Q1: Use the principle of resolution to show that the hypothesis "Chohan works hard", "If Chohan works hard then he is a dull boy", "if Chohan is a dull boy, he will not get a job" imply the conclusion "Chohan will not get the job". **(Marks 3)**

Solution:

P1: P	C1: P	
P2: $P \to Q$	C2: $\neg P \vee Q$	
P3: $Q \to \neg R$	C3: $\neg Q \vee \neg R$	
C: $\neg R$	C4: R	
	C5: Q	From C1 and C2
	C6: $\neg R$	From C3 and C5
	C7: $\square$	From C4 and C6

Q2: Write the negation of the following statements in English using the logical equivalence of $\sim\forall x\, P(x) = \exists x \sim P(x)$ and $\sim\exists x\, P(x) = \forall x \sim P(x)$. No credit will be given if you didn't use these logical equivalences. **(6 marks)**

a. $\forall x \forall y\, (P(x,y) \to \sim Q(x,y))$

Solution: $\neg(\forall x \forall y (P(x,y) \to \neg Q(x,y))) = \exists x \exists y\, (P(x,y) \wedge Q(x,y))$

"There exists an x, for which there exists a y, P(x,y) and Q(x,y)."

b. $\exists x \forall y\, (P(x,y) \vee \sim Q(x,y))$

Solution: $\neg(\exists x \forall y (P(x,y) \vee \neg Q(x,y))) = \forall x \exists y\, (\neg P(x,y) \wedge Q(x,y))$

"For any x, there exist's a y, not P(x,y) and Q(x,y)."

Note: When you are done with simplification of the quantifiers then also use the equivalences of $P \to Q = \sim P \vee Q$ and Demorgan law to simplify further your answer. I will deduct marks if you ignore this.

Scenario 2: Assume all knights to be sad (means the knights will speak lies now)

CASE 1: Knight, Knight $\begin{bmatrix} P = T & \neg P = F \\ Q = T & \neg Q = F \end{bmatrix}$

1) $P \wedge Q$ $= F$

2) $\neg P$ $= F$

↳ CASE DOES NOT HOLD

CASE 2: Knight, Knave $\begin{bmatrix} P = T & \neg P = F \\ Q = F & \neg Q = T \end{bmatrix}$

1) $P \wedge Q$ $= F$

2) $\neg P$ $= F$

↳ CASE HOLDS

CASE 3: Knave, Knight $\begin{bmatrix} P = F & \neg P = T \\ Q = T & \neg Q = F \end{bmatrix}$

1) $P \wedge Q$ $= F$

2) $\neg P$ $= F$

↳ CASE DOES NOT HOLD

CASE 4: Knave, Knave $\begin{bmatrix} P = F & \neg P = T \\ Q = F & \neg Q = T \end{bmatrix}$

1) $P \wedge Q$ $= F$

2) $\neg P$ $= F$

↳ CASE DOES NOT HOLD

Conclusion: A is Knight, B is Knave

Q5: Assume that the statement "if it is sunny day then I will not go to beach" is in contrapositive form. Make the following forms of this statement using English sentences **(3 marks)**

Converse:

"If I will not go to the beach, then it is a sunny day."

$[\neg p \rightarrow \neg q]$

Contrapositive:

"If I will go to the beach, then it is not a sunny day."

$[p \rightarrow q]$

Inverse:

"If it is not a sunny day, then I will go to the beach."

$[q \rightarrow p]$

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Q3: Write a logical expression corresponding to the following using only the predicates, logical conjunctions, disjunction and nothing else. Assume the domain of x,y = {1,2,3} **(Marks 9)**

$\neg\forall x \forall y P(x,y) =$ $[\neg P(1,1) \vee \neg P(1,2) \vee \neg P(1,3)] \vee$
$[\neg P(2,1) \vee \neg P(2,2) \vee \neg P(2,3)] \vee$
$[\neg P(3,1) \vee \neg P(3,2) \vee \neg P(3,3)]$

$\exists x \neg\forall y P(x,y) =$ $[\neg P(1,1) \vee \neg P(1,2) \vee \neg P(1,3)] \vee$
$[\neg P(2,1) \vee \neg P(2,2) \vee \neg P(2,3)] \vee$
$[\neg P(3,1) \vee \neg P(3,2) \vee \neg P(3,3)]$

$\forall x \exists y \neg P(x,y) =$
$[\neg P(1,1) \vee \neg P(1,2) \vee \neg P(1,3)] \wedge$
$[\neg P(2,1) \vee \neg P(2,2) \vee \neg P(2,3)] \wedge$
$[\neg P(3,1) \vee \neg P(3,2) \vee \neg P(3,3)]$

Q4: The following question relates to the inhabitants of the island of knights and knaves created by Smullyan. **The knights only speak the truth when they are happy. The knaves always speak lie regardless they are happy or sad.** You encounter two people A and B. Determine if possible what A and B are if they address you in the way. If you cannot determine what these two people are, can you draw any conclusions? **(Marks 20)**

A says "The two of us are both knights" and
B says "A is a knave"

Solution:

P = A is a knight ~P = A is a knave
Q = B is a knight ~Q = B is a knave

Scenario 1: Assume all knights to be happy

CASE 1: Knight, Knight $\left[\begin{matrix} P = T & \neg P = F \\ Q = T & \neg Q = F \end{matrix}\right]$
1) $P \wedge Q$ = T ✓
2) $\neg P$ = T ✗

↳ CASE DOES NOT HOLD

CASE 2: Knight, Knave $\left[\begin{matrix} P = T & \neg P = F \\ Q = F & \neg Q = T \end{matrix}\right]$
1) $P \wedge Q$ = T ✗
2) $\neg P$ = F

↳ CASE DOES NOT HOLD

CASE 3: Knave, Knight $\left[\begin{matrix} P = F & \neg P = T \\ Q = T & \neg Q = F \end{matrix}\right]$
1) $P \wedge Q$ = F ✓
2) $\neg P$ = T ✓

↳ CASE HOLDS

CASE 4: Knave, Knave $\left[\begin{matrix} P = F & \neg P = T \\ Q = F & \neg Q = T \end{matrix}\right]$
1) $P \wedge Q$ = F ✓
2) $\neg P$ = F ✗

↳ CASE DOES NOT HOLD

Conclusion: A is Knave, B is Knight