Bahria University
Discovering Knowledge

Computer Architecture and Logic Design (CALD)

Lecture 11

Dr. Sorath Hansrajani

Assistant Professor

Department of Software Engineering

Bahria University Karachi Campus

Email: sorathhansrajani.bukc@bahria.edu.pk

Simplification of Boolean Functions

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3-1 Introduction

- Gate-level minimization refers to the design task of finding an optimal gate-level implementation of Boolean functions describing a digital circuit.

3-2 The Map Method

- The complexity of the digital logic gates
  - The complexity of the algebraic expression
- Logic minimization
  - Algebraic approaches: lack specific rules
  - The Karnaugh map
    - A simple straight forward procedure
    - A pictorial form of a truth table
    - Applicable if the # of variables < 7
- A diagram made up of squares
  - Each square represents one minterm

Review of Boolean Function

- Boolean function
  - Sum of minterms
  - Sum of products (or product of sum) in the simplest form
  - A minimum number of terms
  - A minimum number of literals
  - The simplified expression may not be unique

Two-Variable Map

- A two-variable map
  - Four minterms
  - x' = row 0; x = row 1
  - y' = column 0; y = column 1
  - A truth table in square diagram
  - Fig. 3.2(a): $xy = m_3$
  - Fig. 3.2(b): $x+y = x'y+xy' +xy = m_1+m_2+m_3$

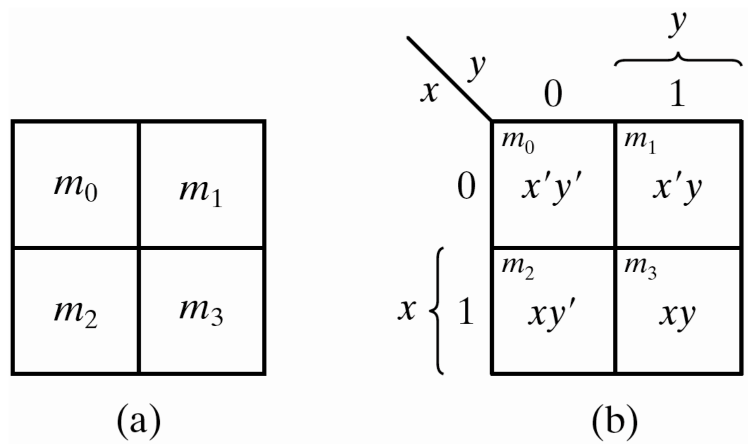


Figure 3.1 Two-variable Map

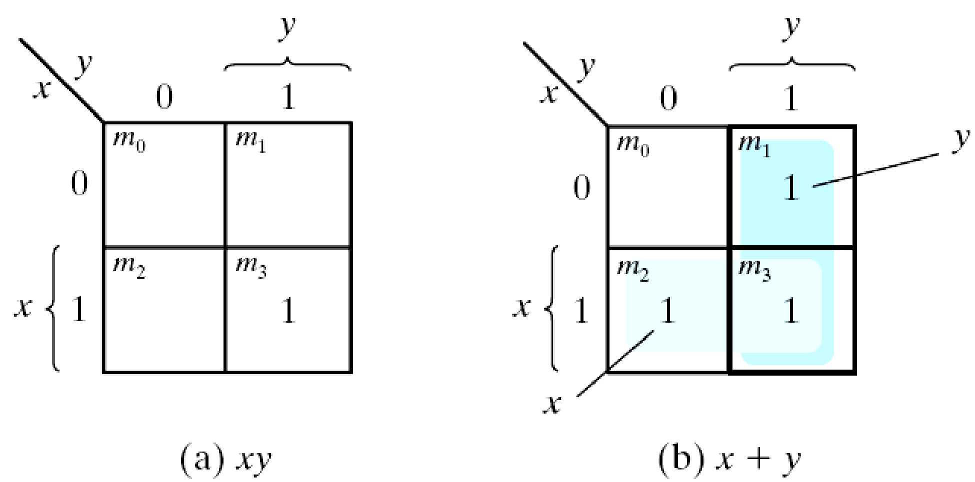


Figure 3.2 Representation of functions in the map

A Three-variable Map

- A three-variable map
  - Eight minterms
  - The Gray code sequence
  - Any two adjacent squares in the map differ by only one variable
    - Primed in one square and unprimed in the other
    - e.g., m_5 and m_7 can be simplified
    - $m_5 + m_7 = xy'z + xyz = xz\,(y'+y) = xz$

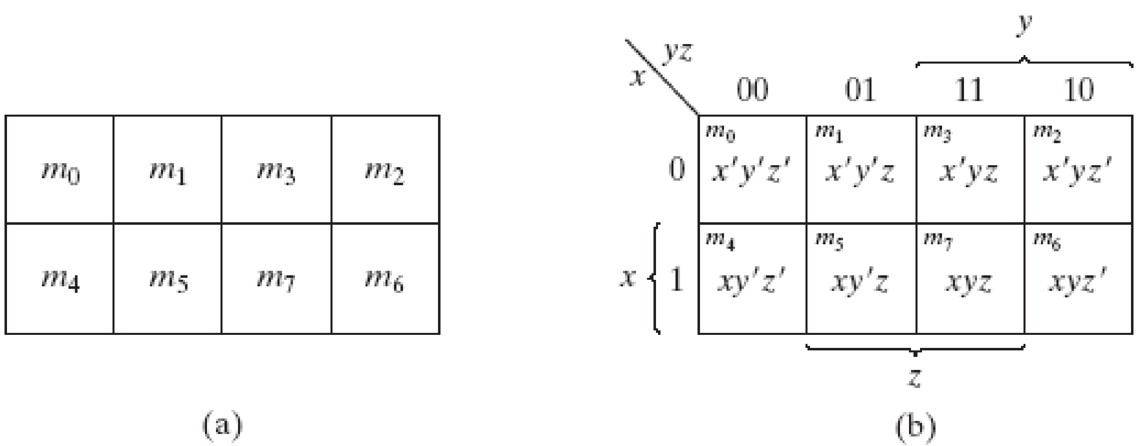


Figure 3.3 Three-variable Map

A Three-variable Map

- m_0 and m_2 (m_4 and m_6) are adjacent
- $m_0 + m_2 = x'y'z' + x'yz' = x'z'(y'+y) = x'z'$
- $m_4 + m_6 = xy'z' + xyz' = xz'(y'+y) = xz'$

m_0	m_1	m_3	m_2
m_4	m_5	m_7	m_6

(a)

$x \backslash yz$	00	01	11	10
0	$x'y'z'$	$x'y'z$	$x'yz$	$x'yz'$
x 1	$xy'z'$	$xy'z$	xyz	xyz'

y (columns 11, 10); z (columns 01, 11)

(b)

Fig. 3-3 Three-variable Map

Example 3.1

- Example 3.1: simplify the Boolean function $F(x, y, z) = \Sigma(2, 3, 4, 5)$
  - $F(x, y, z) = \Sigma(2, 3, 4, 5) = x'y + xy'$

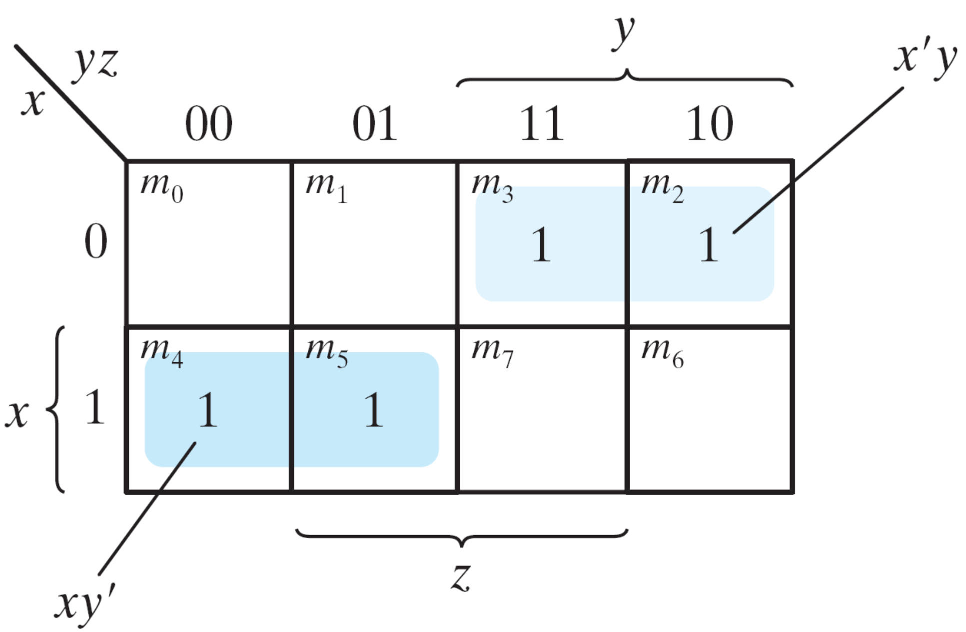


Figure 3.4 Map for Example 3.1, $F(x, y, z) = \Sigma(2, 3, 4, 5) = x'y + xy'$

Example 3.2

- Example 3.2: simplify $F(x, y, z) = \Sigma(3, 4, 6, 7)$
  - $F(x, y, z) = \Sigma(3, 4, 6, 7) = yz + xz'$

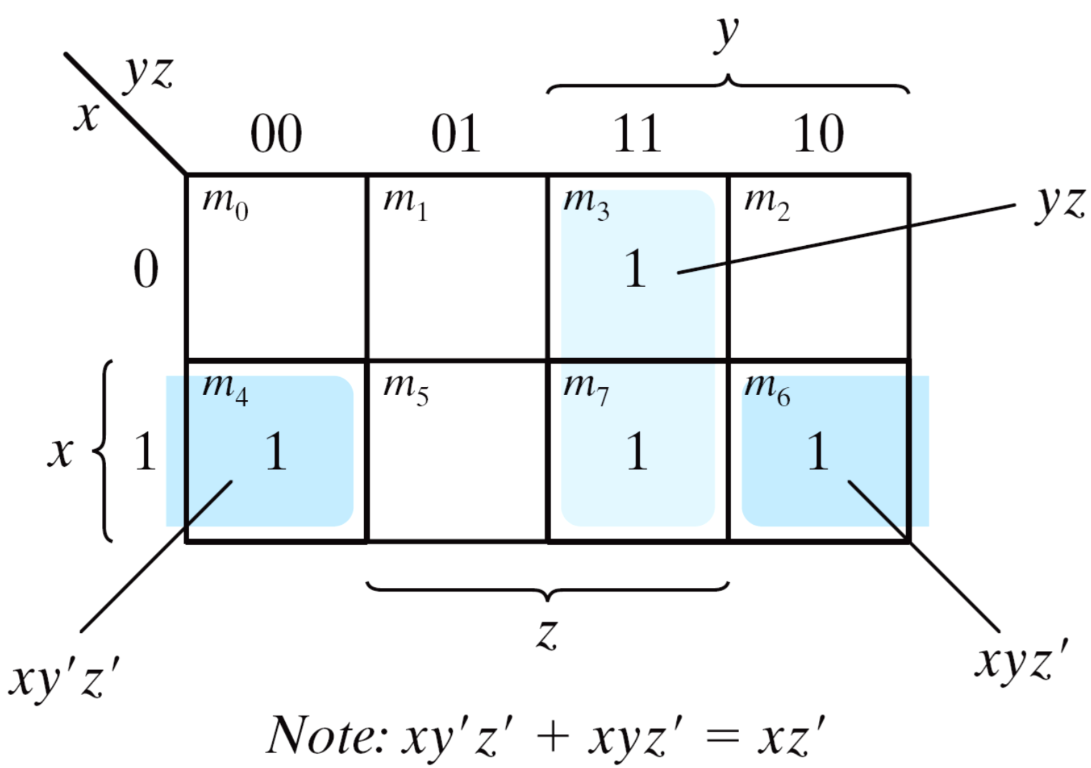


Figure 3.5 Map for Example 3-2; F(x, y, z) = $\Sigma(3, 4, 6, 7) = yz + xz'$

Four adjacent Squares

- Consider four adjacent squares
  - 2, 4, and 8 squares
  - $m_0+m_2+m_4+m_6 = x'y'z'+x'yz'+xy'z'+xyz' = x'z'(y'+y) +xz'(y'+y) = x'z' + xz' = z'$
  - $m_1+m_3+m_5+m_7 = x'y'z+x'yz+xy'z+xyz =x'z(y'+y) + xz(y'+y) =x'z + xz = z$

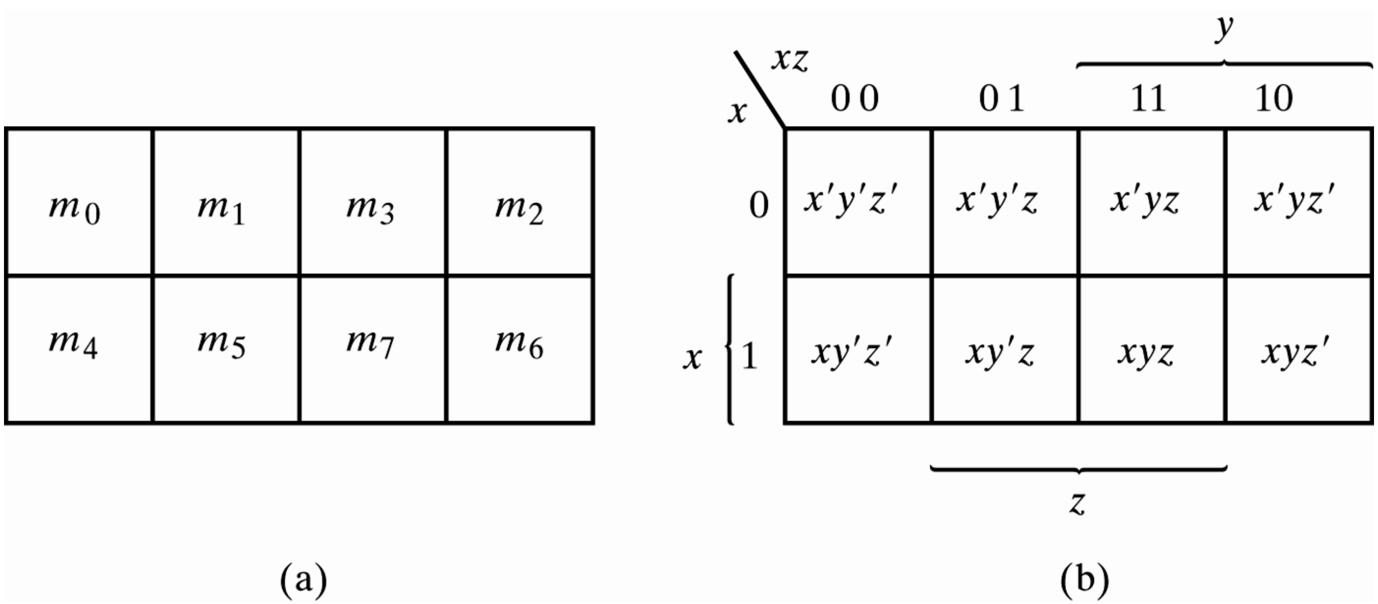


Figure 3.3 Three-variable Map

Example 3.3

- Example 3.3: simplify $F(x, y, z) = \Sigma(0, 2, 4, 5, 6)$
  - $F(x, y, z) = \Sigma(0, 2, 4, 5, 6) = z' + xy'$

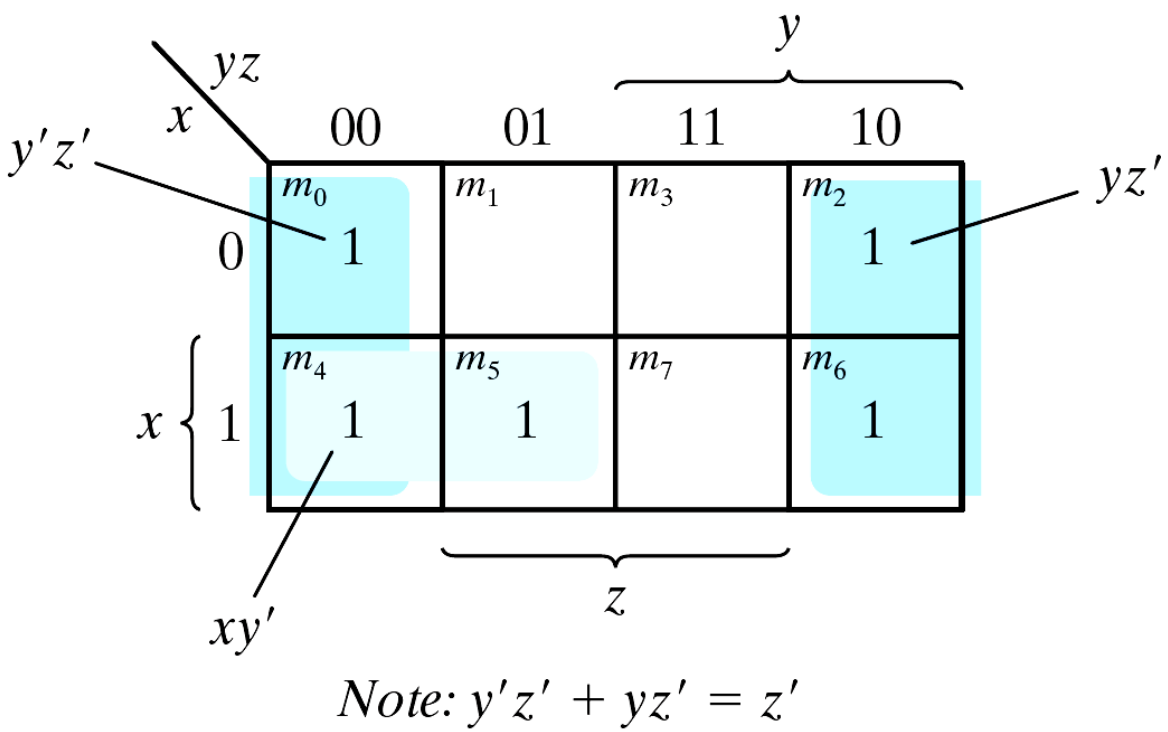


Figure 3.6 Map for Example 3-3, $F(x, y, z) = \Sigma(0, 2, 4, 5, 6) = z' + xy'$

Example 3.4

- Example 3.4: let $F = A'C + A'B + AB'C + BC$
  a) Express it in sum of minterms.
  b) Find the minimal sum of products expression.

  Ans:

  $$F(A, B, C) = \Sigma(1, 2, 3, 5, 7) = C + A'B$$

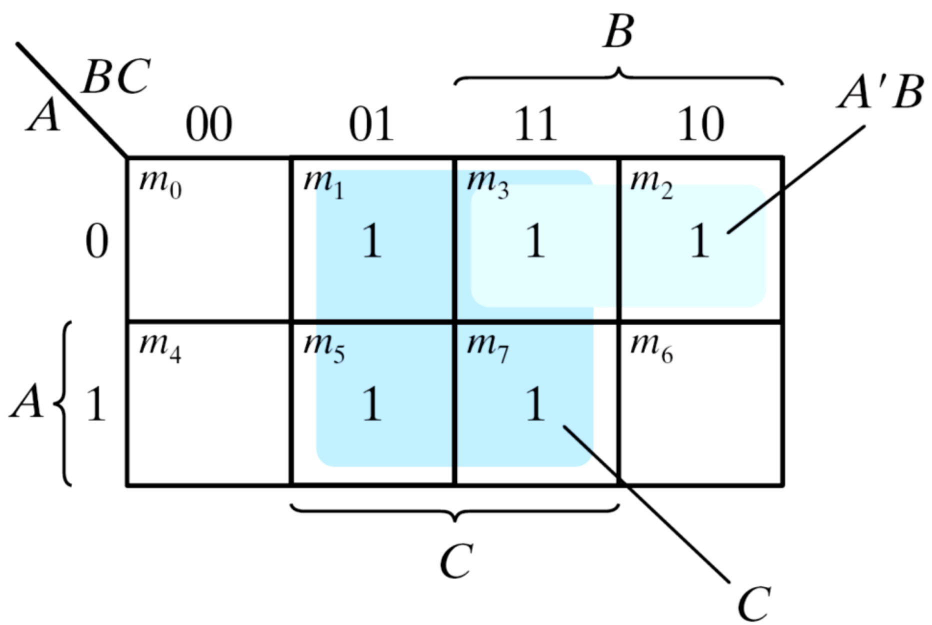


Figure 3.7 Map for Example 3.4, $A'C + A'B + AB'C + BC = C + A'B$

3.3 Four-Variable Map

- The map
  - 16 minterms
  - Combinations of 2, 4, 8, and 16 adjacent squares

m_0	m_1	m_3	m_2
m_4	m_5	m_7	m_6
m_{12}	m_{13}	m_{15}	m_{14}
m_8	m_9	m_{11}	m_{10}

(a)

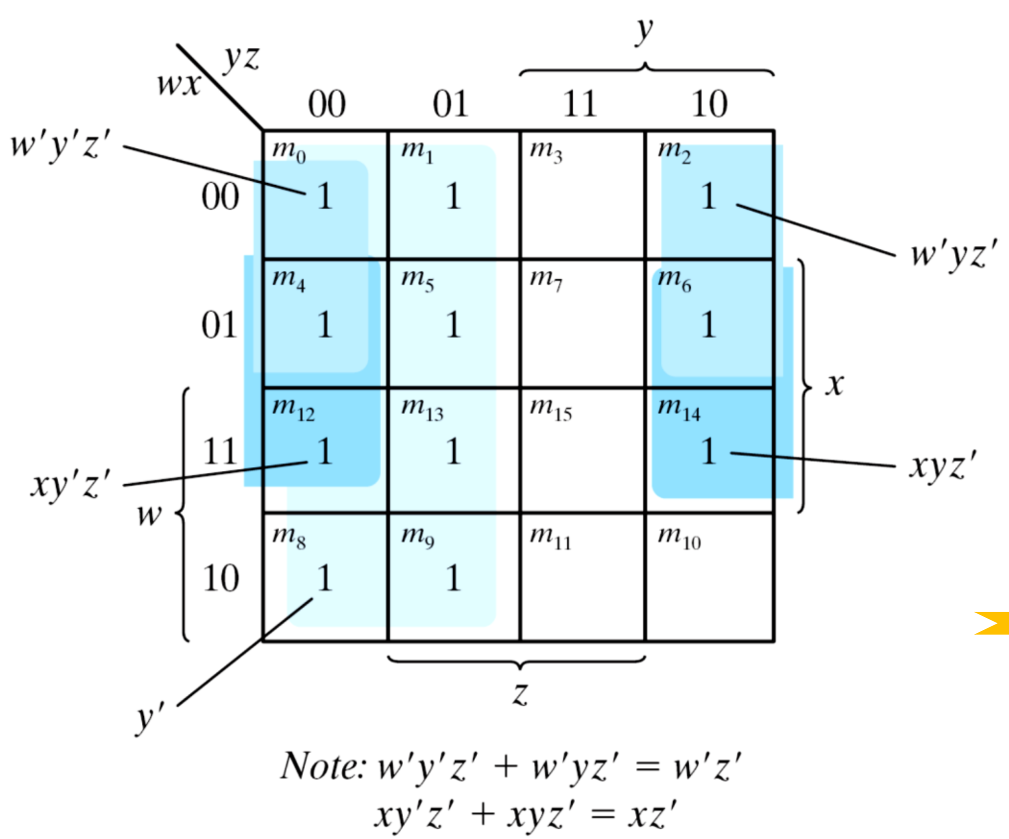
$wx \backslash yz$	00	01	11	10
00	$w'x'y'z'$	$w'x'y'z$	$w'x'yz$	$w'x'yz'$
01	$w'xy'z'$	$w'xy'z$	$w'xyz$	$w'xyz'$
11	$wxy'z'$	$wxy'z$	$wxyz$	$wxyz'$
10	$wx'y'z'$	$wx'y'z$	$wx'yz$	$wx'yz'$

(b)

Figure 3.8 Four-variable Map

Example 3.5

- Example 3.5: simplify $F(w, x, y, z) = \Sigma(0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14)$

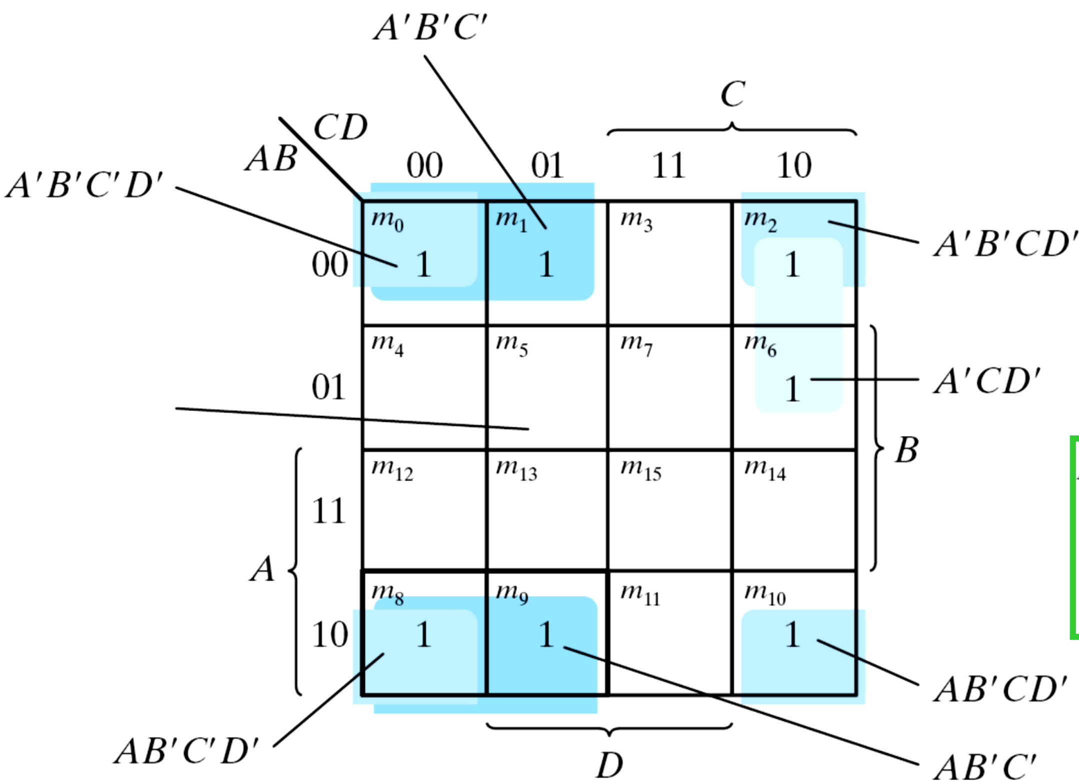


$F = y'+w'z'+xz'$

Figure 3.9 Map for Example 3-5; $F(w, x, y, z) = \Sigma(0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14) = y' + w'z' + xz'$

Example 3.6

- Example 3-6: simplify $F = A'B'C' + B'CD' + A'BCD' + AB'C'$



Note: $A'B'C'D' + A'B'CD' = A'B'D'$
$AB'C'D' + AB'CD' = AB'D'$
$A'B'D' + AB'D' = B'D'$
$A'B'C' + AB'C' = B'C'$

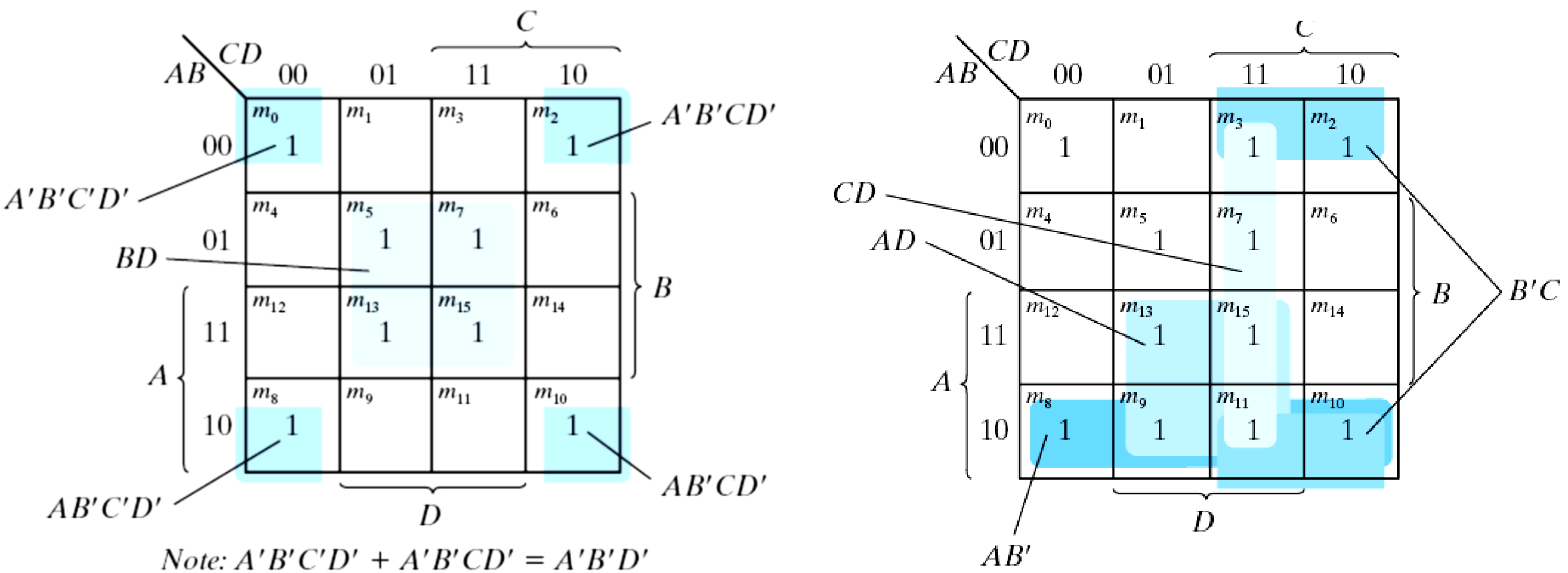
Figure 3.9 Map for Example 3-6; $A'B'C' + B'CD' + A'BCD' + AB'C' = B'D' + B'C' + A'CD'$

Prime Implicants

- Prime Implicants
  - All the minterms are covered.
  - Minimize the number of terms.
  - A prime implicant: a product term obtained by combining the maximum possible number of adjacent squares (combining all possible maximum numbers of squares).
  - Essential P.I.: a minterm is covered by only one prime implicant.
  - The essential P.I. must be included.

Prime Implicants

- Consider $F(A, B, C, D) = \Sigma(0, 2, 3, 5, 7, 8, 9, 10, 11, 13, 15)$
  - The simplified expression may not be unique
  - $F = BD+B'D'+CD+AD = BD+B'D'+CD+AB'$
    $= BD+B'D'+B'C+AD = BD+B'D'+B'C+AB'$



(a) Essential prime implicants BD and $B'D'$

(b) Prime implicants CD, $B'C$, AD, and AB'

Figure 3.11 Simplification Using Prime Implicants

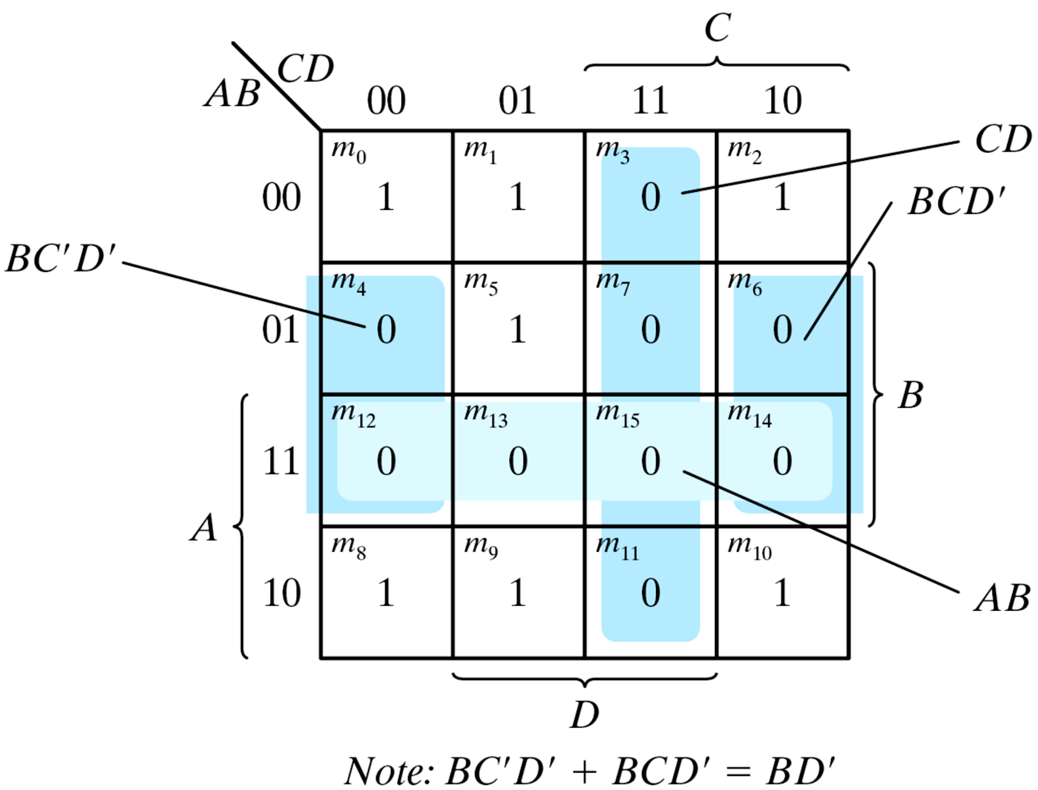
3-5 Product of Sums Simplification

- Approach #1
  - Simplified F' in the form of sum of products
  - Apply DeMorgan's theorem $F = (F')'$
  - F': sum of products $\rightarrow$ F: product of sums
- Approach #2: duality
  - Combinations of maxterms (it was minterms)
  - $M_0M_1 = (A+B+C+D)(A+B+C+D') = (A+B+C)+(DD') = A+B+C$

AB \ CD	00	01	11	10
00	M_0	M_1	M_3	M_2
01	M_4	M_5	M_7	M_6
11	M_{12}	M_{13}	M_{15}	M_{14}
10	M_8	M_9	M_{11}	M_{10}

Example 3.8

- Example 3.8: simplify $F = \Sigma(0, 1, 2, 5, 8, 9, 10)$ into (a) sum-of-products form, and (b) product-of-sums form:



a) $F(A, B, C, D) = \Sigma(0, 1, 2, 5, 8, 9, 10) = B'D' + B'C' + A'C'D$

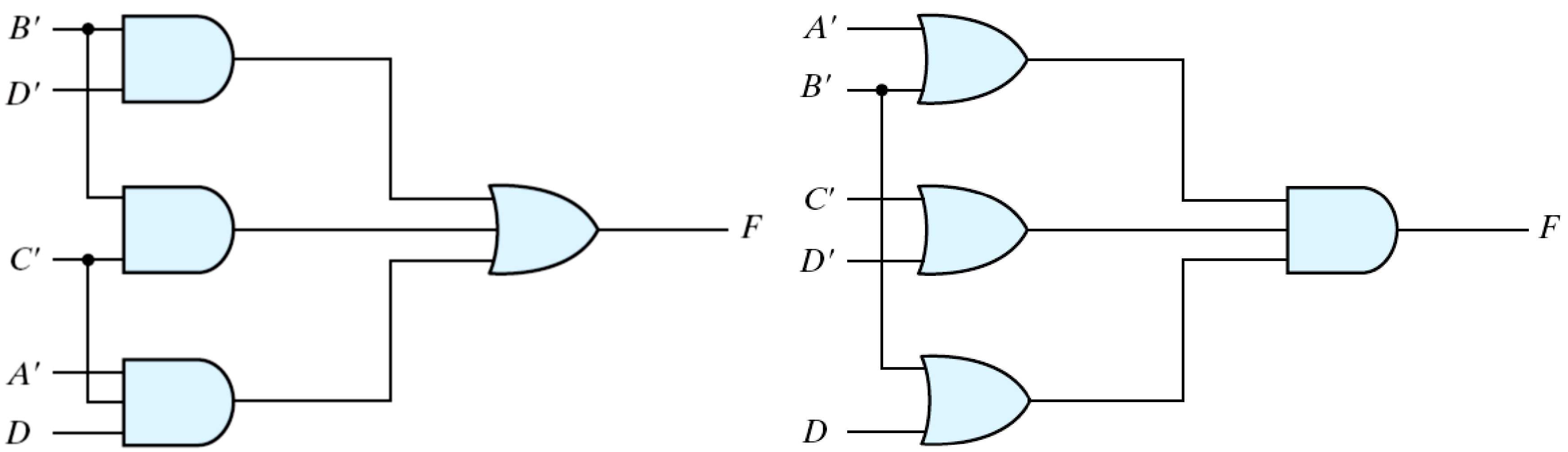
b) $F' = AB + CD + BD'$

  - Apply DeMorgan's theorem; $F = (A' + B')(C' + D')(B' + D)$
  - Or think in terms of maxterms

Figure 3.14 Map for Example 3.8, $F(A, B, C, D) = \Sigma(0, 1, 2, 5, 8, 9, 10) = B'D' + B'C' + A'C'D$

Example 3.8 (cont.)

- Gate implementation of the function of Example 3.8



(a) $F = B'D' + B'C' + A'C'D$

(b) $F = (A' + B')(C' + D')(B' + D)$

Sum-of products form

Product-of sums form

Figure 3.15 Gate Implementation of the Function of Example 3.8

Sum-of-Minterm Procedure

- Consider the function defined in Table 3.2.
  - In sum-of-minterm:

$$\boxed{F(x,y,z)=\sum(1,3,4,6)}$$

  - In sum-of-maxterm:

$$\boxed{F'(x,y,z)=\Pi(0,2,5,7)}$$

  - Taking the complement of F′

$$\boxed{F(x,y,z)=(x'+z')(x+z)}$$

Table 3.2
Truth Table of Function F

x	*y*	*z*	*F*
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

Sum-of-Minterm Procedure

- Consider the function defined in Table 3.2.
  - Combine the 1's:

$$\boxed{F(x,y,z) = x'z + xz'}$$

  - Combine the 0's :

$$\boxed{F'(x,y,z) = xz + x'z'}$$

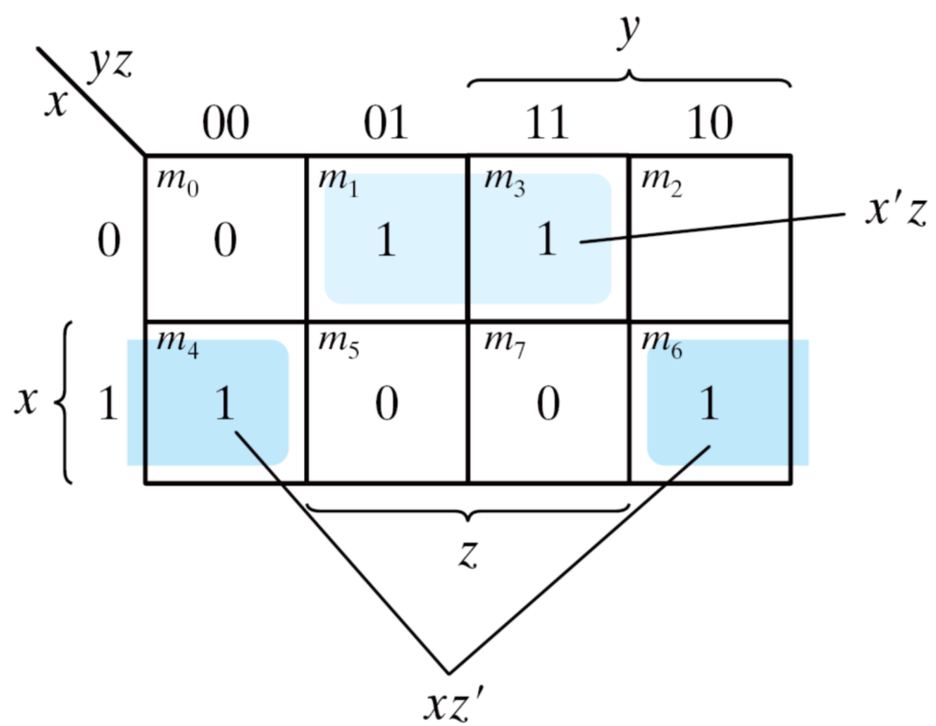


Figure 3.16 Map for the function of Table 3.2

3-6 Don't-Care Conditions

- The value of a function is not specified for certain combinations of variables
  - BCD; 1010-1111: don't care
- The don't-care conditions can be utilized in logic minimization
  - Can be implemented as 0 or 1
- Example 3.9: simplify $F(w, x, y, z) = \Sigma(1, 3, 7, 11, 15)$ which has the don't-care conditions $d(w, x, y, z) = \Sigma(0, 2, 5)$.

Example 3.9 (cont.)

- $F = yz + w'x'$; $F = yz + w'z$
- $F = \Sigma(0, 1, 2, 3, 7, 11, 15)$; $F = \Sigma(1, 3, 5, 7, 11, 15)$
- Either expression is acceptable

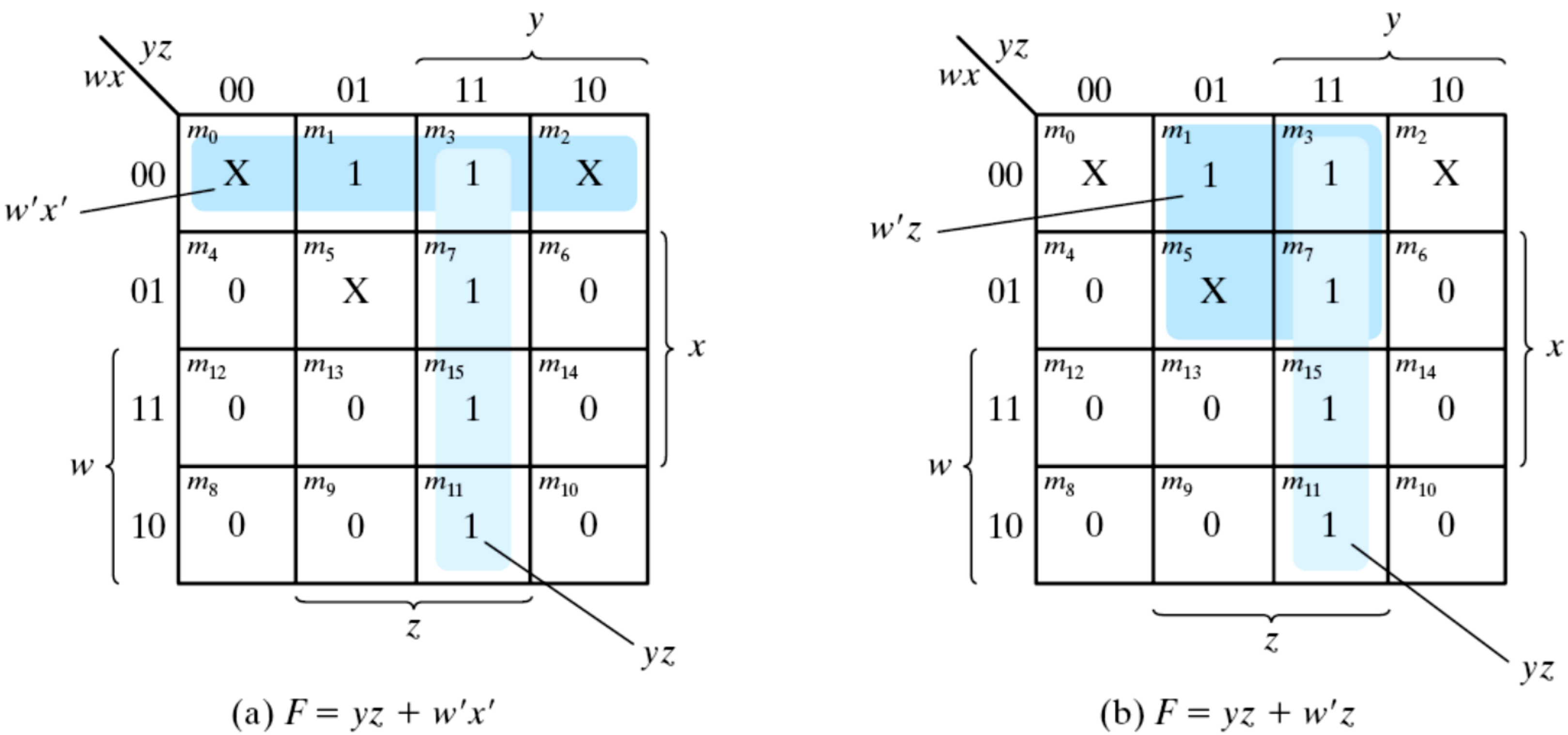


Figure 3.17 Example with don't-care Conditions