Q.NO: 23

MT-284

CAL-2

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Ans $y'^{/8} \frac{dy}{dx} + y^{3/8} = 1$ $y(0) = 4$

(Sol)

$$\frac{dy}{dx} + \frac{y^{3/8}}{y^{1/8}} = y^{-1/8}$$

$$\frac{dy}{dx} + y = y^{-1/8} \longrightarrow \text{②}$$

$$\boxed{\text{Here } n = -1/8, \quad \text{so } y = y^{-1}}$$

Let $u = y^{1-(-1/8)} = y^{3/8}$ $u^{2/3} = y^{3/8 \cdot n^{2/8}}$

or $y = u^{2/3}$ or $y^{3/8} = u$

$$\frac{dy}{dx} = \frac{8}{3} u^{2/3 - 1} \frac{du}{dx} = \frac{8}{3} u^{-1/3} \frac{du}{dx}$$

$$\frac{dy}{dx} = \frac{8}{3} u^{-1/3} \frac{du}{dx}$$

Therefore ②

$$\frac{8}{3} u^{-1/3} \frac{du}{dx} + u^{2/3} = \left(\frac{8/3}{u}\right)^{-1/8}$$

$$\frac{8}{3} u^{-1/3} \frac{du}{dx} + u^{2/3} = u^{-1/3}$$

$$\frac{8}{3}\frac{dy}{dn} + \frac{u^{2/3}}{u^{-1/3}} = \frac{u^{-1/3}}{u^{-1/3}}$$

$$\frac{8}{3}\frac{du}{dn} + u = 1$$

or $$\frac{du}{dn} + \frac{3}{8}u = \frac{3}{8} \rightarrow ③$$

$$u(n) = e^{\int \frac{3}{8}dn} = e^{\frac{3}{8}n} \rightarrow ④$$

x'ing ③ by u given

$$e^{\frac{3}{8}n}\frac{du}{dn} + \frac{3}{8}e^{\frac{3}{8}n}u = \frac{3}{8}e^{\frac{3}{8}n}$$

$$\frac{d}{dn}\left[u\,e^{\frac{3}{8}n}\right] = \frac{3}{8}e^{\frac{3}{8}n}$$

$$u\,e^{\frac{3}{8}n} = \frac{3}{8}\int e^{\frac{3}{8}n}\,dn$$

$$u\,e^{\frac{3}{8}n} = \frac{3}{8}\frac{e^{\frac{3}{8}n}}{\frac{3}{8}} + C \quad \text{or} \quad u = \frac{e^{\frac{3}{8}n}}{e^{\frac{3}{8}n}} + \frac{C}{e^{\frac{3}{8}n}}$$

$$u = 1 + c\,e^{-\frac{3}{8}n}$$

Substituting again $u = y^{3/8}$ we have

$$y^{3/8} = 1 + c\,e^{-\frac{3}{8}n}$$

$$y = \left(1 + c\,e^{\frac{3}{8}n}\right)^{8/3}$$

③

$$y(0) = 4$$

$$4 = (1 + ce^0)^{2/3} \quad \text{or} \quad 1 + c(4)^{3/2} \quad \text{or} \quad$$

$$\text{or} \quad c = -1 + 4^{3/2}$$