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Assignment = 1

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Question: 1

Sol

$$T(N) = \sum_{i=0}^{n-2} \sum_{j=i+1}^{n-1} \sum_{k=i}^{n} (1)$$

$$= \sum_{i=0}^{n-2} \sum_{j=j+1}^{n-1} (n-i+1)$$

$$= \sum_{i=0}^{n-2} \left[n \sum_{j=i+1}^{n-1} (1) - i \sum_{j=i+1}^{n-1} (1) + 1 \sum_{j=i+1}^{n-1} (1) \right]$$

$$= \sum_{i=0}^{n-2} n \{(n-1-i-\cancel{1}+\cancel{1})\} - i \{(n-1-i-1+1)\} + 1\{(n-1-i\ldots$$

$$= \sum_{i=0}^{n-2} \left[\{n(n-i-1)\} - i(n-i-1) + 1(n-i-1) \right]$$

$$= \sum_{i=0}^{n-2} \{(n-i-1)(n-i+1)\}$$

$$= \sum_{i=0}^{n-2} \{(n-i)^2 - (1)^2\}$$

$$= \sum_{i=0}^{n-2} \{(n^2 - 2ni + i^2 - 1)\}$$

$$= n^2 \sum_{i=0}^{n-2} 1 - 2n \sum_{i=0}^{n-2} i + \sum_{i=0}^{n-2} i^2 - \sum_{i=0}^{n-2} (1)$$

$$= n^2(n-2+1) - 2n \cdot \frac{n^2}{2} + \frac{n^3}{3} - (n-2+1)$$

$$= n^2(n-1) - \frac{\cancel{2}n \cdot (n-2)^2}{\cancel{2}} + \frac{(n-2)^3}{3} - (n-1)$$

$$= n^2(n-1) - n(n^2-4n+4) + \frac{n^3 - 3(2)(n)(n-2) - 8}{3} - (n-1)$$

$$= n^3 - n^2 - n^3 + 4n^2 - 4n + (n^3 - 6n^2 + 12n - 8) - n + 1$$

$$= n^3 - n^2 - n^3 + 4n^2 - 4n + \frac{n^3 - 6n^2 + 12n - 8}{3} - n + 1$$

Taking n having highest power

$$T(n) = \frac{n^3}{3} \approx O(n^3)$$

$$\boxed{T(n) = O(n^3)}$$ Ans.

QUESTION : 2

a) $x(n) = 9x(n/3) + 5$ for $n > 1$, $x(1) = 0$

Sol

$a = 9$, $b = 3$, $k = 0$

$\log_b^a = \log_3^9 = 2$

$\because \log_b^a > k$ i.e $2 > 0$

$\therefore O(n^{\log_b^a})$

$\Rightarrow O(n^{\log_3^9})$

$\Rightarrow O(n^2)$ Ans

b) $x(n) = x(n/2) + n$ for $n > 1$, $x(1) = 1$

Sol

$a = 1$, $b = 2$, $k = 1$

$\log_b^a = \log_2^1 = 0$

$\because \log_b^a < k$ and $p \geq 0$

$\therefore O(n^k \log^p n)$

$\Rightarrow O(n^1 \log^0 n)$

$\Rightarrow O(n)$ Ans

c) $x(n) = x(n/3) + 1$ for $n > 1$, $x(1) = 1$

Sol

$a = 1$, $b = 3$, $k = 0$

$\log_b^a = \log_3^1 = 0$

$\because \log_b^a = k$ and $p > -1$

$\therefore O(n^k \log^{p+1} n)$

$\Rightarrow O(n^0 \log n)$

$\Rightarrow O(\log n)$ Ans

d) $x(n) = 4x(n/2) + n^2$ for $n > 1$, $x(1) = 1$

Sol

$a = 4$, $b = 2$, $k = 2$

$\log_2^4 = 2$

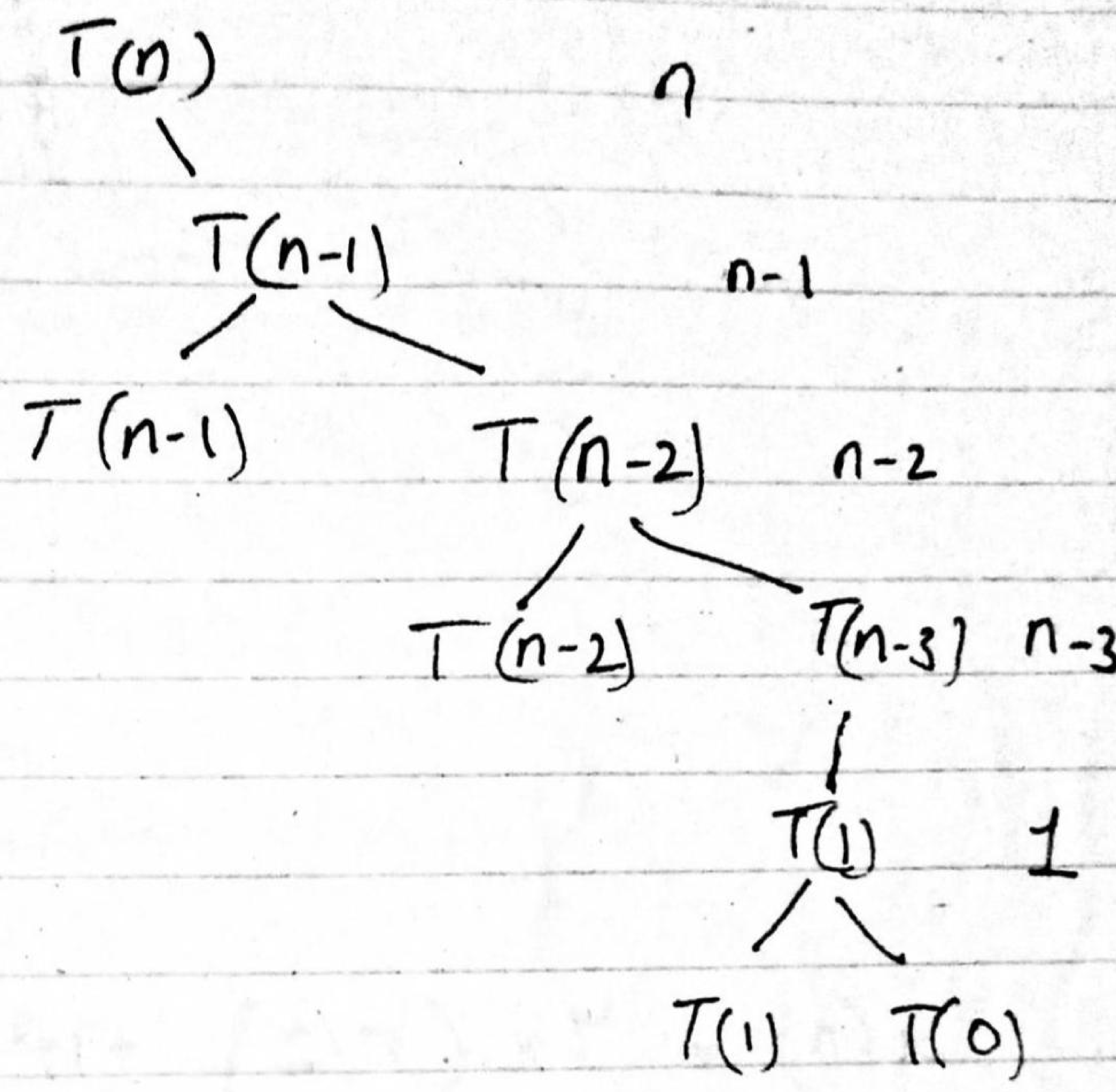
$\because \log_b^a = k$ and $p > -1$

$\therefore O(n^k \log^{p+1} n)$

$\Rightarrow O(n^2 \log n)$ Ans

QUESTION: 3

a) $T(n) = T(n-1) + 1$



$T(n) = n + (n-1) + (n-2) + (n-3) + 2 + 1$

$T(n)$ = sum of natural numbers n

$$T(n) = \frac{n+1}{2}$$

$$T(n) = O(n)$$

b) $T(n) = T(n-1) + 1 \longrightarrow (A)$

put $n = n-1$ in (A)

$T(n) = T(n-1-1) + 1$

$T(n-2) + 1 \longrightarrow (B)$

put $n = n-2$ in eq (A)

$T(n) = T(n-2-1) + 1$

$T(n) = T(n-3) + 1$

putting eq (B) at $T(n-1)$

$T(n) = T(n-2) + 1 + 1$

$T(n) = T(n-2) + 2$

putting eq (2)

$T(n-3) + 1 + 2$

$T(n) = T(n-3) + 3$

$T(n) = T(n-k) + k$

From Base Condition $T(0) = 1$

$n = k$

$T(n) = 1 + n$

$T(n) = O(n)$ Ans.