Name:____ Roll Number:_________________________

Quiz-3

Max. Time: 20 min Max. Points: 20

Note: Solve all parts. Limit your written responses to the provided space.

Q.1. [8] Choose by putting a check mark on the most appropriate option. Note: No cutting/overwriting is allowed.

i. Not every linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$ is a matrix transformation.

(A) True (B) False

ii. A transformation $T: \mathbb{R}^n \to \mathbb{R}^m$ with A as its standard matrix, is one-to-one if and only if the columns of A span $\mathbb{R}^m$.

(A) True (B) False

iii. The columns of the standard matrix for a linear transformation A from $\mathbb{R}^n$ to $\mathbb{R}^m$ are the images of the of the columns of $n \times n$ identity matrix $\boldsymbol{I}$.

(A) True (B) False

iv. When two linear transformations are performed one after another, the combined effect may not always be linear.

(A) True (B) False

v. If A is a 3×5 matrix representing a linear transformation T, then T cannot map $\mathbb{R}^5$ onto $\mathbb{R}^3$.

(A) True (B) False

vi. The codomain of the transformation $\mathbf{x} \mapsto \boldsymbol{A}\mathbf{x}$ is the set of all linear combinations of the columns of $\boldsymbol{A}$.

(A) True (B) False

vii. The transformation $T: \mathbb{R}^n \to \mathbb{R}^m$ is onto $\mathbb{R}^m$ if every vector $\mathbf{x}$ in $\mathbb{R}^n$ maps onto some vector in $\mathbb{R}^m$.

(A) True (B) False

viii. If $T: \mathbb{R}^n \to \mathbb{R}^m$ is a linear transformation and $\mathbf{c}$ is in $\mathbb{R}^m$, then whether or not $\mathbf{c}$ is in the range of T is a uniqueness question.

(A) True (B) False

Q.2. [7+5]

a) For the following transformation $T: \mathbb{R}^4 \to \mathbb{R}^3$, find all vectors in $\mathbb{R}^4$ that get mapped to the origin in $\mathbb{R}^3$.

$$A = \begin{bmatrix} 1 & -4 & 7 & -5 \\ 0 & 1 & -4 & 3 \\ 2 & -6 & 6 & -4 \end{bmatrix}$$

Name:____ Roll Number:_________________________

Solve $A\mathbf{x} = \mathbf{0}$. $\begin{bmatrix} 1 & -4 & 7 & -5 & 0 \\ 0 & 1 & -4 & 3 & 0 \\ 2 & -6 & 6 & -4 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -4 & 7 & -5 & 0 \\ 0 & 1 & -4 & 3 & 0 \\ 0 & 2 & -8 & 6 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -4 & 7 & -5 & 0 \\ 0 & 1 & -4 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

$$\sim \begin{bmatrix} 1 & 0 & -9 & 7 & 0 \\ 0 & 1 & -4 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{array}{rcrcrcl} x_1 & - & 9x_3 & + & 7x_4 & = & 0 \\ x_2 & - & 4x_3 & + & 3x_4 & = & 0, \\ & & & & 0 & = & 0 \end{array} \quad \begin{cases} x_1 = 9x_3 - 7x_4 \\ x_2 = 4x_3 - 3x_4 \\ x_3 \text{ is free} \\ x_4 \text{ is free} \end{cases}$$

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 9x_3 - 7x_4 \\ 4x_3 - 3x_4 \\ x_3 \\ x_4 \end{bmatrix} = x_3 \begin{bmatrix} 9 \\ 4 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -7 \\ -3 \\ 0 \\ 1 \end{bmatrix}$$

b) Consider $T: \mathbb{R}^2 \to \mathbb{R}^2$. Give the standard matrix for the linear transformation A that first reflects points through the horizontal axis x_1 and then reflects points through the line $x_2=x_1$.

$$\mathbf{e}_1 \to \mathbf{e}_1 \to \mathbf{e}_2 \text{ and } \mathbf{e}_2 \to -\mathbf{e}_2 \to -\mathbf{e}_1, \text{ so } A = [\mathbf{e}_2 \quad -\mathbf{e}_1] = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

Name:____ Roll Number:________________________