lecture 12:- Properties of Relations.

1- Reflexive.

2- Symmetric.

$\forall a,b \in A$ if $(a,b) \in R \boxed{\longrightarrow} (b,a) \in R$.

✓ F ✗

A , B

$R \subseteq A \times B$.

$\underline{|A \times B|} = |A| \times |B|$.

Subsets of $A \times B$.

$2^{|A \times B|} = 2^{|A| \times |B|}$

Ex7 P462. $A = \{1,2,3,4\}$.

$R_1 = \{(1,1), (1,2), (2,1), (2,2), (3,4), (4,1), (4,4)\}$ Not Symmetric.

a b. a b. a b. ✓ ✗.

if $(1,1) \in R \rightarrow (1,1) \in R$ ✓.

if $(1,2) \in R \rightarrow (2,1) \in R$ ✓.

if $(2,1) \in R \rightarrow (1,2) \in R$ ✓

$R_4 = \{(1,2)\}$

$R_2 = \{\ \}$. Symmetric = ?

$R_3 = \{(1,1)\}$.

3- Anti Symmetric. if $\forall a,b \in A$ if $\overset{p}{\underset{F}{\boxed{(a,b) \in R \wedge (b,a) \in R}}} \rightarrow \overset{q}{\boxed{a = b}}$

Ex7 P462. $A = \{1,2,3,4\}$.

$R_1 = \{(1,1), (1,2), (2,1), (2,2), (3,4), (4,1), (4,4)\}$. ✗ Anti Symmetric.

a b. a b.

$\underbrace{(1,1)^{\in R} \wedge (1,1) \in R}_{T} \rightarrow \underbrace{1 = 1}_{T}$.

$(1,2) \in R \wedge (2,1) \in R \rightarrow 1 \neq 2$.

$\underbrace{\overset{T}{(1,2) \in R} \wedge \overset{F}{(2,1) \notin R}}_{F}$

$R = \{\ \}$. ✓

$R = \{(1,1), (1,2)\}$.

a b

Do the remaining Examples on P462. Yourself.

Ex12. P463:- Divides Relation on Set of the Integers

1) Symmetric.

2) Antisymmetric.

$\forall a,b \in A$ if $(a,b) \in R \wedge (b,a) \in R \rightarrow a = b$

1) Symmetric .
2) Antisymmetric .

$\forall a,b \in A$ if $(a,b) \in R \wedge (b,a) \in R \rightarrow a = b$

$R = \{(a,b) \mid a \text{ divides } b\}$ . $Z^+$.

$\forall a,b \in A$ if $(a,b) \in R \longrightarrow (b,a) \in R$ .

Symmetric :- $\forall a,b \in A$ if $(a,b) \in R \longrightarrow (b,a) \in R$.

a divided by $b = \frac{a}{b}$. $a \div b$

$\forall a,b \in Z^+$ if a divides b $\longrightarrow$ b divides a.

a divides $b = b \div a = \frac{b}{a}$.

$(1,2) \in R \longrightarrow (2,1) \notin R$ x.
$(2,4) \in R. \longrightarrow (4,2) \notin R$ x.

Anti Symmetric :- $\forall a,b \in A$ if $(a,b) \in R \wedge (b,a) \in R \rightarrow a = b$.

$\forall a,b \in Z^+$ if a divides b $\wedge$ b divides a $\longrightarrow a = b$.

It is Anti Symmetric. but not Symmetric.

4- Transitive. $\forall a,b,c \in A$ if $(a,b) \in R \wedge (b,c) \in R \longrightarrow (a,c) \in R$.

Ex15 P464 :- $R = \{(a,b) \mid a \text{ divides } b\}$ $Z^+$.

$\forall abc \in A$ if $(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R$.

$\forall abc \in Z^+$ if a divides b $\wedge$ b divides c $\rightarrow$ a divides c.

Ex7 $A = \{1,2,3,4\}$.

$R = \{(1,1),(1,2),(2,1),(2,2),(3,4),(4,1),(4,4)\}$. X ,

a b. b a.

$R = \{\}$. $R = \{(1,1),(1,2)\}$.

$R = \{(a,b) \mid a > b\}$. $A = Z$ .

1- $\forall a \in A$ $(a,a) \in R$,

$\forall a \in Z$ $a \geq a$ it is true for all $a \in A$ .

2- $\forall a,b \in A$ if $(a,b) \in R \rightarrow (b,a) \in R$.

$\forall a,b \in Z$ if $a \geq b \rightarrow b \geq a$ .

$(2,1)$
$2 \geq 1 \rightarrow 1 \geq 2$. X.

3. $\forall a,b \in A$ if $(a,b) \in R \wedge (b,a) \in R \rightarrow a = b$.

$\forall a,b \in Z$ if $a \geq b \wedge b \geq a \rightarrow a = b$

3. $\forall a,b \in A$ if $(a,b)\in R \wedge (b,a)\in R \rightarrow a=b.$

$\forall a,b \in Z$ if $a \geq b \wedge b \geq a \rightarrow a=b.$

4- $\forall a,b,c \in A$ if $(a,b)\in R \wedge (b,c)\in R \rightarrow (a,c)\in R.$

$\forall a,b,c \in Z$ if $a \geq b \wedge b \geq c \rightarrow a \geq c.$

$3 \geq 2 \wedge 2 \geq 1 \rightarrow 3 \geq 1$