

DATA STRUCTURES & ALGORITHMS

Hash Tables

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Hash Table

A data structure that provides a mapping from **key to value** using a technique called **hashing**.

The keys are used for **indexing** the values/data.

Effective way of implementing **dictionaries**.

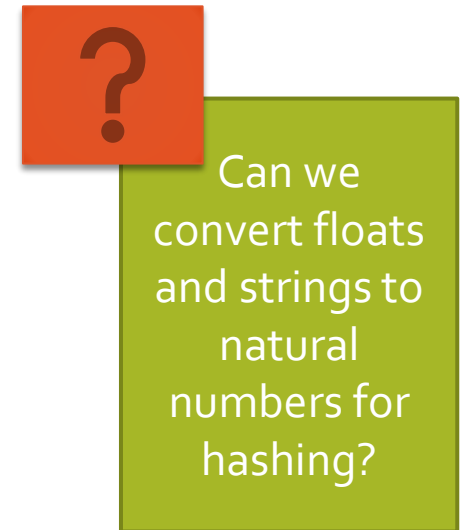
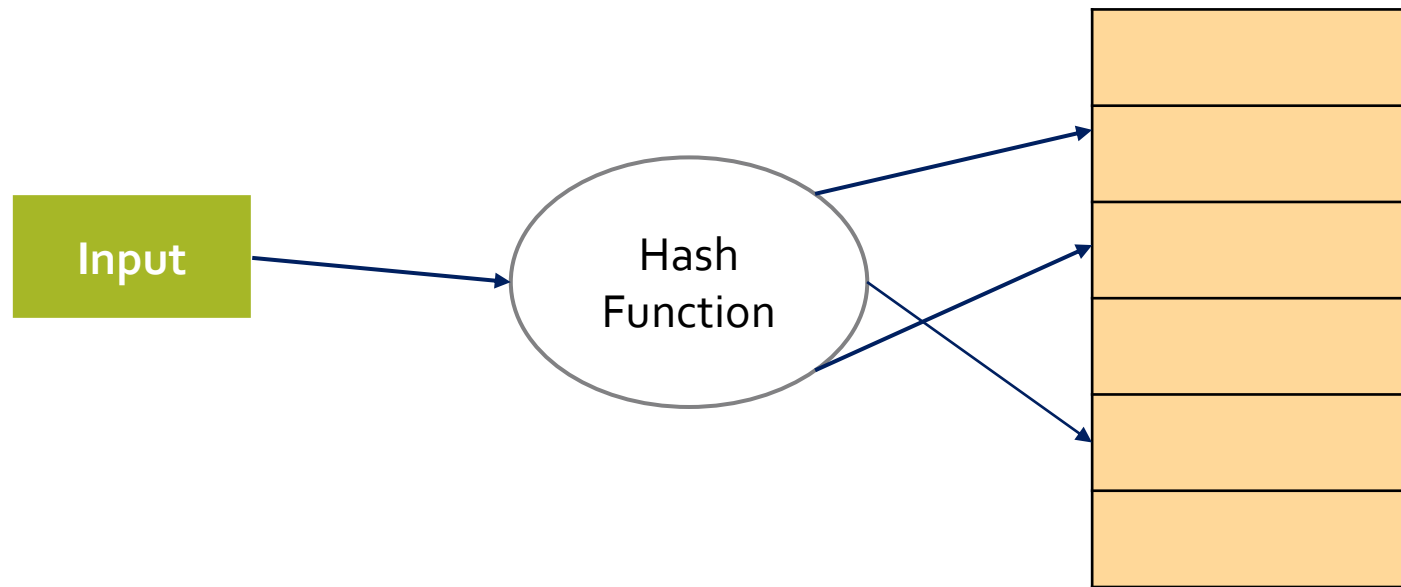
Key	Value
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Key - value pairs must be unique, but values can be repeated.

Hash Function

A **hash function** $H(x)$ is a function that maps a key ' x ' to a whole numbers in fixed range.

Choosing an efficient hash function is a crucial part of creating a good hash table.



Hashing Methods

- ✓ Direct hashing
- ✓ Modulo division
- ✓ Mid square method
- ✓ Folding method
- ✓ Rotation Method
- ✓ Pseudo Random Methods

Modulo Division Method

Map a key into one of the m slots by taking the remainder of k divided by m that is

$$H(k) = k \bmod m$$

Where, m can be the list size

Example:

$$m = 31 \text{ and } k = 78$$

$$78 \bmod 31$$

$$H(K) = 16$$

With Strings

Cat

ASCII : $c = 99, a = 97, t = 116$

$$99 + 95 + 116 = 312$$

If, $m = 11$ and $k = 312$

$$\text{Then, } H(K) = 4$$

Mid Square Method

The key k is multiplied by itself and the address is obtained by selecting an appropriate number of digits from the middle of the square.

Example:

$$9452 * 9452 = 89340304$$

$$H(k) = 3403$$

Folding Method

The key is **partitioned** into a number of parts, each of which has the **same length** as the required address with the possible exception of the last part.

Example:

123456789

$$123 + 456 + 789 = \text{1368}$$

$$H(k) = \text{368}$$

Digit Exaction Method

Selected digits are **extracted** from the key and used as the address.

Address = extracted digits from key

Example:

121267

$H(k) = 112$

Rotation Method

Usually used with the combination of other hashing mechanisms.

Example:

*250089***1**

$H(k) = 1250089$

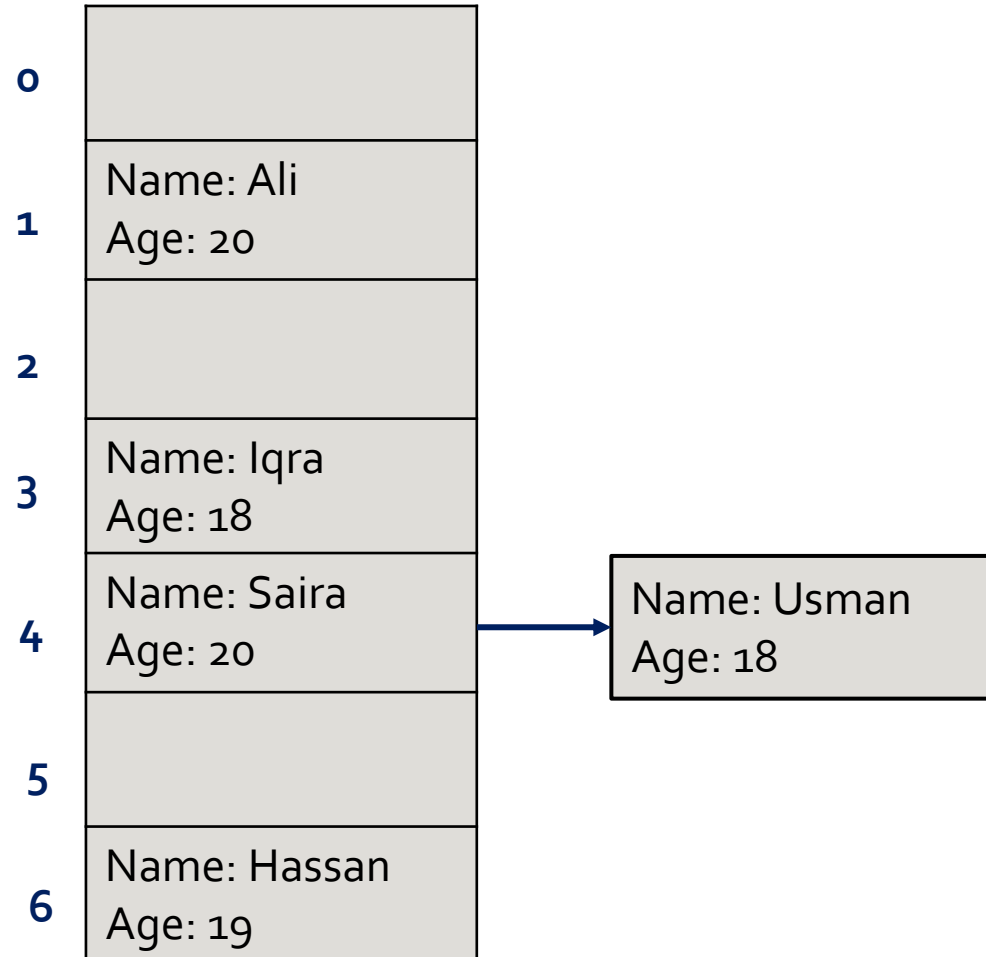
Collision

Hash function gets us a small number for a key which is a big integer or string, there is a possibility that two keys result in the same value. The situation where a newly inserted key maps to an already occupied slot in the hash table is called **collision** and must be handled using some **collision handling** techniques.

Collision Resolutions

Separate Chaining

Deals with collisions by maintaining a data structure to hold all the different values which hashed to a particular value.



Collision Resolutions

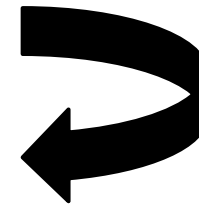
Open Addressing

Deals with hash collisions the hash table for all the object to go by offsetting if from the position to which it hashed to.

Types

- Linear probing
- Quadratic probing
- Double Hashing

0	
1	Name: Ali Age: 20
2	
3	Name: Iqra Age: 18
4	Name: Saira Age: 20
5	Name: Usman Age: 18
6	Name: Hassan Age: 19



Collision Resolutions

Open addressing

- ✓ Linear Probing

- If collision occurs, we linearly probe for next slot

- ✓ Quadratic Probing

- If collision occurs:

- $(\text{hash value} + 1^2) \% \text{table size}$

- if again collision occurs

- $(\text{hash value} + 2^2) \% \text{table size}$

- If again

- $(\text{hash value} + 3^2) \% \text{table size}$

- In general : $(\text{hash value} + i^2) \% \text{table size}$

Insert {89, 18, 49, 58, 69, 78} with $h(x) = x \bmod 10$ using separate chaining.

$x = 89$

$$h(89) = 89 \bmod 10 \Rightarrow 9$$

$x = 18$

$$h(18) = 18 \bmod 10 \Rightarrow 8$$

$x = 49$

$$h(49) = 49 \bmod 10 \Rightarrow 9 \text{ (C)}$$

$x = 58$

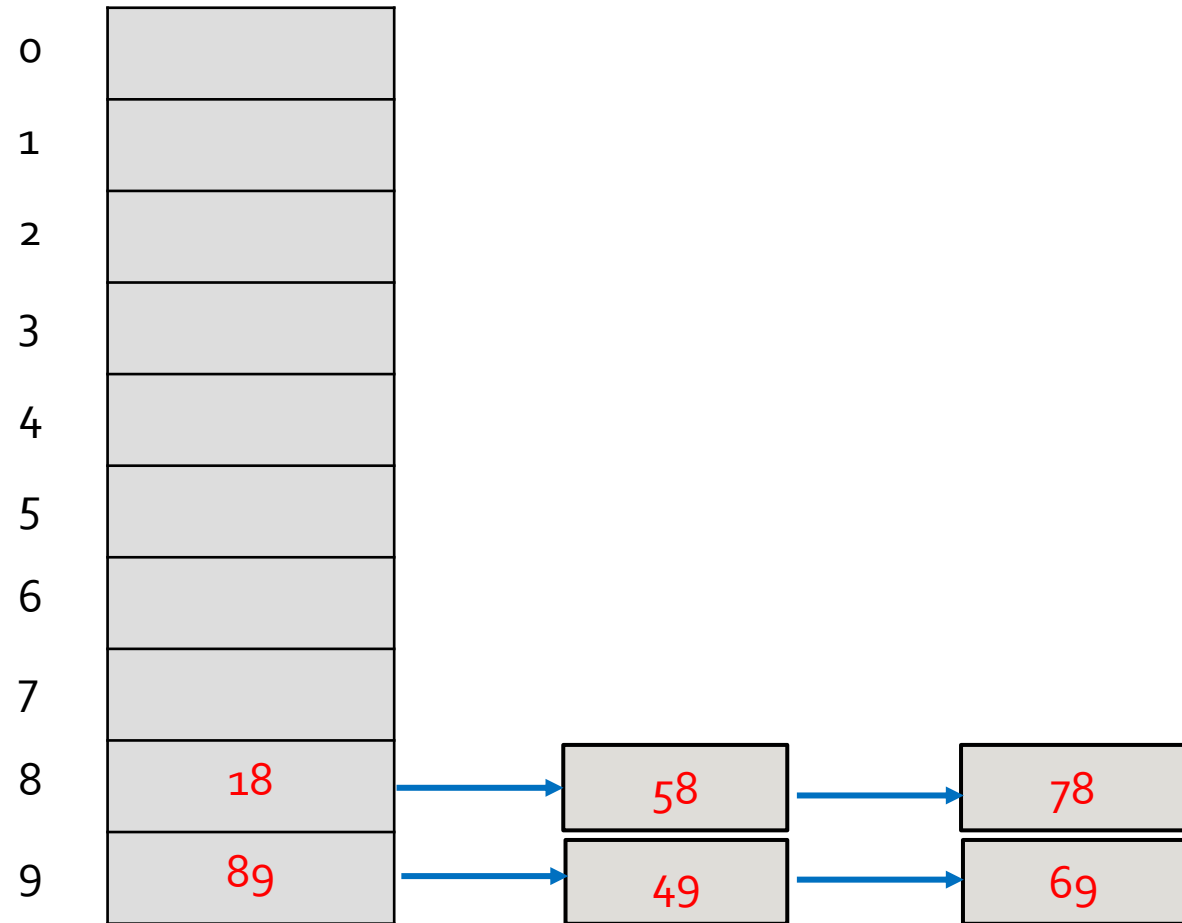
$$h(58) = 58 \bmod 10 \Rightarrow 8 \text{ (C)}$$

$x = 69$

$$h(69) = 69 \bmod 10 \Rightarrow 9 \text{ (C)}$$

$x = 78$

$$h(78) = 78 \bmod 10 \Rightarrow 8 \text{ (C)}$$



Insert {89, 18, 49, 58, 69, 78} with $h(x) = x \bmod 10$ using linear probing.

$x = 89$

$$h(89) = 89 \bmod 10 \Rightarrow 9$$

$x = 18$

$$h(18) = 18 \bmod 10 \Rightarrow 8$$

$x = 49$

$$h(49) = 49 \bmod 10 \Rightarrow 9 \text{ (C)}$$

$x = 58$

$$h(58) = 58 \bmod 10 \Rightarrow 8 \text{ (C)}$$

$x = 69$

$$h(69) = 69 \bmod 10 \Rightarrow 9 \text{ (C)}$$

$x = 78$

$$h(78) = 78 \bmod 10 \Rightarrow 8 \text{ (C)}$$

0	49
1	58
2	69
3	78
4	
5	
6	
7	
8	18
9	89



Insert {89, 18, 49, 58, 69, 78} with $h(x) = x \bmod 10$ using quadratic probing.

$x = 89$

$$h(89) = 89 \bmod 10 \Rightarrow 9$$

$x = 18$

$$h(18) = 18 \bmod 10 \Rightarrow 8$$

$x = 49$

$$h(49) = 49 \bmod 10 \Rightarrow 9 \text{ (collision)}$$

$$h_1(49) = (9 + 1^2) \bmod 10 \Rightarrow 0$$

$x = 58$

$$h(58) = 58 \bmod 10 \Rightarrow 8 \text{ (collision)}$$

$$h_1(58) = (8 + 1^2) \bmod 10 \Rightarrow 9 \text{ (collision)}$$

$$h_2(58) = (8 + 2^2) \bmod 10 \Rightarrow 2$$

$x = 69$

$$h(69) = 69 \bmod 10 \Rightarrow 9 \text{ (collision)}$$

$$h_1(69) = (9 + 1^2) \bmod 10 \Rightarrow 0 \text{ (collision)}$$

$$h_2(69) = (9 + 2^2) \bmod 10 \Rightarrow 3$$

$x = 78$

$$h(78) = 78 \bmod 10 \Rightarrow 8 \text{ (collision)}$$

$$h_1(78) = (8 + 1^2) \bmod 10 \Rightarrow 9 \text{ (collision)}$$

$$h_2(78) = (8 + 2^2) \bmod 10 \Rightarrow 2 \text{ (collision)}$$

$$h_3(78) = (8 + 3^2) \bmod 10 \Rightarrow 7$$

0	49
1	
2	58
3	69
4	
5	
6	
7	78
8	18
9	89

Exercise

Map all integer key to the range (0,9), $h(x) = (x^2 - 6x + 9) \bmod 10$

$$x = 4$$

$$h(4) = (4^2 - 6(4) + 9) \bmod 10 \Rightarrow 1$$

$$x = 7$$

???

$$x = 0$$

???

$$x = 2$$

???

$$x = 8$$

???

Applications

- File management
- Comparing complex values
- Cryptography
- Security systems

Hash Table Complexity

Search	$O(n)$
Insertion	$O(n)$
Deletion	$O(n)$