 & 0 & 6 & 10 \\ -1 & 8 & 5 & 3 \\ 1 & -2 & 1 & 3 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 3 & 5 \\ -1 & 8 & 5 & 3 \\ 1 & -2 & 1 & 3 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 3 & 5 \\ 0 & 8 & 8 & 8 \\ 0 & -2 & -2 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 3 & 5 \\ 0 & 8 & 8 & 8 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Yes, \mathbf{b} is in the span of the columns of A because the system in echelon form is consistent.