lecture #5 :-

A says "I am Knight" → "A is a Knight"

B says " I am Knight". → " B is a knight".

p = —— $\neg p$ = ——

q = —— $\neg q$ = ——

CASE 1 :- Knight, Knight.

$p = T$ $p = T$ $\neg p = F$

$q = T$ $q = T$ $\neg q = F$

$T = T$

$T = T$.

Predicates :-

Quantifiers :

$P(x) = ?$ → Predicate.

$p(x) = x > 3$.

↓

Subject.

$x = \{1, 2, 3, 4\}$.

$P(1) = 1 > 3 = F$.

$P(2) = 2 > 3 = F$

$P(3) = 3 > 3 = F$

$P(4) = 4 > 3 = T$.

Ex 1. P 31. $P(x) = x > 3$.

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$P(2)$ $P(4)$.

$P(2) = 2 > 3 = F$

$P(4) = 4 > 3 = T$.

Ex 2: P 31.

$A(x) =$ Computer x is under attack by an intruder.

out of all Computers only CS2 and Math1 are under attack.

$A(CS1) = ? = $ Computer CS1 is under attack. $= F$

$A(CS2) = ? = ?$

$A(Math1) = ? = ?$

do it.

Ex 3: p 31: $Q(x,y) = (x = y+3)$

$Q(1,2) = 1 = 2+3$

$1 = 5 = F$.

$Q(3,0) = ?$ Do it.

Ex 4: P 31: Homework.

Ex 5: P 31: $R(x,y,z) = x+y=z$.

$R(1,2,3) = 1+2=3$

$\Rightarrow 3 = 3 \quad = T.$

Quantifiers.

Universal Quantifier:- $\forall$

all of the **for all**, for every, for each.

$\forall_x\, p(x).$ $\qquad p(x) = x > 3.$

$\forall_x\, p(x) = \boxed{p(1) \land p(2) \land p(3) \land p(4)} \quad x = \{1,2,3,4\}.$

Existential. $\exists x\, p(x)$

for some.

$p(x) = \quad x > 3 \quad x = \{1,2,3,4\}.$ atleast one.

there is

there exist.

$\exists_x p(x). = p(1) \lor p(2) \lor p(3) \lor p(4)$

Ex8:- P33. $\qquad p(x) = x+1 > x.$

$\forall_x p(x). = T \qquad x \in \mathbb{R}.$

Ex9:- P34. $\qquad p(x) = x < 2$

$x \in \mathbb{R}.$

Counter Example:-

{An Example which make $\forall$ false}

$\forall x\, P(x) = ? = F.$

Ex 10 – 12 :- Do it.

Ex 13 :- P 34 :- $\forall x (x^2 \geq x)$. $x \in \mathbb{R}$.

let $P(x) = x^2 \geq x$.

$\forall x P(x) = \forall x (x^2 \geq x)$.

$x = 0.5$

$(0.5)^2 \geq 0.5$

$0.25 \not\geq 0.5$

$= F$.

Ex 14 : P 35 $P(x) = x > 3$. $x \in \mathbb{R}$.

$\exists x P(x) = ?$

$P(4) = T$

$P(4) = 4 > 3$.

$\exists x P(x) = T$

Ex 16 :- P 35. $P(x) = x^2 > 10$. $x \in \{1, 2, 3, 4\}$

$\exists x P(x) = ?$

$\exists x P(x) = P(1) \vee P(2) \vee P(3) \vee P(4)$

$= F \vee F \vee F \vee T = T$

Ex 17 :- $\forall x < 0 \, (x^2 > 0)$. $x \in \mathbb{R}$

let $P(x) = (x^2 > 0)$

$\forall x\, P(x) = \forall x\, (x^2 > 0)$. $x < 0 \wedge x \in \mathbb{R}$.

$$\forall x\, P(x) = T$$

P37:- Important Property.

$$\forall x\,(P(x) \wedge Q(x)) = \forall x\, P(x) \wedge \forall x\, Q(x).$$

Negating Quantifiers. $x \in \{1,2,3,\ldots N\}$.

$$\neg \forall x P(x) = \neg\left(P(1) \wedge P(2) \wedge P(3) \wedge \cdots\cdots \wedge P(N)\right)$$

$$= \neg P(1) \vee \neg P(2) \vee \neg P(3) \vee \cdots\cdots \vee \neg P(N).$$

$$= \exists x\, \neg P(x).$$

$$\neg \exists x P(x) = \neg\left(P(1) \vee P(2) \vee P(3) \vee \cdots\cdots \vee P(N)\right)$$

$$= \neg P(1) \wedge \neg P(2) \wedge \neg P(3) \wedge \cdots\cdots \wedge \neg P(N).$$

$$= \forall x\, \neg P(x).$$

$$\neg \forall x\, P(x) = \exists x\, \neg P(x).$$

$$\neg \exists x\, P(x) = \forall x\, \neg P(x).$$

$$\forall x\, \forall y\, \exists z\; P(x,y,z).$$ find Negation.

$$\neg\left(\forall x\, \forall y\, \exists z\, P(x,y,z)\right).$$

$$\neg \forall x\, \underbrace{\left(\forall y\, \exists z\; P(x,y,z)\right)}_{P(x)}.$$

$$\neg \forall x\, P(x).$$

$$= \exists x\, \neg P(x).$$

$$\exists x \, \neg \, \underbrace{\forall y}\big(\underbrace{\exists z \, P(x,y,z)}_{P(x)} \big).$$

$$\exists x \, \exists y \, \underbrace{\neg \exists z} \, \underbrace{P(x,y,z)}_{P(x)}.$$

$$\neg \exists_x P(x).$$
$$= \forall x \, \neg P(x).$$

$$\exists x \exists y \, \forall z \, \neg P(x,y,z).$$

$$x, y, z = \{0, 1\}.$$

→ Processing of English Statements including Quantifiers.

→. Values inside Predicates with Quantifiers.