(Number System)

⇛ $(D_{n-1}, D_{n-2} \cdots D_2 D_1 D_0)_r$ → radix (base)

- $r = 10$ Decimal
- $r = 16$ Hexa-decimal
- $r = 8$ Octal
- $r = 4$ Nibble
- $r = 2$ Binary

when $r \neq 10$

$$N = (r^{n-1} D_{n-1} + r^{n-2} D_{n-2} + \cdots + r^1 D_1 + r^0 D_0)$$

$$= (\quad)_{10}$$

$$\overset{D_2 D_1 D_0}{(3\,5\,2)_7} = (\quad)_{10}$$

$$= 7^2 \cdot 3 + 7 \cdot 5 + 2 = (184)_{10}$$

⇛ $(D_{n-1} \cdot D_{n-2} \cdots D_1 D_0 \,.\, D_1' D_2' \cdots)_r$

radix point

$$= r^{n-1} D_{n-1} + r^{n-2} D_{n-2} + \cdots + r D_1 + D_0 + r^{-1} D_1' + r^{-2} D_2 + \cdots$$

convert decimal to radix:-

$(73)_{10} = (\quad)_r$

$= (Q_{lst} \ldots R_2 R_1)_r$

r	73	
	Q_1	R_1
	⋮	R_2
	⋮	⋮
	Q_{lst}	R_{lst}

$Q_1 \not< r$

$Q_{lst} < r$

for $r = 4$

4	73	
4	18	1
4	4	2
	1	0

so, $(73)_{10} = (1021)_4$

for $r = 5$

5	73	
5	14	3
	2	4

so, $(73)_{10} = (243)_5$

$(0.16)_{10} = (.I_1 I_2 \ldots)_r$

$0.16 \times r = I_1 . \underline{\text{fraction } (f_1)}$ $\quad f_1 \neq 0$

$f_1 \times r = I_2 . \underline{f_2}$

⋮

$f_n \times r = I_n . \underline{00\ldots}$ stop

∘. for $r = 4$

$(0.16)_{10} = (.)_4$

$0.16 \times 4 = 0.64$
$0.64 \times 4 = 2.56$
$0.56 \times 4 = 2.24$
$0.24 \times 4 = 0.96$
$0.96 \times 4 = 3.84$
$0.84 \times 4 = 3.36$
⋮

Eg:- $(73.16)_{10}$ convert it into $r = 16, 8, 4, 2, 5$

for $r=16$

16	73	
16	~~464~~	9
	~~4~~	

stop

$0.16 \times 16 = 2.56$
$0.56 \times 16 = 8.96$
$0.96 \times 16 = 15.36$

$(73.16)_{10} = (49.28F)_{16}$

for $r = 8$

8	73	
8	9	1
	1	1

$(111.121)_8$

$0.16 \times 8 = 1.28$
$0.28 \times 8 = 2.24$
$0.24 \times 8 = 1.92$

Similarly $(1021.023)_4$, $(1001001.00101)_2$

$(243.04)_5$

$$(\underline{0100}\,\underline{1001} \cdot \underline{0010}\,\underline{1000})_2 = (49.28)_{16}$$

$$(\underline{001}\,\underline{001}\,\underline{001} \cdot \underline{001}\,\underline{010})_2 = (111.12)_8$$

Q. $$(\underline{02}\,\underline{12}\,\underline{22}\,\underline{21} \cdot \underline{20}\,\underline{21}\,\underline{22}\,\underline{20})_3 = (2587.6786)_9$$

2 5 8 7 6 7 8 6

Q. $$(6845.854)_{10} = (\quad)_2$$

$$(6845.854)_{10} \rightarrow (1A\cdot\ )_{16} \rightarrow (\quad)_2$$

16	6845	
16	428	6
16	26	12 (C)
	1	10 (A)

$0.854 \times 16 = 13.664$

$0.664 \times 16 = 10.624$

$0.624 \times 16 = 9.984$

1AC6 . DA9

NOW $(1AC6.DA9)_{16} \rightarrow (\quad)_2$

In any number system there are $(r-1)$'s compliment, and r's compliment.

Compliments are required in any number system to simplify the substraction opn.

Q. Find 2's compliment of the given binary number.

$(0\,1101.01)_2$

$N \xrightarrow{1's} N' \xrightarrow{+1} \underbrace{N''}$ 2's compliment

$$
\begin{array}{r}
01101.01 \\
10010.10 \\
+1 \\
\hline
10010.11
\end{array}
$$

1's

2's

Alterate way:-

0 1101.01 scan for 1

0 1101.01 perfor bitwise compliment after 1'st one.

$\Downarrow$

$\boxed{10010.11}$ 2's compliment

Q. Find 6's compliment of a given number:

(a) $(3245)_6$

$$\begin{array}{r} 5555 \\ -\ 3245 \\ \hline 2310 \\ +1 \\ \hline 2311 \end{array}$$

2310 — 5's compliment

2311 — 6's compliment

(b) $(4230)_6$

$$\begin{array}{r} 5555 \\ -\ 4230 \\ \hline 1325 \\ +1 \\ \hline 1330 \end{array}$$

1325 — 5's

1330 — 6's

when result ≥ radix then carry is placed to next radix place.

Q. Find the sum of two hexadecimal number:

$(7CB9)_{16}$

$(5EAB)_{16}$

$$\begin{array}{r} 1\ \ 1\ \ 1\ \ \ \\ 7\ \ C\ \ B\ \ 9 \\ +\ 5\ \ E\ \ A\ \ B \\ \hline D\ \ B\ \ 6\ \ 4 \end{array}$$

$= (DB64)_{16}$

$r = 16$

$9 + B = 20$

$20 > r$

$$\begin{array}{r} 20 \\ -16 \\ \hline 4 \end{array}$$

$9 + A = 19$

$19 > r$

$$\begin{array}{r} 19 \\ -16 \\ \hline 3 \end{array}$$

Signed Binary numbers :-

Sign Magnitude notation	Sign 1's notation	Sign 2's notation
+ve MSB = 0	Mag ~~…~~ 1's compliment	Mag ~~…~~ 2's compliment
−ve MSB = 1	+ MSB = 0	+ MSB = 0
	− MSB = 1	− MSB = 1

$\underbrace{R_{n-1}}_{\text{Sign}}\ \underbrace{R_{n-2}\ \cdots R_1 R_0}_{\text{True magnitude}}$

Q. The register R_1, R_2, R_3 are given below:

0	1	1	0	1	1

R_1

1	1	0	1	1

R_2

1	1	1	0	1

R_3

what are decimal value of these register :

(a) sign magnitude form

(b) sign 1's

(c) sign 2's

(a) $R_1 = +13$, $R_2 = -13$, $R_3 = -13$

(b) $R_1 = 01011$ so, $R_1 = \underline{+11}$

MSB
+ve

$R_2 = 11011$ so, $R_2 = 00100$ (1's)
$= \underline{-4}$

MSB
-ve

$R_3 = 11101$ so, $R_3 = 00010$ (1's)
$= \underline{-2}$

MSB
-ve

(c) $R_1 = +11$, $R_2 = -5$, $R_3 = -3$

Range of signed numbers :-

(i) sign magnitude $-(2^{n-1}-1)$ to $(2^{n-1}-1)$
-7 $+7$

(ii) sign 1's $-(2^{n-1}-1)$ to $(2^{n-1}-1)$
-7 $+7$

(iii) sign 2's -2^{n-1} to $(2^{n-1}-1)$
-8 $+7$

R_3	R_2	R_1	R_0	Sign Mag	Sign 1's	Sign 2's
0	0	0	0	+0	+0	+0
0	0	0	1	+1	+1	+1
0	0	1	0	+2	+2	+2
0	0	1	1	+3	+3	+3
0	1	0	0	+4	+4	+4
0	1	0	1	+5	+5	+5
0	1	1	0	+6	+6	+6
0	1	1	1	+7	+7	+7
1	0	0	0	$\boxed{-0}$	-7	-8
1	0	0	1	-1	-6	-7
1	0	1	0	-2	-5	-6
1	0	1	1	-3	-4	-5
1	1	0	0	-4	-3	-4
1	1	0	1	-5	-2	-3
1	1	1	0	-6	-1	-2
1	1	1	1	-7	$\boxed{-0}$	-1

- Binary 0 is having unique representation in sign 2's compliment notation, due to which we have larger range.

Decimal equivalent of 2's form:-

Let 2's form be $a_{n-1}\, a_{n-2} \cdots a_1\, a_0$

then

$$\left(\text{decimal value} = -2^{n-1}\cdot a_{n-1} + 2^{n-2} a_{n-2} + \cdots \cdots 2\cdot a_1 + a_0\right)$$

eg:-

$(N)_2 = 101010$ 2's form

$$(N)_{10} = -2^5\cdot 1 + 2^3\cdot 1 + 2\cdot 1$$

$$= -32 + 8 + 2$$

$$= -22$$

Q. What are the maximum number of function created using n variably.

(a) 2^n (b) 2^{2^n} ✓ (c) $2^{2^{n-1}}$ (d) None

Total number of ~~point~~ minterm $= 2^n$

Now each ~~term~~ minterm has two choices whether it is present in funcⁿ or not.

so, for 2^n terms, total possible cases are

$$\underbrace{(2 \times 2 \times 2 \times \cdots \cdots 2)}_{2^n \text{ times}} = \left(2^{2^n}\right)$$