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MOTOWOTOFOS H/W C/W Dated:...../...../20.....

DOP-8147

Assignment 1

Due 8th October, 2021

Sec-3B

Q No 1: The problems below relate to inhabitants of an island on which there are three kinds of people: knights who always tells lie, knaves who always speak truth and spies (called normals by smullyan [sm78]) who can either lie or tell the truth. You encounter three people, A, B and C, You know two of them are knights and one is a knave. For each of the situations mentioned below, if possible, determine whether there is a unique solution and determine who the knights and knave are. When There is no ~~no~~ no unique solution, list all possible solutions or state that there are no solutions

⇒ There will be total of nine propositions that you will need to assume. Call these propositions as $p_1, p_2, p_3, q_1, q_2, q_3$ and r_1, r_2, r_3 where p's are the proposition corresponding to persons A, q's are the proposition corresponding

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to person B and q's are the propasition corrasponding to person C.

(a) A says "C is the knowe"
B says "A is knight"
C says "I am the spy"

(b) A says "I am the knight"
B says "I am the knowe"
C says "B is the knight"

(c) A says "I am the knowe"
B says "I am the knowe"
C says "I am the knowe"

(d) A says "I am the knight"
B says "A is telling truth"
C says "I am the spy"

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(Solution)

(a) A says "C is the knave"
B says "A is the knight"
C says "I am the spy"

There will be total 9 propositions

p_1 = A is knight
p_2 = A is knave
p_3 = A is spy

q_1 = B is knight
q_2 = B is knave
q_3 = B is spy

r_1 = C is knight
r_2 = C is knave
r_3 = C is spy.

(Now we will make conditions)

(i) r_2
(ii) p_1
(iii) r_3

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Now we will make the truth table there will be 27 possible cases but we take only those in which two of them are knight and one is knave.

K	K	K	K
K	K	K	N
K	K	K	S
K	K	N	K
K	K	N	N
K	K	N	S
K	K	S	K
K	K	S	N
K	K	S	S
K	N	K	K
K	N	K	N
K	N	K	S
K	N	N	K
K	N	N	N
K	N	N	S
K	N	S	K
K	N	S	N
K	N	S	S
K	S	K	K
K	S	K	N
K	S	K	S
K	S	N	K
K	S	N	N
K	S	N	S
K	S	S	K
K	S	S	N
K	S	S	S
N	K	K	K
N	K	K	N
N	K	K	S
N	K	N	K
N	K	N	N
N	K	N	S
N	K	S	K
N	K	S	N
N	K	S	S
N	N	K	K
N	N	K	N
N	N	K	S
N	N	N	K
N	N	N	N
N	N	N	S
N	N	S	K
N	N	S	N
N	N	S	S
N	S	K	K
N	S	K	N
N	S	K	S
N	S	N	K
N	S	N	N
N	S	N	S
N	S	S	K
N	S	S	N
N	S	S	S
S	K	K	K
S	K	K	N
S	K	K	S
S	K	N	K
S	K	N	N
S	K	N	S
S	K	S	K
S	K	S	N
S	K	S	S
S	N	K	K
S	N	K	N
S	N	K	S
S	N	N	K
S	N	N	N
S	N	N	S
S	N	S	K
S	N	S	N
S	N	S	S
S	S	K	K
S	S	K	N
S	S	K	S
S	S	N	K
S	S	N	N
S	S	N	S
S	S	S	K
S	S	S	N
S	S	S	S

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Case (i)

A	B	C
Knight	Knight	knave

(i) $\gamma_2 = F$
(ii) $p_1 = F$
(iii) $\gamma_3 = T$

$p_1 = T$	$p_2 = F$	$p_3 = F$
$q_1 = T$	$q_2 = F$	$q_3 = F$
$\gamma_1 = F$	$\gamma_2 = T$	$\gamma_3 = T$

⇒ "Now we will check the conditions by putting the values of the γ_2, p_2, γ_3. in eq (i), ii, iii.

(i) $\gamma_2 = F$
$T = F$

This case does not holds because $T \neq F$.

Case (ii)

A	B	C
Knight	Knave	knight

(i) $\gamma_2 = F$
(ii) $p_1 = T$
(iii) $\gamma_3 = F$

$$\begin{bmatrix} p_1 = T & p_2 = F & p_3 = F \\ q_1 = F & q_2 = T & q_3 = F \\ \gamma_1 = T & \gamma_2 = F & \gamma_3 = F \end{bmatrix}$$

Now put the values in the equation.

(i) $F = F$ ✓

(ii) $T = T$ ✓

(iii) $F = F$ ✓

Hence "A is Knight", "B is Knave & and "C is knight".

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case (iii)

A	B	C
Knave	knight	knight

(i) $r_2 = T$

(ii) $p_1 = F$

(iii) $r_3 = F$

$p_1 = F$	$p_2 = T$	$p_3 = F$
$q_1 = T$	$q_2 = F$	$q_3 = F$
$r_1 = T$	$r_2 = F$	$r_3 = F$

(i) $F = T$ X

This case does not holds.

b) A says "I am knight"
B says "I am knave"
C says "B is knight"

Now we will make the 9 propositions.

p1 = A is knight
p2 = A is knave
p3 = A is spy

q1 = B is knight
q2 = B is knave
q3 = B is spy

r1 = C is knight
r2 = C is knave
r3 = C is spy.

"Now we will make the equations"

(i) p1
(ii) q2
(iii) q1

Now we will make its truth table.

K	K	K
K	K	N
K	K	S
K	N	K
K	N	N
K	N	S
K	S	K
K	S	N
K	S	S
N	K	K
N	K	N
N	K	S
N	N	K
N	N	N
N	N	S
N	S	K
N	S	N
N	S	S
S	K	K
S	K	N
S	K	S
S	N	K
S	N	N
S	N	S
S	S	K
S	S	N
S	S	S

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Case (i)

A Knight B Knight C knave

(i) $p_1 = F$
(ii) $q_2 = F$
(iii) $q_1 = T$

$p_1 = T$	$p_2 = F$	$p_3 = F$
$q_1 = T$	$q_2 = F$	$q_3 = F$
$r_1 = F$	$r_2 = T$	$r_3 = F$

(i) $T = F$

This case does not holds

$T \neq F$

M□T□W□T□F□S H/W□-C/W□ Dated:...../...../20.....

case (ii)

A	B	C
knight	knave	knight

(i) $p_1 = F$
(ii) $q_2 = T$
(iii) $q_1 = F$

$p_1 = T$	$p_2 = F$	$p_3 = F$
$q_1 = F$	$q_2 = T$	$q_3 = F$
$r_1 = T$	$r_2 = F$	$r_3 = F$

(i) $T = F$

This case does not hold

$T \neq F$ ✗

case iii

A knave B knight C knight

$p1 = \overline{T}$

$q\delta = F$

$q1 = F$

$$\begin{bmatrix} p1=F & p\delta=\overline{T} & p3=\overline{F} \\ q1=\overline{T} & q\delta=F & q3=F \\ \delta1=\overline{T} & \delta\delta=F & \delta3=F \end{bmatrix}$$

(i) $F = \overline{T}$

This case does not holds becase

$F \neq \overline{T}$

M□T□W□T□F□S H/W□-C/W□ Dated:...../...../20.....

c) A says "I am knave"
B says "I am knave"
C says "I am knave"

ons.

⇒ Now we will make its 9 propositions.

p_1 = A is knave
p_2 = A is knight
p_3 = A is spy

q_1 = B is knave
q_2 = B is knight
q_3 = B is spy

r_1 = C is knave
r_2 = C is knight
r_3 = C is spy.

⇒ Now we will make its equations.

(i) p_1
(ii) q_1
(iii) r_1

M□T□W□T□F□S H/W□-C/W□ Dated:...../...../20.....

Case (i)

A	B	C
Knight	Knight	knave

(i) $p_1 = F$
(ii) $q_1 = F$
(iii) $r_1 = T$

$p_1 = F$	$p_2 = T$	$p_3 = F$
$q_1 = F$	$q_2 = T$	$q_3 = F$
$r_1 = T$	$r_2 = F$	$r_3 = F$

(i) $F = F$ ✓

(ii) $F = F$ ✓

(iii) $\overline{T} = \overline{T}$ ✓

Hence A is knight, B is knight and C is knave.

case (ii)

A	B	C
Knight	knowe	knight

(i) $p1 = F$
(ii) $q1 = T$
(iii) $r1 = F$

$p1=F$	$p2=T$	$p3=F$
$q1=T$	$q2=F$	$q3=F$
$r1=F$	$r2=T$	$r3=F$

(i) $F = F$ ✓

(ii) $T = F$ ✓

(iii) $F = F$ ✓

Hence "A is knight", "B is knowe" and "C is knight"

M□T□W□T□F□S H/W□-C/W□ Dated:...../...../20.....

case (iii)

A	B	C
Knave	knight	knight

$p_1 = T$
$q_1 = F$
$r_1 = F$

$p_1 = T$	$p_2 = F$	$p_3 = F$
$q_1 = F$	$q_2 = T$	$q_3 = F$
$r_1 = F$	$r_2 = T$	$r_3 = F$

(i) $T = T$ ✓

(ii) $F = F$ ✓

(iii) $F = F$

Hence A is knave, B is knight and C is knight.

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d) A says "I am knight"
B says "A is telling the truth"
C says "I am spy"

Now we will make 9 propositions.

p_1 = A is knight
p_2 = A is knave
p_3 = A is spy

q_1 = B is knight
q_2 = B is knave
q_3 = B is spy

r_1 = C is knight
r_2 = C is knave
r_3 = C is spy

Now we make equations.

(i) p_1
(ii) p_1
(iii) r_3

Case (i)

A	B	C
Knight	knight	knave

(i) $p_1 = F$
(ii) $p_1 = F$
(iii) $\gamma_3 = T$

$p_1 = T$	$p_2 = F$	$p_3 = F$
$q_1 = T$	$q_2 = F$	$q_3 = F$
$r_1 = F$	$r_2 = T$	$r_3 = F$

(i) $T = F$ X

This case does not hold.

M T O W T F S

case (ii)

A Knight B knave C knight

$p_1 = F$

$p_1 = T$

$\gamma_3 = F$

$$\begin{bmatrix} p_1=T & p_2=F & p_3=F \\ q_1=F & q_2=T & q_3=F \\ \gamma_1=T & \gamma_2=F & \gamma_3=F \end{bmatrix}$$

(i) $T = F$ X

This case does not hold.

M□T□W□T□F□S S/W□-C/W□ Dated:...../...../20.....

Case (iii)

A	B	C
Knave	knight	knight

(i) $p_1 = T$

(ii) $p_1 = F$

(iii) $\gamma_3 = F$

$$\begin{bmatrix} p_1=F & p_2=T & p_3=F \\ q_1=T & q_2=F & q_3=F \\ \gamma_1=T & \gamma_2=F & \gamma_3=F \end{bmatrix}$$

(i) $F = T$

Hence this case does not hold.