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Quiz-2

Max. Time: 20 min Max. Points: 20

Note: Solve all parts. Limit your written responses to the provided space.

Q.1. [8] Choose by putting a check mark on the most appropriate option. Note: No cutting/overwriting is allowed.

1. A linear system $\mathbf{Ax} = \mathbf{b}$ can have a trivial solution.

(A) True (B) False

2. Having pivots in every row is a sufficient condition for independence of the columns of an $m \times n$ coefficient matrix **A**.

(A) True (B) False

3. Column vectors of an $m \times n$ matrix **A** will always span $\mathbb{R}^m$ if $m < n$.

(A) True (B) False

4. If $n \leq m$, the columns of **A** may not span $\mathbb{R}^m$.

(A) True (B) False

5. An $m \times n$ matrix **A** representing a linear system having fewer pivots than m, must necessarily have infinite solutions.

(A) True (B) False

6. The equation $\mathbf{x} = \mathbf{p} + t\mathbf{v}$ describes a line through **v** parallel to **p**.

(A) True (B) False

7. Parametric vector form gives an explicit representation of the underlying linear structure.

(A) True (B) False

8. A homogeneous linear system can have a nontrivial unique solution.

(A) True (B) False

Q.2. [7+5]

a) Determine if the following planes in $\mathbb{R}^3$ intersect. If so, give a geometric description of their intersection.

$$x + 4y - 5z = 0$$

$$2x - y + 8z = 9$$

Solution:

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Row reduce the augmented matrix:

$$\begin{bmatrix} 1 & 4 & -5 & 0 \\ 2 & -1 & 8 & 9 \end{bmatrix} \sim \begin{bmatrix} 1 & 4 & -5 & 0 \\ 0 & -9 & 18 & 9 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 3 & 4 \\ 0 & 1 & -2 & -1 \end{bmatrix}$$

$$\begin{aligned} x \quad + 3z &= 4 \\ y - 2z &= -1 \end{aligned}$$

Thus $x = 4 - 3z$, $y = -1 + 2z$, with z free. The general solution in parametric vector form is

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 - 3z \\ -1 + 2z \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -1 \\ 0 \end{bmatrix} + z \begin{bmatrix} -3 \\ 2 \\ 1 \end{bmatrix}$$

$$\mathbf{x} = \mathbf{p} + t\,\mathbf{v}$$

The intersection of the two planes is the line through $\mathbf{p}$ in the direction of $\mathbf{v}$.

b) Find the value of h for which the following vectors are linearly dependent.

$$\begin{bmatrix} 1 \\ 5 \\ -3 \end{bmatrix}, \quad \begin{bmatrix} -2 \\ -9 \\ 6 \end{bmatrix}, \quad \begin{bmatrix} 3 \\ h \\ -9 \end{bmatrix}$$

Solution:

To study the linear dependence of three vectors, say $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$, row reduce the augmented matrix $[\mathbf{v}_1 \ \ \mathbf{v}_2 \ \ \mathbf{v}_3 \ \ \mathbf{0}]$:

$$\begin{bmatrix} 1 & -2 & 3 & 0 \\ 5 & -9 & h & 0 \\ -3 & 6 & -9 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & 3 & 0 \\ 0 & 1 & h-15 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The equation $x_1\mathbf{v}_1 + x_2\mathbf{v}_2 + x_3\mathbf{v}_3 = \mathbf{0}$ has a free variable and hence a nontrivial solution no matter what the value of h. So the vectors are linearly dependent for all values of h.

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