

## lecture 14:-

Ex 5  
463, 9.

class Question.

Symmetric  $R = \{(a,b) \mid a=b \text{ or } a=-b\}$ .  $A \subseteq \mathbb{Z}$ .  
 $\forall a,b \in A$  if  $(a,b) \in R \rightarrow (b,a) \in R$ .

$\forall a,b \in \mathbb{Z}$  if  $a=b \text{ or } a=-b \rightarrow b=a \text{ or } b=-a$ .

## SOME ADDITIONAL OPERATIONS ON RELATIONS.

$$\bar{R} = \{(a,b) \mid (a,b) \notin R\}$$

$$\bar{R} = A \times A - R$$

$$R^{-1} = \{(b,a) \mid (a,b) \in R\}$$

$$= \{(a,b) \mid (b,a) \in R\}$$

$$= \{(a,b) \mid b < a\}$$

Ex:  $A = \{1,2,3,4\}$

$$R = \{(a,b) \mid a < b\}$$

$$R = \{(1,2), (1,3), (1,4), (2,3), (2,4), (3,4)\}$$

$$\bar{R} = \{(1,1), (2,1), (2,2), (3,1), (3,2), (3,3), (4,1), (4,2), (4,3), (4,4)\}$$

$$R^{-1} = \{(2,1), (3,1), (4,1), (3,2), (4,2), (4,3)\}$$

$$A \times A = \{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4), (4,1), (4,2), (4,3), (4,4)\}$$

## N-ARY RELATIONS.

R

$$A \times A \times A$$

$A \subseteq \mathbb{N}$

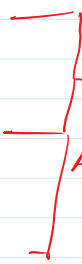
$$2^{|A| \times |A| \times |A|}$$

$$= 2^{|A|^3} = 2^{n^3}$$

How many Binary Relations.

How many ternary

How many 4-ary



$$A = \{1,2,3\}$$

$$A \times A \times A = \{(1,1,1), (1,1,2), (1,2,1), (1,2,2), (2,1,1), (2,1,2), (2,2,1), (2,2,2)\}$$

Ex 1 :-  
469

R

$\mathbb{N} \times \mathbb{N} \times \mathbb{N}$

$$R = \{(a,b,c) \mid a < b < c\}$$

$$(1,2,3) \in R \quad ?$$

$$(2,4,3) \notin R \quad -$$

Ex 2 :-  
469

R

$\mathbb{Z} \times \mathbb{Z} \times \mathbb{Z}$

$$R = \{(a,b,c) \mid b = a+k, c = a+2k\}$$

$$k \in \mathbb{Z}$$

$$(1,3,5) \in R$$

$$= 1 + 2 \dots$$

$$(1, 3, 5) \in R.$$

$$\downarrow \downarrow \downarrow$$

$$a \quad b \quad c.$$

$$3 = 1 + k \Rightarrow k = 2.$$

$$5 = 1 + 2 \cdot 2 \Rightarrow 5 = 1 + 4 \Rightarrow 5 = 5.$$

$$(2, 5, 9) \notin R \text{ ?}$$

$$9 \equiv 5 \pmod{4}.$$

Ex3.

469.

$$\mathbb{Z} \times \mathbb{Z} \times \mathbb{Z}^+$$

$$R = \{(a, b, m) \mid a \equiv b \pmod{m}\}.$$

$$(-1, 9, 5) \in R \checkmark \text{ ?}$$

$$(14, 0, 7) \in R \checkmark$$

$$(7, 2, 3) \in R \cdot X.$$

$$(-2, -8, 5) \in R. \quad \text{? HW.}$$

$$(11, 0, 6) \in R \quad \text{? HW.}$$

$$9 \pmod{4} = 1.$$

$$5 \pmod{4} = 1.$$

$$4 \overline{) \frac{1}{5}}$$

$$\frac{4}{1}$$

Ex4

470.

$$(PIA, PK169, Karachi, Dubai, 12:00am).$$

$$A \times N \times S \times D \times T.$$

$$A = \text{Set of Airlines.}$$

$$N = \text{Flight Number.}$$

$$S = \text{Starting Point.}$$

$$D = \text{Destination.}$$

$$T = \text{departure time.}$$

REPRESENTING RELATIONS.

MATRICES.

$R$

$$A \times B.$$

$$A = \{a_1, a_2, a_3, \dots, a_m\}.$$

$$B = \{b_1, b_2, b_3, \dots, b_n\}.$$

$$M_R = [m_{ij}].$$

$$m_{ij} = \begin{cases} 1 & \text{if } (a_i, b_j) \in R. \\ 0 & \text{if } (a_i, b_j) \notin R. \end{cases}$$

$$\text{rows} = |A| = m$$

$$\text{col} = |B| = n.$$

Ex1:-  
476.

$$A = \{ \overset{a_1}{1}, \overset{a_2}{2}, \overset{a_3}{3} \}.$$

$$B = \{ \overset{b_1}{1}, \overset{b_2}{2} \}.$$

$$R = \{(2, 1), (3, 1), (3, 2)\}.$$

$$M_R = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$$

$$M_{12} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \\ m_{31} & m_{32} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$$

