$$(I * k) = \begin{bmatrix} 1 & 2 & 2 \\ -1 & -2 & -2 \\ -1 & -1 & -1 \end{bmatrix} * \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 1 \\ 1 & -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 2 \\ -1 & -2 & -2 \\ -1 & -1 & -1 \end{bmatrix} \circ \begin{bmatrix} 1 & -2 & 1 \\ 1 & 2 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

where $\circ$ denotes the correlation operation

$$= (1 \times 1) + (2 \times -2) + (2 \times 1) + (-1 \times 1) + (-2 \times 2)$$
$$+ (-2 \times 2) + (-1 \times 1) + (-1 \times 1) + (-1 \times 1)$$

$$= \cancel{1} - 4 + 2 \cancel{-1} - 4 - 4 - 1 - 1 - 1$$

$$= -13$$

$$I = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 6 & 2 & 3 & 1 & 0 \\ 0 & -1 & 2 & -2 & -1 & 0 \\ 0 & -1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\text{Max-Pool}(I, \text{filter\_size} = 3, \text{stride} = 3) = \begin{bmatrix} 6 & 3 \\ 2 & 1 \end{bmatrix}$$

①
$$(I * k) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 2 & 1 & 0 \\ 0 & -1 & -2 & -2 & -1 & 0 \\ 0 & -1 & -1 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} * \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 1 \\ 1 & -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 2 & 1 & 0 \\ 0 & -1 & -2 & -2 & -1 & 0 \\ 0 & -1 & -1 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \circ \begin{bmatrix} 1 & -2 & 1 \\ 1 & 2 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\text{where } \circ \text{ denotes the cross-correlation operation}$$

$$(I * k) = \begin{bmatrix} 7 & 10 & 10 & 6 \\ 2 & 4 & 3 & 0 \\ -8 & -13 & -12 & -6 \\ -4 & -4 & -4 & -3 \end{bmatrix}$$

② $\text{ReLU}(I * k) = \max(0, I * k)$

$$= \begin{bmatrix} 7 & 10 & 10 & 6 \\ 2 & 4 & 3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

③

$$\text{Max-Pool}(I, \text{filter\_size} = 2, \text{stride} = 2) = \begin{bmatrix} 10 & 10 \\ 0 & 0 \end{bmatrix}$$

④ $\text{Flatten}(I) = I_{\text{flattened}} = \begin{bmatrix} 10 & 10 & 0 & 0 \end{bmatrix}^T$

⑤ $\text{Fully-Connected}(I_{\text{flattened}}, W) = W \times I^T$

$$= \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{bmatrix} \begin{bmatrix} 10 \\ 10 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 30 \\ 110 \end{bmatrix}$$

⑥ $\text{Softmax}(z_j) = \dfrac{e^{z_j}}{\sum_i e^{z_i}}$

$$\text{Softmax}\left(\begin{bmatrix} 30 \\ 110 \end{bmatrix}\right) = \begin{bmatrix} 1.8 \times 10^{-35} \\ 1.0 \end{bmatrix}$$