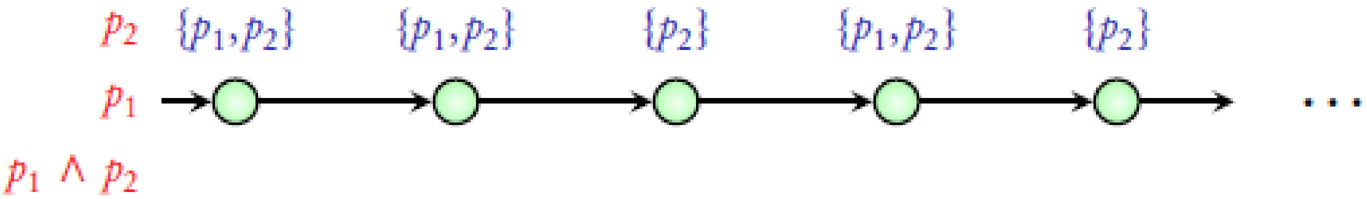
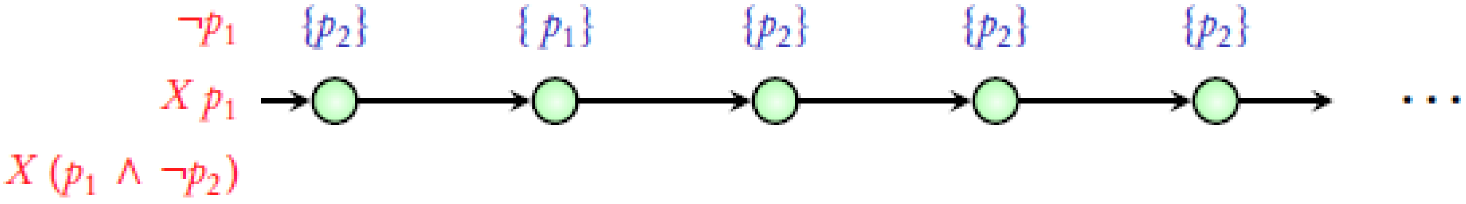
Lecture # 13

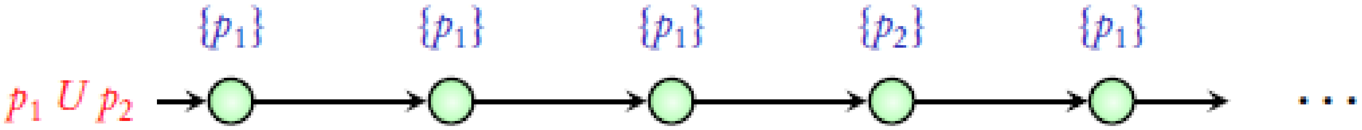
LTL | Büchi Automata | LTL to NBA

Linear Temporal Logic

LTL formulas that satisfy the set of worlds:

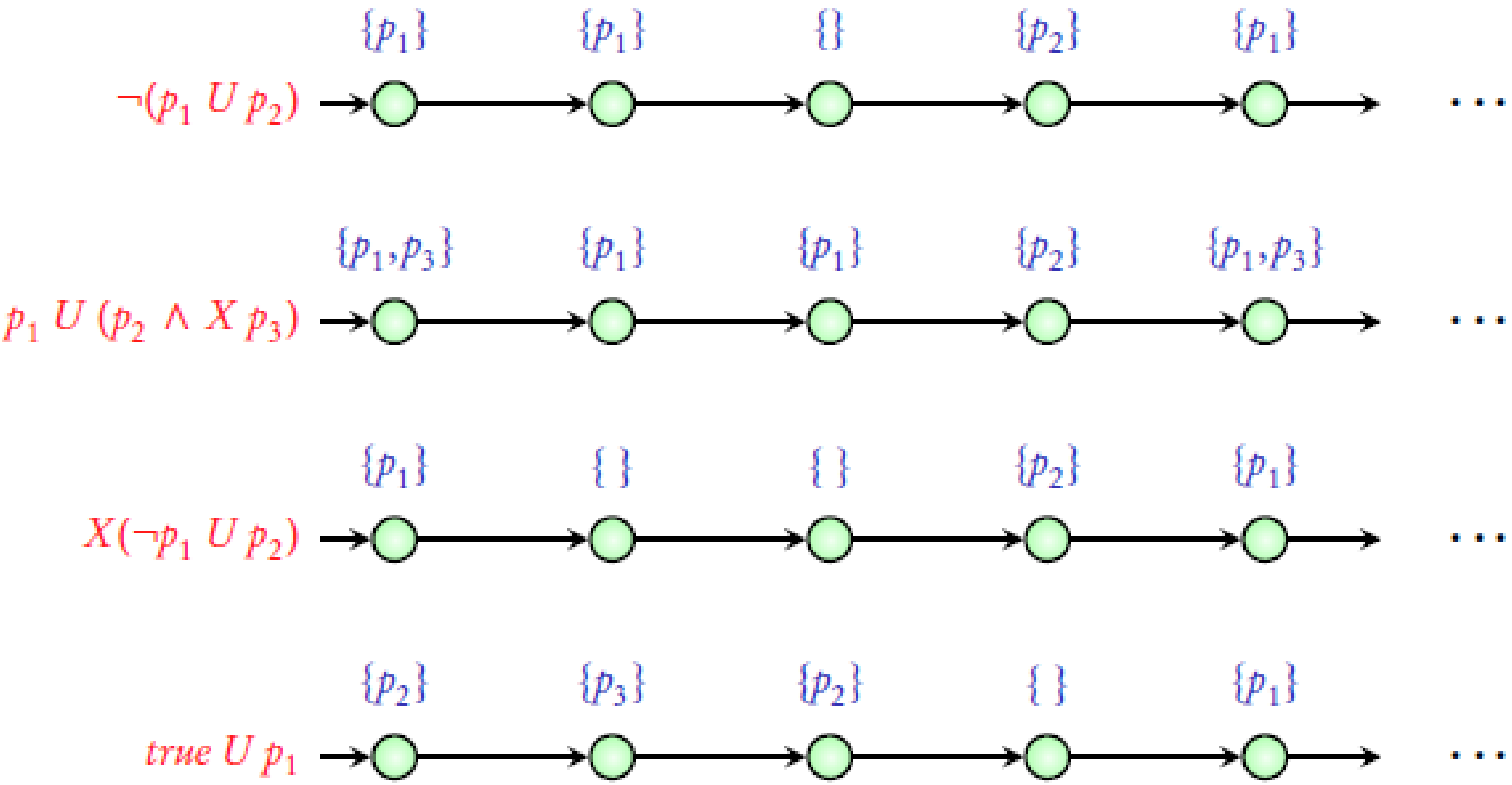






Linear Temporal Logic

LTL formulas that satisfy the set of worlds:



AUTOMATA

The term "Automata" is derived from the Greek word "αὐτόματα" which means "self-acting". An automaton (Automata in plural) is an abstract self-propelled computing device which follows a predetermined sequence of operations automatically.

An automaton with a finite number of states is called a **Finite Automaton** (FA) or **Finite State Machine** (FSM).

Here: Finite state automata to describe sets of finite words

AUTOMATA

Σ : finite alphabet $\quad \Sigma^* =$ set of all words over Σ

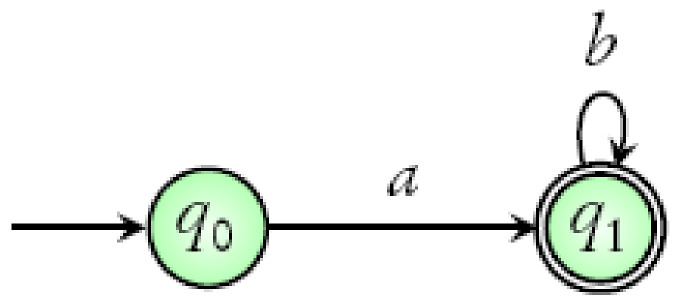
Language: A set of finite words

$ab(ab)^*$	$\{ab, abab, ababab, \ldots\}$
$a\Sigma^*$	finite words starting with an a
$b\Sigma^*$	finite words starting with a b
b^*	$\{\epsilon, b, bb, bbb, \ldots\}$
$(ab)^*$	$\{\epsilon, ab, abab, ababab, \ldots\}$
$(bbb)^*$	$\{\epsilon, bbb, bbbbbb, (bbb)^3, \ldots\}$
$a\Sigma^*a$	words starting and ending with an a

Alphabet: $\{a, b\}$

$$ab^* = \{a, ab, ab^2, ab^3, ab^4, \ldots\}$$

Design a Finite automaton for ab^*



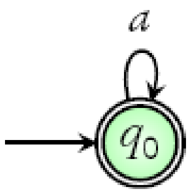
AUTOMATA

Alphabet: $\{ a, b \}$

$$a^* = \{ \epsilon, a, aa, aaa, aaaa, a^5, \dots \}$$

a^* is the set of all words having only a

Design a Finite automaton for a^*

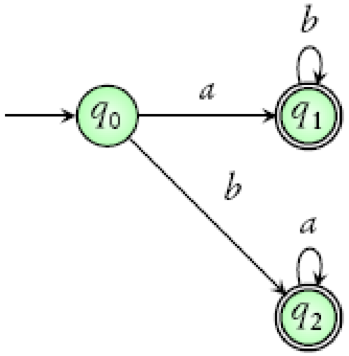


Alphabet: $\{ a, b \}$

$$ab^* = \{ a, ab, ab^2, ab^3, ab^4, \dots \}$$

$$ba^* = \{ b, ba, ba^2, ba^3, ba^4, \dots \}$$

Design a Finite automaton for $ab^* \cup ba^*$



Alphabet: $\{ a, b \}$

$$ab^* = \{ a, ab, ab^2, ab^3, ab^4, \dots \}$$

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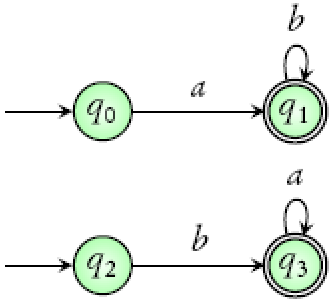
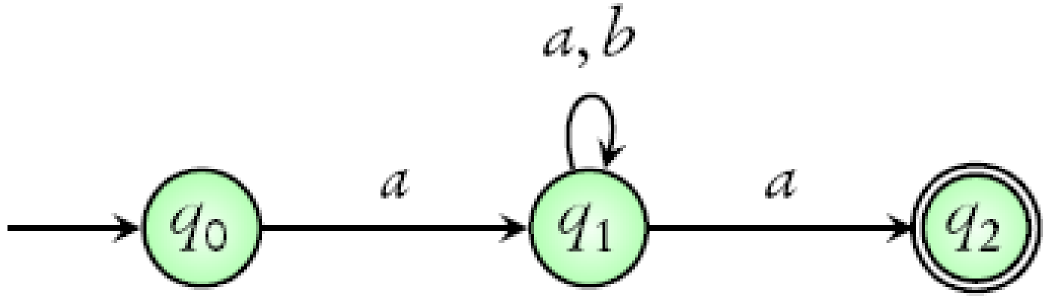
Design a Finite automaton for $ab^* \cup ba^*$



Multiple initial states: **non-deterministic** automaton

AUTOMATA

What is the language of the following automaton?



Answer: $a\Sigma^*a$

words starting and ending with a

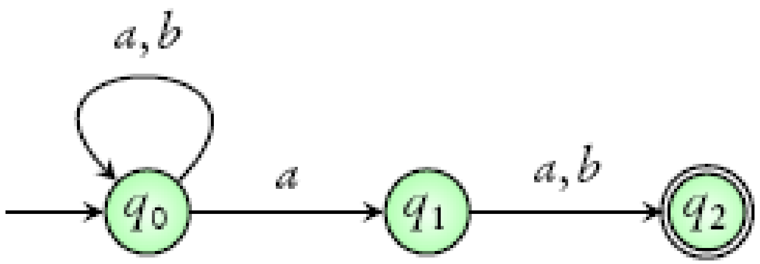
Languages over finite words

$a \quad b \quad b \quad a \quad a \quad b \quad a \quad b$

Runs:

$$q_0 \xrightarrow{a} q_0 \xrightarrow{b} q_0 \xrightarrow{b} q_0 \xrightarrow{a} q_0 \xrightarrow{a} q_0 \xrightarrow{b} q_0 \xrightarrow{a} q_0 \xrightarrow{b} q_0$$

$$q_0 \xrightarrow{a} q_0 \xrightarrow{b} q_0 \xrightarrow{b} q_0 \xrightarrow{a} q_0 \xrightarrow{a} q_0 \xrightarrow{b} q_0 \xrightarrow{a} q_1 \xrightarrow{b} q_2$$ accepting run



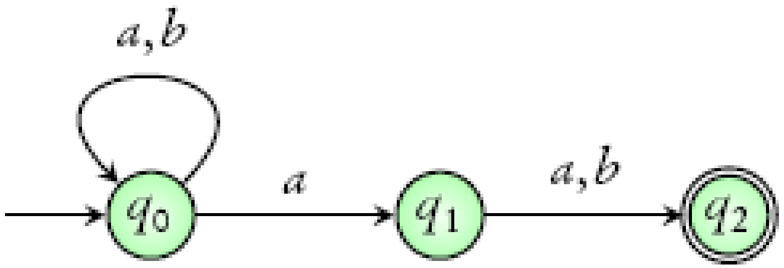
Language: set of words for which there exists an accepting run

Languages over finite words

$a \quad b \quad b \quad b \quad a$

Runs:

$q_0 \xrightarrow{a} q_0 \xrightarrow{b} q_0 \xrightarrow{b} q_0 \xrightarrow{b} q_0 \xrightarrow{a} q_0$ Not accepted



Language: set of words for which there exists an accepting run

Languages over infinite words

$$\Sigma = \{ a,\ b \}$$

Example 1: Infinite word consisting only of a $\qquad a^\omega$

$$\{ aaaaaaaaaaaaaaaa\ldots \}$$

Example 2: Infinite words containing only a or only b $\quad a^\omega + b^\omega$

$$\{ aaaaaaaaaaaaaaa\ldots,\ bbbbbbbbbbb\ldots \}$$

Example 3: a word in $aa\Sigma^*aa$ followed by only b-s $\quad aa\Sigma^*aa \cdot b^\omega$

$$\{ aaaabbbbbb\ldots,\ aababaabbbbb\ldots,\ aabbbaabbbbbb\ldots,\ \ldots \}$$

Non Deterministic Büchi Automata

In finite words, there is an **end**

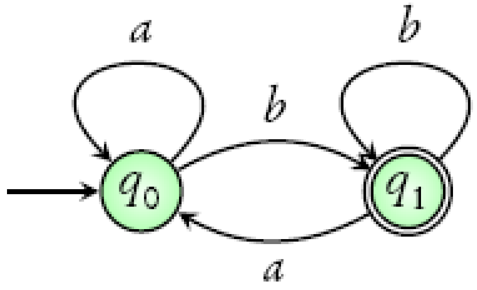
A run is accepting if it **ends in an accepting state**

How do we define **accepting runs** for **infinite words?**

Non Deterministic Büchi Automata

$$a \; b \; a \; b \; a \; a \; b \; b \; b \; b \; b \; b \; \ldots$$

$$q_0 \xrightarrow{a} q_0 \xrightarrow{b} q_1 \xrightarrow{a} q_0 \xrightarrow{b} q_1 \xrightarrow{a} q_0 \xrightarrow{a} q_0 \xrightarrow{b} q_1 \xrightarrow{b} q_1 \xrightarrow{b} q_1 \xrightarrow{b} q_1 \cdots$$



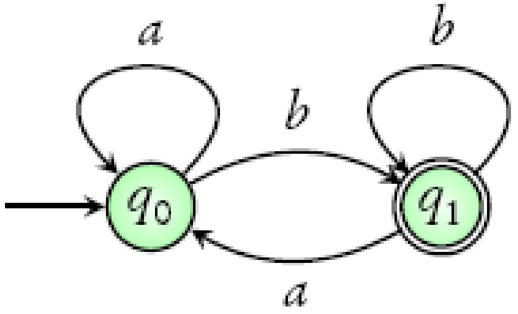
Above word is accepted by this automaton

Run is accepting if **some accepting state occurs infinitely often**

Non Deterministic Büchi Automata

$a \; b \; a \; b \; a \; a \; a \; a \; a \; a \; a \; a \; \ldots$

$$q_0 \xrightarrow{a} q_0 \xrightarrow{b} q_1 \xrightarrow{a} q_0 \xrightarrow{b} q_1 \xrightarrow{a} q_0 \xrightarrow{a} q_0 \xrightarrow{a} q_0 \xrightarrow{a} q_0 \xrightarrow{a} q_0 \cdots$$



Above word is **not accepted** by this automaton

Run is accepting if **some accepting state occurs infinitely often**

Non Deterministic Büchi Automata

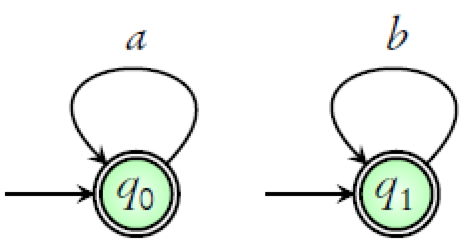
Non-deterministic Büchi Automata

- States, transitions, initial and accepting states like an NFA
- Difference in accepting condition

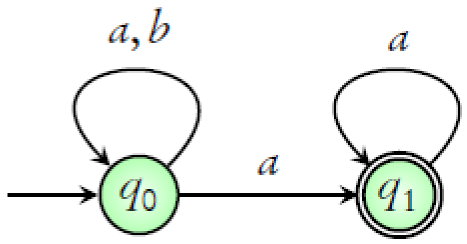
Word is accepted if it has a run in which **some accepting state occurs infinitely often**

Non Deterministic Büchi Automata

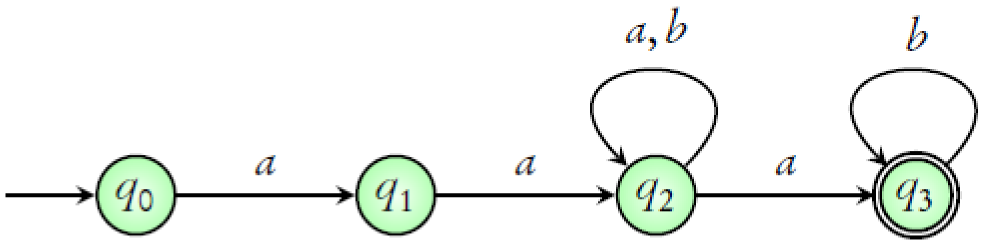
Example: $a^\omega + b^\omega$



Example: $(a + b)^* a^\omega$



Example: $aa(a+b)^* ab^\omega$



Non-deterministic Büchi Automaton

Accepting state occurs infinitely often

LTL to NBA

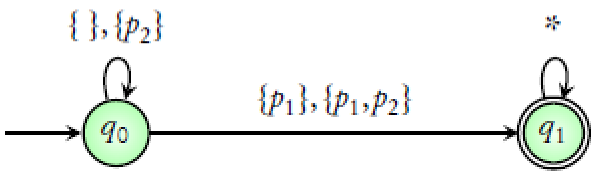
Converting LTL formula to Non Deterministic Büchi Automata

$\mathbf{F}\ p_1$ Words where p_1 occurs sometime

$\{p_2\}\{\}\{p_2\}\{p_1,p_2\}\{p_2\}\{p_2\}\{p_2\}\ldots$

$\{p_1,p_2\}\{\}\{\ \}\{p_1\}\{p_1\}\{p_1,p_2\}\ldots$

$\vdots$

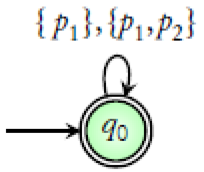


$\mathbf{G}\ p_1$ Words where p_1 occurs always

$\{p_1\}\{p_1,p_2\}\{p_1\}\{p_1,p_2\}\{p_1\}\{p_1\}\{p_1\}\ldots$

$\{p_1,p_2\}\{p_1,p_2\}\{p_1\}\{p_1\}\{p_1\}\{p_1,p_2\}\ldots$

$\vdots$

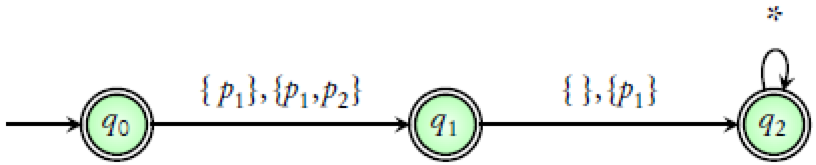


$p_1 \wedge \mathbf{X} \neg p_2$

$\{p_1\}\{\}\{p_2\}\{p_1,p_2\}\{p_2\}\{p_2\}\{p_2\}\ldots$

$\{p_1,p_2\}\{p_1\}\{\ \}\{p_1\}\{p_1\}\{p_1,p_2\}\ldots$

$\vdots$



LTL to NBA

Converting LTL formula to Non Deterministic Büchi Automata

$p_1 \wedge \neg p_2$ Words starting with $\{p_1\}$

$\{p_1\}\{\}\{p_2\}\{p_1,p_2\}\{p_2\}\{p_2\}\{p_2\}\ldots$

$\{p_1\}\{\}\{\ \}\{p_1\}\{p_1\}\{p_1,p_2\}\ldots$

$\vdots$

$XX(p_1 \wedge X\neg p_2)$

$\{\}\{\}\{p_1\}\{\}\{p_2\}\{p_1,p_2\}\{p_2\}\{p_2\}\{p_2\}\ldots$

$\{p_2\}\{p_1\}\{p_1,p_2\}\{p_1\}\{\ \}\{p_1\}\{p_1\}\{p_1,p_2\}\ldots$

$\vdots$

$p_1\ \mathbf{U}\ p_2$

$\{p_1\}\{p_1\}\{p_2\}\{p_1,p_2\}\{p_2\}\{p_2\}\{p_2\}\ldots$

$\{p_1,p_2\}\{\}\{\ \}\{p_1\}\{p_1\}\{p_1,p_2\}\ldots$

$\vdots$