lecture 6 :- Ex Q31 - P-45.

$x = \{0,1\}$.
$y = \{0,1\}$
$z = \{0,1\}$.

(a) $\forall y \; Q(0, y, 0)$.

$= Q(0,0,0) \wedge Q(0,1,0)$

$x = \{0, 1, 2, \cdots N\}$

$\forall x P(x)$

$= P(0) \wedge P(1) \wedge P(2) \wedge \cdots \wedge P(N)$.

(d) $\exists x \neg Q(x, 0, 1)$.

$= \neg Q(0,0,1) \vee \neg Q(1,0,1)$.

$\exists x P(x)$

$= P(0) \vee P(1) \vee P(2) \vee \cdots \vee P(N)$.

Q32:- a) All dogs have fleas.

$\underbrace{\text{All}}_{\forall}$ $\underbrace{\text{dogs}}_{\text{subject}}$ have $\underbrace{\text{fleas}}_{\text{predicate}}$.

$\underbrace{\text{for all } x,}_{\forall}$ $\underbrace{x \text{ is a dog,}}_{\text{subject}}$ $\underbrace{x \text{ have fleas,}}$

let $P(x) =$ x have fleas.

$\neg(\forall x \, P(x))$. $x \in$ Set of dogs.

$= \underline{\exists x} \, \neg P(x)$.

there exist x, x is dog, x does not has fleas.

<u>Nested</u> <u>Quantifier</u>:-

$x \in \{1, 2, 3, \cdots N\}$.

$\forall x \, P(x)$

$= P(1) \wedge P(2) \wedge \cdots P(N)$

$x \; \boxed{\forall x} \forall y \; P(x, y)$.

$\forall (P(x,1) \wedge P(x,2) \wedge \cdots \wedge P(x,N))$

$$= \forall x \left(P(x,1) \wedge P(x,2) \wedge \cdots \wedge P(x,N) \right) \qquad x, y \in \{1, 2, 3 \cdots N\}$$

$$= \forall x P(x,1) \wedge \forall x P(x,2) \wedge \forall x P(x,3) \wedge \cdots \wedge \forall x P(x,N).$$

$$= \left(P(1,1) \wedge P(2,1) \wedge P(3,1) \wedge \cdots \wedge P(N,1) \right) \wedge$$

$$\left(P(1,2) \wedge P(2,2) \wedge P(3,2) \wedge \cdots \wedge P(N,2) \right) \wedge$$

$$\vdots$$

$$\left(P(1,N) \wedge P(2,N) \wedge P(3,N) \wedge \cdots \wedge P(N,N) \right).$$

$$\boxed{\forall x} \exists y \, P(x,y).$$

$$\forall x \left(P(x,1) \vee \underline{P(x,2)} \vee P(x,3) \vee \cdots \vee P(x,N) \right).$$

$$\left(P(1,1) \wedge P(2,1) \wedge P(3,1) \wedge \cdots \wedge P(N,1) \right) \vee$$

$$\left(P(1,2) \wedge P(2,2) \wedge P(3,2) \wedge \cdots \wedge P(N,2) \right) \vee$$

$$\vdots$$

$$\left(P(1,N) \wedge P(2,N) \wedge P(3,N) \wedge \cdots \wedge P(N,N) \right).$$

$$\exists x \forall y \, P(x,y) = \,?$$

$$\exists x \exists y \, P(x,y) = \,?$$

$\neg(\forall x \forall y\, P(x,y)) = \exists x \exists y \neg P(x,y)$.

$\neg(\forall x \exists y\, P(x,y))$

$\neg(\exists x \forall y\, P(x,y))$.

$\neg(\exists x \exists y\, P(x,y))$.

Ex1. P47:- $\forall x \forall y\, (x+y = y+x)$. $x,y \in \mathbb{R}$.

Ex4 P48 $\exists y \forall x\, (x+y = 0) = F$ $x,y \in \mathbb{R}$.

$\forall x \exists y\, (x+y = 0) = T$. "

Ex5 P49 $\forall x \forall y \exists z\, Q(x,y,z) = x+y = z$

for all x, for all y, there exist z $x,y,z \in \mathbb{R}$.
such that $x+y = z$.

$\exists z \forall x \forall y\, Q(x,y,z) = x+y = z$.

there exist z for all x, for all y,
$x+y = z$.

Ex6:- the sum of two positive integers is always positive.

positive.

for all x, for all y, (x, y are positive Integers).

$x+y \geq 0$.

$\forall x \forall x \, P(x, y)$

let $P(x, y) = x+y \geq 0$.

$x, y \in \mathbb{Z}^+$.

Ex 9. P-51. $C(x) =$ x has a Computer.

$F(x,y) =$ x and y are friends.

$\underline{\forall x} (C(x) \vee \exists y (C(y) \wedge F(x,y)))$.

$x, y \in$ Students in your School.

for all x, x is a Student, x has a Computer, or there exist y, y is a student, y has a Computer and x and y are friends.

for all x, there exist y, x has a Computer or y has a computer and x and y are friends.

Ex 10:- P51. $F(a, b) =$ a and b are friends.

$\exists x \forall y \forall z \, ((F(x,y) \wedge F(x,z) \wedge (y \neq z)) \xrightarrow[\text{then}]{} \neg F(y,z))$.

Ex 11:- If a person is a female and is a parent then this person is Someone's mother.

this person is someone's mother.

for all x, x ~~is a~~ belongs to. person, if x is female $x, y \in$ Persons and x is a parent, then there exist y, x is a mother y.

let $F(x)$: x is a female.

$P(x)$: x is a parent.

$M(x,y)$: x is the mother of y.

$$\forall x \exists y ((F(x) \wedge P(x)) \rightarrow M(x,y))$$

Every student in this class has taken atleast one Computer Science Course.

for all x, x is a student, there exist y, y is a Computer Science Course, such that x has taken y.

let $x \in$ Student

$T(x,y)$: x has taken y. $y \in$ CS Course.

$$\forall x \exists y\, T(x,y)$$