Lab 9

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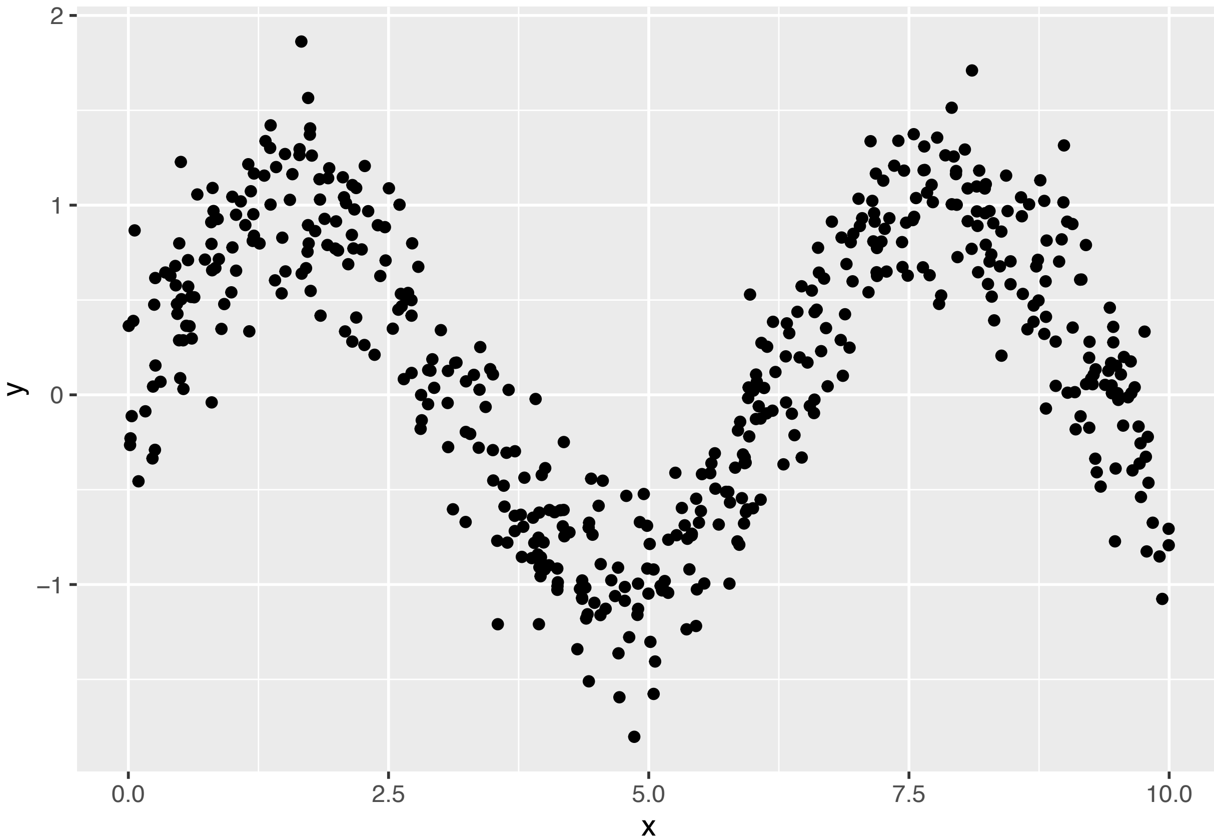
Here we will learn about trees, bagged trees and random forests. You can use the `YARF` package if it works, otherwise, use the `randomForest` package (the standard).

Let's take a look at the simulated sine curve data from practice lecture 12. Below is the code for the data generating process:

```
rm(list = ls())
n = 500
sigma = 0.3
x_min = 0
x_max = 10
f_x = function(x){sin(x)}
y_x = function(x, sigma){f_x(x) + rnorm(n, 0, sigma)}
x_train = runif(n, x_min, x_max)
y_train = y_x(x_train, sigma)
```

Plot an example dataset of size 500:

```
pacman::p_load(ggplot2)
ggplot(data.frame(x=x_train,y=y_train)) +
  geom_point(aes(x=x,y=y))
```



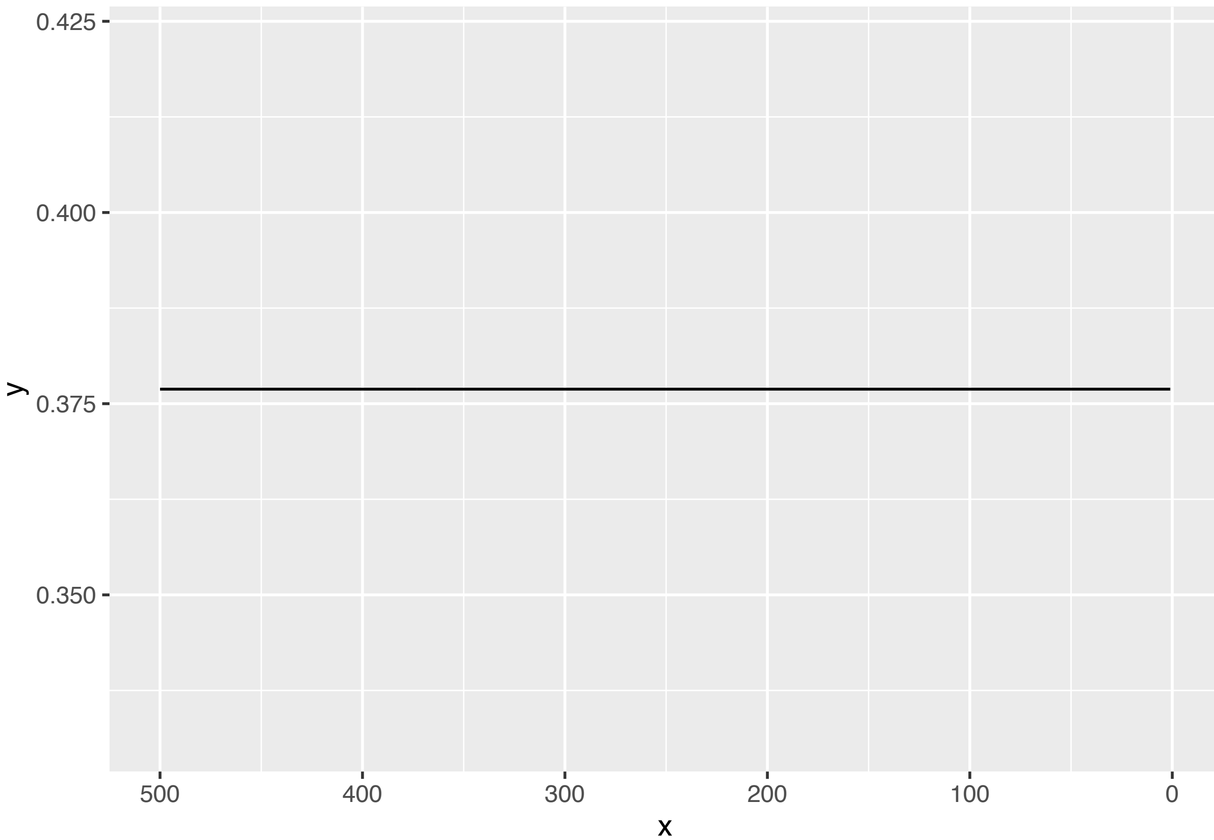
Create a test set of size 500 as well

```
x_test = runif(n, x_min, x_max)
y_test = y_x(x_test, sigma)
```

Locate the optimal node size hyperparameter for the regression tree model. I believe you can use `randomForest` here by setting `ntree = 1`, `replace = FALSE`, `sampsize = n` (`mtry` is already set to be 1 because there is only one feature) and then you can set `nodesize`. plot node size out of sample

```
pacman ::p_load (randomForest)

node_sizes = 1:n
se_by_node_sizes = array(NA, length(node_sizes))
for (i in 1:length(node_sizes)){
  rf_mod = randomForest(x= data.frame (x= x_train),y =y_train,ntree=1,replace =FALSE, sampsize = n, node
  y_hat_test = predict(rf_mod,data.frame(x= x_test))
  se_by_node_sizes[i] = sd(y_test - y_hat_test)
}
ggplot(data.frame(x= node_sizes,y = se_by_node_sizes)) +
  geom_line(aes(x= x, y= y)) +
scale_x_reverse ()
```

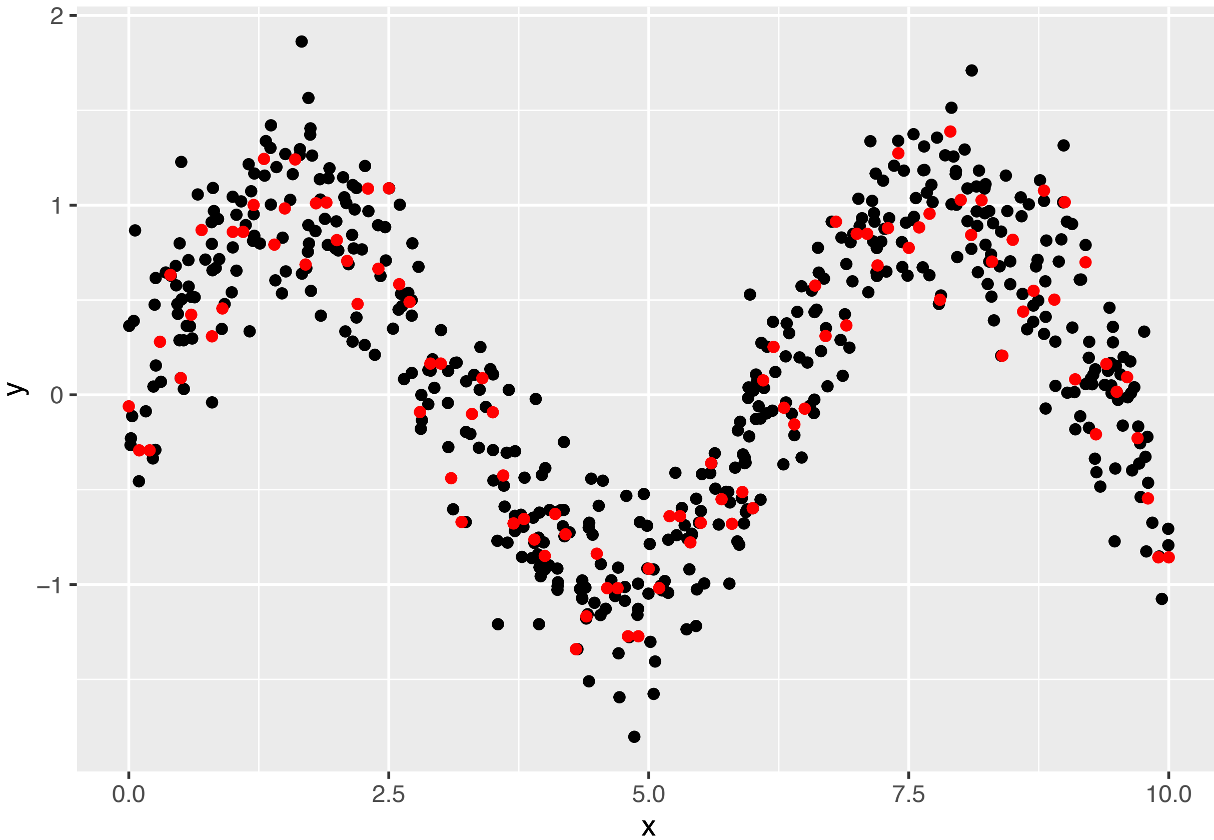



```
which.min(se_by_node_sizes)
```

```
## [1] 1
```

Plot the regression tree model with the optimal node size.

```
 rf_mod = randomForest(x= data.frame (x= x_train),y =y_train,ntree=1,replace =FALSE, sampsize = n, node
resolution = 0.1
x_grid = seq(from = x_min , to = x_max , by = resolution)
g_x = predict(rf_mod,data.frame(x= x_grid))
ggplot(data.frame(x= x_grid ,y = g_x)) +
  (aes(x= x, y=y)) +
  geom_point(data= data.frame(x=x_train,y=y_train)) +
geom_point(color = "red")
```



Provide the bias-variance decomposition of this DGP fit with this model. It is a lot of code, but it is in the practice lectures. If your three numbers don't add up within two significant digits, increase your resolution.

```
#TO-DO
```

```
rm(list = ls())
```

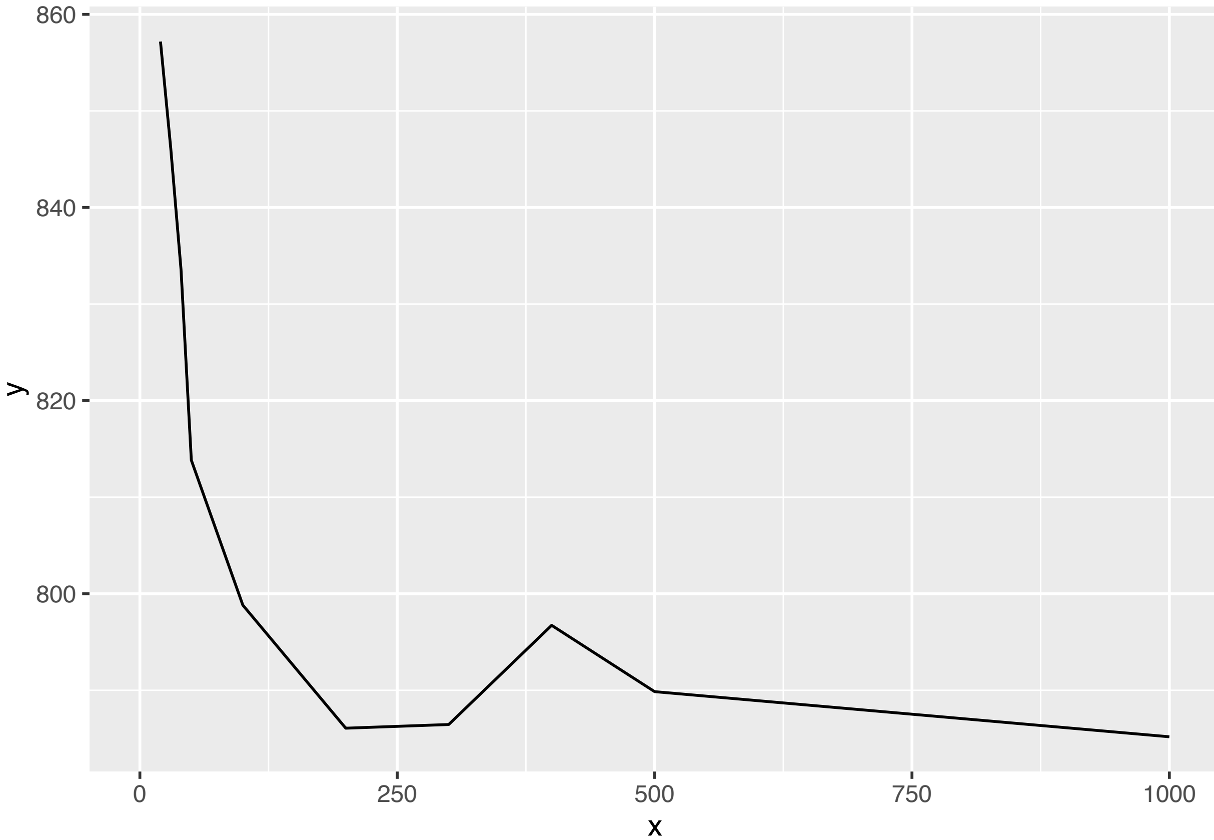
Take a sample of n = 2000 observations from the diamonds data.

```
pacman :: p_load (dplyr)
diamonds_samp = diamonds %>%
  sample_n(2000)
```

find the oob s_e for a RF model using 1, 2, 5, 10, 20, 30, 40, 50, 100, 200, 300, 400, 500, 1000 trees. If you are using the `randomForest` package, you can calculate oob residuals via `e_oob = y_train - rf_mod$predicted`.

```
num_trees = c(1, 2, 5, 10, 20, 30, 40, 50, 100, 200, 300, 400, 500, 1000)
oob_se_by_num_trees = array(NA,length(num_trees))
 for (i in 1:length(num_trees)) {
   rf_mod = randomForest(price~., data= diamonds_samp, ntree = num_trees[i])
   oob_se_by_num_trees[i] = sd(diamonds_samp$price - rf_mod$predicted)
 }
ggplot(data.frame(x= num_trees,y= oob_se_by_num_trees)) +
  geom_line(aes(x= x,y=y))
```

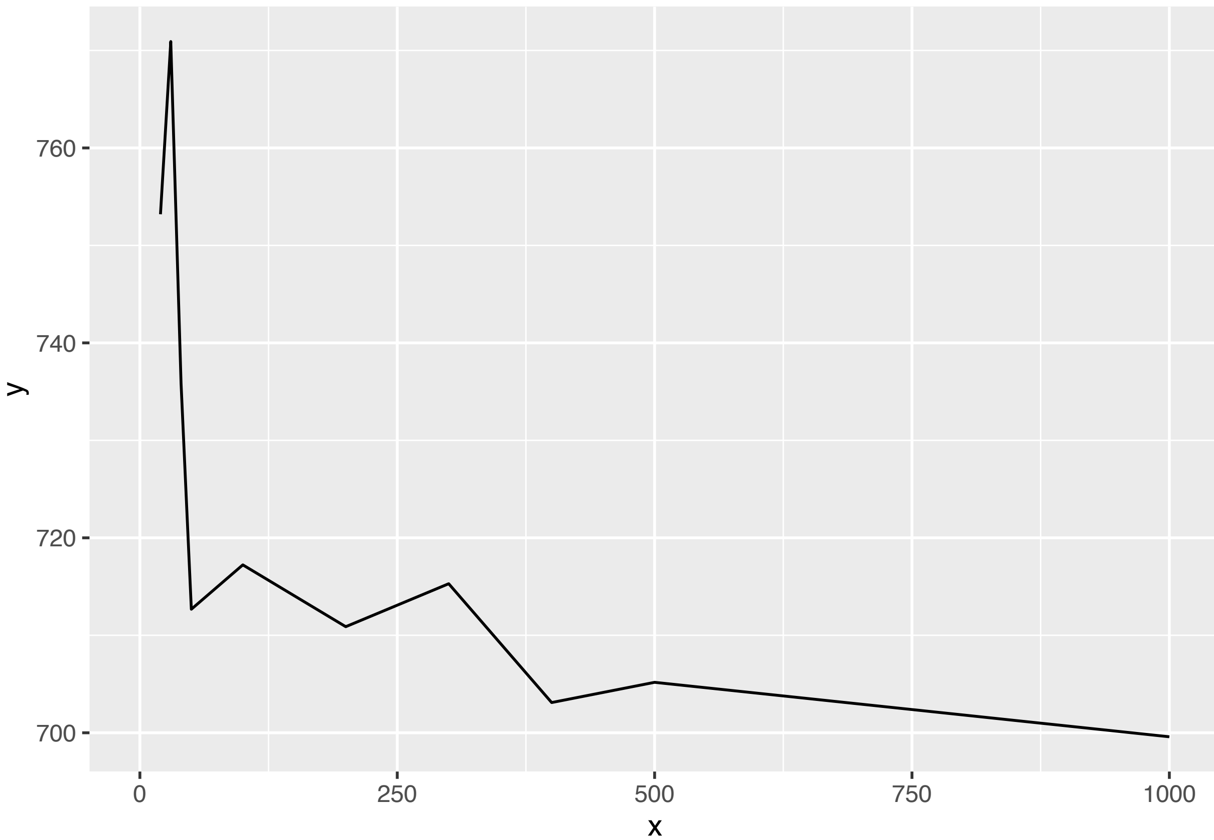
```
## Warning: Removed 4 row(s) containing missing values (geom_path).
```



Using the diamonds data, find the oob s_e for a bagged-tree model using 1, 2, 5, 10, 20, 30, 40, 50, 100, 200, 300, 400, 500, 1000 trees. If you are using the `randomForest` package, you can create the bagged tree model via setting an argument within the RF constructor function.

```
num_trees = c(1, 2, 5, 10, 20, 30, 40, 50, 100, 200, 300, 400, 500, 1000)
oob_se_by_num_bag = array(NA,length(num_trees))
 for (i in 1:length(num_trees)) {
   rf_mod = randomForest(price~., data= diamonds_samp, ntree = num_trees[i], mtry =ncol(diamonds_samp)-
   oob_se_by_num_bag[i] = sd(diamonds_samp$price - rf_mod$predicted)
 }
ggplot(data.frame(x= num_trees,y= oob_se_by_num_bag)) +
  geom_line(aes(x= x,y=y))
```

```
## Warning: Removed 4 row(s) containing missing values (geom_path).
```



What is the percentage gain / loss in performance of the RF model vs bagged trees model?

```
#(oob_se_by_num_trees - oob_se_by_num_bag) / oob_se_by_num_bag *100
## this worked, now it doesnt work, i dont know why
```

Plot bootstrap s_e by number of trees for both RF and bagged trees.

```
#ggplot(rbind(data.frame(num_trees = num_trees,value = oob_se_by_num_bag ,model = "RF"), data.frame (nu
#geom_line (aes(x= num_trees, y = value, color = model))
## this worked, now it doesnt work, i dont know why
```

Build RF models for 500 trees using different `mtry` values: 1, 2, . . . the maximum. That maximum will be the number of features assuming that we do not binarize categorical features if you are using `randomForest` or the number of features assuming binarization of the categorical features if you are using `YARF`. Calculate oob s_e for all mtry values.

```
#mtrys = 1:(ncol(diamonds_samp)-1)
#oob_se_by_mtrys = array(NA,length(mtrys))
# for (i in 1:length(mtrys)) {
  # rf_mod - randomForest(price~. , data - diamonds_samp, mtry - mtrys[i])
  # oob_se_by_mtrys[i] = sd(diamonds_samp$price - rf_mod$predicted)
 #}
#gplot(data.frame(x= mtrys,y= oob_se_by_mtrys)) +
 # geom_line(aes(x= x,y=y))
## this worked, now it doesnt work, i dont know why
```

```
rm(list = ls())
```

Take a sample of n = 2000 observations from the adult data.

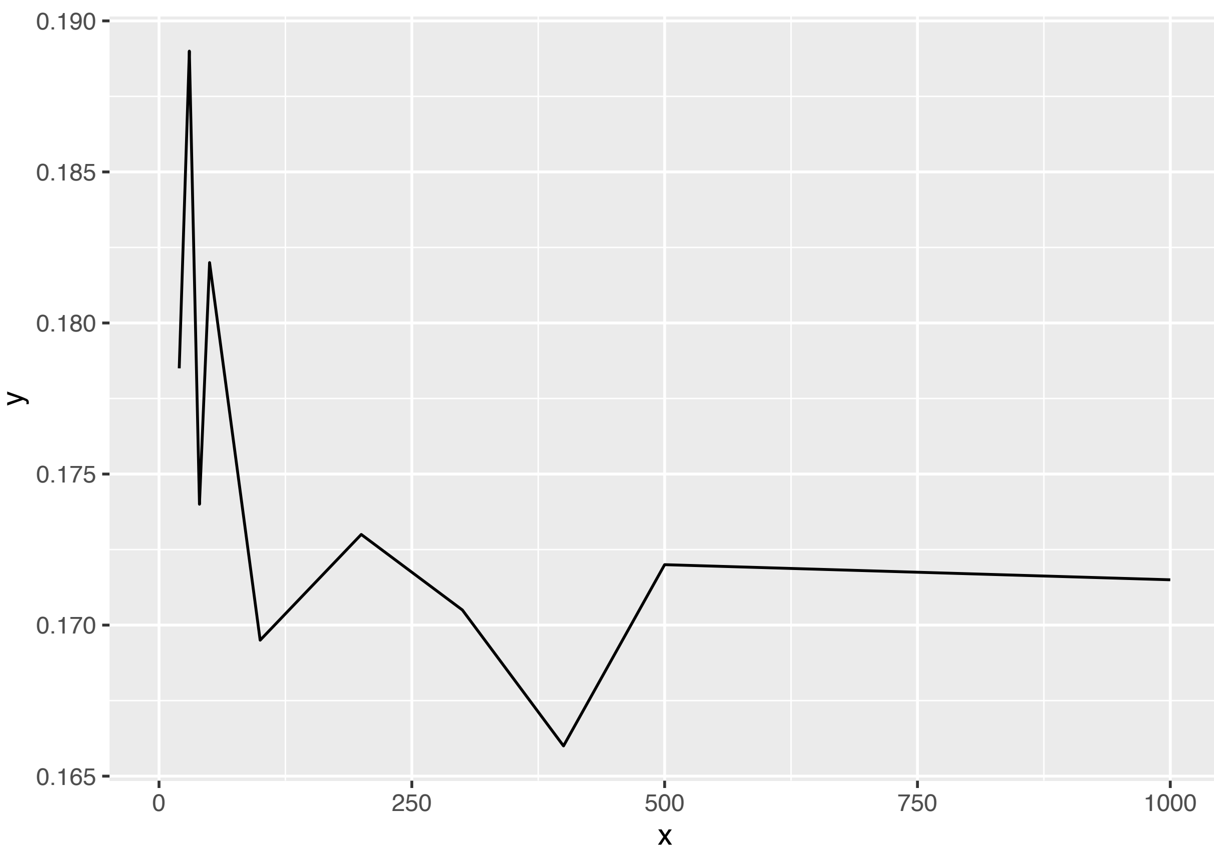
```
pacman::p_load_gh("coatless/ucidata")
data(adult)
adult = na.omit(adult) #kill any observations with missingness

adult_samp = adult %>%
  sample_n(2000)
```

Using the adult data, find the bootstrap misclassification error for an RF model using 1, 2, 5, 10, 20, 30, 40, 50, 100, 200, 300, 400, 500, 1000 trees.

```
num_trees = c(1, 2, 5, 10, 20, 30, 40, 50, 100, 200, 300, 400, 500, 1000)
oob_me_by_num_trees = array(NA,length(num_trees))
 for (i in 1:length(num_trees)) {
   rf_mod = randomForest(income~., data= adult_samp, ntree = num_trees[i])
   oob_me_by_num_trees[i] = mean(adult_samp$income!= rf_mod$predicted)
 }
ggplot(data.frame(x= num_trees,y= oob_me_by_num_trees)) +
  geom_line(aes(x= x,y=y))
```

```
## Warning: Removed 4 row(s) containing missing values (geom_path).
```

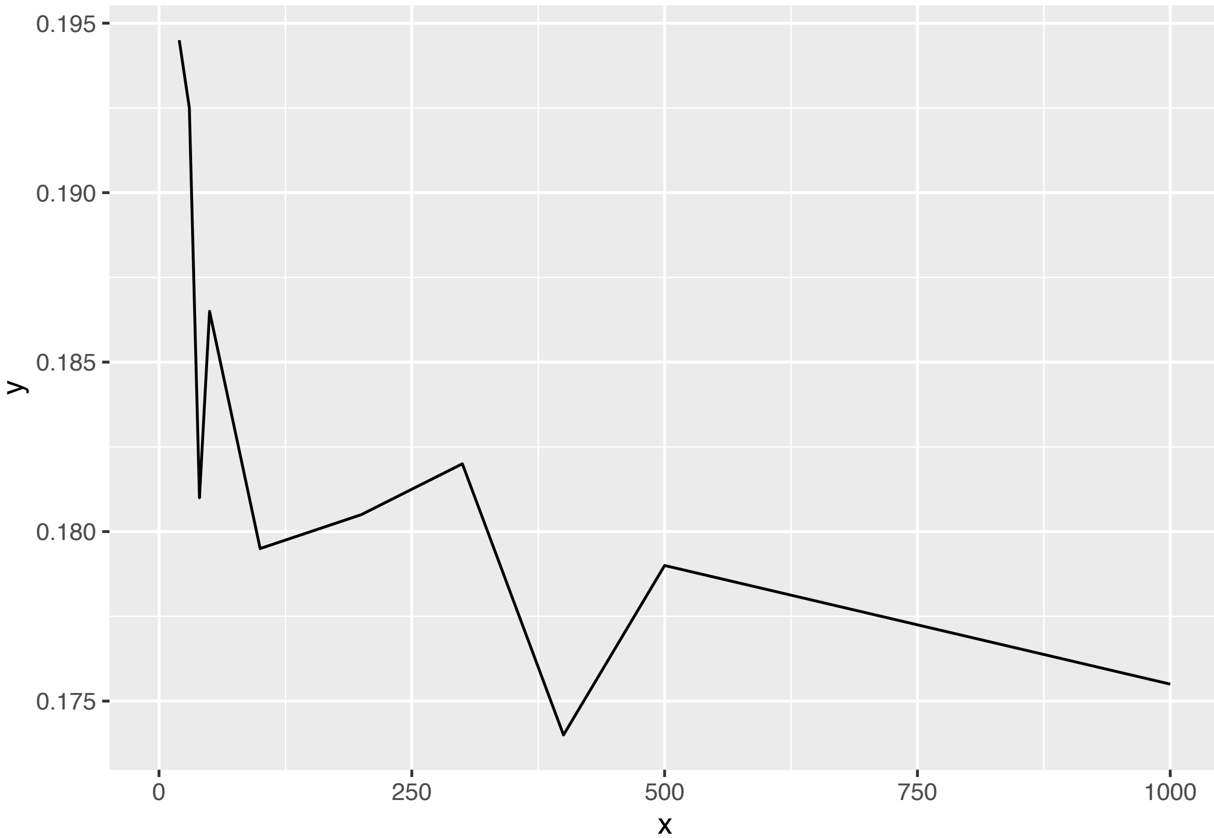


Using the adult data, find the bootstrap misclassification error for a bagged-tree model using 1, 2, 5, 10, 20, 30, 40, 50, 100, 200, 300, 400, 500, 1000 trees.

```
oob_me_by_num_trees_bag = array(NA,length(num_trees))
 for (i in 1:length(num_trees)) {
   rf_mod = randomForest(income~., data= adult_samp, ntree = num_trees[i],mtry = ncol(adult)-1)
   oob_me_by_num_trees_bag[i] = mean(adult_samp$income!= rf_mod$predicted)
 }
```

```
ggplot(data.frame(x= num_trees,y= oob_me_by_num_trees_bag)) +
  geom_line(aes(x= x,y=y))
```

```
## Warning: Removed 4 row(s) containing missing values (geom_path).
```

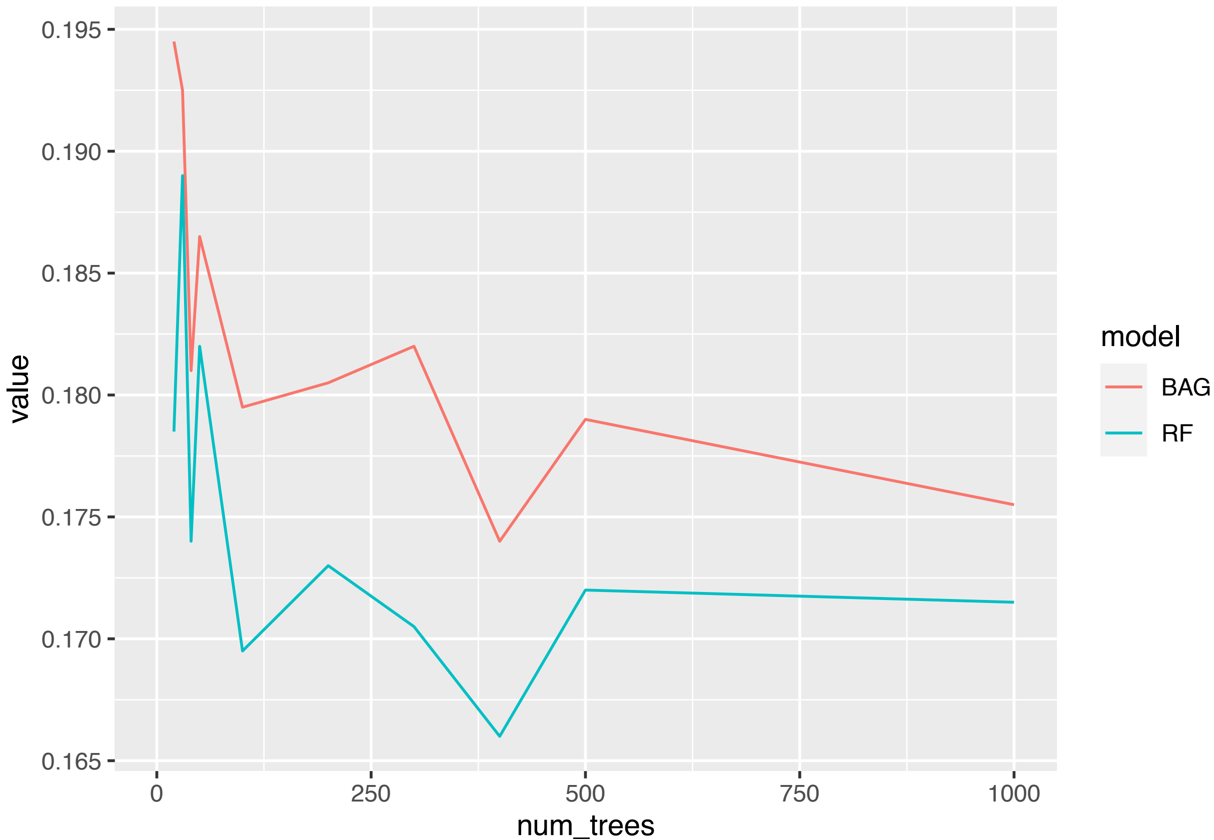


What is the percentage gain / loss in performance of the RF model vs bagged trees model?

Plot bootstrap misclassification error by number of trees for both RF and bagged trees.

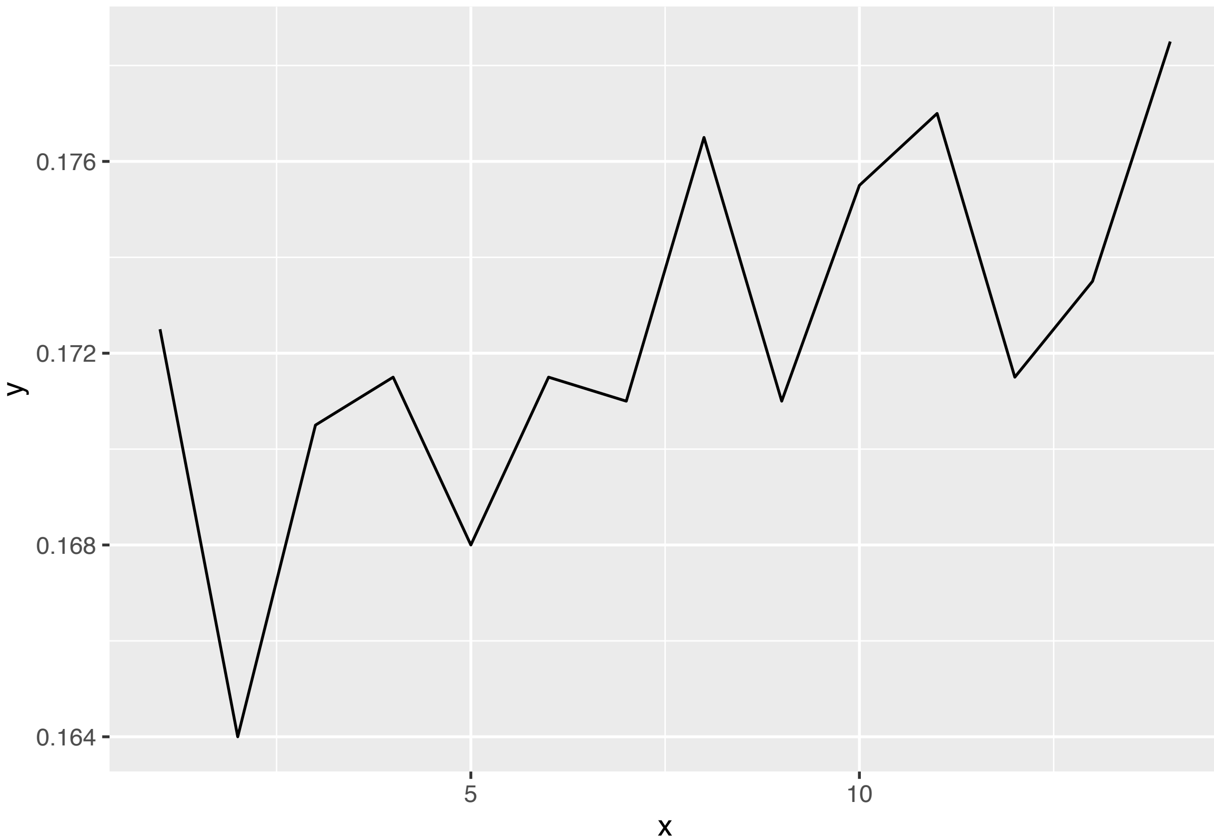
```
ggplot(rbind(data.frame(num_trees = num_trees,value = oob_me_by_num_trees,model = "RF"),data.frame (num_
geom_line (aes(x= num_trees, y = value, color = model))
```

```
## Warning: Removed 8 row(s) containing missing values (geom_path).
```



Build RF models for 500 trees using different `mtry` values: 1, 2, . . . the maximum (see above as maximum is defined by the specific RF algorithm implementation).

```
mtrys = 1:(ncol(adult_samp)-1)
oob_me_by_mtrys = array(NA,length(mtrys))
 for (i in 1:length(mtrys)) {
   rf_mod = randomForest(income~. , data = adult_samp, mtry = mtrys[i])
   oob_me_by_mtrys[i] = mean(adult_samp$income!= rf_mod$predicted)
 }
ggplot(data.frame(x= mtrys,y= oob_me_by_mtrys)) +
  geom_line(aes(x= x,y=y))
```




```
rm(list = ls())
```

Write a function `random_bagged_ols` which takes as its arguments `X` and `y` with further arguments `num_ols_models` defaulted to 100 and `mtry` defaulted to NULL which then gets set within the function to be 50% of available features. This argument builds an OLS on a bootstrap sample of the data and uses only `mtry < p` of the available features. The function then returns all the `lm` models as a list with size `num_ols_models`.

```
#to do
```

Load up the Boston Housing Data and separate into `X` and `y`.

```
library(MASS)
```

```
##
## Attaching package: 'MASS'

## The following object is masked from 'package:dplyr':
##
##     select
```

```
data(Boston)
Boston
```

```
##          crim    zn indus chas    nox    rm   age     dis rad tax ptratio  black
## 1     0.00632  18.0  2.31    0 0.5380 6.575  65.2  4.0900   1 296    15.3 396.90
## 2     0.02731   0.0  7.07    0 0.4690 6.421  78.9  4.9671   2 242    17.8 396.90
## 3     0.02729   0.0  7.07    0 0.4690 7.185  61.1  4.9671   2 242    17.8 392.83
## 4     0.03237   0.0  2.18    0 0.4580 6.998  45.8  6.0622   3 222    18.7 394.63
## 5     0.06905   0.0  2.18    0 0.4580 7.147  54.2  6.0622   3 222    18.7 396.90
## 6     0.02985   0.0  2.18    0 0.4580 6.430  58.7  6.0622   3 222    18.7 394.12
## 7     0.08829  12.5  7.87    0 0.5240 6.012  66.6  5.5605   5 311    15.2 395.60
```

```
## 8    0.14455  12.5  7.87    0 0.5240 6.172  96.1  5.9505   5 311    15.2 396.90
## 9    0.21124  12.5  7.87    0 0.5240 5.631 100.0  6.0821   5 311    15.2 386.63
## 10   0.17004  12.5  7.87    0 0.5240 6.004  85.9  6.5921   5 311    15.2 386.71
## 11   0.22489  12.5  7.87    0 0.5240 6.377  94.3  6.3467   5 311    15.2 392.52
## 12   0.11747  12.5  7.87    0 0.5240 6.009  82.9  6.2267   5 311    15.2 396.90
## 13   0.09378  12.5  7.87    0 0.5240 5.889  39.0  5.4509   5 311    15.2 390.50
## 14   0.62976   0.0  8.14    0 0.5380 5.949  61.8  4.7075   4 307    21.0 396.90
## 15   0.63796   0.0  8.14    0 0.5380 6.096  84.5  4.4619   4 307    21.0 380.02
## 16   0.62739   0.0  8.14    0 0.5380 5.834  56.5  4.4986   4 307    21.0 395.62
## 17   1.05393   0.0  8.14    0 0.5380 5.935  29.3  4.4986   4 307    21.0 386.85
## 18   0.78420   0.0  8.14    0 0.5380 5.990  81.7  4.2579   4 307    21.0 386.75
## 19   0.80271   0.0  8.14    0 0.5380 5.456  36.6  3.7965   4 307    21.0 288.99
## 20   0.72580   0.0  8.14    0 0.5380 5.727  69.5  3.7965   4 307    21.0 390.95
## 21   1.25179   0.0  8.14    0 0.5380 5.570  98.1  3.7979   4 307    21.0 376.57
## 22   0.85204   0.0  8.14    0 0.5380 5.965  89.2  4.0123   4 307    21.0 392.53
## 23   1.23247   0.0  8.14    0 0.5380 6.142  91.7  3.9769   4 307    21.0 396.90
## 24   0.98843   0.0  8.14    0 0.5380 5.813 100.0  4.0952   4 307    21.0 394.54
## 25   0.75026   0.0  8.14    0 0.5380 5.924  94.1  4.3996   4 307    21.0 394.33
## 26   0.84054   0.0  8.14    0 0.5380 5.599  85.7  4.4546   4 307    21.0 303.42
## 27   0.67191   0.0  8.14    0 0.5380 5.813  90.3  4.6820   4 307    21.0 376.88
## 28   0.95577   0.0  8.14    0 0.5380 6.047  88.8  4.4534   4 307    21.0 306.38
## 29   0.77299   0.0  8.14    0 0.5380 6.495  94.4  4.4547   4 307    21.0 387.94
## 30   1.00245   0.0  8.14    0 0.5380 6.674  87.3  4.2390   4 307    21.0 380.23
## 31   1.13081   0.0  8.14    0 0.5380 5.713  94.1  4.2330   4 307    21.0 360.17
## 32   1.35472   0.0  8.14    0 0.5380 6.072 100.0  4.1750   4 307    21.0 376.73
## 33   1.38799   0.0  8.14    0 0.5380 5.950  82.0  3.9900   4 307    21.0 232.60
## 34   1.15172   0.0  8.14    0 0.5380 5.701  95.0  3.7872   4 307    21.0 358.77
## 35   1.61282   0.0  8.14    0 0.5380 6.096  96.9  3.7598   4 307    21.0 248.31
## 36   0.06417   0.0  5.96    0 0.4990 5.933  68.2  3.3603   5 279    19.2 396.90
## 37   0.09744   0.0  5.96    0 0.4990 5.841  61.4  3.3779   5 279    19.2 377.56
## 38   0.08014   0.0  5.96    0 0.4990 5.850  41.5  3.9342   5 279    19.2 396.90
## 39   0.17505   0.0  5.96    0 0.4990 5.966  30.2  3.8473   5 279    19.2 393.43
## 40   0.02763  75.0  2.95    0 0.4280 6.595  21.8  5.4011   3 252    18.3 395.63
## 41   0.03359  75.0  2.95    0 0.4280 7.024  15.8  5.4011   3 252    18.3 395.62
## 42   0.12744   0.0  6.91    0 0.4480 6.770   2.9  5.7209   3 233    17.9 385.41
## 43   0.14150   0.0  6.91    0 0.4480 6.169   6.6  5.7209   3 233    17.9 383.37
## 44   0.15936   0.0  6.91    0 0.4480 6.211   6.5  5.7209   3 233    17.9 394.46
## 45   0.12269   0.0  6.91    0 0.4480 6.069  40.0  5.7209   3 233    17.9 389.39
## 46   0.17142   0.0  6.91    0 0.4480 5.682  33.8  5.1004   3 233    17.9 396.90
## 47   0.18836   0.0  6.91    0 0.4480 5.786  33.3  5.1004   3 233    17.9 396.90
## 48   0.22927   0.0  6.91    0 0.4480 6.030  85.5  5.6894   3 233    17.9 392.74
## 49   0.25387   0.0  6.91    0 0.4480 5.399  95.3  5.8700   3 233    17.9 396.90
## 50   0.21977   0.0  6.91    0 0.4480 5.602  62.0  6.0877   3 233    17.9 396.90
## 51   0.08873  21.0  5.64    0 0.4390 5.963  45.7  6.8147   4 243    16.8 395.56
## 52   0.04337  21.0  5.64    0 0.4390 6.115  63.0  6.8147   4 243    16.8 393.97
## 53   0.05360  21.0  5.64    0 0.4390 6.511  21.1  6.8147   4 243    16.8 396.90
## 54   0.04981  21.0  5.64    0 0.4390 5.998  21.4  6.8147   4 243    16.8 396.90
## 55   0.01360  75.0  4.00    0 0.4100 5.888  47.6  7.3197   3 469    21.1 396.90
## 56   0.01311  90.0  1.22    0 0.4030 7.249  21.9  8.6966   5 226    17.9 395.93
## 57   0.02055  85.0  0.74    0 0.4100 6.383  35.7  9.1876   2 313    17.3 396.90
## 58   0.01432 100.0  1.32    0 0.4110 6.816  40.5  8.3248   5 256    15.1 392.90
## 59   0.15445  25.0  5.13    0 0.4530 6.145  29.2  7.8148   8 284    19.7 390.68
## 60   0.10328  25.0  5.13    0 0.4530 5.927  47.2  6.9320   8 284    19.7 396.90
## 61   0.14932  25.0  5.13    0 0.4530 5.741  66.2  7.2254   8 284    19.7 395.11
```

```
## 62   0.17171  25.0  5.13    0 0.4530 5.966  93.4  6.8185   8 284    19.7 378.08
## 63   0.11027  25.0  5.13    0 0.4530 6.456  67.8  7.2255   8 284    19.7 396.90
## 64   0.12650  25.0  5.13    0 0.4530 6.762  43.4  7.9809   8 284    19.7 395.58
## 65   0.01951  17.5  1.38    0 0.4161 7.104  59.5  9.2229   3 216    18.6 393.24
## 66   0.03584  80.0  3.37    0 0.3980 6.290  17.8  6.6115   4 337    16.1 396.90
## 67   0.04379  80.0  3.37    0 0.3980 5.787  31.1  6.6115   4 337    16.1 396.90
## 68   0.05789  12.5  6.07    0 0.4090 5.878  21.4  6.4980   4 345    18.9 396.21
## 69   0.13554  12.5  6.07    0 0.4090 5.594  36.8  6.4980   4 345    18.9 396.90
## 70   0.12816  12.5  6.07    0 0.4090 5.885  33.0  6.4980   4 345    18.9 396.90
## 71   0.08826   0.0 10.81    0 0.4130 6.417   6.6  5.2873   4 305    19.2 383.73
## 72   0.15876   0.0 10.81    0 0.4130 5.961  17.5  5.2873   4 305    19.2 376.94
## 73   0.09164   0.0 10.81    0 0.4130 6.065   7.8  5.2873   4 305    19.2 390.91
## 74   0.19539   0.0 10.81    0 0.4130 6.245   6.2  5.2873   4 305    19.2 377.17
## 75   0.07896   0.0 12.83    0 0.4370 6.273   6.0  4.2515   5 398    18.7 394.92
## 76   0.09512   0.0 12.83    0 0.4370 6.286  45.0  4.5026   5 398    18.7 383.23
## 77   0.10153   0.0 12.83    0 0.4370 6.279  74.5  4.0522   5 398    18.7 373.66
## 78   0.08707   0.0 12.83    0 0.4370 6.140  45.8  4.0905   5 398    18.7 386.96
## 79   0.05646   0.0 12.83    0 0.4370 6.232  53.7  5.0141   5 398    18.7 386.40
## 80   0.08387   0.0 12.83    0 0.4370 5.874  36.6  4.5026   5 398    18.7 396.06
## 81   0.04113  25.0  4.86    0 0.4260 6.727  33.5  5.4007   4 281    19.0 396.90
## 82   0.04462  25.0  4.86    0 0.4260 6.619  70.4  5.4007   4 281    19.0 395.63
## 83   0.03659  25.0  4.86    0 0.4260 6.302  32.2  5.4007   4 281    19.0 396.90
## 84   0.03551  25.0  4.86    0 0.4260 6.167  46.7  5.4007   4 281    19.0 390.64
## 85   0.05059   0.0  4.49    0 0.4490 6.389  48.0  4.7794   3 247    18.5 396.90
## 86   0.05735   0.0  4.49    0 0.4490 6.630  56.1  4.4377   3 247    18.5 392.30
## 87   0.05188   0.0  4.49    0 0.4490 6.015  45.1  4.4272   3 247    18.5 395.99
## 88   0.07151   0.0  4.49    0 0.4490 6.121  56.8  3.7476   3 247    18.5 395.15
## 89   0.05660   0.0  3.41    0 0.4890 7.007  86.3  3.4217   2 270    17.8 396.90
## 90   0.05302   0.0  3.41    0 0.4890 7.079  63.1  3.4145   2 270    17.8 396.06
## 91   0.04684   0.0  3.41    0 0.4890 6.417  66.1  3.0923   2 270    17.8 392.18
## 92   0.03932   0.0  3.41    0 0.4890 6.405  73.9  3.0921   2 270    17.8 393.55
## 93   0.04203  28.0 15.04    0 0.4640 6.442  53.6  3.6659   4 270    18.2 395.01
## 94   0.02875  28.0 15.04    0 0.4640 6.211  28.9  3.6659   4 270    18.2 396.33
## 95   0.04294  28.0 15.04    0 0.4640 6.249  77.3  3.6150   4 270    18.2 396.90
## 96   0.12204   0.0  2.89    0 0.4450 6.625  57.8  3.4952   2 276    18.0 357.98
## 97   0.11504   0.0  2.89    0 0.4450 6.163  69.6  3.4952   2 276    18.0 391.83
## 98   0.12083   0.0  2.89    0 0.4450 8.069  76.0  3.4952   2 276    18.0 396.90
## 99   0.08187   0.0  2.89    0 0.4450 7.820  36.9  3.4952   2 276    18.0 393.53
## 100  0.06860   0.0  2.89    0 0.4450 7.416  62.5  3.4952   2 276    18.0 396.90
## 101  0.14866   0.0  8.56    0 0.5200 6.727  79.9  2.7778   5 384    20.9 394.76
## 102  0.11432   0.0  8.56    0 0.5200 6.781  71.3  2.8561   5 384    20.9 395.58
## 103  0.22876   0.0  8.56    0 0.5200 6.405  85.4  2.7147   5 384    20.9  70.80
## 104  0.21161   0.0  8.56    0 0.5200 6.137  87.4  2.7147   5 384    20.9 394.47
## 105  0.13960   0.0  8.56    0 0.5200 6.167  90.0  2.4210   5 384    20.9 392.69
## 106  0.13262   0.0  8.56    0 0.5200 5.851  96.7  2.1069   5 384    20.9 394.05
## 107  0.17120   0.0  8.56    0 0.5200 5.836  91.9  2.2110   5 384    20.9 395.67
## 108  0.13117   0.0  8.56    0 0.5200 6.127  85.2  2.1224   5 384    20.9 387.69
## 109  0.12802   0.0  8.56    0 0.5200 6.474  97.1  2.4329   5 384    20.9 395.24
## 110  0.26363   0.0  8.56    0 0.5200 6.229  91.2  2.5451   5 384    20.9 391.23
## 111  0.10793   0.0  8.56    0 0.5200 6.195  54.4  2.7778   5 384    20.9 393.49
## 112  0.10084   0.0 10.01    0 0.5470 6.715  81.6  2.6775   6 432    17.8 395.59
## 113  0.12329   0.0 10.01    0 0.5470 5.913  92.9  2.3534   6 432    17.8 394.95
## 114  0.22212   0.0 10.01    0 0.5470 6.092  95.4  2.5480   6 432    17.8 396.90
## 115  0.14231   0.0 10.01    0 0.5470 6.254  84.2  2.2565   6 432    17.8 388.74
```

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## 116  0.17134   0.0 10.01    0 0.5470 5.928  88.2  2.4631   6 432    17.8 344.91
## 117  0.13158   0.0 10.01    0 0.5470 6.176  72.5  2.7301   6 432    17.8 393.30
## 118  0.15098   0.0 10.01    0 0.5470 6.021  82.6  2.7474   6 432    17.8 394.51
## 119  0.13058   0.0 10.01    0 0.5470 5.872  73.1  2.4775   6 432    17.8 338.63
## 120  0.14476   0.0 10.01    0 0.5470 5.731  65.2  2.7592   6 432    17.8 391.50
## 121  0.06899   0.0 25.65    0 0.5810 5.870  69.7  2.2577   2 188    19.1 389.15
## 122  0.07165   0.0 25.65    0 0.5810 6.004  84.1  2.1974   2 188    19.1 377.67
## 123  0.09299   0.0 25.65    0 0.5810 5.961  92.9  2.0869   2 188    19.1 378.09
## 124  0.15038   0.0 25.65    0 0.5810 5.856  97.0  1.9444   2 188    19.1 370.31
## 125  0.09849   0.0 25.65    0 0.5810 5.879  95.8  2.0063   2 188    19.1 379.38
## 126  0.16902   0.0 25.65    0 0.5810 5.986  88.4  1.9929   2 188    19.1 385.02
## 127  0.38735   0.0 25.65    0 0.5810 5.613  95.6  1.7572   2 188    19.1 359.29
## 128  0.25915   0.0 21.89    0 0.6240 5.693  96.0  1.7883   4 437    21.2 392.11
## 129  0.32543   0.0 21.89    0 0.6240 6.431  98.8  1.8125   4 437    21.2 396.90
## 130  0.88125   0.0 21.89    0 0.6240 5.637  94.7  1.9799   4 437    21.2 396.90
## 131  0.34006   0.0 21.89    0 0.6240 6.458  98.9  2.1185   4 437    21.2 395.04
## 132  1.19294   0.0 21.89    0 0.6240 6.326  97.7  2.2710   4 437    21.2 396.90
## 133  0.59005   0.0 21.89    0 0.6240 6.372  97.9  2.3274   4 437    21.2 385.76
## 134  0.32982   0.0 21.89    0 0.6240 5.822  95.4  2.4699   4 437    21.2 388.69
## 135  0.97617   0.0 21.89    0 0.6240 5.757  98.4  2.3460   4 437    21.2 262.76
## 136  0.55778   0.0 21.89    0 0.6240 6.335  98.2  2.1107   4 437    21.2 394.67
## 137  0.32264   0.0 21.89    0 0.6240 5.942  93.5  1.9669   4 437    21.2 378.25
## 138  0.35233   0.0 21.89    0 0.6240 6.454  98.4  1.8498   4 437    21.2 394.08
## 139  0.24980   0.0 21.89    0 0.6240 5.857  98.2  1.6686   4 437    21.2 392.04
## 140  0.54452   0.0 21.89    0 0.6240 6.151  97.9  1.6687   4 437    21.2 396.90
## 141  0.29090   0.0 21.89    0 0.6240 6.174  93.6  1.6119   4 437    21.2 388.08
## 142  1.62864   0.0 21.89    0 0.6240 5.019 100.0  1.4394   4 437    21.2 396.90
## 143  3.32105   0.0 19.58    1 0.8710 5.403 100.0  1.3216   5 403    14.7 396.90
## 144  4.09740   0.0 19.58    0 0.8710 5.468 100.0  1.4118   5 403    14.7 396.90
## 145  2.77974   0.0 19.58    0 0.8710 4.903  97.8  1.3459   5 403    14.7 396.90
## 146  2.37934   0.0 19.58    0 0.8710 6.130 100.0  1.4191   5 403    14.7 172.91
## 147  2.15505   0.0 19.58    0 0.8710 5.628 100.0  1.5166   5 403    14.7 169.27
## 148  2.36862   0.0 19.58    0 0.8710 4.926  95.7  1.4608   5 403    14.7 391.71
## 149  2.33099   0.0 19.58    0 0.8710 5.186  93.8  1.5296   5 403    14.7 356.99
## 150  2.73397   0.0 19.58    0 0.8710 5.597  94.9  1.5257   5 403    14.7 351.85
## 151  1.65660   0.0 19.58    0 0.8710 6.122  97.3  1.6180   5 403    14.7 372.80
## 152  1.49632   0.0 19.58    0 0.8710 5.404 100.0  1.5916   5 403    14.7 341.60
## 153  1.12658   0.0 19.58    1 0.8710 5.012  88.0  1.6102   5 403    14.7 343.28
## 154  2.14918   0.0 19.58    0 0.8710 5.709  98.5  1.6232   5 403    14.7 261.95
## 155  1.41385   0.0 19.58    1 0.8710 6.129  96.0  1.7494   5 403    14.7 321.02
## 156  3.53501   0.0 19.58    1 0.8710 6.152  82.6  1.7455   5 403    14.7  88.01
## 157  2.44668   0.0 19.58    0 0.8710 5.272  94.0  1.7364   5 403    14.7  88.63
## 158  1.22358   0.0 19.58    0 0.6050 6.943  97.4  1.8773   5 403    14.7 363.43
## 159  1.34284   0.0 19.58    0 0.6050 6.066 100.0  1.7573   5 403    14.7 353.89
## 160  1.42502   0.0 19.58    0 0.8710 6.510 100.0  1.7659   5 403    14.7 364.31
## 161  1.27346   0.0 19.58    1 0.6050 6.250  92.6  1.7984   5 403    14.7 338.92
## 162  1.46336   0.0 19.58    0 0.6050 7.489  90.8  1.9709   5 403    14.7 374.43
## 163  1.83377   0.0 19.58    1 0.6050 7.802  98.2  2.0407   5 403    14.7 389.61
## 164  1.51902   0.0 19.58    1 0.6050 8.375  93.9  2.1620   5 403    14.7 388.45
## 165  2.24236   0.0 19.58    0 0.6050 5.854  91.8  2.4220   5 403    14.7 395.11
## 166  2.92400   0.0 19.58    0 0.6050 6.101  93.0  2.2834   5 403    14.7 240.16
## 167  2.01019   0.0 19.58    0 0.6050 7.929  96.2  2.0459   5 403    14.7 369.30
## 168  1.80028   0.0 19.58    0 0.6050 5.877  79.2  2.4259   5 403    14.7 227.61
## 169  2.30040   0.0 19.58    0 0.6050 6.319  96.1  2.1000   5 403    14.7 297.09
```

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## 170  2.44953   0.0 19.58    0 0.6050 6.402  95.2  2.2625   5 403    14.7 330.04
## 171  1.20742   0.0 19.58    0 0.6050 5.875  94.6  2.4259   5 403    14.7 292.29
## 172  2.31390   0.0 19.58    0 0.6050 5.880  97.3  2.3887   5 403    14.7 348.13
## 173  0.13914   0.0  4.05    0 0.5100 5.572  88.5  2.5961   5 296    16.6 396.90
## 174  0.09178   0.0  4.05    0 0.5100 6.416  84.1  2.6463   5 296    16.6 395.50
## 175  0.08447   0.0  4.05    0 0.5100 5.859  68.7  2.7019   5 296    16.6 393.23
## 176  0.06664   0.0  4.05    0 0.5100 6.546  33.1  3.1323   5 296    16.6 390.96
## 177  0.07022   0.0  4.05    0 0.5100 6.020  47.2  3.5549   5 296    16.6 393.23
## 178  0.05425   0.0  4.05    0 0.5100 6.315  73.4  3.3175   5 296    16.6 395.60
## 179  0.06642   0.0  4.05    0 0.5100 6.860  74.4  2.9153   5 296    16.6 391.27
## 180  0.05780   0.0  2.46    0 0.4880 6.980  58.4  2.8290   3 193    17.8 396.90
## 181  0.06588   0.0  2.46    0 0.4880 7.765  83.3  2.7410   3 193    17.8 395.56
## 182  0.06888   0.0  2.46    0 0.4880 6.144  62.2  2.5979   3 193    17.8 396.90
## 183  0.09103   0.0  2.46    0 0.4880 7.155  92.2  2.7006   3 193    17.8 394.12
## 184  0.10008   0.0  2.46    0 0.4880 6.563  95.6  2.8470   3 193    17.8 396.90
## 185  0.08308   0.0  2.46    0 0.4880 5.604  89.8  2.9879   3 193    17.8 391.00
## 186  0.06047   0.0  2.46    0 0.4880 6.153  68.8  3.2797   3 193    17.8 387.11
## 187  0.05602   0.0  2.46    0 0.4880 7.831  53.6  3.1992   3 193    17.8 392.63
## 188  0.07875  45.0  3.44    0 0.4370 6.782  41.1  3.7886   5 398    15.2 393.87
## 189  0.12579  45.0  3.44    0 0.4370 6.556  29.1  4.5667   5 398    15.2 382.84
## 190  0.08370  45.0  3.44    0 0.4370 7.185  38.9  4.5667   5 398    15.2 396.90
## 191  0.09068  45.0  3.44    0 0.4370 6.951  21.5  6.4798   5 398    15.2 377.68
## 192  0.06911  45.0  3.44    0 0.4370 6.739  30.8  6.4798   5 398    15.2 389.71
## 193  0.08664  45.0  3.44    0 0.4370 7.178  26.3  6.4798   5 398    15.2 390.49
## 194  0.02187  60.0  2.93    0 0.4010 6.800   9.9  6.2196   1 265    15.6 393.37
## 195  0.01439  60.0  2.93    0 0.4010 6.604  18.8  6.2196   1 265    15.6 376.70
## 196  0.01381  80.0  0.46    0 0.4220 7.875  32.0  5.6484   4 255    14.4 394.23
## 197  0.04011  80.0  1.52    0 0.4040 7.287  34.1  7.3090   2 329    12.6 396.90
## 198  0.04666  80.0  1.52    0 0.4040 7.107  36.6  7.3090   2 329    12.6 354.31
## 199  0.03768  80.0  1.52    0 0.4040 7.274  38.3  7.3090   2 329    12.6 392.20
## 200  0.03150  95.0  1.47    0 0.4030 6.975  15.3  7.6534   3 402    17.0 396.90
## 201  0.01778  95.0  1.47    0 0.4030 7.135  13.9  7.6534   3 402    17.0 384.30
## 202  0.03445  82.5  2.03    0 0.4150 6.162  38.4  6.2700   2 348    14.7 393.77
## 203  0.02177  82.5  2.03    0 0.4150 7.610  15.7  6.2700   2 348    14.7 395.38
## 204  0.03510  95.0  2.68    0 0.4161 7.853  33.2  5.1180   4 224    14.7 392.78
## 205  0.02009  95.0  2.68    0 0.4161 8.034  31.9  5.1180   4 224    14.7 390.55
## 206  0.13642   0.0 10.59    0 0.4890 5.891  22.3  3.9454   4 277    18.6 396.90
## 207  0.22969   0.0 10.59    0 0.4890 6.326  52.5  4.3549   4 277    18.6 394.87
## 208  0.25199   0.0 10.59    0 0.4890 5.783  72.7  4.3549   4 277    18.6 389.43
## 209  0.13587   0.0 10.59    1 0.4890 6.064  59.1  4.2392   4 277    18.6 381.32
## 210  0.43571   0.0 10.59    1 0.4890 5.344 100.0  3.8750   4 277    18.6 396.90
## 211  0.17446   0.0 10.59    1 0.4890 5.960  92.1  3.8771   4 277    18.6 393.25
## 212  0.37578   0.0 10.59    1 0.4890 5.404  88.6  3.6650   4 277    18.6 395.24
## 213  0.21719   0.0 10.59    1 0.4890 5.807  53.8  3.6526   4 277    18.6 390.94
## 214  0.14052   0.0 10.59    0 0.4890 6.375  32.3  3.9454   4 277    18.6 385.81
## 215  0.28955   0.0 10.59    0 0.4890 5.412   9.8  3.5875   4 277    18.6 348.93
## 216  0.19802   0.0 10.59    0 0.4890 6.182  42.4  3.9454   4 277    18.6 393.63
## 217  0.04560   0.0 13.89    1 0.5500 5.888  56.0  3.1121   5 276    16.4 392.80
## 218  0.07013   0.0 13.89    0 0.5500 6.642  85.1  3.4211   5 276    16.4 392.78
## 219  0.11069   0.0 13.89    1 0.5500 5.951  93.8  2.8893   5 276    16.4 396.90
## 220  0.11425   0.0 13.89    1 0.5500 6.373  92.4  3.3633   5 276    16.4 393.74
## 221  0.35809   0.0  6.20    1 0.5070 6.951  88.5  2.8617   8 307    17.4 391.70
## 222  0.40771   0.0  6.20    1 0.5070 6.164  91.3  3.0480   8 307    17.4 395.24
## 223  0.62356   0.0  6.20    1 0.5070 6.879  77.7  3.2721   8 307    17.4 390.39
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## 224 0.61470  0.0  6.20    0 0.5070 6.618  80.8 3.2721   8 307    17.4 396.90
## 225 0.31533  0.0  6.20    0 0.5040 8.266  78.3 2.8944   8 307    17.4 385.05
## 226 0.52693  0.0  6.20    0 0.5040 8.725  83.0 2.8944   8 307    17.4 382.00
## 227 0.38214  0.0  6.20    0 0.5040 8.040  86.5 3.2157   8 307    17.4 387.38
## 228 0.41238  0.0  6.20    0 0.5040 7.163  79.9 3.2157   8 307    17.4 372.08
## 229 0.29819  0.0  6.20    0 0.5040 7.686  17.0 3.3751   8 307    17.4 377.51
## 230 0.44178  0.0  6.20    0 0.5040 6.552  21.4 3.3751   8 307    17.4 380.34
## 231 0.53700  0.0  6.20    0 0.5040 5.981  68.1 3.6715   8 307    17.4 378.35
## 232 0.46296  0.0  6.20    0 0.5040 7.412  76.9 3.6715   8 307    17.4 376.14
## 233 0.57529  0.0  6.20    0 0.5070 8.337  73.3 3.8384   8 307    17.4 385.91
## 234 0.33147  0.0  6.20    0 0.5070 8.247  70.4 3.6519   8 307    17.4 378.95
## 235 0.44791  0.0  6.20    1 0.5070 6.726  66.5 3.6519   8 307    17.4 360.20
## 236 0.33045  0.0  6.20    0 0.5070 6.086  61.5 3.6519   8 307    17.4 376.75
## 237 0.52058  0.0  6.20    1 0.5070 6.631  76.5 4.1480   8 307    17.4 388.45
## 238 0.51183  0.0  6.20    0 0.5070 7.358  71.6 4.1480   8 307    17.4 390.07
## 239 0.08244 30.0  4.93    0 0.4280 6.481  18.5 6.1899   6 300    16.6 379.41
## 240 0.09252 30.0  4.93    0 0.4280 6.606  42.2 6.1899   6 300    16.6 383.78
## 241 0.11329 30.0  4.93    0 0.4280 6.897  54.3 6.3361   6 300    16.6 391.25
## 242 0.10612 30.0  4.93    0 0.4280 6.095  65.1 6.3361   6 300    16.6 394.62
## 243 0.10290 30.0  4.93    0 0.4280 6.358  52.9 7.0355   6 300    16.6 372.75
## 244 0.12757 30.0  4.93    0 0.4280 6.393   7.8 7.0355   6 300    16.6 374.71
## 245 0.20608 22.0  5.86    0 0.4310 5.593  76.5 7.9549   7 330    19.1 372.49
## 246 0.19133 22.0  5.86    0 0.4310 5.605  70.2 7.9549   7 330    19.1 389.13
## 247 0.33983 22.0  5.86    0 0.4310 6.108  34.9 8.0555   7 330    19.1 390.18
## 248 0.19657 22.0  5.86    0 0.4310 6.226  79.2 8.0555   7 330    19.1 376.14
## 249 0.16439 22.0  5.86    0 0.4310 6.433  49.1 7.8265   7 330    19.1 374.71
## 250 0.19073 22.0  5.86    0 0.4310 6.718  17.5 7.8265   7 330    19.1 393.74
## 251 0.14030 22.0  5.86    0 0.4310 6.487  13.0 7.3967   7 330    19.1 396.28
## 252 0.21409 22.0  5.86    0 0.4310 6.438   8.9 7.3967   7 330    19.1 377.07
## 253 0.08221 22.0  5.86    0 0.4310 6.957   6.8 8.9067   7 330    19.1 386.09
## 254 0.36894 22.0  5.86    0 0.4310 8.259   8.4 8.9067   7 330    19.1 396.90
## 255 0.04819 80.0  3.64    0 0.3920 6.108  32.0 9.2203   1 315    16.4 392.89
## 256 0.03548 80.0  3.64    0 0.3920 5.876  19.1 9.2203   1 315    16.4 395.18
## 257 0.01538 90.0  3.75    0 0.3940 7.454  34.2 6.3361   3 244    15.9 386.34
## 258 0.61154 20.0  3.97    0 0.6470 8.704  86.9 1.8010   5 264    13.0 389.70
## 259 0.66351 20.0  3.97    0 0.6470 7.333 100.0 1.8946   5 264    13.0 383.29
## 260 0.65665 20.0  3.97    0 0.6470 6.842 100.0 2.0107   5 264    13.0 391.93
## 261 0.54011 20.0  3.97    0 0.6470 7.203  81.8 2.1121   5 264    13.0 392.80
## 262 0.53412 20.0  3.97    0 0.6470 7.520  89.4 2.1398   5 264    13.0 388.37
## 263 0.52014 20.0  3.97    0 0.6470 8.398  91.5 2.2885   5 264    13.0 386.86
## 264 0.82526 20.0  3.97    0 0.6470 7.327  94.5 2.0788   5 264    13.0 393.42
## 265 0.55007 20.0  3.97    0 0.6470 7.206  91.6 1.9301   5 264    13.0 387.89
## 266 0.76162 20.0  3.97    0 0.6470 5.560  62.8 1.9865   5 264    13.0 392.40
## 267 0.78570 20.0  3.97    0 0.6470 7.014  84.6 2.1329   5 264    13.0 384.07
## 268 0.57834 20.0  3.97    0 0.5750 8.297  67.0 2.4216   5 264    13.0 384.54
## 269 0.54050 20.0  3.97    0 0.5750 7.470  52.6 2.8720   5 264    13.0 390.30
## 270 0.09065 20.0  6.96    1 0.4640 5.920  61.5 3.9175   3 223    18.6 391.34
## 271 0.29916 20.0  6.96    0 0.4640 5.856  42.1 4.4290   3 223    18.6 388.65
## 272 0.16211 20.0  6.96    0 0.4640 6.240  16.3 4.4290   3 223    18.6 396.90
## 273 0.11460 20.0  6.96    0 0.4640 6.538  58.7 3.9175   3 223    18.6 394.96
## 274 0.22188 20.0  6.96    1 0.4640 7.691  51.8 4.3665   3 223    18.6 390.77
## 275 0.05644 40.0  6.41    1 0.4470 6.758  32.9 4.0776   4 254    17.6 396.90
## 276 0.09604 40.0  6.41    0 0.4470 6.854  42.8 4.2673   4 254    17.6 396.90
## 277 0.10469 40.0  6.41    1 0.4470 7.267  49.0 4.7872   4 254    17.6 389.25
```

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## 278 0.06127 40.0  6.41    1 0.4470 6.826 27.6 4.8628  4 254    17.6 393.45
## 279 0.07978 40.0  6.41    0 0.4470 6.482 32.1 4.1403  4 254    17.6 396.90
## 280 0.21038 20.0  3.33    0 0.4429 6.812 32.2 4.1007  5 216    14.9 396.90
## 281 0.03578 20.0  3.33    0 0.4429 7.820 64.5 4.6947  5 216    14.9 387.31
## 282 0.03705 20.0  3.33    0 0.4429 6.968 37.2 5.2447  5 216    14.9 392.23
## 283 0.06129 20.0  3.33    1 0.4429 7.645 49.7 5.2119  5 216    14.9 377.07
## 284 0.01501 90.0  1.21    1 0.4010 7.923 24.8 5.8850  1 198    13.6 395.52
## 285 0.00906 90.0  2.97    0 0.4000 7.088 20.8 7.3073  1 285    15.3 394.72
## 286 0.01096 55.0  2.25    0 0.3890 6.453 31.9 7.3073  1 300    15.3 394.72
## 287 0.01965 80.0  1.76    0 0.3850 6.230 31.5 9.0892  1 241    18.2 341.60
## 288 0.03871 52.5  5.32    0 0.4050 6.209 31.3 7.3172  6 293    16.6 396.90
## 289 0.04590 52.5  5.32    0 0.4050 6.315 45.6 7.3172  6 293    16.6 396.90
## 290 0.04297 52.5  5.32    0 0.4050 6.565 22.9 7.3172  6 293    16.6 371.72
## 291 0.03502 80.0  4.95    0 0.4110 6.861 27.9 5.1167  4 245    19.2 396.90
## 292 0.07886 80.0  4.95    0 0.4110 7.148 27.7 5.1167  4 245    19.2 396.90
## 293 0.03615 80.0  4.95    0 0.4110 6.630 23.4 5.1167  4 245    19.2 396.90
## 294 0.08265  0.0 13.92    0 0.4370 6.127 18.4 5.5027  4 289    16.0 396.90
## 295 0.08199  0.0 13.92    0 0.4370 6.009 42.3 5.5027  4 289    16.0 396.90
## 296 0.12932  0.0 13.92    0 0.4370 6.678 31.1 5.9604  4 289    16.0 396.90
## 297 0.05372  0.0 13.92    0 0.4370 6.549 51.0 5.9604  4 289    16.0 392.85
## 298 0.14103  0.0 13.92    0 0.4370 5.790 58.0 6.3200  4 289    16.0 396.90
## 299 0.06466 70.0  2.24    0 0.4000 6.345 20.1 7.8278  5 358    14.8 368.24
## 300 0.05561 70.0  2.24    0 0.4000 7.041 10.0 7.8278  5 358    14.8 371.58
## 301 0.04417 70.0  2.24    0 0.4000 6.871 47.4 7.8278  5 358    14.8 390.86
## 302 0.03537 34.0  6.09    0 0.4330 6.590 40.4 5.4917  7 329    16.1 395.75
## 303 0.09266 34.0  6.09    0 0.4330 6.495 18.4 5.4917  7 329    16.1 383.61
## 304 0.10000 34.0  6.09    0 0.4330 6.982 17.7 5.4917  7 329    16.1 390.43
## 305 0.05515 33.0  2.18    0 0.4720 7.236 41.1 4.0220  7 222    18.4 393.68
## 306 0.05479 33.0  2.18    0 0.4720 6.616 58.1 3.3700  7 222    18.4 393.36
## 307 0.07503 33.0  2.18    0 0.4720 7.420 71.9 3.0992  7 222    18.4 396.90
## 308 0.04932 33.0  2.18    0 0.4720 6.849 70.3 3.1827  7 222    18.4 396.90
## 309 0.49298  0.0  9.90    0 0.5440 6.635 82.5 3.3175  4 304    18.4 396.90
## 310 0.34940  0.0  9.90    0 0.5440 5.972 76.7 3.1025  4 304    18.4 396.24
## 311 2.63548  0.0  9.90    0 0.5440 4.973 37.8 2.5194  4 304    18.4 350.45
## 312 0.79041  0.0  9.90    0 0.5440 6.122 52.8 2.6403  4 304    18.4 396.90
## 313 0.26169  0.0  9.90    0 0.5440 6.023 90.4 2.8340  4 304    18.4 396.30
## 314 0.26938  0.0  9.90    0 0.5440 6.266 82.8 3.2628  4 304    18.4 393.39
## 315 0.36920  0.0  9.90    0 0.5440 6.567 87.3 3.6023  4 304    18.4 395.69
## 316 0.25356  0.0  9.90    0 0.5440 5.705 77.7 3.9450  4 304    18.4 396.42
## 317 0.31827  0.0  9.90    0 0.5440 5.914 83.2 3.9986  4 304    18.4 390.70
## 318 0.24522  0.0  9.90    0 0.5440 5.782 71.7 4.0317  4 304    18.4 396.90
## 319 0.40202  0.0  9.90    0 0.5440 6.382 67.2 3.5325  4 304    18.4 395.21
## 320 0.47547  0.0  9.90    0 0.5440 6.113 58.8 4.0019  4 304    18.4 396.23
## 321 0.16760  0.0  7.38    0 0.4930 6.426 52.3 4.5404  5 287    19.6 396.90
## 322 0.18159  0.0  7.38    0 0.4930 6.376 54.3 4.5404  5 287    19.6 396.90
## 323 0.35114  0.0  7.38    0 0.4930 6.041 49.9 4.7211  5 287    19.6 396.90
## 324 0.28392  0.0  7.38    0 0.4930 5.708 74.3 4.7211  5 287    19.6 391.13
## 325 0.34109  0.0  7.38    0 0.4930 6.415 40.1 4.7211  5 287    19.6 396.90
## 326 0.19186  0.0  7.38    0 0.4930 6.431 14.7 5.4159  5 287    19.6 393.68
## 327 0.30347  0.0  7.38    0 0.4930 6.312 28.9 5.4159  5 287    19.6 396.90
## 328 0.24103  0.0  7.38    0 0.4930 6.083 43.7 5.4159  5 287    19.6 396.90
## 329 0.06617  0.0  3.24    0 0.4600 5.868 25.8 5.2146  4 430    16.9 382.44
## 330 0.06724  0.0  3.24    0 0.4600 6.333 17.2 5.2146  4 430    16.9 375.21
## 331 0.04544  0.0  3.24    0 0.4600 6.144 32.2 5.8736  4 430    16.9 368.57
```

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## 332  0.05023  35.0  6.06    0 0.4379 5.706  28.4  6.6407   1 304    16.9 394.02
## 333  0.03466  35.0  6.06    0 0.4379 6.031  23.3  6.6407   1 304    16.9 362.25
## 334  0.05083   0.0  5.19    0 0.5150 6.316  38.1  6.4584   5 224    20.2 389.71
## 335  0.03738   0.0  5.19    0 0.5150 6.310  38.5  6.4584   5 224    20.2 389.40
## 336  0.03961   0.0  5.19    0 0.5150 6.037  34.5  5.9853   5 224    20.2 396.90
## 337  0.03427   0.0  5.19    0 0.5150 5.869  46.3  5.2311   5 224    20.2 396.90
## 338  0.03041   0.0  5.19    0 0.5150 5.895  59.6  5.6150   5 224    20.2 394.81
## 339  0.03306   0.0  5.19    0 0.5150 6.059  37.3  4.8122   5 224    20.2 396.14
## 340  0.05497   0.0  5.19    0 0.5150 5.985  45.4  4.8122   5 224    20.2 396.90
## 341  0.06151   0.0  5.19    0 0.5150 5.968  58.5  4.8122   5 224    20.2 396.90
## 342  0.01301  35.0  1.52    0 0.4420 7.241  49.3  7.0379   1 284    15.5 394.74
## 343  0.02498   0.0  1.89    0 0.5180 6.540  59.7  6.2669   1 422    15.9 389.96
## 344  0.02543  55.0  3.78    0 0.4840 6.696  56.4  5.7321   5 370    17.6 396.90
## 345  0.03049  55.0  3.78    0 0.4840 6.874  28.1  6.4654   5 370    17.6 387.97
## 346  0.03113   0.0  4.39    0 0.4420 6.014  48.5  8.0136   3 352    18.8 385.64
## 347  0.06162   0.0  4.39    0 0.4420 5.898  52.3  8.0136   3 352    18.8 364.61
## 348  0.01870  85.0  4.15    0 0.4290 6.516  27.7  8.5353   4 351    17.9 392.43
## 349  0.01501  80.0  2.01    0 0.4350 6.635  29.7  8.3440   4 280    17.0 390.94
## 350  0.02899  40.0  1.25    0 0.4290 6.939  34.5  8.7921   1 335    19.7 389.85
## 351  0.06211  40.0  1.25    0 0.4290 6.490  44.4  8.7921   1 335    19.7 396.90
## 352  0.07950  60.0  1.69    0 0.4110 6.579  35.9 10.7103   4 411    18.3 370.78
## 353  0.07244  60.0  1.69    0 0.4110 5.884  18.5 10.7103   4 411    18.3 392.33
## 354  0.01709  90.0  2.02    0 0.4100 6.728  36.1 12.1265   5 187    17.0 384.46
## 355  0.04301  80.0  1.91    0 0.4130 5.663  21.9 10.5857   4 334    22.0 382.80
## 356  0.10659  80.0  1.91    0 0.4130 5.936  19.5 10.5857   4 334    22.0 376.04
## 357  8.98296   0.0 18.10    1 0.7700 6.212  97.4  2.1222  24 666    20.2 377.73
## 358  3.84970   0.0 18.10    1 0.7700 6.395  91.0  2.5052  24 666    20.2 391.34
## 359  5.20177   0.0 18.10    1 0.7700 6.127  83.4  2.7227  24 666    20.2 395.43
## 360  4.26131   0.0 18.10    0 0.7700 6.112  81.3  2.5091  24 666    20.2 390.74
## 361  4.54192   0.0 18.10    0 0.7700 6.398  88.0  2.5182  24 666    20.2 374.56
## 362  3.83684   0.0 18.10    0 0.7700 6.251  91.1  2.2955  24 666    20.2 350.65
## 363  3.67822   0.0 18.10    0 0.7700 5.362  96.2  2.1036  24 666    20.2 380.79
## 364  4.22239   0.0 18.10    1 0.7700 5.803  89.0  1.9047  24 666    20.2 353.04
## 365  3.47428   0.0 18.10    1 0.7180 8.780  82.9  1.9047  24 666    20.2 354.55
## 366  4.55587   0.0 18.10    0 0.7180 3.561  87.9  1.6132  24 666    20.2 354.70
## 367  3.69695   0.0 18.10    0 0.7180 4.963  91.4  1.7523  24 666    20.2 316.03
## 368 13.52220   0.0 18.10    0 0.6310 3.863 100.0  1.5106  24 666    20.2 131.42
## 369  4.89822   0.0 18.10    0 0.6310 4.970 100.0  1.3325  24 666    20.2 375.52
## 370  5.66998   0.0 18.10    1 0.6310 6.683  96.8  1.3567  24 666    20.2 375.33
## 371  6.53876   0.0 18.10    1 0.6310 7.016  97.5  1.2024  24 666    20.2 392.05
## 372  9.23230   0.0 18.10    0 0.6310 6.216 100.0  1.1691  24 666    20.2 366.15
## 373  8.26725   0.0 18.10    1 0.6680 5.875  89.6  1.1296  24 666    20.2 347.88
## 374 11.10810   0.0 18.10    0 0.6680 4.906 100.0  1.1742  24 666    20.2 396.90
## 375 18.49820   0.0 18.10    0 0.6680 4.138 100.0  1.1370  24 666    20.2 396.90
## 376 19.60910   0.0 18.10    0 0.6710 7.313  97.9  1.3163  24 666    20.2 396.90
## 377 15.28800   0.0 18.10    0 0.6710 6.649  93.3  1.3449  24 666    20.2 363.02
## 378  9.82349   0.0 18.10    0 0.6710 6.794  98.8  1.3580  24 666    20.2 396.90
## 379 23.64820   0.0 18.10    0 0.6710 6.380  96.2  1.3861  24 666    20.2 396.90
## 380 17.86670   0.0 18.10    0 0.6710 6.223 100.0  1.3861  24 666    20.2 393.74
## 381 88.97620   0.0 18.10    0 0.6710 6.968  91.9  1.4165  24 666    20.2 396.90
## 382 15.87440   0.0 18.10    0 0.6710 6.545  99.1  1.5192  24 666    20.2 396.90
## 383  9.18702   0.0 18.10    0 0.7000 5.536 100.0  1.5804  24 666    20.2 396.90
## 384  7.99248   0.0 18.10    0 0.7000 5.520 100.0  1.5331  24 666    20.2 396.90
## 385 20.08490   0.0 18.10    0 0.7000 4.368  91.2  1.4395  24 666    20.2 285.83
```

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## 386 16.81180   0.0 18.10    0 0.7000 5.277  98.1  1.4261  24 666    20.2 396.90
## 387 24.39380   0.0 18.10    0 0.7000 4.652 100.0  1.4672  24 666    20.2 396.90
## 388 22.59710   0.0 18.10    0 0.7000 5.000  89.5  1.5184  24 666    20.2 396.90
## 389 14.33370   0.0 18.10    0 0.7000 4.880 100.0  1.5895  24 666    20.2 372.92
## 390  8.15174   0.0 18.10    0 0.7000 5.390  98.9  1.7281  24 666    20.2 396.90
## 391  6.96215   0.0 18.10    0 0.7000 5.713  97.0  1.9265  24 666    20.2 394.43
## 392  5.29305   0.0 18.10    0 0.7000 6.051  82.5  2.1678  24 666    20.2 378.38
## 393 11.57790   0.0 18.10    0 0.7000 5.036  97.0  1.7700  24 666    20.2 396.90
## 394  8.64476   0.0 18.10    0 0.6930 6.193  92.6  1.7912  24 666    20.2 396.90
## 395 13.35980   0.0 18.10    0 0.6930 5.887  94.7  1.7821  24 666    20.2 396.90
## 396  8.71675   0.0 18.10    0 0.6930 6.471  98.8  1.7257  24 666    20.2 391.98
## 397  5.87205   0.0 18.10    0 0.6930 6.405  96.0  1.6768  24 666    20.2 396.90
## 398  7.67202   0.0 18.10    0 0.6930 5.747  98.9  1.6334  24 666    20.2 393.10
## 399 38.35180   0.0 18.10    0 0.6930 5.453 100.0  1.4896  24 666    20.2 396.90
## 400  9.91655   0.0 18.10    0 0.6930 5.852  77.8  1.5004  24 666    20.2 338.16
## 401 25.04610   0.0 18.10    0 0.6930 5.987 100.0  1.5888  24 666    20.2 396.90
## 402 14.23620   0.0 18.10    0 0.6930 6.343 100.0  1.5741  24 666    20.2 396.90
## 403  9.59571   0.0 18.10    0 0.6930 6.404 100.0  1.6390  24 666    20.2 376.11
## 404 24.80170   0.0 18.10    0 0.6930 5.349  96.0  1.7028  24 666    20.2 396.90
## 405 41.52920   0.0 18.10    0 0.6930 5.531  85.4  1.6074  24 666    20.2 329.46
## 406 67.92080   0.0 18.10    0 0.6930 5.683 100.0  1.4254  24 666    20.2 384.97
## 407 20.71620   0.0 18.10    0 0.6590 4.138 100.0  1.1781  24 666    20.2 370.22
## 408 11.95110   0.0 18.10    0 0.6590 5.608 100.0  1.2852  24 666    20.2 332.09
## 409  7.40389   0.0 18.10    0 0.5970 5.617  97.9  1.4547  24 666    20.2 314.64
## 410 14.43830   0.0 18.10    0 0.5970 6.852 100.0  1.4655  24 666    20.2 179.36
## 411 51.13580   0.0 18.10    0 0.5970 5.757 100.0  1.4130  24 666    20.2   2.60
## 412 14.05070   0.0 18.10    0 0.5970 6.657 100.0  1.5275  24 666    20.2  35.05
## 413 18.81100   0.0 18.10    0 0.5970 4.628 100.0  1.5539  24 666    20.2  28.79
## 414 28.65580   0.0 18.10    0 0.5970 5.155 100.0  1.5894  24 666    20.2 210.97
## 415 45.74610   0.0 18.10    0 0.6930 4.519 100.0  1.6582  24 666    20.2  88.27
## 416 18.08460   0.0 18.10    0 0.6790 6.434 100.0  1.8347  24 666    20.2  27.25
## 417 10.83420   0.0 18.10    0 0.6790 6.782  90.8  1.8195  24 666    20.2  21.57
## 418 25.94060   0.0 18.10    0 0.6790 5.304  89.1  1.6475  24 666    20.2 127.36
## 419 73.53410   0.0 18.10    0 0.6790 5.957 100.0  1.8026  24 666    20.2  16.45
## 420 11.81230   0.0 18.10    0 0.7180 6.824  76.5  1.7940  24 666    20.2  48.45
## 421 11.08740   0.0 18.10    0 0.7180 6.411 100.0  1.8589  24 666    20.2 318.75
## 422  7.02259   0.0 18.10    0 0.7180 6.006  95.3  1.8746  24 666    20.2 319.98
## 423 12.04820   0.0 18.10    0 0.6140 5.648  87.6  1.9512  24 666    20.2 291.55
## 424  7.05042   0.0 18.10    0 0.6140 6.103  85.1  2.0218  24 666    20.2   2.52
## 425  8.79212   0.0 18.10    0 0.5840 5.565  70.6  2.0635  24 666    20.2   3.65
## 426 15.86030   0.0 18.10    0 0.6790 5.896  95.4  1.9096  24 666    20.2   7.68
## 427 12.24720   0.0 18.10    0 0.5840 5.837  59.7  1.9976  24 666    20.2  24.65
## 428 37.66190   0.0 18.10    0 0.6790 6.202  78.7  1.8629  24 666    20.2  18.82
## 429  7.36711   0.0 18.10    0 0.6790 6.193  78.1  1.9356  24 666    20.2  96.73
## 430  9.33889   0.0 18.10    0 0.6790 6.380  95.6  1.9682  24 666    20.2  60.72
## 431  8.49213   0.0 18.10    0 0.5840 6.348  86.1  2.0527  24 666    20.2  83.45
## 432 10.06230   0.0 18.10    0 0.5840 6.833  94.3  2.0882  24 666    20.2  81.33
## 433  6.44405   0.0 18.10    0 0.5840 6.425  74.8  2.2004  24 666    20.2  97.95
## 434  5.58107   0.0 18.10    0 0.7130 6.436  87.9  2.3158  24 666    20.2 100.19
## 435 13.91340   0.0 18.10    0 0.7130 6.208  95.0  2.2222  24 666    20.2 100.63
## 436 11.16040   0.0 18.10    0 0.7400 6.629  94.6  2.1247  24 666    20.2 109.85
## 437 14.42080   0.0 18.10    0 0.7400 6.461  93.3  2.0026  24 666    20.2  27.49
## 438 15.17720   0.0 18.10    0 0.7400 6.152 100.0  1.9142  24 666    20.2   9.32
## 439 13.67810   0.0 18.10    0 0.7400 5.935  87.9  1.8206  24 666    20.2  68.95
```

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## 440  9.39063   0.0 18.10    0 0.7400 5.627  93.9  1.8172 24 666    20.2 396.90
## 441 22.05110   0.0 18.10    0 0.7400 5.818  92.4  1.8662 24 666    20.2 391.45
## 442  9.72418   0.0 18.10    0 0.7400 6.406  97.2  2.0651 24 666    20.2 385.96
## 443  5.66637   0.0 18.10    0 0.7400 6.219 100.0  2.0048 24 666    20.2 395.69
## 444  9.96654   0.0 18.10    0 0.7400 6.485 100.0  1.9784 24 666    20.2 386.73
## 445 12.80230   0.0 18.10    0 0.7400 5.854  96.6  1.8956 24 666    20.2 240.52
## 446 10.67180   0.0 18.10    0 0.7400 6.459  94.8  1.9879 24 666    20.2  43.06
## 447  6.28807   0.0 18.10    0 0.7400 6.341  96.4  2.0720 24 666    20.2 318.01
## 448  9.92485   0.0 18.10    0 0.7400 6.251  96.6  2.1980 24 666    20.2 388.52
## 449  9.32909   0.0 18.10    0 0.7130 6.185  98.7  2.2616 24 666    20.2 396.90
## 450  7.52601   0.0 18.10    0 0.7130 6.417  98.3  2.1850 24 666    20.2 304.21
## 451  6.71772   0.0 18.10    0 0.7130 6.749  92.6  2.3236 24 666    20.2   0.32
## 452  5.44114   0.0 18.10    0 0.7130 6.655  98.2  2.3552 24 666    20.2 355.29
## 453  5.09017   0.0 18.10    0 0.7130 6.297  91.8  2.3682 24 666    20.2 385.09
## 454  8.24809   0.0 18.10    0 0.7130 7.393  99.3  2.4527 24 666    20.2 375.87
## 455  9.51363   0.0 18.10    0 0.7130 6.728  94.1  2.4961 24 666    20.2   6.68
## 456  4.75237   0.0 18.10    0 0.7130 6.525  86.5  2.4358 24 666    20.2  50.92
## 457  4.66883   0.0 18.10    0 0.7130 5.976  87.9  2.5806 24 666    20.2  10.48
## 458  8.20058   0.0 18.10    0 0.7130 5.936  80.3  2.7792 24 666    20.2   3.50
## 459  7.75223   0.0 18.10    0 0.7130 6.301  83.7  2.7831 24 666    20.2 272.21
## 460  6.80117   0.0 18.10    0 0.7130 6.081  84.4  2.7175 24 666    20.2 396.90
## 461  4.81213   0.0 18.10    0 0.7130 6.701  90.0  2.5975 24 666    20.2 255.23
## 462  3.69311   0.0 18.10    0 0.7130 6.376  88.4  2.5671 24 666    20.2 391.43
## 463  6.65492   0.0 18.10    0 0.7130 6.317  83.0  2.7344 24 666    20.2 396.90
## 464  5.82115   0.0 18.10    0 0.7130 6.513  89.9  2.8016 24 666    20.2 393.82
## 465  7.83932   0.0 18.10    0 0.6550 6.209  65.4  2.9634 24 666    20.2 396.90
## 466  3.16360   0.0 18.10    0 0.6550 5.759  48.2  3.0665 24 666    20.2 334.40
## 467  3.77498   0.0 18.10    0 0.6550 5.952  84.7  2.8715 24 666    20.2  22.01
## 468  4.42228   0.0 18.10    0 0.5840 6.003  94.5  2.5403 24 666    20.2 331.29
## 469 15.57570   0.0 18.10    0 0.5800 5.926  71.0  2.9084 24 666    20.2 368.74
## 470 13.07510   0.0 18.10    0 0.5800 5.713  56.7  2.8237 24 666    20.2 396.90
## 471  4.34879   0.0 18.10    0 0.5800 6.167  84.0  3.0334 24 666    20.2 396.90
## 472  4.03841   0.0 18.10    0 0.5320 6.229  90.7  3.0993 24 666    20.2 395.33
## 473  3.56868   0.0 18.10    0 0.5800 6.437  75.0  2.8965 24 666    20.2 393.37
## 474  4.64689   0.0 18.10    0 0.6140 6.980  67.6  2.5329 24 666    20.2 374.68
## 475  8.05579   0.0 18.10    0 0.5840 5.427  95.4  2.4298 24 666    20.2 352.58
## 476  6.39312   0.0 18.10    0 0.5840 6.162  97.4  2.2060 24 666    20.2 302.76
## 477  4.87141   0.0 18.10    0 0.6140 6.484  93.6  2.3053 24 666    20.2 396.21
## 478 15.02340   0.0 18.10    0 0.6140 5.304  97.3  2.1007 24 666    20.2 349.48
## 479 10.23300   0.0 18.10    0 0.6140 6.185  96.7  2.1705 24 666    20.2 379.70
## 480 14.33370   0.0 18.10    0 0.6140 6.229  88.0  1.9512 24 666    20.2 383.32
## 481  5.82401   0.0 18.10    0 0.5320 6.242  64.7  3.4242 24 666    20.2 396.90
## 482  5.70818   0.0 18.10    0 0.5320 6.750  74.9  3.3317 24 666    20.2 393.07
## 483  5.73116   0.0 18.10    0 0.5320 7.061  77.0  3.4106 24 666    20.2 395.28
## 484  2.81838   0.0 18.10    0 0.5320 5.762  40.3  4.0983 24 666    20.2 392.92
## 485  2.37857   0.0 18.10    0 0.5830 5.871  41.9  3.7240 24 666    20.2 370.73
## 486  3.67367   0.0 18.10    0 0.5830 6.312  51.9  3.9917 24 666    20.2 388.62
## 487  5.69175   0.0 18.10    0 0.5830 6.114  79.8  3.5459 24 666    20.2 392.68
## 488  4.83567   0.0 18.10    0 0.5830 5.905  53.2  3.1523 24 666    20.2 388.22
## 489  0.15086   0.0 27.74    0 0.6090 5.454  92.7  1.8209  4 711    20.1 395.09
## 490  0.18337   0.0 27.74    0 0.6090 5.414  98.3  1.7554  4 711    20.1 344.05
## 491  0.20746   0.0 27.74    0 0.6090 5.093  98.0  1.8226  4 711    20.1 318.43
## 492  0.10574   0.0 27.74    0 0.6090 5.983  98.8  1.8681  4 711    20.1 390.11
## 493  0.11132   0.0 27.74    0 0.6090 5.983  83.5  2.1099  4 711    20.1 396.90
```

```
## 494  0.17331   0.0  9.69    0 0.5850 5.707  54.0  2.3817   6 391    19.2 396.90
## 495  0.27957   0.0  9.69    0 0.5850 5.926  42.6  2.3817   6 391    19.2 396.90
## 496  0.17899   0.0  9.69    0 0.5850 5.670  28.8  2.7986   6 391    19.2 393.29
## 497  0.28960   0.0  9.69    0 0.5850 5.390  72.9  2.7986   6 391    19.2 396.90
## 498  0.26838   0.0  9.69    0 0.5850 5.794  70.6  2.8927   6 391    19.2 396.90
## 499  0.23912   0.0  9.69    0 0.5850 6.019  65.3  2.4091   6 391    19.2 396.90
## 500  0.17783   0.0  9.69    0 0.5850 5.569  73.5  2.3999   6 391    19.2 395.77
## 501  0.22438   0.0  9.69    0 0.5850 6.027  79.7  2.4982   6 391    19.2 396.90
## 502  0.06263   0.0 11.93    0 0.5730 6.593  69.1  2.4786   1 273    21.0 391.99
## 503  0.04527   0.0 11.93    0 0.5730 6.120  76.7  2.2875   1 273    21.0 396.90
## 504  0.06076   0.0 11.93    0 0.5730 6.976  91.0  2.1675   1 273    21.0 396.90
## 505  0.10959   0.0 11.93    0 0.5730 6.794  89.3  2.3889   1 273    21.0 393.45
## 506  0.04741   0.0 11.93    0 0.5730 6.030  80.8  2.5050   1 273    21.0 396.90
##     lstat medv
## 1    4.98 24.0
## 2    9.14 21.6
## 3    4.03 34.7
## 4    2.94 33.4
## 5    5.33 36.2
## 6    5.21 28.7
## 7   12.43 22.9
## 8   19.15 27.1
## 9   29.93 16.5
## 10  17.10 18.9
## 11  20.45 15.0
## 12  13.27 18.9
## 13  15.71 21.7
## 14   8.26 20.4
## 15  10.26 18.2
## 16   8.47 19.9
## 17   6.58 23.1
## 18  14.67 17.5
## 19  11.69 20.2
## 20  11.28 18.2
## 21  21.02 13.6
## 22  13.83 19.6
## 23  18.72 15.2
## 24  19.88 14.5
## 25  16.30 15.6
## 26  16.51 13.9
## 27  14.81 16.6
## 28  17.28 14.8
## 29  12.80 18.4
## 30  11.98 21.0
## 31  22.60 12.7
## 32  13.04 14.5
## 33  27.71 13.2
## 34  18.35 13.1
## 35  20.34 13.5
## 36   9.68 18.9
## 37  11.41 20.0
## 38   8.77 21.0
## 39  10.13 24.7
## 40   4.32 30.8
```

```
## 41   1.98 34.9
## 42   4.84 26.6
## 43   5.81 25.3
## 44   7.44 24.7
## 45   9.55 21.2
## 46  10.21 19.3
## 47  14.15 20.0
## 48  18.80 16.6
## 49  30.81 14.4
## 50  16.20 19.4
## 51  13.45 19.7
## 52   9.43 20.5
## 53   5.28 25.0
## 54   8.43 23.4
## 55  14.80 18.9
## 56   4.81 35.4
## 57   5.77 24.7
## 58   3.95 31.6
## 59   6.86 23.3
## 60   9.22 19.6
## 61  13.15 18.7
## 62  14.44 16.0
## 63   6.73 22.2
## 64   9.50 25.0
## 65   8.05 33.0
## 66   4.67 23.5
## 67  10.24 19.4
## 68   8.10 22.0
## 69  13.09 17.4
## 70   8.79 20.9
## 71   6.72 24.2
## 72   9.88 21.7
## 73   5.52 22.8
## 74   7.54 23.4
## 75   6.78 24.1
## 76   8.94 21.4
## 77  11.97 20.0
## 78  10.27 20.8
## 79  12.34 21.2
## 80   9.10 20.3
## 81   5.29 28.0
## 82   7.22 23.9
## 83   6.72 24.8
## 84   7.51 22.9
## 85   9.62 23.9
## 86   6.53 26.6
## 87  12.86 22.5
## 88   8.44 22.2
## 89   5.50 23.6
## 90   5.70 28.7
## 91   8.81 22.6
## 92   8.20 22.0
## 93   8.16 22.9
## 94   6.21 25.0
```

```
## 95  10.59 20.6
## 96   6.65 28.4
## 97  11.34 21.4
## 98   4.21 38.7
## 99   3.57 43.8
## 100  6.19 33.2
## 101  9.42 27.5
## 102  7.67 26.5
## 103 10.63 18.6
## 104 13.44 19.3
## 105 12.33 20.1
## 106 16.47 19.5
## 107 18.66 19.5
## 108 14.09 20.4
## 109 12.27 19.8
## 110 15.55 19.4
## 111 13.00 21.7
## 112 10.16 22.8
## 113 16.21 18.8
## 114 17.09 18.7
## 115 10.45 18.5
## 116 15.76 18.3
## 117 12.04 21.2
## 118 10.30 19.2
## 119 15.37 20.4
## 120 13.61 19.3
## 121 14.37 22.0
## 122 14.27 20.3
## 123 17.93 20.5
## 124 25.41 17.3
## 125 17.58 18.8
## 126 14.81 21.4
## 127 27.26 15.7
## 128 17.19 16.2
## 129 15.39 18.0
## 130 18.34 14.3
## 131 12.60 19.2
## 132 12.26 19.6
## 133 11.12 23.0
## 134 15.03 18.4
## 135 17.31 15.6
## 136 16.96 18.1
## 137 16.90 17.4
## 138 14.59 17.1
## 139 21.32 13.3
## 140 18.46 17.8
## 141 24.16 14.0
## 142 34.41 14.4
## 143 26.82 13.4
## 144 26.42 15.6
## 145 29.29 11.8
## 146 27.80 13.8
## 147 16.65 15.6
## 148 29.53 14.6
```

```
## 149 28.32 17.8
## 150 21.45 15.4
## 151 14.10 21.5
## 152 13.28 19.6
## 153 12.12 15.3
## 154 15.79 19.4
## 155 15.12 17.0
## 156 15.02 15.6
## 157 16.14 13.1
## 158  4.59 41.3
## 159  6.43 24.3
## 160  7.39 23.3
## 161  5.50 27.0
## 162  1.73 50.0
## 163  1.92 50.0
## 164  3.32 50.0
## 165 11.64 22.7
## 166  9.81 25.0
## 167  3.70 50.0
## 168 12.14 23.8
## 169 11.10 23.8
## 170 11.32 22.3
## 171 14.43 17.4
## 172 12.03 19.1
## 173 14.69 23.1
## 174  9.04 23.6
## 175  9.64 22.6
## 176  5.33 29.4
## 177 10.11 23.2
## 178  6.29 24.6
## 179  6.92 29.9
## 180  5.04 37.2
## 181  7.56 39.8
## 182  9.45 36.2
## 183  4.82 37.9
## 184  5.68 32.5
## 185 13.98 26.4
## 186 13.15 29.6
## 187  4.45 50.0
## 188  6.68 32.0
## 189  4.56 29.8
## 190  5.39 34.9
## 191  5.10 37.0
## 192  4.69 30.5
## 193  2.87 36.4
## 194  5.03 31.1
## 195  4.38 29.1
## 196  2.97 50.0
## 197  4.08 33.3
## 198  8.61 30.3
## 199  6.62 34.6
## 200  4.56 34.9
## 201  4.45 32.9
## 202  7.43 24.1
```

```
## 203  3.11 42.3
## 204  3.81 48.5
## 205  2.88 50.0
## 206 10.87 22.6
## 207 10.97 24.4
## 208 18.06 22.5
## 209 14.66 24.4
## 210 23.09 20.0
## 211 17.27 21.7
## 212 23.98 19.3
## 213 16.03 22.4
## 214  9.38 28.1
## 215 29.55 23.7
## 216  9.47 25.0
## 217 13.51 23.3
## 218  9.69 28.7
## 219 17.92 21.5
## 220 10.50 23.0
## 221  9.71 26.7
## 222 21.46 21.7
## 223  9.93 27.5
## 224  7.60 30.1
## 225  4.14 44.8
## 226  4.63 50.0
## 227  3.13 37.6
## 228  6.36 31.6
## 229  3.92 46.7
## 230  3.76 31.5
## 231 11.65 24.3
## 232  5.25 31.7
## 233  2.47 41.7
## 234  3.95 48.3
## 235  8.05 29.0
## 236 10.88 24.0
## 237  9.54 25.1
## 238  4.73 31.5
## 239  6.36 23.7
## 240  7.37 23.3
## 241 11.38 22.0
## 242 12.40 20.1
## 243 11.22 22.2
## 244  5.19 23.7
## 245 12.50 17.6
## 246 18.46 18.5
## 247  9.16 24.3
## 248 10.15 20.5
## 249  9.52 24.5
## 250  6.56 26.2
## 251  5.90 24.4
## 252  3.59 24.8
## 253  3.53 29.6
## 254  3.54 42.8
## 255  6.57 21.9
## 256  9.25 20.9
```

```
## 257  3.11 44.0
## 258  5.12 50.0
## 259  7.79 36.0
## 260  6.90 30.1
## 261  9.59 33.8
## 262  7.26 43.1
## 263  5.91 48.8
## 264 11.25 31.0
## 265  8.10 36.5
## 266 10.45 22.8
## 267 14.79 30.7
## 268  7.44 50.0
## 269  3.16 43.5
## 270 13.65 20.7
## 271 13.00 21.1
## 272  6.59 25.2
## 273  7.73 24.4
## 274  6.58 35.2
## 275  3.53 32.4
## 276  2.98 32.0
## 277  6.05 33.2
## 278  4.16 33.1
## 279  7.19 29.1
## 280  4.85 35.1
## 281  3.76 45.4
## 282  4.59 35.4
## 283  3.01 46.0
## 284  3.16 50.0
## 285  7.85 32.2
## 286  8.23 22.0
## 287 12.93 20.1
## 288  7.14 23.2
## 289  7.60 22.3
## 290  9.51 24.8
## 291  3.33 28.5
## 292  3.56 37.3
## 293  4.70 27.9
## 294  8.58 23.9
## 295 10.40 21.7
## 296  6.27 28.6
## 297  7.39 27.1
## 298 15.84 20.3
## 299  4.97 22.5
## 300  4.74 29.0
## 301  6.07 24.8
## 302  9.50 22.0
## 303  8.67 26.4
## 304  4.86 33.1
## 305  6.93 36.1
## 306  8.93 28.4
## 307  6.47 33.4
## 308  7.53 28.2
## 309  4.54 22.8
## 310  9.97 20.3
```

```
## 311 12.64 16.1
## 312  5.98 22.1
## 313 11.72 19.4
## 314  7.90 21.6
## 315  9.28 23.8
## 316 11.50 16.2
## 317 18.33 17.8
## 318 15.94 19.8
## 319 10.36 23.1
## 320 12.73 21.0
## 321  7.20 23.8
## 322  6.87 23.1
## 323  7.70 20.4
## 324 11.74 18.5
## 325  6.12 25.0
## 326  5.08 24.6
## 327  6.15 23.0
## 328 12.79 22.2
## 329  9.97 19.3
## 330  7.34 22.6
## 331  9.09 19.8
## 332 12.43 17.1
## 333  7.83 19.4
## 334  5.68 22.2
## 335  6.75 20.7
## 336  8.01 21.1
## 337  9.80 19.5
## 338 10.56 18.5
## 339  8.51 20.6
## 340  9.74 19.0
## 341  9.29 18.7
## 342  5.49 32.7
## 343  8.65 16.5
## 344  7.18 23.9
## 345  4.61 31.2
## 346 10.53 17.5
## 347 12.67 17.2
## 348  6.36 23.1
## 349  5.99 24.5
## 350  5.89 26.6
## 351  5.98 22.9
## 352  5.49 24.1
## 353  7.79 18.6
## 354  4.50 30.1
## 355  8.05 18.2
## 356  5.57 20.6
## 357 17.60 17.8
## 358 13.27 21.7
## 359 11.48 22.7
## 360 12.67 22.6
## 361  7.79 25.0
## 362 14.19 19.9
## 363 10.19 20.8
## 364 14.64 16.8
```

```
## 365  5.29 21.9
## 366  7.12 27.5
## 367 14.00 21.9
## 368 13.33 23.1
## 369  3.26 50.0
## 370  3.73 50.0
## 371  2.96 50.0
## 372  9.53 50.0
## 373  8.88 50.0
## 374 34.77 13.8
## 375 37.97 13.8
## 376 13.44 15.0
## 377 23.24 13.9
## 378 21.24 13.3
## 379 23.69 13.1
## 380 21.78 10.2
## 381 17.21 10.4
## 382 21.08 10.9
## 383 23.60 11.3
## 384 24.56 12.3
## 385 30.63  8.8
## 386 30.81  7.2
## 387 28.28 10.5
## 388 31.99  7.4
## 389 30.62 10.2
## 390 20.85 11.5
## 391 17.11 15.1
## 392 18.76 23.2
## 393 25.68  9.7
## 394 15.17 13.8
## 395 16.35 12.7
## 396 17.12 13.1
## 397 19.37 12.5
## 398 19.92  8.5
## 399 30.59  5.0
## 400 29.97  6.3
## 401 26.77  5.6
## 402 20.32  7.2
## 403 20.31 12.1
## 404 19.77  8.3
## 405 27.38  8.5
## 406 22.98  5.0
## 407 23.34 11.9
## 408 12.13 27.9
## 409 26.40 17.2
## 410 19.78 27.5
## 411 10.11 15.0
## 412 21.22 17.2
## 413 34.37 17.9
## 414 20.08 16.3
## 415 36.98  7.0
## 416 29.05  7.2
## 417 25.79  7.5
## 418 26.64 10.4
```

```
## 419 20.62  8.8
## 420 22.74  8.4
## 421 15.02 16.7
## 422 15.70 14.2
## 423 14.10 20.8
## 424 23.29 13.4
## 425 17.16 11.7
## 426 24.39  8.3
## 427 15.69 10.2
## 428 14.52 10.9
## 429 21.52 11.0
## 430 24.08  9.5
## 431 17.64 14.5
## 432 19.69 14.1
## 433 12.03 16.1
## 434 16.22 14.3
## 435 15.17 11.7
## 436 23.27 13.4
## 437 18.05  9.6
## 438 26.45  8.7
## 439 34.02  8.4
## 440 22.88 12.8
## 441 22.11 10.5
## 442 19.52 17.1
## 443 16.59 18.4
## 444 18.85 15.4
## 445 23.79 10.8
## 446 23.98 11.8
## 447 17.79 14.9
## 448 16.44 12.6
## 449 18.13 14.1
## 450 19.31 13.0
## 451 17.44 13.4
## 452 17.73 15.2
## 453 17.27 16.1
## 454 16.74 17.8
## 455 18.71 14.9
## 456 18.13 14.1
## 457 19.01 12.7
## 458 16.94 13.5
## 459 16.23 14.9
## 460 14.70 20.0
## 461 16.42 16.4
## 462 14.65 17.7
## 463 13.99 19.5
## 464 10.29 20.2
## 465 13.22 21.4
## 466 14.13 19.9
## 467 17.15 19.0
## 468 21.32 19.1
## 469 18.13 19.1
## 470 14.76 20.1
## 471 16.29 19.9
## 472 12.87 19.6
```

```
## 473 14.36 23.2
## 474 11.66 29.8
## 475 18.14 13.8
## 476 24.10 13.3
## 477 18.68 16.7
## 478 24.91 12.0
## 479 18.03 14.6
## 480 13.11 21.4
## 481 10.74 23.0
## 482  7.74 23.7
## 483  7.01 25.0
## 484 10.42 21.8
## 485 13.34 20.6
## 486 10.58 21.2
## 487 14.98 19.1
## 488 11.45 20.6
## 489 18.06 15.2
## 490 23.97  7.0
## 491 29.68  8.1
## 492 18.07 13.6
## 493 13.35 20.1
## 494 12.01 21.8
## 495 13.59 24.5
## 496 17.60 23.1
## 497 21.14 19.7
## 498 14.10 18.3
## 499 12.92 21.2
## 500 15.10 17.5
## 501 14.33 16.8
## 502  9.67 22.4
## 503  9.08 20.6
## 504  5.64 23.9
## 505  6.48 22.0
## 506  7.88 11.9
```

```
Boston = Boston [sample(1:nrow(Boston)),]
Boston_train = Boston [1:380, ]
Boston_test = Boston[380 : nrow(Boston),]
```

Similar to lab 1, write a function that takes a matrix and punches holes (i.e. sets entries equal to NA) randomly with an argument prob_missing.

```
#TO-DO
```

Create a matrix Xmiss which is X but has missingness with probability of 10%.

```
#TO-DO
```

Use a random forest modeling procedure to iteratively fill in the NA's by predicting each feature of X using every other feature of X. You need to start by filling in the holes to use RF. So fill them in with the average of the feature.

```
#TO-DO
```