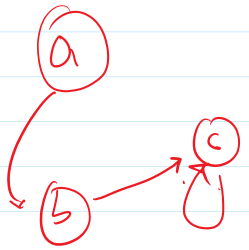
Lecture 17:-

Definition 485: R is a Relation on A.

the Connectivity Relation R^* Consists of pairs (a,b) such that $\exists$ a path from a to b in R.



$R = \{(a,b), (c,c), (b,c)\}$.

$R^* = \{(a,b), (c,c), (b,c), (a,c)\}$.

$$R^* = \bigcup_{n=1}^{\infty} R^n.$$

Ex4 485:- R set of all people.

$R = \{(a,b) \mid a \text{ has met } b\}$.

What is $R^n = ?$ $(n \geq 2)$. What is R^*.

Solution:- $R^2 = R \circ R$.

Revision.

R (a,b) $a \in A, b \in B$.

R S (b,c) $b \in B, c \in C$.

$(a,c) \in S \circ R$ $\exists b$ $(a,b) \in R \wedge (b,c) \in S$.

$(a,c) \in R \circ R$ $\exists b.$ $(a,b) \in R \wedge (b,c) \in R$

R^2

$(a,b) \in R^2$ $\exists x_1$ $(a,x_1) \in R \wedge (x_1,b) \in R$.

$(a,b) \in R^3$ $\exists x_1, x_2$ $(a,x_1) \in R \wedge (x_1,x_2) \in R \wedge (x_2,b) \in R$.

$\underline{(a,b) \in R^n}$ $\exists x_1, x_2, \ldots, x_{n-1}$ $(a,x_1) \in R \wedge$
$(x_1,x_2) \in R \wedge$
$\vdots$
$(x_{n-2}, x_{n-1}) \in R \wedge$
$(x_{n-1}, b) \in R$.

R^*.

Ex6 486:- R Set of states in United States.

$R = \{(a,b) \mid a \text{ and } b \text{ has a Common border}\}$.

What is $R^n = ?$ HW

$R^* = ?$ HW.

theorem :- the Connectivity Relation R^* is transitive.
the transitive Closure equals the Connectivity Relation R^*.

WARSHAL ALGO:

EQUIVELENCE RELATION.

A relation which is
- Reflexive.
- Symmetric
- Transitive.

$(a,b) \in R$.

$a \sim_R b$.

Ex 1 / 494 $R = \{(a,b) \mid a = b \text{ or } a = -b\}$. $A = \mathbb{Z}$.
is R Equivalence.

Reflexive $\forall a \in A$ $(a,a) \in R$.
$\forall a \in \mathbb{Z}$ $a = a$ or $a = -a$.

Symmetric $\forall a,b \in A$ if $(a,b) \in R \rightarrow (b,a) \in R$.
$\forall a,b \in \mathbb{Z}$ if $a = b$ or $a = -b$ $\rightarrow$ $b = a$ or $b = -a$.

Transitive $\forall a,b,c \in A$ if $(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R$.
$\forall a,b,c \in \mathbb{Z}$ if $a = b$ or $a = -b$ $\wedge$ $b = c$ or $b = -c$ $\rightarrow a = c$ or $a = -c$

Ex 2. :- / 494 $R = \{(a,b) \mid a - b \in \mathbb{Z}\}$. $A = \mathbb{R}$.

Reflexive $\forall a \in A$ $(a,a) \in R$.
$\forall a \in \mathbb{R}$. $a - a \in \mathbb{Z}$ ✓.

Symmetric $\forall a,b \in A$ if $(a,b) \in R \rightarrow (b,a) \in R$.
$\forall a,b \in \mathbb{R}$. if $a - b \in \mathbb{Z}$ $\rightarrow$ $b - a \in \mathbb{Z}$. ✓

Transitive $\forall a,b,c \in A$ if $(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \in R$.
$\forall a,b,c \in$ if $a - b \in \mathbb{Z} \wedge b - c \in \mathbb{Z} \rightarrow a - c \in \mathbb{Z}$ ✓.

$= c$

Hence Equivalence Relation.

Ex 4 :- p494. $R = \{(a,b) \mid a \equiv b \bmod m\}$. $m > 1$. $m > \mathbb{Z}^+$. $A = \mathbb{Z}$.

Reflexive $\forall a \in A$ $(a,a) \in R$.
$\forall a \in \mathbb{Z}$ $a \equiv a \bmod m$. ✓

Symmetric $\forall a,b \in A$ if $(a,b) \in R \to (b,a) \in R$.
$\forall a,b \in \mathbb{Z}$ if $a \equiv b \bmod m \to b \equiv a \bmod m$. ✓

Transitive $\forall a,b,c \in A$ if $(a,b) \in R \wedge (b,c) \in R \to (a,c) \in R$.
$\forall a,b,c \in \mathbb{Z}$ if $a \equiv b \bmod m \wedge b \equiv c \bmod m \to$ ✓.
$a \equiv c \bmod m$.

Ex 6 :- 495 $R = \{(a,b) \mid a \text{ divides } b\}$. $A = \mathbb{Z}$.

Reflexive $\forall a \in A$ $(a,a) \in R$.
$\forall a \in \mathbb{Z}$ a divides a ✓

Symmetric $\forall a,b \in A$ if $(a,b) \in R \to (b,a) \in R$.
$\forall a,b \in \mathbb{Z}$ if a divides b $\to$ b divides a ✗.

Transitive $\forall a,b,c \in A$ if $(a,b) \in R \wedge (b,c) \in R \to (a,c) \in R$.
$\forall a,b,c \in \mathbb{Z}$ if

Ex 7 :- 495 $R = \{(x,y) \mid |x-y| < 1\}$. $A = \mathbb{R}$.

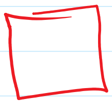
Reflexive $\forall a \in A$ $(a,a) \in R$.
$\forall a \in \mathbb{Z}$ $|a-a| < 1$. ✓

Symmetric $\forall a,b \in A$ if $(a,b) \in R \to (b,a) \in R$.
$\forall a,b \in \mathbb{Z}$ if $|a-b| < 1 \to |b-a| < 1$. ✓

Transitive $\forall a,b,c \in A$ if $(a,b) \in R \wedge (b,c) \in R \to (a,c) \in R$.
$\forall a,b,c \in \mathbb{Z}$ if $|a-b| < 1 \wedge |b-c| < 1 \to |a-c| < 1$. $a = c$
$|0.4 - 1.3| < 1 \wedge |1.3 - 2.0| < 1 \to |0.4 - 2.0| \not< 1$

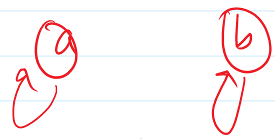
Not Equivalence Relation.

Checking for Equivalence Relation Given Graphs.



✓

✗.



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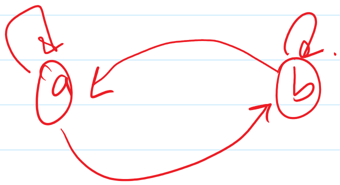

✓

Checking Equivalence Relation Given Matrix.

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$ ✗.

$[\ \ ]_{0\times 0}$ ✓

Ex P500 - 503. Q1-30.