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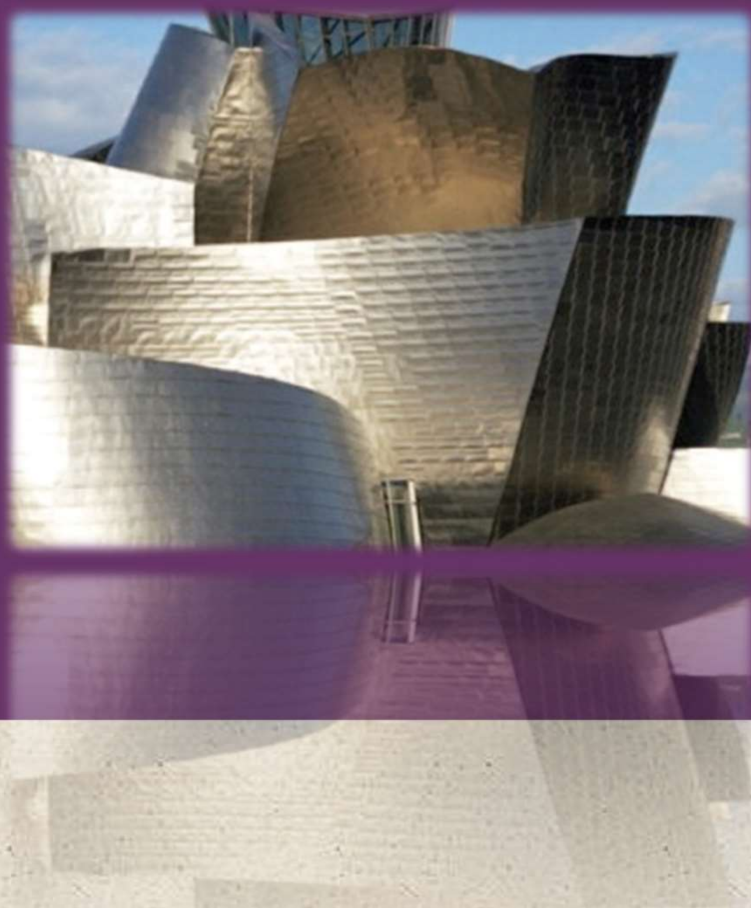

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Computer Organization and Architecture
10th Edition

Chapter 9

Number Systems

The Decimal System

- System based on decimal digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9) to represent numbers
- For example the number 83 means eight tens plus three:

$$83 = (8 * 10) + 3$$

- The number 4728 means four thousands, seven hundreds, two tens, plus eight:

$$4728 = (4 * 1000) + (7 * 100) + (2 * 10) + 8$$

- The decimal system is said to have a ***base***, or ***radix***, of 10. This means that each digit in the number is multiplied by 10 raised to a power corresponding to that digit's position:

$$83 = (8 * 10^1) + (3 * 10^0)$$

$$4728 = (4 * 10^3) + (7 * 10^2) + (2 * 10^1) + (8 * 10^0)$$

Decimal Fractions

- The same principle holds for decimal fractions, but negative powers of 10 are used. Thus, the decimal fraction 0.256 stands for 2 tenths plus 5 hundredths plus 6 thousandths:

$$0.256 = (2 * 10^{-1}) + (5 * 10^{-2}) + (6 * 10^{-3})$$

- A number with both an integer and fractional part has digits raised to both positive and negative powers of 10:

$$442.256 = (4 * 10^{2}) + (4 + 10^{1}) + (2 * 10^{0}) + (2 * 10^{-1}) + (5 * 10^{-2})$$
$$+ (6 * 10^{-3})$$

- ***Most significant digit***
  - The leftmost digit (carries the highest value)
- ***Least significant digit***
  - The rightmost digit

Table 9.1
Positional Interpretation of a Decimal Number

4	7	2	2	5	6
100s 10^2 position 2	10s 10^1 position 1	1s 10^0 position 0	tenths 10^{-1} position −1	hundredths 10^{-2} position −2	thousandths 10^{-3} position −3

Positional Number Systems

- Each number is represented by a string of digits in which each digit position i has an associated weight r^i, where r is the *radix*, or *base,* of the number system.

- The general form of a number in such a system with radix r is

$$(\ldots a_3 a_2 a_1 a_0 . a_{-1} a_{-2} a_{-3} \ldots)_r$$

where the value of any digit a_i *is an integer in the range* $0 \leq a_i < r$. The dot between a_0 *and* a_{-1} is called the ***radix point.***

Table 9.2
Positional Interpretation of a Number in Base 7

Position	4	3	2	1	0	−1
Value in exponential form	7^4	7^3	7^2	7^1	7^0	7^{-1}
Decimal value	2401	343	49	7	1	1/7

+ The Binary System

- Only two digits, 1 and 0
- Represented to the base 2
- The digits 1 and 0 in binary notation have the same meaning as in decimal notation:

$$0_2 = 0_{10}$$

$$1_2 = 1_{10}$$

- To represent larger numbers each digit in a binary number has a value depending on its position:

$$10_2 = (1 * 2^1) + (0 * 2^0) = 2_{10}$$

$$11_2 = (1 * 2^1) + (1 * 2^0) = 3_{10}$$

$$100_2 = (1 * 2^2) + (0 * 2^1) + (0 * 2^0) = 4_{10}$$

and so on. Again, fractional values are represented with negative powers of the radix:

$$1001.101 = 2^3 + 2^0 + 2^{-1} + 2^{-3} = 9.625_{10}$$

Converting Between Binary and Decimal

Binary notation to decimal notation:

- Multiply each binary digit by the appropriate power of 2 and add the results

Decimal notation to binary notation:

- Integer and fractional parts are handled separately

Integers

For the integer part, recall that in binary notation, an integer represented by

$$b_{m-1}b_{m-2}\ldots b_2b_1b_0 \qquad b_i = 0 \text{ or } 1$$

has the value

$$(b_{m-1} * 2^{m-1}) + (b_{m-2} * 2^{m-2}) + \ldots + (b_1 * 2^1) + b_0$$

Suppose it is required to convert a decimal integer N into binary form. If we divide N by 2, in the decimal system, and obtain a quotient N_1 and a remainder R_0, we may write

$$N = 2 * N_1 + R_0 \qquad R_0 = 0 \text{ or } 1$$

Next, we divide the quotient N_1 by 2. Assume that the new quotient is N_2 and the new remainder R_1. Then

$$N_1 = 2 * N_2 + R_1 \qquad R_1 = 0 \text{ or } 1$$

so that

$$N = 2(2N_2 + R_1) + R_0 = (N_2 * 2^2) + (R_1 * 2^1) + R_0$$

If next

$$N_2 = 2N_3 + R_2$$

we have

$$N = (N_3 * 2^3) + (R_2 * 2^2) + (R_1 * 2^1) + R_0$$

Continued . . .