Truth Table to Boolean Expression

- Read off of table
  - For 1, write variable name
  - For 0, write complement of variable
- *Sum of Products (SoP)*
  - Take rows with 1's in output column, sum products of inputs
  - $C = \overline{A}B + \overline{B}A$
- *Product of Sums (PoS)*
  - Take rows with 0's in output column, product the sum of the *complements of the inputs*
  - $C = (A + B) \cdot (\overline{A} + \overline{B})$

a	b	c
0	0	0
0	1	1
1	0	1
1	1	0

We can show that these are equivalent!

Basic Boolean Identities

- $X + 0 = X$
- $X \cdot 1 = X$
- $X + 1 = 1$
- $X \cdot 0 = 0$
- $X + X = X$
- $X \cdot X = X$
- $X + \overline{X} = 1$
- $X \cdot \overline{X} = 0$
- $\overline{\overline{X}} = X$

Basic Boolean Algebra Laws

- Commutative Law:

$X + Y = Y + X$ $\qquad$ $X \cdot Y = Y \cdot X$

- Associative Law:

$X+(Y+Z) = (X+Y)+Z$ $\qquad$ $X \cdot (Y \cdot Z) = (X \cdot Y) \cdot Z$

- Distributive Law:

$X{\cdot}(Y+Z) = X{\cdot}Y+X{\cdot}Z$ $\qquad$ $X+YZ = (X+Y){\cdot}(X+Z)$

Advanced Laws (Absorption)

- $X + XY = X$
- $XY + X\overline{Y} = X$
- $X + \overline{X}Y = X + Y$
- $X(X + Y) = X$
- $(X + Y)(X + \overline{Y}) = X$
- $X(\overline{X} + Y) = XY$

Practice Problem

- Boolean Function: $\mathrm{F} = \overline{\mathrm{X}}\mathrm{YZ} + \mathrm{XZ}$

Truth Table:

X	Y	Z	F
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

Simplification:

Lecture Outline

- Course Logistics
- Course Motivation
- Combinational Logic Review
- Combinational Logic in the Lab

Why Is This Useful?

- Logic minimization: reduce complexity at gate level
  - Allows us to build smaller and faster hardware
  - Care about both # of gates, # of literals (gate inputs), # of gate levels, and types of logic gates

Why Is This Useful?

- Faster hardware?
  - Fewer inputs implies faster gates in some technologies
  - Fan-ins (# of gate inputs) are limited in some technologies
  - Fewer levels of gates implies reduced signal propagation delays
  - # of gates (or gate packages) influences manufacturing costs
  - Simpler Boolean expressions → smaller transistor networks → smaller circuit delays → faster hardware
  - Does the type of gate matter?

Does the Type of Gate Matter?

- Yes!

2-Input Gate Type	# of transistors
NOT	2
AND	6
OR	6
NAND	4
NOR	4
XOR	8
XNOR	8

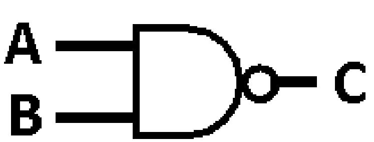
- Can recreate all other gates using only NAND or only NOR gates
  - Called "universal" gates
  - *e.g.* A NAND $\mathrm{A} = \overline{\mathrm{A}}$, B NOR $\mathrm{B} = \overline{\mathrm{B}}$
  - DeMorgan's Law helps us here!

DeMorgan's Law

- $\overline{X+Y} = \bar{X} \cdot \bar{Y}$
- $\overline{X \cdot Y} = \bar{X} + \bar{Y}$

				NOR		NAND	
X	Y	$\bar{X}$	$\bar{Y}$	$\overline{X+Y}$	$\bar{X} \cdot \bar{Y}$	$\overline{X \cdot Y}$	$\bar{X} + \bar{Y}$
0	0	1	1	1		1	
0	1	1	0	0		1	
1	0	0	1	0		1	
1	1	0	0	0		0	

- In Boolean Algebra, converts between NAND/NOR and OR/AND expressions
  - $Z = \overline{(A + B + \bar{C}) \cdot (A + \bar{B} + \bar{C}) \cdot (\bar{A} + B + \bar{C})}$
  - $Z = \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}C$
  - At gate level, can convert from NAND/NOR to OR/AND gates
  - "Flip" all input/output bubbles and "switch" gate



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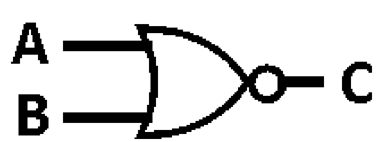
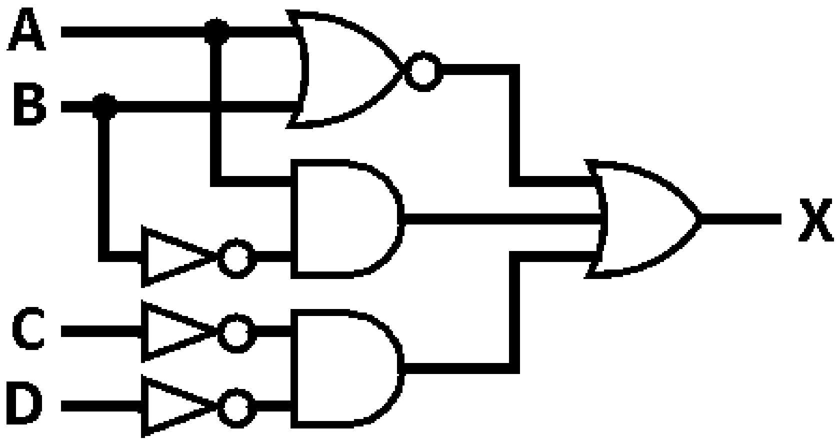


$\Leftrightarrow$

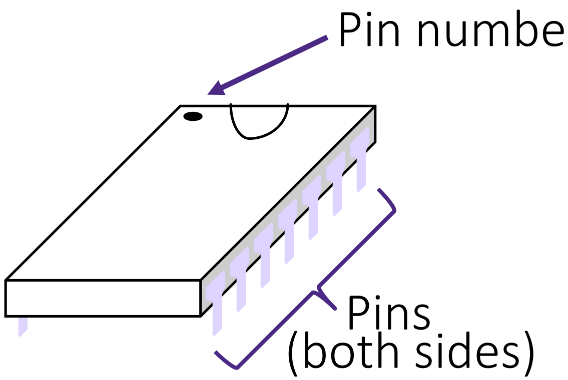
DeMorgan's Law Practice Problem

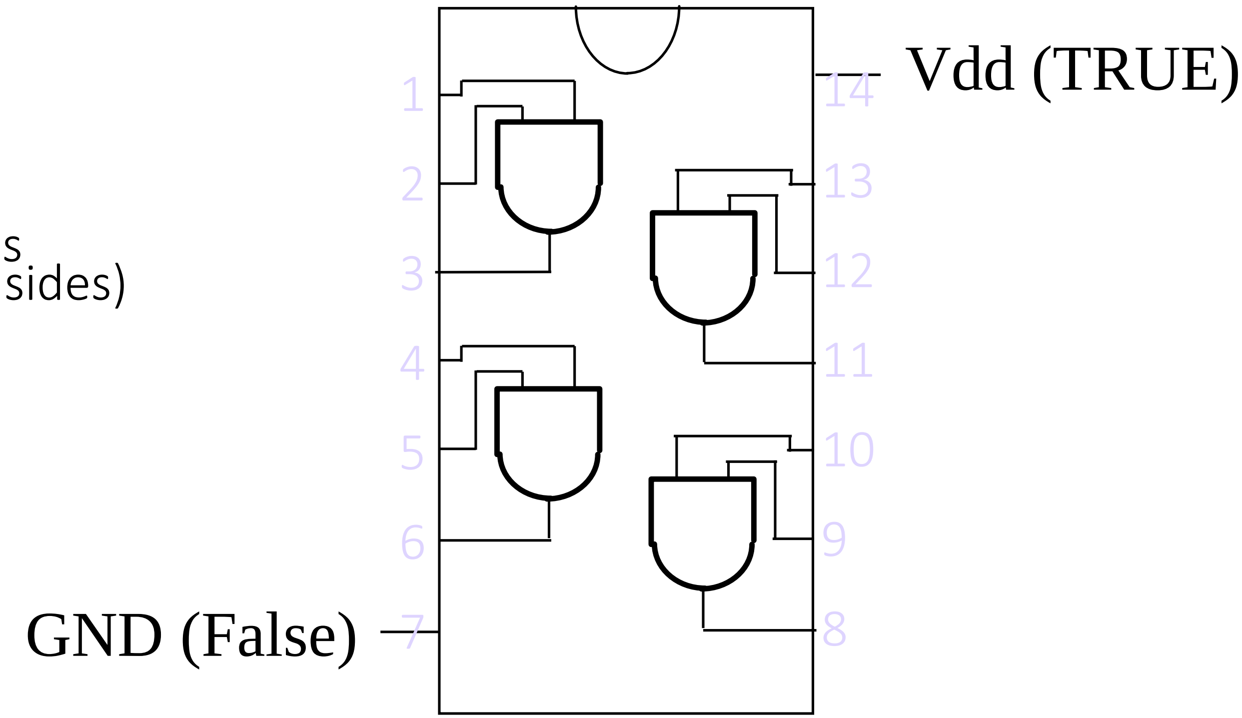
- Simplify the following diagram:



- Then implement with only NAND gates:

Transistor-Transistor Logic (TTL) Packages

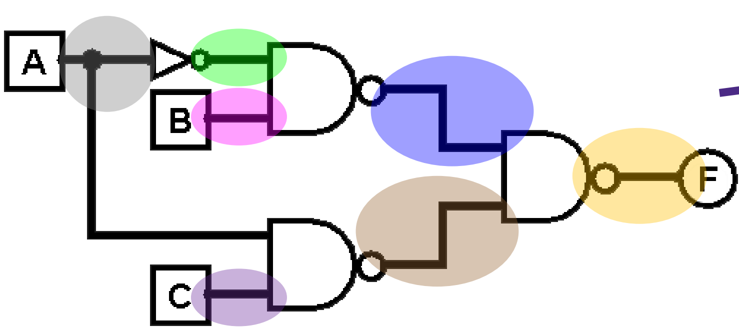




- Diagrams like these and other useful/helpful information can be found on part data sheets
  - It's really useful to learn how to read these

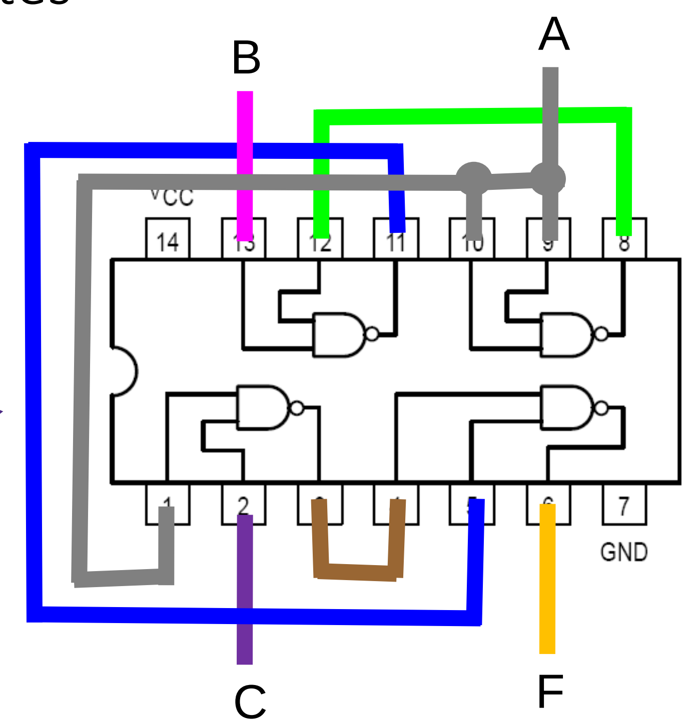
Mapping truth tables to logic gates

- Given a truth table:
  1) Write the Boolean expression
  2) Minimize the Boolean expression
  3) Draw circuit diagram with gates
  4) Map to available gates
  5) Determine # of packages
     and their connections

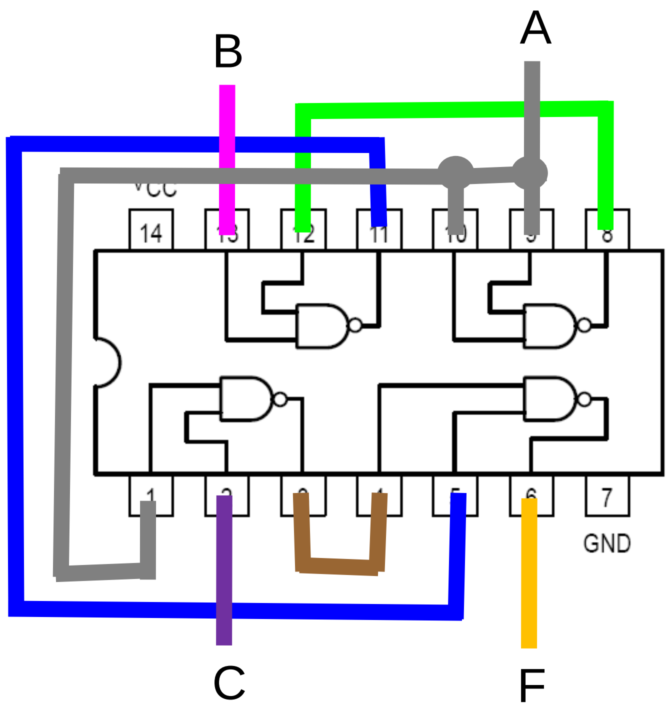


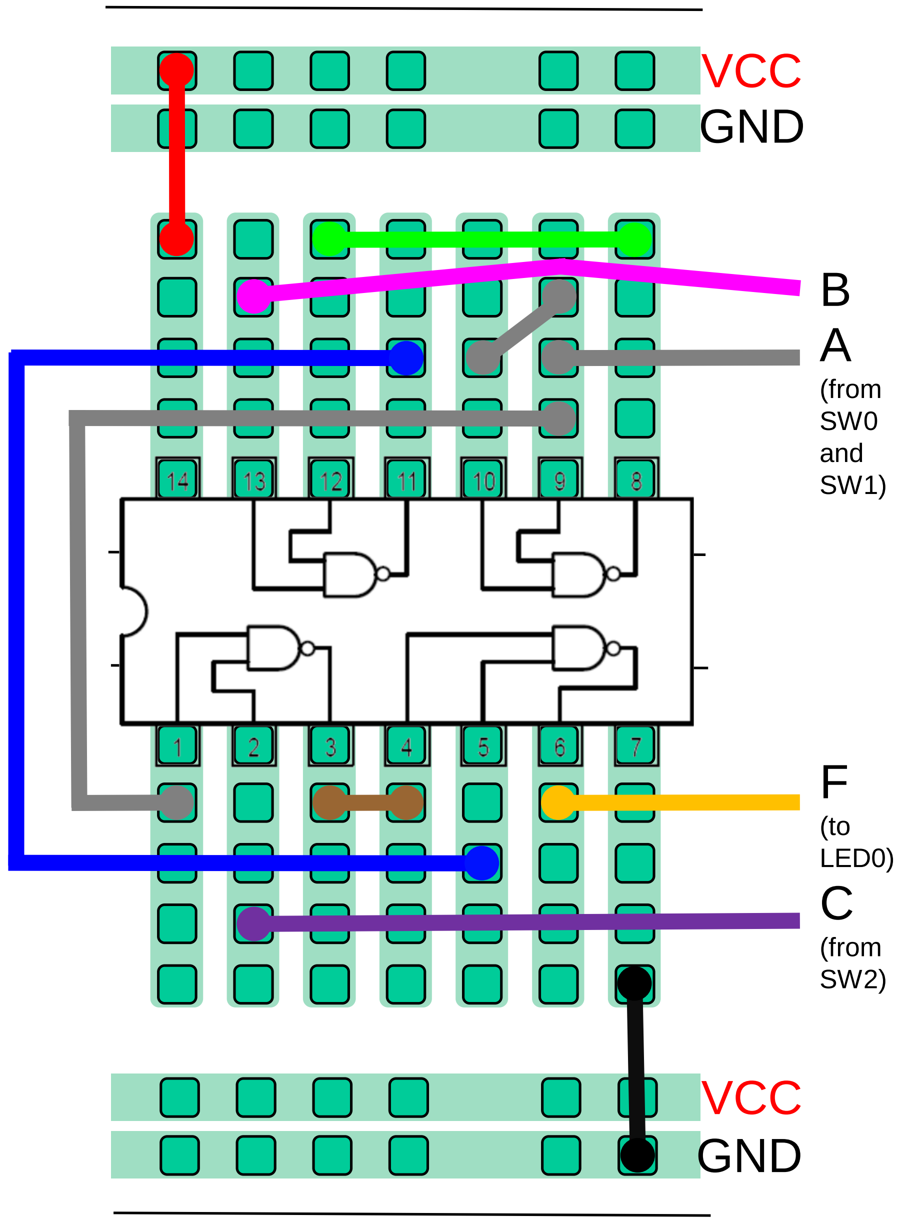
7 nets (wires) in this design

(4)



Breadboarding circuits





Summary

- Digital systems are constructed from Combinational and Sequential Logic
- Logic minimization to create smaller and faster hardware
- Gates come in TTL packages that require careful wiring