Lecture # 18

Binary Decision Diagram | Reduction rules for BDDs

Boolean Functions

x, y : *Boolean* variables

$f(x,y) = x + y$ *Boolean function*

$$0+0 = 0$$
$$0+1 = 1$$
$$1+0 = 1$$
$$1+1 = 1$$

Logical OR

x	y	$f(x,y)$
0	0	0
0	1	1
1	0	1
1	1	1

Truth table

Boolean Functions

x, y : *Boolean* variables

$f(x,y) = x \cdot y$ *Boolean function*

$0 \cdot 0 = 0$

$0 \cdot 1 = 0$ *Logical AND*

$1 \cdot 0 = 0$

$1 \cdot 1 = 1$

x	y	$f(x,y)$
0	0	0
0	1	0
1	0	0
1	1	1

Truth table

Boolean Functions

x : *Boolean* variable

$f(x) = \overline{x}$ *Boolean function*

$$\overline{0} = 1$$
$$\overline{1} = 0$$

Logical NOT

x	$f(x)$
0	1
1	0

Truth table

Boolean Functions

$x_1, x_2, \ldots, x_n$: *Boolean* variables

$$f : \{x_1, x_2, \ldots, x_n\} \rightarrowtail \{0, 1\}$$

Boolean function

$+ \quad \cdot \quad -$

Boolean operations

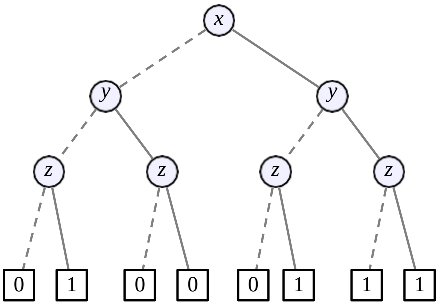
Examples: $f_1(x, y) = \overline{x} + y$, $f_2(x, y, z) = x \cdot y + \overline{y} \cdot z$, $f_3(x, y, z) = \overline{x + \overline{y} \cdot z}$

Representing Boolean functions

$$f(x,y,z) = x \cdot y + \bar{y} \cdot z$$

x	y	z	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Truth table



Binary Decision Tree

Operations on truth tables

$$g(x,y,z) = f\,\overline{(x,y,z)} = \overline{x \cdot y + \overline{y} \cdot z}$$

x	y	z	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

x	y	z	g
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	0

Operations on truth tables

x	y	z	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

$$f(x,y,z) = x \cdot y + \overline{y} \cdot z$$

$$g(x,y,z) = \overline{x} \cdot \overline{y}$$

$$h(x,y,z) = f(x,y,z) + g(x,y,z)$$

x	y	z	g
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

x	y	z	h
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Operations on truth tables

$$f(x,y,z) = x \cdot y + \overline{y} \cdot z$$

$$g(x,y,z) = x$$

$$h(x,y,z) = f(x,y,z) \cdot g(x,y,z)$$

x	y	z	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

x	y	z	g
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

x	y	z	h
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

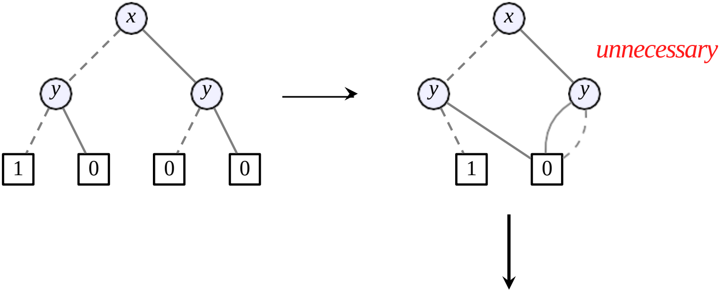
Truth table representation for boolean functions

- **Space:** For n variables, needs to store $2^n \cdot (n+1)$ bits
- **Operations:** Visit each entry of truth table

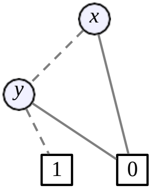
- Sequential circuits can be modeled using boolean functions

If boolean functions are represented using truth tables, a circuit with 100 variables needs more than 2^{100} bits!

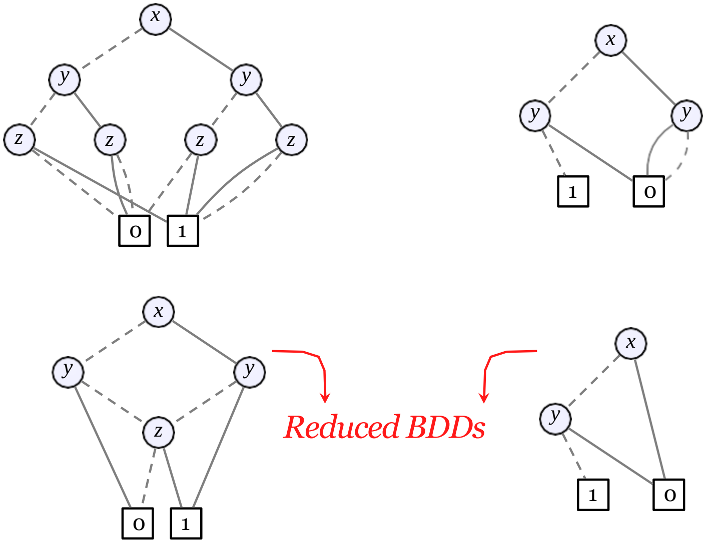
$$\bar{x} \cdot \bar{y}$$



Binary Decision Diagram



Binary Decision Diagrams



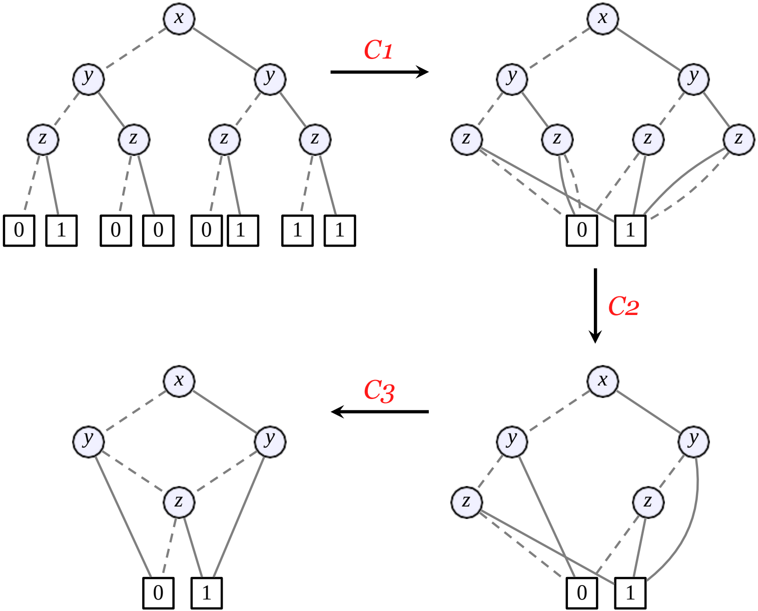
Reduction rules for BDDs

- C1: Removal of duplicate leaves

- C2: Removal of redundant tests

- C3: Removal of duplicate sub-trees

Reduction rules for BDDs



$$\bar{x} \cdot \bar{y}$$