Probability & Statistics

Sessional I Solution

Fall 2021, BS-SE-19

Problem # 01 **Marks =10**

(a) There are 20 computers in a store. Among them, 15 are brand new and 5 are refurbished. Six computers are purchased for a student lab. From the first look, they are indistinguishable, so the six computers are selected at random. Compute the probability that among the chosen computers, two are refurbished.

Sol (a) Total Computers $= 20$, $B \cdot N = 15$ & $R = 5$

$\binom{20}{6} = \frac{20!}{6!\,14!} = \boxed{38760}$ is the total number of elements in the sample space.

Let E is the event that among choosen computers two are refurbished, then number of elements in E is

$$\binom{5}{2} \times \binom{15}{4} = \frac{5!}{2!\,3!} \times \frac{15!}{4!\,11!} = 10 \times 1365 = \boxed{13650}$$

Thus $P(E) = \dfrac{\text{No. of elements in } E}{\text{Total No. of elements in } S}$

$$= \frac{13650}{38760}$$

$$\Rightarrow \boxed{P(E) = 0.3522} \text{ Ans}$$

(b) If n people are present in a room, then what is the probability that no two of them will celebrate their birthday on the same day of the year?

Sol:

$$\frac{(365)(364)(363)\cdots(365-n+1)}{(365)^n}$$

(Detail in slides)

Problem # 02 **Marks =10**

(a) In an experiment to study the relationship of hypertension and smoking habits, the following data are collected for **180** individuals:

	Nonsmokers	Moderate Smokers	Heavy Smokers
H	21	36	30
NH	48	26	19

Favourable = 30

Total = 49

where **H** and **NH** in the table stand for Hypertension and Nonhypertension, respectively. If one of these individuals is selected at random, find the probability that the person is experiencing hypertension, given that the person is a heavy smoker.

Sol (shortcut) If we know that a given person is a heavy smoker then the probability that he/she has hypertension is $30/49 = 0.6122$ Ans

Formal Method

$$P(\text{Hyp.} \mid \text{Heavy Smoker})$$

$$= \frac{P(\text{HYP} \cap \text{Heavy Smoker})}{P(\text{Heavy Smoker})}$$

$$= \frac{30/180}{49/180} = \frac{30}{49} = 0.6122$$

(b) What information does the **"odds of an event"** tell us about the event? Give example.

Sol

The *odds* of an event A is defined by

$$\frac{P(A)}{P(A^c)} = \frac{P(A)}{1 - P(A)}$$

Thus the odds of an event A tells how much more likely it is that A occurs than that it does not occur. For instance, if $P(A) = 3/4$, then $P(A)/(1 - P(A)) = 3$, so the odds are 3. Consequently, it is 3 times as likely that A occurs as it is that it does not.

Note: One can describe the above idea in some other words.
That is also OK.

Problem # 03 **Marks =10**

The following data are the blood types of 50 volunteers at a blood plasma donation clinic:

O A O AB A A O O B A O A AB B O O O A B A A O A A O

B A O AB A O O A B A A A O B O O A O A B O AB A O B

(a) Represent these data in a frequency table.

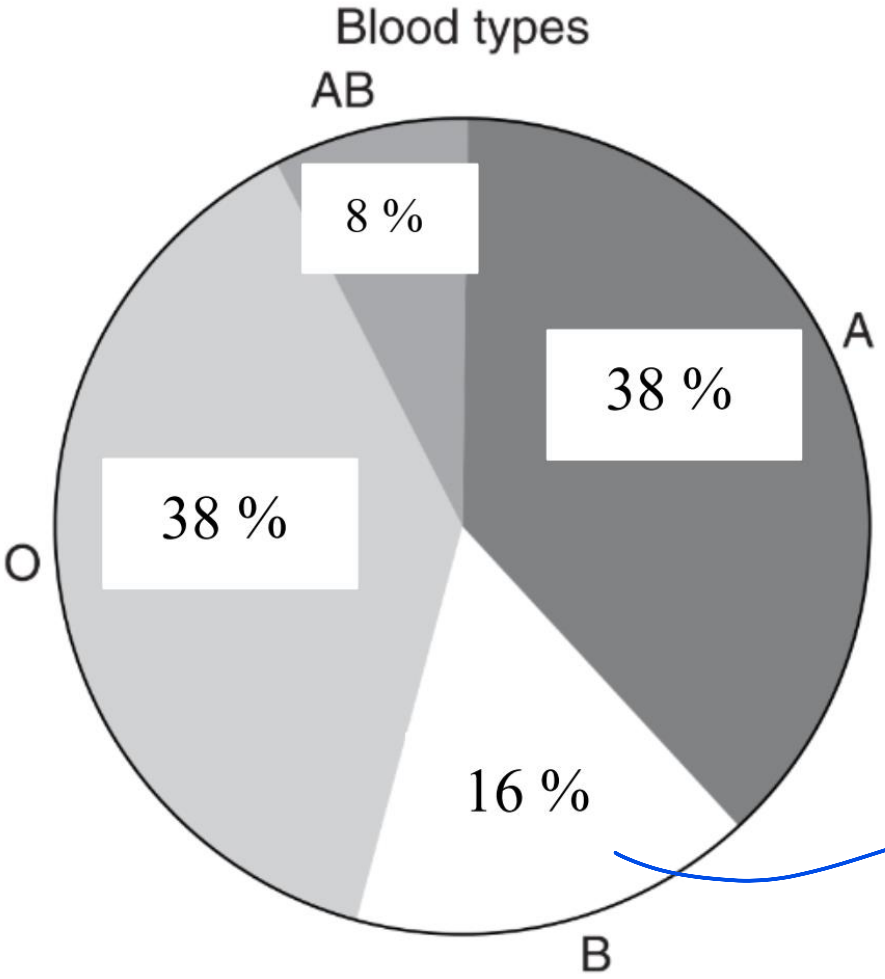
(b) Represent them in a pie chart.

Sol

(a)

Blood type	Frequency
A	19
B	8
O	19
AB	4

(b)



$\frac{8}{949} \times 100$

Problem # 04 **Marks =10**

The following stem-and-leaf plot records the diastolic blood pressure of a sample of 30 men.

9	3, 5, 8
8	6, 7, 8, 9, 9, 9
7	0, 1, 2, 2, 4, 5, 5, 6, 7, 8
6	0, 1, 2, 2, 3, 4, 5, 5
5	4, 6, 8

Find the sample standard deviation and quartiles of the data.

Sol

$$s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{\sum x_i^2 - n\bar{x}^2}{n-1}}$$

Here $n = 30$, $\bar{x} = 74.13$, $\sum x_i^2 = 169514$

So $s = \sqrt{\dfrac{169514 - 30(74.13^2)}{29}} \approx 12.67$ Ans

Next to find Q_1 we have $p = \frac{1}{4}$ so $np = 30\left(\frac{1}{4}\right) = 7.5$

So $\boxed{Q_1 = 63}$ Ans

For Q_2 we have $p = \frac{1}{2}$ So $np = \frac{30}{2} = 15$, so

$Q_2 = \frac{72+74}{2} = 73 \Rightarrow \boxed{Q_2 = 73}$ Ans

Finally for Q_3, we have $P = 3/4$ so $np = 30\left(\frac{3}{4}\right) = 22.5$ Thus $\boxed{Q_3 = 87}$ Ans

The End