lecture 15 :-

Ex 477

$A = \{a_1, a_2, a_3\}$. $B = \{b_1, b_2, b_3, b_4, b_5\}$

$$M = \begin{array}{c} \\ a_1 \\ a_2 \\ a_3 \end{array} \begin{array}{c} \begin{array}{ccccc} b_1 & b_2 & b_3 & b_4 & b_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

KEY OBSERVATION
RELATION $\longleftrightarrow$ MATRIX.
Interchangeable.

$R = \{(a_1, b_2), (a_2, b_1), (a_2, b_3), (a_2, b_4), (a_3, b_1), (a_3, b_3), (a_3, b_5)\}$.

PROPERTIES OF RELATIONS (when we have MATRIX).

1- REFLEXIVE $\forall a \in A$ $(a, a) \in R$. $A = \{a_1, a_2, \dots a_n\}$.
$\forall i$ $m_{ii} = 1$. $i = 1, 2, \dots n$.

2- Symmetric $\forall a, b \in A$ if $(a, b) \in R \rightarrow (b, a) \in R$. $A = \{a_1, a_2, a_3 \dots a_n\}$.
$\forall a_i, a_j \in A$ if $(a_i, a_j) \in R \rightarrow (a_j, a_i) \in R$.

$\forall i, j \in \{1, 2, 3 \dots n\}$. if $m_{ij} = 1 \rightarrow m_{ji} = 1$.

$$\begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 \end{bmatrix}$$

$(M^T = M)$ Symmetric.

3- Anti Symmetric :- $\forall a, b$ if $(a, b) \in R \land (b, a) \in R \rightarrow a = b$.
$\forall a_i, a_j$ if $(a_i, a_j) \in R \land (a_j, a_i) \in R \rightarrow a_i = a_j$. $A = \{a_1, a_2, \dots a_n\}$.
$\forall i, j \in \{1, 2, 3 \dots n\}$ if $m_{ij} = 1 \land m_{ji} = 1 \rightarrow i = j$

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

4- Transitive :- $\forall a, b, c \in A$ if $(a, b) \in R \land (b, c) \in R \rightarrow (a, c) \in R$.
$\forall a_i, a_j, a_k \in A$ if $(a_i, a_j) \in R \land (b_j, a_k) \in R \rightarrow (a_i, a_k) \in R$.

$\forall i j k \in \{1, 2, 3 \dots n\}$ if $m_{ij} = 1 \land m_{jk} = 1 \rightarrow m_{ik} = 1$.

COMPLEMENT
$\overline{R}$ = Subtract ones matrix from M_R.

INVERSE

R^{-1} = By taking transpose.

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$$M_{R_1} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \qquad M_{R_2} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$R_1 \cup R_2 = ?$

$$M_{R_1 \cup R_2} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$R_1 \cap R_2$.

$$M_{R_1 \cap R_2} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$M_{R_1 - R_2}$ = HW.

$M_{R_2 - R_1}$ = HW.

COMPOSITE.

R	(a,b)	$A \times B$	$a \in A$	$b \in B$
S	(b,c)	$B \times C$.	$b \in B$	$c \in C$.

$S \circ R$ (a,c) if $\exists_b$ $(a,b) \in R \wedge (b,c) \in S$.

$M_R = [r_{ij}]$ $m \times n$.

$M_S = [s_{ij}]$. $n \times p$.

$M_{S \circ R} = [t_{ij}]$. $m \times p$.

$S \circ R$. (a_i, c_j) if $\exists_{b_K}$ $(a_i, b_k) \in R \wedge (b_k, c_j) \in S$.

$t_{ij} = 1$ if $\exists_k$ $r_{ik} = 1 \wedge s_{kj} = 1$. for some k.

HW.

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$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad M_S = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$M_{S \circ R} = \begin{bmatrix} t_{11} & t_{12} & t_{13} \\ t_{21} & t_{22} & t_{23} \\ t_{31} & t_{32} & t_{33} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ HW & HW & HW \\ HW & HW & HW \end{bmatrix}$$

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