lecture 14:-

$R = \{(a,b) \mid a = b \text{ or } a = -b\}$. $A = \mathbb{Z}$.

Ex 5 / 463.

Symmetric $\forall a,b \in A$ if $(a,b) \in R \rightarrow (b,a) \in R$.

class Question. $\forall a,b \in \mathbb{Z}$ if $a = b$ or $a = -b \rightarrow b = a$ or $b = -a$.

SOME ADDITIONAL OPERATIONS ON RELATIONS.

$\bar{R} = \{(a,b) \mid (a,b) \notin R\}$.

$\bar{R} = A \times A - R$.

$R^{-1} = \{(b,a) \mid (a,b) \in R\}$.

$= \{(a,b) \mid (b,a) \in R\}$.

$= \{(a,b) \mid b < a\}$.

Ex: $A = \{1,2,3,4\}$.

$R = \{(a,b) \mid a < b\}$.

$R = \{(1,2),(1,3),(1,4),(2,3),(2,4),(3,4)\}$.

$\bar{R} = \{(1,1),(2,1),(2,2),(3,1),(3,2),(3,3),(4,1),(4,2),(4,3),(4,4)\}$.

$R^{-1} = \{(2,1),(3,1),(4,1),(3,2),(4,2),(4,3)\}$.

$A \times A = \{(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(2,3),(2,4),(3,1),(3,2),(3,3),(3,4),(4,1),(4,2),(4,3),(4,4)\}$.

N-ARY RELATIONS. R

$A \times A \times A$ $\quad |A| \times |A| \times |A| = |A|^3 = 2^{n^3}$. $\quad A = n$.

How many Binary Relations. $A = \{1,2\}$.

How many ternary $A \times A \times A = \{(1,1,1),(1,1,2),(1,2,1),(1,2,2),(2,1,1),(2,1,2),(2,2,1),(2,2,2)\}$.

How many 4-ary

Ex 1 / 469: R $\quad \mathbb{N} \times \mathbb{N} \times \mathbb{N}$.

$R = \{(a,b,c) \mid a < b < c\}$.

$(1,2,3) \in R$?

$(2,4,3) \notin R$

Ex 2 / 469: R $\quad \mathbb{Z} \times \mathbb{Z} \times \mathbb{Z}$.

$R = \{(a,b,c) \mid b = a+k, c = a+2k\}$. $k \in \mathbb{Z}$.

$(1,3,5) \in R$.

$(1,3,5) \in R$.
$\downarrow\ \downarrow\ \downarrow$
$a\ \ b\ \ c$.

$3 = 1+k \Rightarrow k = 2$.
$5 = 1+2\cdot 2 \Rightarrow 5 = 1+4 \Rightarrow 5=5$.

$(2,5,9) \in R$?

$9 \equiv 5 \mod 4$.

Ex3
469.

$Z \times Z \times Z^+$

$R = \{(a,b,m) \mid a \equiv b \mod m\}$.

$9 \bmod 4 = 1$.

$5 \bmod 4 = 1$.

$4 \overline{)\ 5\ }\ 1$
$\quad\ 4$
$\quad\ (1)$

$(-1,9,5) \in R$ ✓ ?

$(14,0,7) \in R$. ✓

$(7,2,3) \in R$. ✗

$(-2,-8,5) \in R$. = HW.

$(11,0,6) \in R$ = HW.

Ex4
470.

(PIA, PK102, Karachi, Dubai, 12:00am).

$A \times N \times S \times D \times T$.
A = Set of Airlines.
N = Flight Number.
S = Starting Point.
D = Destination.
T = departure time.

REPRESENTING RELATIONS.

MATRICES. R

$A \times B$.

$A = \{a_1, a_2, a_3, \dots, a_m\}$.
$B = \{b_1, b_2, b_3, \dots, b_n\}$.

$M_R = [m_{ij}]$.

$$m_{ij} = \begin{cases} 1 & \text{if } (a_i, b_j) \in R. \\ 0 & \text{if } (a_i, b_j) \notin R. \end{cases}$$

rows = $|A| = m$
Col = $|B| = n$.

Ex1 :-
476.

$A = \{1,2,3\}$ (a_1, a_2, a_3).

$B = \{1,2\}$ (b_1, b_2).

$R = \{(2,1),(3,1),(3,2)\}$.

$$M_R = \begin{bmatrix} m_{11} & m_{12} \\ \vdots & \vdots \end{bmatrix} \quad \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

$$M_{12} = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \\ m_{31} & m_{32} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$$