

FUNCTIONAL DEPENDENCIES

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Functional Dependency

- Functional dependency (FD) is a set of constraints between two attributes in a relation.
- Functional dependency says that if two tuples have same values for attributes A_1, A_2, \dots, A_n , then those two tuples must have to have same values for attributes B_1, B_2, \dots, B_n .
 - $A \longrightarrow B$
- It does not mean that A derives B, although it may be the case sometime
- It means that if we know value of A then we can precisely determine a unique value of B
- Normalization is based on functional dependencies (FDs)

Functionally Dependency

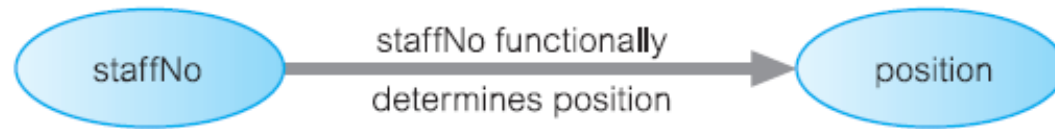
- Attribute of set of attributes on the left side are called **determinant** and on the right are called **dependents**

- Like R (a, b, c, d, e)

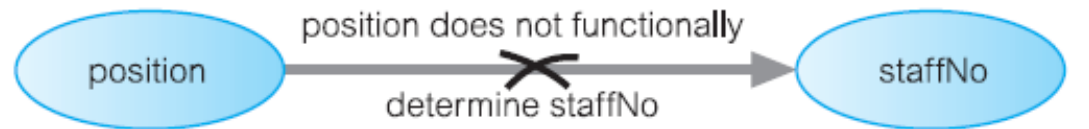
$a \rightarrow b, c, d$

$d \rightarrow d, e$

Functional dependency example



Staff number SL21 → Manager
(a)



Manager → Staff number SL21
Manager → Staff number SG5
(b)

Functional dependency

Fully Functional dependency

Indicates that if A and B are attributes of a relation, B is fully functionally dependent on A if B is functionally dependent on A, but not on any proper subset of A.

Partial dependency

Indicates that if there is some attribute that can be removed from A and yet the dependency still holds.

EMP(eld, eName, eAdr, eDept, prId, prSal)

$\text{eld} \rightarrow \text{eName}, \text{eDept}, \text{eAdr}$

$\text{eld}, \text{prId} \rightarrow \text{prSal}$

Inference Rules

- Called inference axioms or Armstrong axioms
- These are rules that establish certain FDs from a given set of FDs

Reflexivity

If B is a subset of A then $A \rightarrow B$, it also implies that A always hold A, that is

stName, stAdr \rightarrow stName

Or stName \rightarrow stName

Augmentation

If we have $A \longrightarrow B$ then

$AC \longrightarrow BC$ that is

if $stId \longrightarrow stName$ then

$stId, stAdr \longrightarrow stName, stAdr$

Transitivity

If $A \rightarrow B$ and $B \rightarrow C$ then $A \rightarrow C$

that is

If $stId \rightarrow prName$ and $prName \rightarrow credits$

Then

$stId \rightarrow credits$

Additivity or Union

If $A \rightarrow B$ and $A \rightarrow C$ then $A \rightarrow BC$

if $\text{empld} \rightarrow \text{eName}$ and $\text{empld} \rightarrow \text{qual}$

Then we can write it as

$\text{empld} \rightarrow \text{eName}, \text{qual}$

Projectivity or Decomposition

If $A \twoheadrightarrow BC$ then $A \twoheadrightarrow B$ and $A \twoheadrightarrow C$

if $\text{empld} \twoheadrightarrow \text{eName, qual}$

Then we can write it as

$\text{empld} \twoheadrightarrow \text{eName}$ and $\text{empld} \twoheadrightarrow \text{qual}$

Pseudo transitivity

If $A \rightarrow B$ and $CB \rightarrow D$ then $AC \rightarrow D$

if $stId \rightarrow stName$ and

$stName, fName \rightarrow stAdr$

Then we can write it as

$stId, fName \rightarrow stAdr$

Types of FDs

1. Trivial functional dependency
2. Non-Trivial functional dependency
3. Multivalued functional dependency
4. Transitive functional dependency

Trivial functional dependency

A dependent is always a subset of the determinant.

If $X \rightarrow Y$ and Y is the subset of X , then it is called trivial functional dependency

$\{\text{roll_no}, \text{name}\} \rightarrow \text{name}$ is a trivial functional dependency, since the dependent name is a subset of determinant set $\{\text{roll_no}, \text{name}\}$
Similarly, $\text{roll_no} \rightarrow \text{roll_no}$ is also an example of trivial functional dependency.

Roll_no	Name	Age
12	Ali	17
13	Osama	18
14	Faraz	18

Non-Trivial functional dependency

The dependent is strictly not a subset of the determinant.

If $X \rightarrow Y$ and Y is not a subset of X , then it is called Non-trivial functional dependency.

roll_no \rightarrow **name** is a non-trivial functional dependency, since the dependent name is not a subset of determinant **roll_no**

Similarly, **{roll_no, name}** \rightarrow **age** is also a non-trivial functional dependency, since age is not a subset of **{roll_no, name}**

Roll_no	Name	Age
12	Ali	17
13	Osama	18
14	Faraz	18

Multivalued functional dependency

Entities of the dependent set are not dependent on each other.

If $a \rightarrow \{b, c\}$ and there exists no functional dependency between b and c , then it is called a multivalued functional dependency.

Here, $\text{roll_no} \rightarrow \{\text{name}, \text{age}\}$ is a multivalued functional dependency, since the dependents name & age are not dependent on each other(i.e. $\text{name} \rightarrow \text{age}$ or $\text{age} \rightarrow \text{name}$ doesn't exist !)

Roll_no	Name	Age
12	Ali	17
13	Osama	18
14	Faraz	18

Transitive functional dependency

Dependent is indirectly dependent on determinant.

If $a \rightarrow b$ & $b \rightarrow c$, then according to axiom of transitivity, $a \rightarrow c$. This is a transitive functional dependency

Here, $\text{Roll_no} \rightarrow \text{dept}$ and $\text{dept} \rightarrow \text{buildingno}$,
Hence, according to the axiom of transitivity, $\text{Roll_no} \rightarrow \text{buildingno}$ is a valid functional dependency. This is an indirect functional dependency, hence called Transitive functional dependency.

Roll_no	name	dept	buildingno
12	Ali	SE	4
13	Osama	EE	2
14	Faraz	CS	1
15	Sara	EE	2