Lecture 8:- Principle of Resolution.

Literal:- A Variable of its negation. p $\neg p$

Clause:- A disjunction of literals. → $p \vee q$ $\neg p \vee \neg q$ $p \vee \neg q$ $\neg p \vee q$ $p \vee \neg q \vee r$ ...

" Given a Set of clauses a deduction of C from S is a finite sequence $C_1, C_2, \ldots, C_k$ of clauses.
A deduction of $\square$ empty clause is called refutation or a proof of S".

$$\begin{array}{ll} p_1 & \rightarrow C_1 \\ p_2 & \rightarrow C_2 \\ \vdots & \vdots \\ p_n & C_k \\ \hline \therefore C. & \neg C_{k+1} \end{array}$$

$$\begin{array}{l} p \vee q \\ \neg p \vee r \\ \hline \therefore p \vee r. \end{array}$$

Ex P65. p_1 P $\quad$ p_2 $P \rightarrow Q$ $\quad$ C $\therefore Q.$

(C1) P ⌐ $\quad$ (C2) $\neg P \vee Q$ ⌐ $\quad$ (C3) $\neg Q$ ⌐

$P \vee Q.$ $\quad$ $\neg P \vee r.$ $\quad$ $\therefore Q \vee r.$

from (C1, C3) Q — (4) ⌐.

from (4, C3) $\square$

$P \rightarrow Q = \neg P \vee Q.$

Ex 11-P65. p_1 $T \rightarrow M \vee E$ $\quad$ p_2 $S \rightarrow \neg E$ $\quad$ p_3 $T \wedge S.$

C1 → $\neg T \vee M \vee E$ ⌐

C2 → $\neg S \vee \neg E$ ⌐

C3 → T ⌐

p3 $T \wedge S$.

c. $\therefore M$.

C3 → T ✓

C4 → S ✓

C5 → ¬M. ✓

from C1, C2 $\neg T \vee M \vee \neg S$. — 6 ✓

fom C3, 6. $M \vee \neg S$. — 7 ✓

fom C4, 7 M — 8. ✓

fom C5, 8 □

Ex6 P-62.

P1 $\neg p \wedge q$

p2 $\gamma \rightarrow p$

p3 $\neg\gamma \rightarrow s$

p4 $s \rightarrow t$

c. $\therefore t$.

C1 $\neg p$. ✓

C2. q

C3 $\neg\gamma \vee p$. ✓

C4 $\gamma \vee s$ ✓

C5 $\neg s \vee t$ ✓

C6 $\neg t$ ✓

from C1, C3 $\neg\gamma$ (7) ✓

from C4, 7. s (8) ✓

from C5, 8 t (9) ✓

from C6, 9 □ (10)

Ex7: P 63:-

P1 $p \rightarrow q$

P2 $\neg p \rightarrow \gamma$

P3 $\gamma \rightarrow s$

c. $\therefore \neg q \rightarrow s$.

C1 $\neg p \vee q$ ✓

C2 $p \vee \gamma$ ✓

C3 $\neg\gamma \vee s$ ✓

C4 $\neg q$ ✓

C5 $\neg s$. ✓

from C1, C2 $q \vee \gamma$ — 6 ✓

from C1,C2 q,VR — 6 ✓

from C3,6 q,VS — 7 ✓

from C4,7 S — 8 ✓

from C5,8 □

Ex 9 P65.

P1 L → A

P2 E → ¬I

P3 A → E

C. ∴ L → ¬I.

C1 ¬L VA ✓

C2 ¬E V ¬I ✓

C3 ¬A V E ✓

C4 L ✓

C5 I ✓

from C1,C3 ¬L V E — 6 ✓

from C2,6 ¬I V ¬L — 7. ✓

from C4,7 ¬I — 8 ✓

from C5,8 □