DATA STRUCTURES & ALGORITHMS

Complexity Analysis

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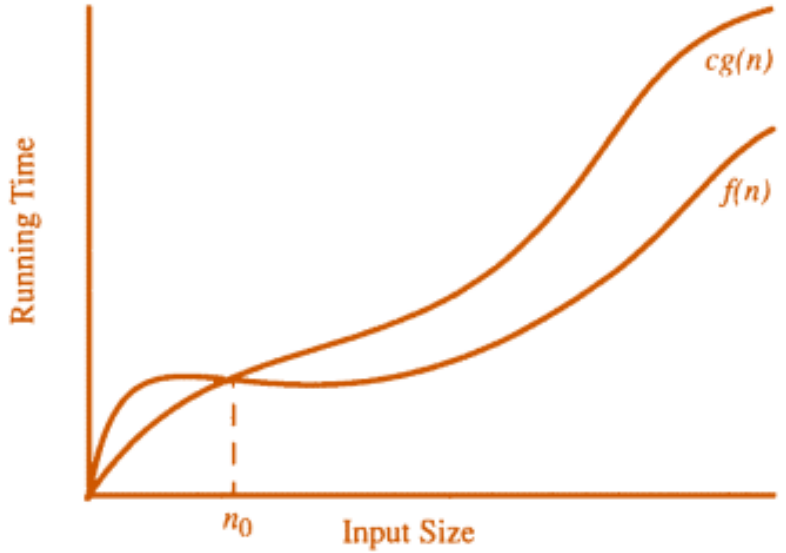
Big Oh Notation

Simplified analysis of an algorithm's efficiency.

- O is called Landaus's symbol, comes from the inventors name Edmund Landau.
- The letter O is used because the rate of growth of a function is also called its order
- Used in complexity theory, computer science and mathematics to describe the behavior of functions.
- It determines how fast a function grows or declines.

Big Oh Notation

- Let f (n) and g(n) be functions mapping non-negative numbers to non-negative numbers.
- Big-Oh. f (n) is O(g(n)) if there is a constant c > 0 and a constant $n_0 \geq 1$ such that f (n) ≤ c · g(n) for every number $n \geq n_0$.



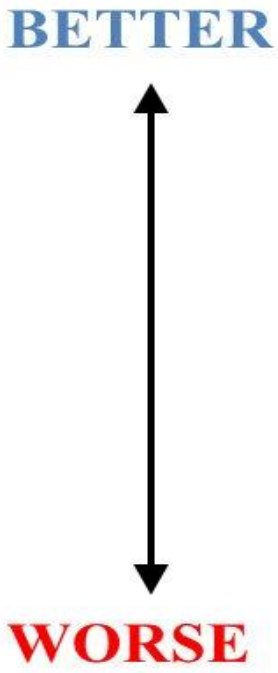
Example

The function $f(n) = 3 \cdot n + 17$ is $O(n)$. .....(Here $g(n) = n$.)

Proof.

Take $c = 4$ and $n_0 = 17$. Then $f(n) = 3n + 17 \leq c \cdot g(n)$ for every $n \geq$ no. because $3 \cdot n + 17 \leq 4 \cdot n = 3 \cdot n + n$ for every $n \geq 17$.

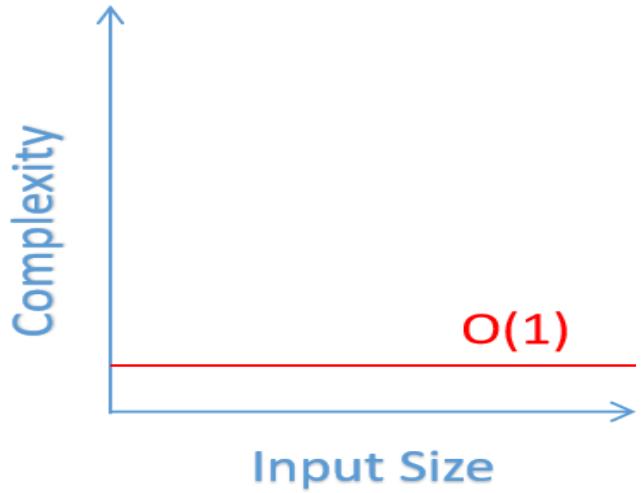
Big Oh



- $O(1)$ constant time
- $O(\log n)$ log time
- $O(n)$ linear time
- $O(n \log n)$ log linear time
- $O(n^2)$ quadratic time
- $O(n^3)$ cubic time
- $O(2^n)$ exponential time

Constant Time: O(1)

Run in constant time if it requires the same amount of time regardless of the input size.



Daily life Example???

Example: accessing any element in array

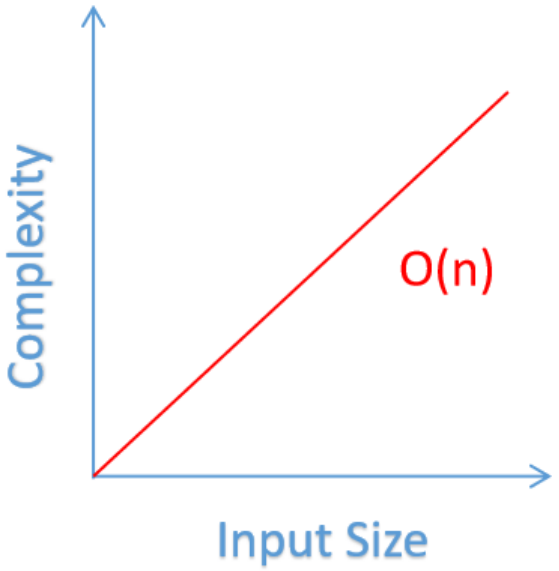
Linear Time: O(n)

n= number of items

What will be the worst case???

best

average



Worst case
need 'n' steps
for 'n' items

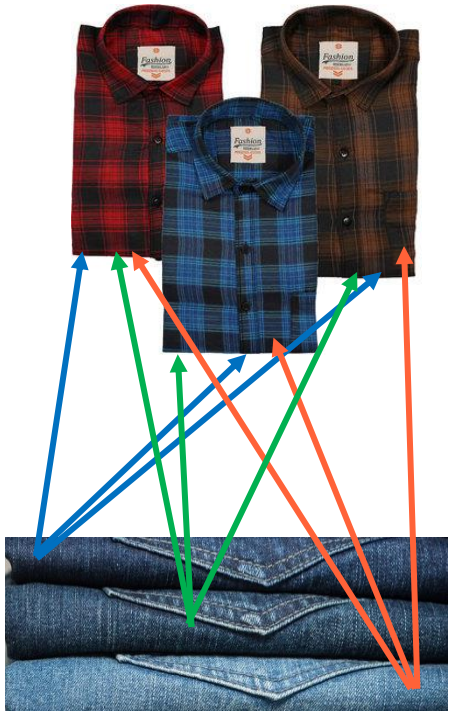
Example: traversing an array

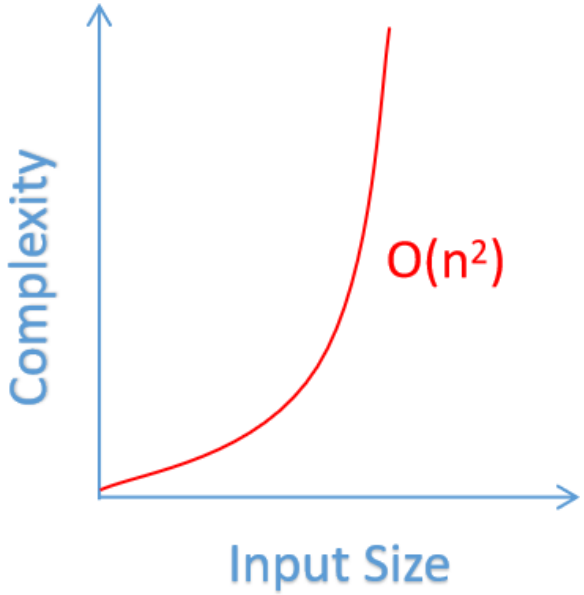
Quadratic Time: $O(n^2)$

- n= number of items

What will be the worst case???

Worst case
need 'n*n' steps for desired output

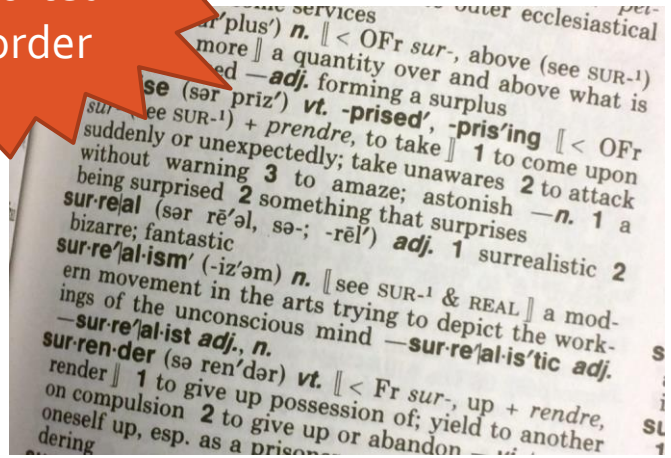




Example: bubble sort, selection sort, insertion sort

Logarithmic Time: O(log n)

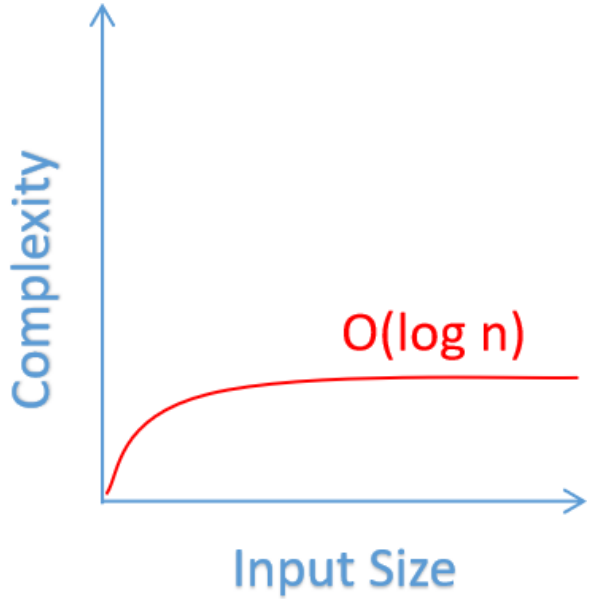
Sorted order



log 10 = ?

Log 20 =?

Log 100 = ?



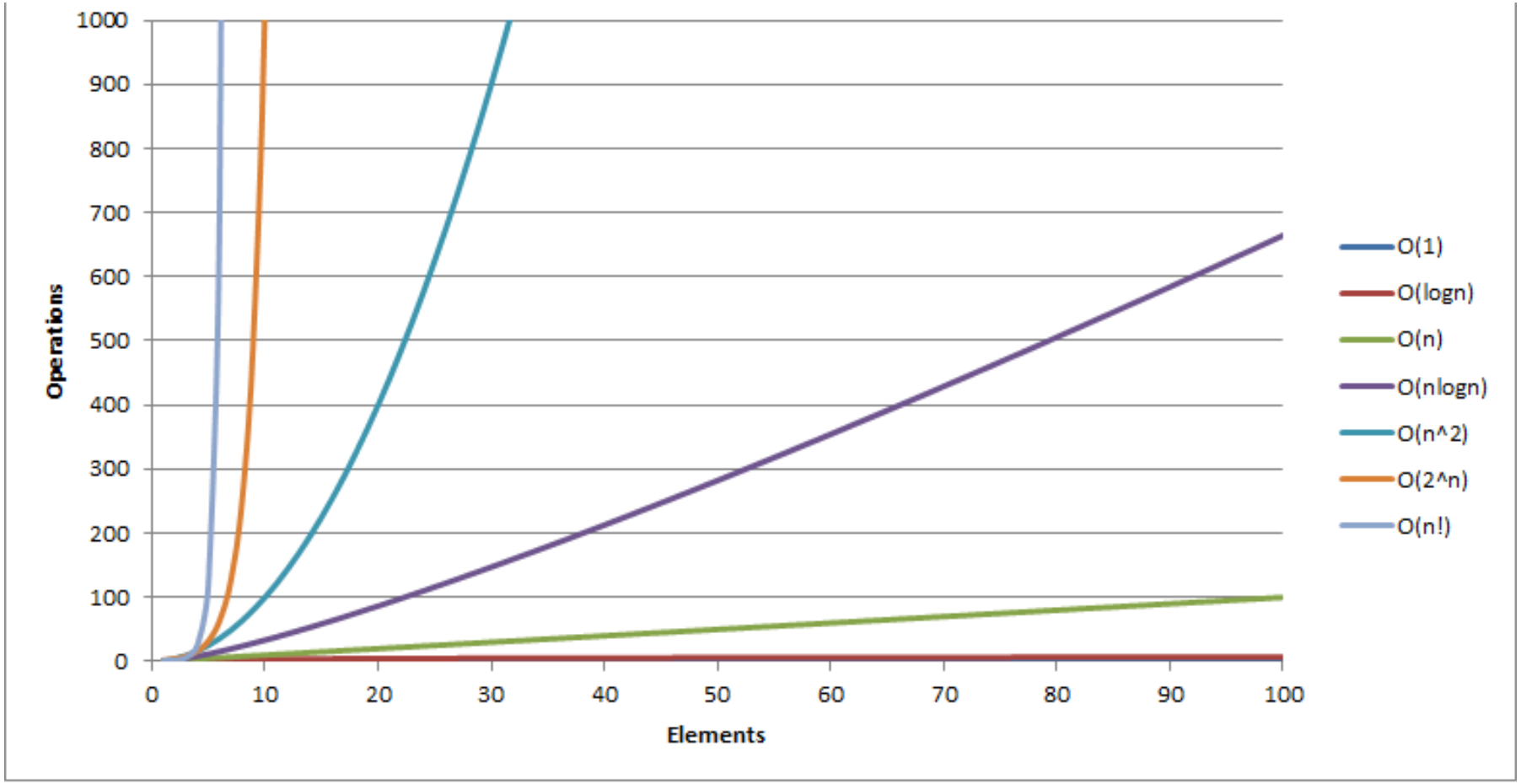
Example: binary search

O(nlog n)

- Growth rate is faster as compared to linear and log functions

Example: merge sort

Complexity graph



Rules for analysis

- Ignore constants

6n –> O(N) ← 'n' gets larger & constant no longer matter

- Certain terms dominate others

$O(1) < O(\log n) < O(n) < O(n \log n) < O(n^2) < O(2^n) < O(n!)$ ← Ignore lower order terms

Rules for analysis

Loops	Number of iterations
Nested loops	Complexity of inner loop * outer loop
Consecutive statements	Addition
If/else	Block which take long time
Switch case	Block which take long time

Example

```
x= 5 + (15 * 30);          // O(1)
```

(independent of input size)

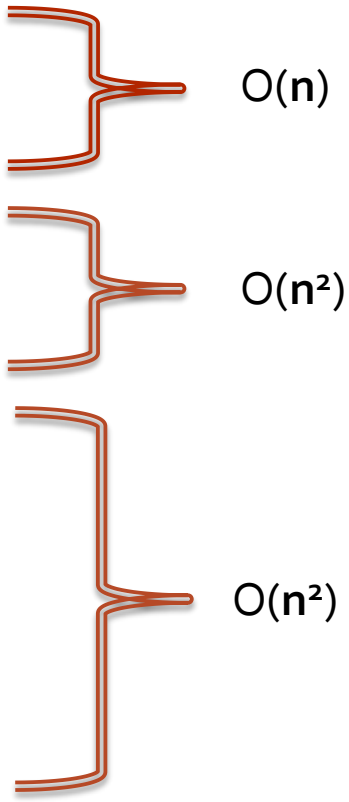
```
x= 5 + (15 * 30);          // O(1)
y= 6-4;                    // O(1)
print x+y;                 // O(1)
```

Total Time = O(1) + O(1) + O(1) = > O(1)

(drop constant)

Example

```
for (int i = 0; i < n; i ++)
        sum = sum – i;
for (int i = 0; i < n * n ; i ++)
        sum = sum + i;
sum = 0
for (int i = 0; i < n; i ++)
        for (int j = 0; j < n; j ++)
        sum + =  i * j;
```



Example

```
int sum( int n )
{
        int partialSum;

        partialSum = 0;
        for( int i = 1; i <= n; ++i )
                partialSum += i * i * i;
        return partialSum;
}
```