Lecture #7: Rules of Inference.

"If (you have a correct password) then (you can log onto the network)"
p q

"(You have the password)".
p

Therefore

"you can log onto the network".
q

Argument $\begin{cases} p_1 :- p \to q \\ p_2 :- p \\ \hline \therefore q \end{cases}$ Modus ponens

Conclusion.

$(p_1 \wedge p_2 \wedge \cdots \wedge p_n) \to q$.

tautology.

Valid.

$\begin{cases} \text{Premise 1} \to p_1 \\ \text{Premise 2} \to p_2 \\ \vdots \quad \vdots \\ \text{Premise n} \to p_n \\ \hline \therefore \text{Conclusion} \to q \end{cases}$

$\begin{cases} p_1 :- p \to q \\ p_2 :- p \\ \hline \end{cases}$

$((p \to q) \wedge (p)) \to q$.

$$\frac{q_1 q_2 :- p}{\therefore q} \quad \{$$

p1 → p
p2 → p → q
c → ∴ q

$$((p \wedge (p \to q)) \to q) \text{ Tautology.}$$

Rules of Inference.

1). Modus Ponens.
$$\frac{p \qquad p \to q}{\therefore q}$$

2) Modus tollens.
$$\frac{\neg q \qquad p \to q}{\therefore \neg p}$$

3). Hypothetical Syllogism.
$$\frac{p \to q \qquad q \to r}{\therefore p \to r}$$

4) Disjunction Syllogism
$$\frac{p \vee q \qquad \neg p}{\therefore q}$$

5) Addition.
$$\frac{p}{\therefore p \vee q}$$

6). Simplication.
$$\frac{p \wedge q}{\therefore p}$$

7) Conjunction.
$$\frac{p \qquad q}{\therefore p \wedge q}$$

8) Resolution ☆.
$$\frac{p \vee q \qquad \neg p \vee r}{\therefore q \vee r}$$

Ex6:- P 62:- "It is not Sunny ($\neg p$) this afternoon and (q) it is colder than yesterday. We will go Swimming (r) only if it is Sunny (p). If we do not go swimming ($\neg r$) then we will take a Canoe trip (s). if we take a Canoe trip (s), then we will be home by Sunset (t)." leads to Conclusion.

"we will be home by Sun Set". (t)

P1 $\neg p \wedge q$ ✓

P2 $\gamma \rightarrow p$ ✓

P3 $\neg\gamma \rightarrow s$ ✓

P4. $s \rightarrow t$ ✓

C. $\therefore t$.

1). $\dfrac{p \quad p \rightarrow q}{\therefore q}$ Modus Ponens.

2) $\dfrac{\neg q \quad p \rightarrow q}{\therefore \neg p}$ Modus tollens.

3). $\dfrac{p \rightarrow q \quad q \rightarrow r}{\therefore p \rightarrow r}$ Hypothetical Syllogism.

4) $\dfrac{p \vee q \quad \neg p}{\therefore q}$ Disjunction Syllogism

5) $\dfrac{p}{\therefore p \vee q}$ Addition.

6). $\dfrac{p \wedge q}{\therefore p}$ Simplication.

7) $\dfrac{p \quad q}{\therefore p \wedge q}$ Conjunction.

8) $\dfrac{p \vee q \quad \neg p \vee r}{\therefore q \vee r}$ Resuntion ★.

from (P1) $\neg p$ ✓ → (5) ✓ by Using Simplification.

from (P2) & (5) $\neg\gamma$ → (6) ✓ by using modus tollens.

from (P3) & (6) s → (7) ✓ by Using modus ponens.

from (P4) & (7) t → (8) by a 4.

which is the Conclusion.

Ex7 p62 :- (P1) $p \rightarrow q$ ✓

(P2) $\neg p \rightarrow r$ ✓

(P3) $\gamma \rightarrow s$. ✓

(C) $\therefore \neg q \rightarrow s$

1). $\dfrac{p \quad p \rightarrow q}{\therefore q}$ Modus Ponens.

2) $\dfrac{\neg q \quad p \rightarrow q}{\therefore \neg p}$ Modus tollens.

3). $\dfrac{p \rightarrow q \quad q \rightarrow r}{\therefore p \rightarrow r}$ Hypothetical Syllogism.

4) $\dfrac{p \vee q \quad \neg p}{\therefore q}$ Disjunction Syllogism

5) $\dfrac{p}{\therefore p \vee q}$ Addition.

6). $\dfrac{p \wedge q}{\therefore p}$ Simplication.

7) $\dfrac{p \quad q}{\therefore p \wedge q}$ Conjunction.

8) $\dfrac{p \vee q \quad \neg p \vee r}{\therefore q \vee r}$ Resuntion ★.

from (P1) $\neg q \rightarrow \neg p$ → (4) ✓ → CP.

from (2) (4) $\neg q \rightarrow r$ → (5) ✓ → HS.

from (3) (5) $\neg q \rightarrow s$ → (6) HS.

which is Conclusion.

P1 :- We may need to remember Logical Equivalences.

✓

Ex8:- (P1) $T \rightarrow (M \vee E)$

(P2) $S \rightarrow \neg E \wedge L$

(P3) $T \wedge S$

(C) $\therefore M.$

1). $\dfrac{P \quad P \rightarrow q}{\therefore q}$ Modus Ponens.

2) $\dfrac{\neg q \quad P \rightarrow q}{\therefore \neg P}$ Modus tollens.

3). $\dfrac{P \rightarrow q \quad q \rightarrow r}{\therefore P \rightarrow r}$ Hypothetical Syllogism.

4) $\dfrac{P \vee q \quad \neg P}{\therefore q}$ Disjunctive Syllogism

5) $\dfrac{P}{\therefore P \vee q}$ Addition.

6). $\dfrac{P \wedge q}{\therefore P}$ Simplification.

7) $\dfrac{P \quad q}{\therefore P \wedge q}$ Conjunction.

8) $\dfrac{P \vee q \quad \neg P \vee r}{\therefore q \vee r}$ Resolution*.

from (P3) T — (4) by $\wedge$ P.

from (P3) S — (5) by $\wedge$ L.

from (4) & (P1) $M \vee E$ — (6) by MP.

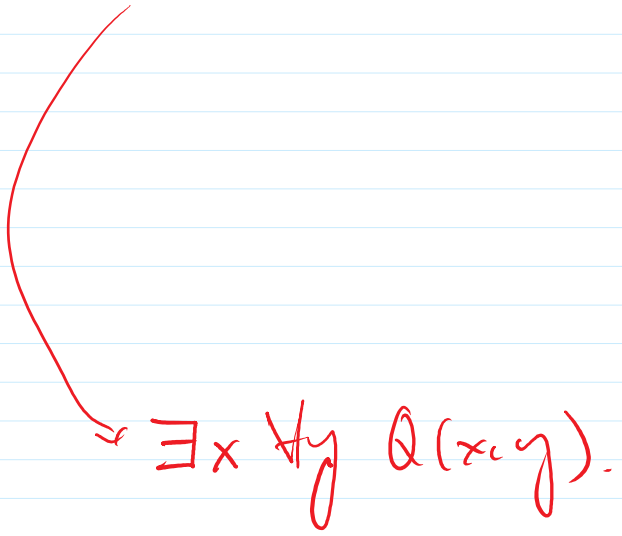
from (5) & (P2) $\neg E$ — (7) by MP.

from (6, 7) M — (8) by DS.

Problem 2.
the sequence may be broken.

Problem 3.
we need to remember all Rules of inference.

(one formula) all your problems will be solved.


$\exists x \forall y\ Q(x,y)$.