Student names: ... (please update)

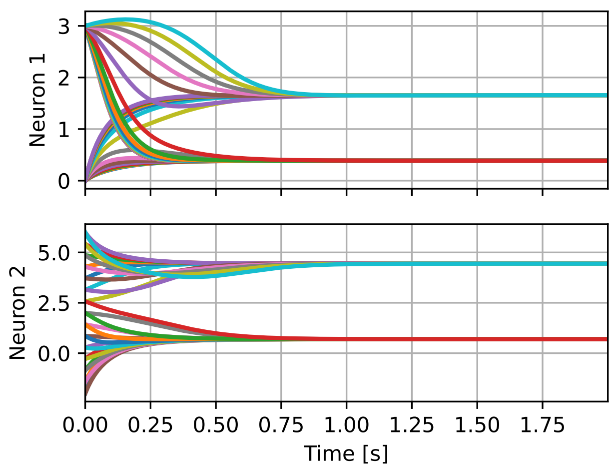
Instructions: Update this file (or recreate a similar one, e.g. in Word) to prepare your answers to the questions. Feel free to add text, equations and figures as needed. Hand-written notes, e.g. for the development of equations, can also be included e.g. as pictures (from your cell phone or from a scanner). ***This lab is not graded. However, the lab exercises are meant as a way to familiarise with dynamical systems and to study them using Python to prepare you for the final project.*** *This file does not need to be submitted and is provided for your own benefit. The graded exercises will have a similar format.*

The file `lab#.py` *is provided to run all exercises in Python. When a file is run, message logs will be printed to indicate information such as what is currently being run and and what is left to be implemented. All warning messages are only present to guide you in the implementation, and can be deleted whenever the corresponding code has been implemented correctly.*

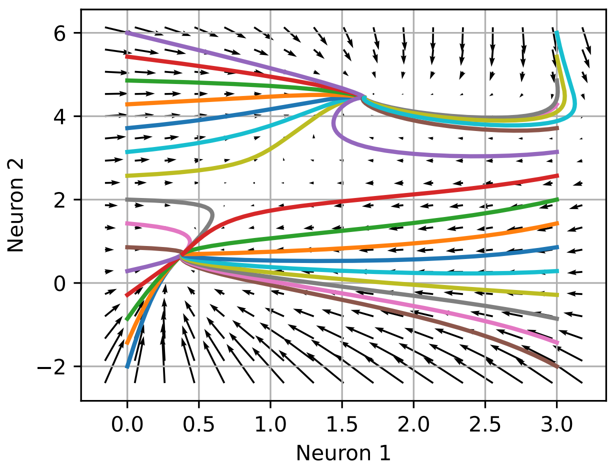
Coupled leaky integrator neurons

The lab of today is based on a network of two coupled leaky-integrator neurons with self- connections as seen in the lecture, and as analyzed in the paper: Beer, R.D. (1995). On the dynamics of small continuous-time recurrent neural networks. Adaptive Behavior 3(4):469-509. By looking at the Figures 4a-4d of that paper, you should be able to reproduce several interesting dynamical regimes and answer the questions below. See the file `lab3.py`

5.a Set the parameters of the network such as to create a dynamical system with *three* fixed points: two stable fixed points and one saddle node (check Beer 1995 for ideas and parameter values). Show figures that illustrate that behavior. Show or demonstrate the stability of the fixed points.



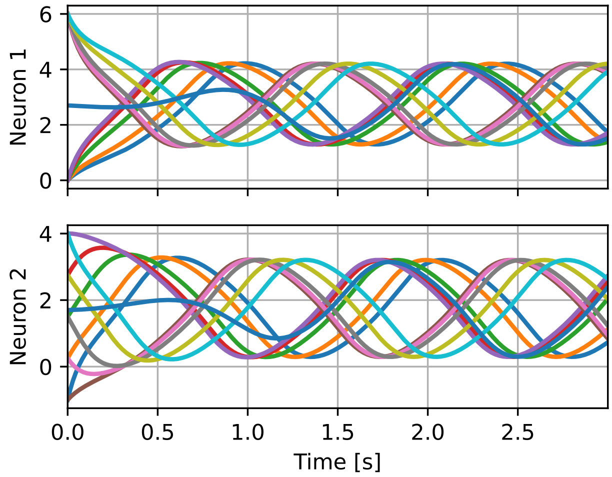
(a) *Time evolution*



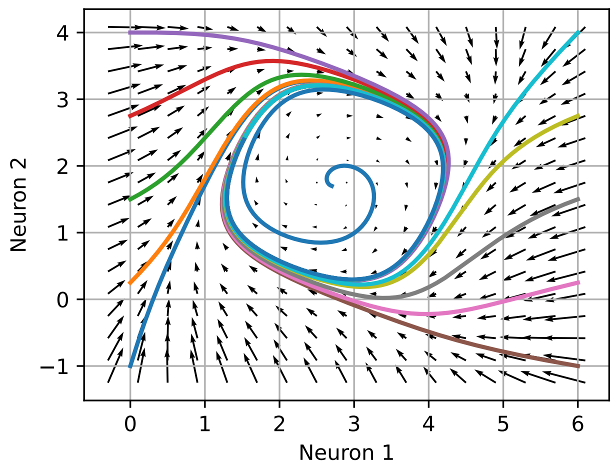
(b) *Phase portrait*

Figure 1: This Figure corresponds to the regime shown in figure 4a, bottom left in (Beer 1995) (`b = [-3.4, -2.5]`*,* `w = [[5.25, 1], [-1, 5.25]]`*). There are two stable fixed points, and one saddle node. The trajectories around the [1, 2.5] area approach towards the saddle node, and then convergence to the two different stable fixed points. All trajectories converge to one of these stable fixed points.*

5.b Set the parameters of the network such as to create a dynamical system with a limit cycle behavior and a single unstable fixed point. Show figures that illustrate that behavior. Look also at the behavior of the crossing of a Poincaré map (a line in this case). Discuss similarities and differences of this neural oscillator with the Hopf oscillator.



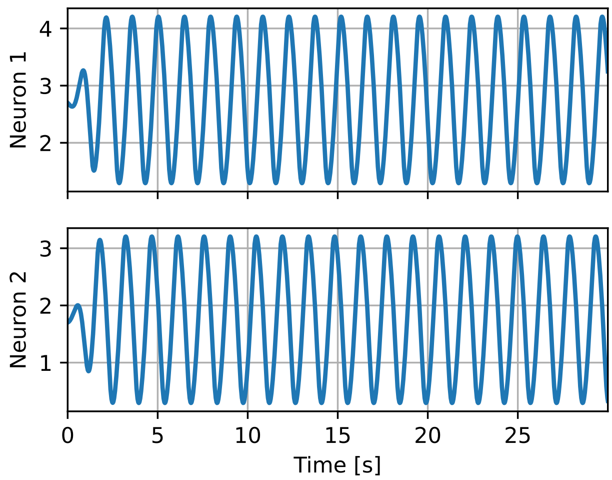
(a) Time evolution



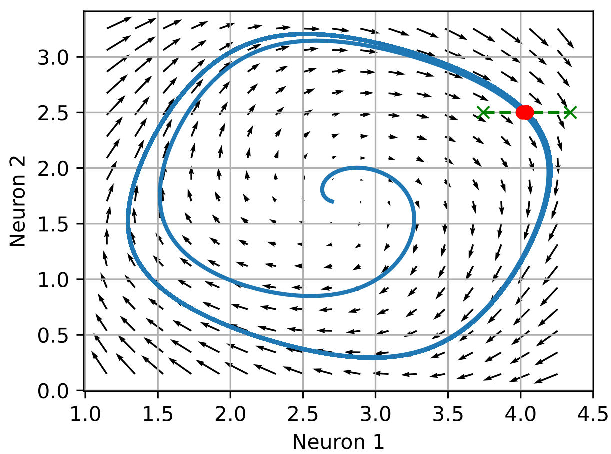
(b) Phase portrait

Figure 2: This Figure corresponds to the regime shown in figure 4a, middle left in (Beer 1995) (`b = [-2.75, -1.75]`, `w = [[4.5, 1], [-1, 4.5]]`). There is a stable limit cycle, with an unstable fixed point inside (as in any limit cycle). From different initial conditions, we see convergence to the stable limit cycle.

We can look at a Poincaré map to see the convergence to the limit cycle:

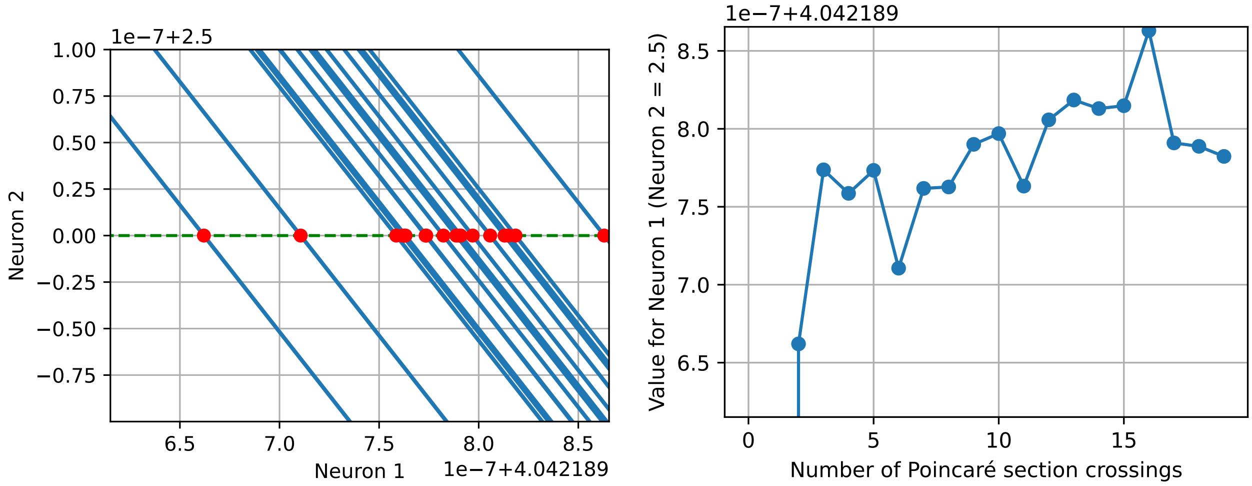


(a) Time evolution



(b) Phase portrait

Figure 3: System run with initial point `[2.7, 1.7]`



(a) Crossings of the Poincaré section

(b) Poincaré map

Figure 4: Crossings of the Poincaré section (left) lead to a Poincaré map (right) in which the states rapidly converge to an almost constant value and oscillates around it. The numerical integration leads to these negligible numerical artefacts (error $\sim 10^{-7}$). To have a better way to detect crossings we can use interpolation and/or finer integration steps. However, at steady state, a similar error would appear.

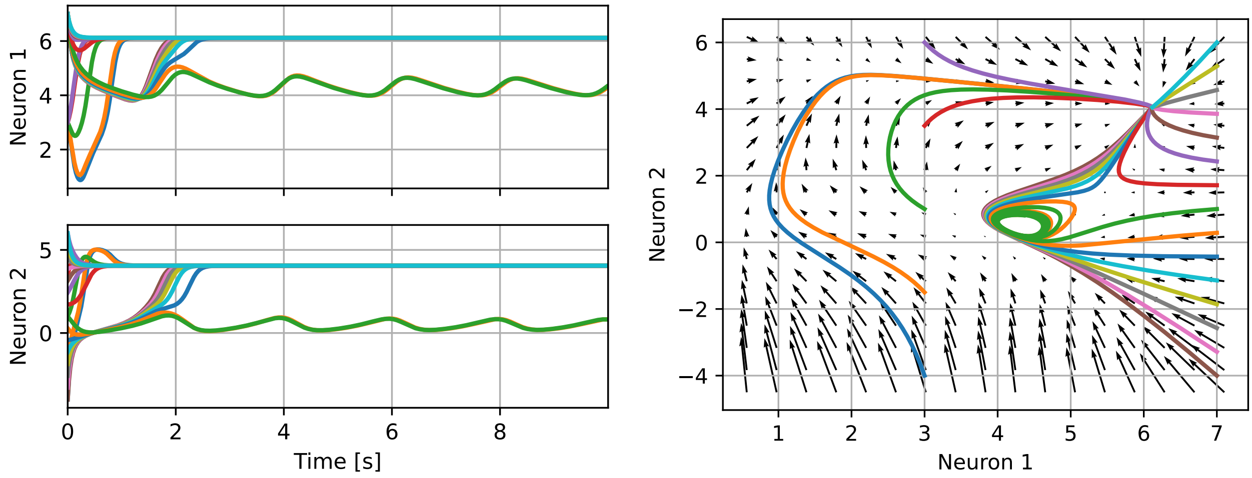
Similarities with Hopf:

- Stable limit cycle behavior
- Basin of attraction = whole state space (except unstable fixed point within the limit cycle)
- Two dimensional oscillators

Differences:

- The shape of the limit cycle is different (harmonic, i.e. sin and cos, circle shape for Hopf, more complex shape here)
- Vector field is not symmetric, convergence strength varies dependent on angle
- Angle is not the phase
- Not as simple to make a bifurcation between between single point attractor and limit cycle (but must be possible)

5.c Set the parameters of the network such as to create a dynamical system with a limit cycle behavior and three fixed points: a single unstable fixed point, a single saddle node, and a single stable fixed point. Show figures that illustrate that behavior. Discuss similarities and differences with the Hopf oscillator. Discuss whether such a system could have interesting properties for motor control.

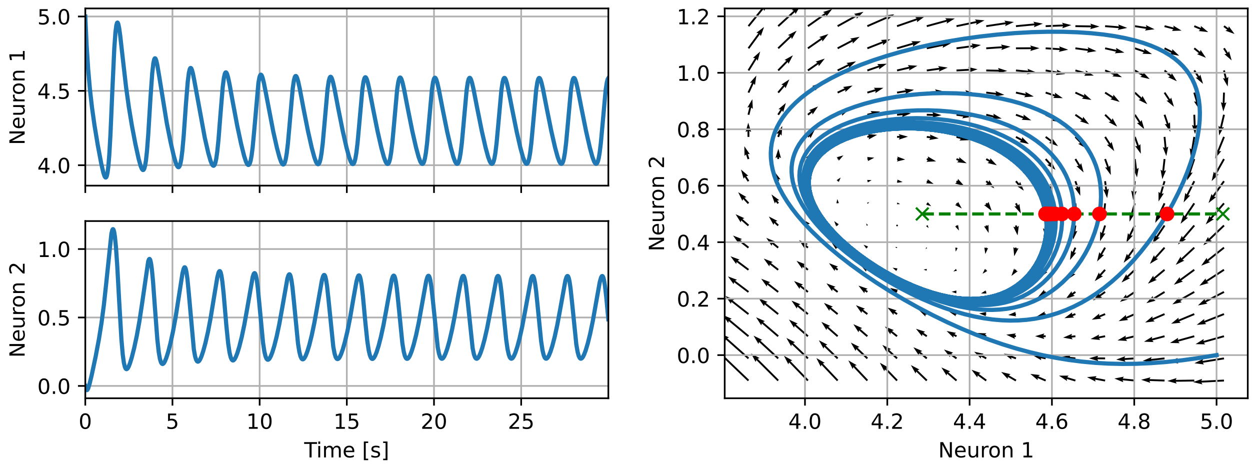


(a) Time evolution *(b) Phase portrait*

Figure 5: This Figure corresponds to the regime shown in figure 4b, middle left in (Beer 1995) (`b = [-3.233, -1.75]`, `w = [[5.5, 1], [-1, 5.5]]`).

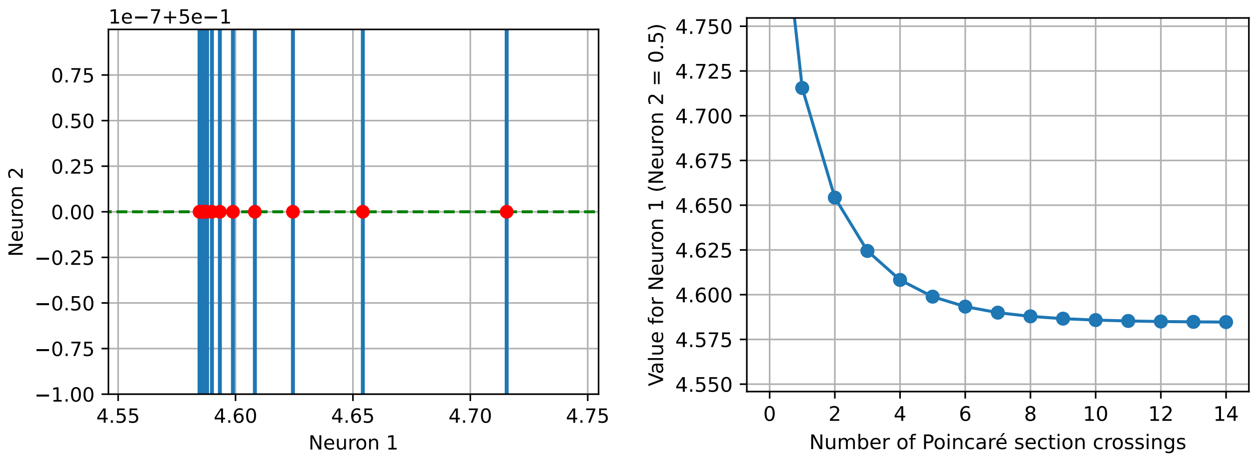
Similarities and differences:

Like 5b. but additional difference: basin of attraction to limit cycle is not the whole space anymore, only a subpart, because of the added stable fixed point which has its own, larger, basin of attraction.



(a) Time evolution *(b) Phase portrait*

Figure 6: System run with initial point `[5, 0.0]`



(a) Crossings of the Poincaré section *(b) Poincaré map*

Figure 7: Poincaré analysis