lecture 1o.   Relations.

Q:- why we left chapters.
Ans:-   Discrete Math metres.   Vs   Discrete Structures.
                 Counting Part.

Note:- Those who missed the class will take
        the lecture on teachable.

SET :- ① { } → Syntax.

        ② Collection of Distinct objects. ——→ Semantics.

        { Tomato, potato, lemon, juice } -
        { Tomato, potato, lemon, juice, Tomato }. X. Semantic Error.

        [Tomato, potato, lemon, juice] X Syntax Error.

    REVISION OF PREVIOUS CONCEPTS.
→ Finding Product of two Sets.
          $A = \{ a, b, c \}$ $B = \{ 1, 2 \}$.

    $A \times B = \{ (a, 1), (a, 2), (b, 1), (b, 2), (c, 1), (c, 2) \}$.

    → Cardinality of a Set
         Number of elements in a Set → Semantics. $-14 \div 4$.
         $|A|$ → Syntax.

                                        $4\sqrt{-14}$ HW.
        $|A| = 3$ $|A \times B| = |A| \times |B|$
        $|B| = 2$. $= 3 \times 2$
                                      $= 6$.

    BINARY RELATION.
        SET A and B. A binary relation from A to B
                      is a Subset of $A \times B$.

        $A = \{ [a], b, c \}$ $B = \{ 1, 2 \}$.

$$A \times B = \{ \boxed{(a,1)}, (a,2), (b,1), (b,2), (c,1), (c,2) \}.$$

$$pow(A) = \{ \phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{a,c\}, \{a,b,c\} \}.$$
$$pow(B) = \{ \phi, \{1\}, \{2\}, \{1,2\} \}.$$

$$|pow(A)| = 2^{|A|} = 2^3 = 8.$$
$$|pow(B)| = 2^{|B|} = 2^2 = 4.$$

Power Set :-   - the Set of all possible Subsets of a Set → Semantics.
Cardinality of Power set of a Set $|A| = 2^{|A|}.$

$$2^{|A \times B|} = 2^{|A| \wedge |B|} = 2^{3 \times 2} = 2^6 = 64.$$

$$pow(A \times B) = \boxed{\phi} \, \phi, \boxed{\{(a,1)\}}, \{(a,2)\}, \{(b,1)\}, \{(b,2)\} \, \{(c,1)\}, \{(c,2)\},$$
$$\{(a,1),(a,2)\}, \{(a,1),(b,1)\}, \{(a,1),(b,2)\} - - - -$$

! Complete it at Home.

$$\{\}$$

Ex3 P460:- $A = \{0,1,2\}$ $B = \{a,b\}.$

$$\{(0,a), (0,b), (1,a), (2,b)\}.$$

is it a Relation from $\boxed{A} \boxed{B}$ = True.
is it a Relation from $B \times A$ = ?

Ex4 P461:- $A = \{1,2,3,4\}$

$$R = \{(a,b) \mid a \text{ divides } b\}. \quad → \text{Set builder Notation.}$$

$$A \times A = \{ (1,1), (1,2), (1,3), (1,4),$$
$$(2,1), (2,2), (2,3), (2,4),$$
$$(3,1), (3,2), (3,3), (3,4),$$
$$(4,1), (4,2), (4,3), (4,4)\}.$$

$1 \div 2$
1 divides $2. = 2 \div 1.$

$$R = \{ (1,1), (1,2), (1,3), (1,4), (2,2), (2,4), (3,3), (4,4)\}$$

Ex5:- $A = \mathbb{Z} = \{-\infty, \cdots \quad +\infty\}.$

$$R_1 = \{ (a,b) \mid a \leq b \}. \qquad (1,1), (1,2), (2,1), (1,-1), (2,2)$$

Exo.   A = Z ...   (-∞).

$R_1 = \{(a,b) \mid a \leq b\}$. $(1,1), (1,2), (2,1), (1,-1), (2,2)$

$R_2 = \{(a,b) \mid a \geq b\}$

$R_3 = \{(a,b) \mid a = b \text{ or } a = -b\}$.

$R_4 = \{(a,b) \mid a = b+1\}$. ?

$R_5 = \{(a,b) \mid a+b \leq 3\}$. ?

PROPERTIES   OF   RELATION.

1- REFLEXIVE.   2- SYMETRIC   3- AntiSymmetric 4-Transitive.

REFLEXIVE :-   A Relation R on a Set A is reflexive.

if $\boxed{\forall_{a \in A} (a,a) \in R}$.

Ex7 :-   A = $\{1,2,3,4\}$.

$\overset{\text{⊤}}{(1,1)} \in R \wedge \overset{\text{⊤}}{(2,2)} \in R \wedge$
$(3,3) \in R \wedge (4,4) \in R$.
F.   ⊤

$R_1 = \{(1,1), (1,2), (2,1), (2,2), (3,4), (4,1), (4,4)\}$ ✗.

$R_2 = \{\emptyset\}$. ✗

$R_3 = \{(1,2), (2,1), (1,1)\}$. ✗.

R4 ?
R5 ?   Do it yourself.
R6 ?

$(-\infty, \infty) \in R \wedge$ - - - - -
$\wedge (1,1), (0,0), (2,2)$

Ex8   Do it yourself.

Ex9 :- $R = \{(a,b) \mid a \text{ divides } b\}$ $A = \dot{Z}$.
                    reflexive ✓. $Z \times \dot{Z}$.

→ Symmetric, AntiSymmetric. Transitive (Next class).