Maxima & Minima in single variable :-

$$f(x) = a\cos x + b\sin x + c$$

Min value $= c - \sqrt{a^2+b^2}$

Max value $= c + \sqrt{a^2+b^2}$

Q. $5\cos x + 3\cos(x + \pi/3) + 3$

$= 5\cos x + 3\left(\cos x \cos \pi/3 - \sin x \sin \pi/3\right) + 3$

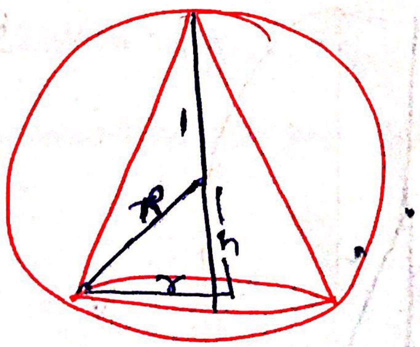
$= 5\cos x + \frac{3}{2}\cos x - \frac{3\sqrt{3}}{2}\sin x + 3$

$= \frac{13}{2}\cos x - \frac{3\sqrt{3}}{2}\sin x + 3$

$= 3 \pm \sqrt{\left(\frac{13}{2}\right)^2 + \left(\frac{3\sqrt{3}}{2}\right)^2}$ $= (-4)$ Min value

$= (10)$ Max value

Q. Find the height of the cone of maximum ~~value~~ volume that can be inscribed in a sphere of $R = 1m$.



$(R = 1)$; $(H = 1 + h)$

$h^2 + r^2 = 1$

$r^2 = 1 - h^2$

$V = \frac{1}{3}\pi r^2 H$

$= \frac{1}{3}\pi (1-h^2)(1+h)$

$$V = \frac{\pi}{3}\left(1+h-h^2-h^3\right)$$

for maximum, $\frac{dV}{dh} = 0$

$$\frac{\pi}{3}\left(1-2h-3h^2\right) = 0$$

$$3h^2+2h-1 = 0$$

$$3h^2+3h-h-1 = 0$$

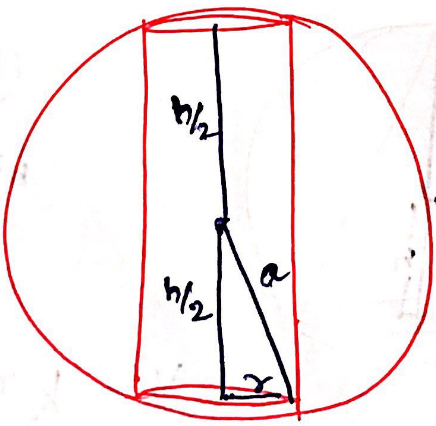
$$3h(h+1)-1(h+1) = 0$$

$$(3h-1)\cancel{(h+1)} = 0 \qquad \left(h = \tfrac{1}{3}\right)$$

$$H = 1+h = 1+\tfrac{1}{3} = \tfrac{4}{3}$$

$$\left(H = \tfrac{4}{3}\ m\right)$$

Q. Find the Height of cylinder of maximum volume that can be inscribed in a sphere of radius a units $(R = a)$.



$$\frac{h^2}{4} + r^2 = a^2$$

$$r^2 = a^2 - \frac{h^2}{4}$$

$$V = \pi r^2 h$$

$$= \pi\left(a^2 - \frac{h^2}{4}\right)h$$

$$V = \pi\left(ha^2 - \frac{h^3}{4}\right)$$

$$\Rightarrow \frac{dV}{dh} = 0$$

$$\Rightarrow \pi\left(a^2 - \frac{3h^2}{4}\right) = 0 \ ; \quad \frac{3h^2}{4} = a^2$$

$$h^2 = \frac{4a^2}{3}$$

$$h = \frac{2a}{\sqrt{3}}$$

⇒ Maxima or Minima of a function in two variables :-

$z = f(x,y)$

$$p = \frac{\partial z}{\partial y}, \quad q = \frac{\partial z}{\partial y}, \quad r = \frac{\partial^2 z}{\partial x^2}, \quad s = \frac{\partial^2 z}{\partial x \partial y}, \quad t^2 = \frac{\partial^2 z}{\partial t^2}$$

For maxima or minima :-

$(p = 0)$
$(q = 0)$

- Since p and q are $f(x,y)$ so we have two eq in (x,y). Solve the eq and get relation b/w x and y.

- Put this relation (substitute) in eq. p and q, then the equation transformed to purely x or y.

- roots of eq(x) : $x_0, x_1, x_2, \ldots\ldots$

  roots of eq(y) : $y_0, y_1, y_2, \ldots\ldots$

(x_0, y_0), (x_1, y_1), (x_2, y_2), ...

These are stationary points which are substitute in the r, s, t.

Case-1: if $(rt - s^2 > 0)$ and $(r > 0)$ Min value

Min-value $= f(x_0, y_0)$

Case-2: if $(rt - s^2 > 0)$ and $(r < 0)$ Max value

Max-value $= f(x_0, y_0)$

Case-3: if $(rt - s^2 < 0)$ No Extreme value

then (x_0, y_0) is saddle point

Case-4: if $(rt - s^2 = 0)$ Needs further investigation

NOTE :-

'stationary point' :- where max or min can be present

'Saddle point' :- where neither maxima nor minima are present.

Q. $f(x,y) = x^2+y^2+6x+12$

has,

✓(a) min at $(-3,0)$

(b) max at $(-3,0)$

(c) no extreme at $(-3,0)$

(d) None

$p = 2x+6$

$q = 2y$

at $(-3,0)$ $p=0$, $q=0$

so it is a stationary point

$r = 2$

$s = 0$

$t = 2$

$rt-s^2 = 4 > 0$

$r = 2 > 0$

so, $f(x,y)$ is minimum at $(-3,0)$

Q. $f(x,y) = 1-x^2-y^2$ has

(a) min at $(0,0)$

✓(b) max at $(0,0)$

(c) saddle point

(d) None

$p = -2x$

$q = -2y$

at $(0,0)$ $p=0$, $q=0$

so it is stationary point

$r = -2$

$t = -2$

$s = 0$

$rt-s^2 = 4 > 0$

$r = -2 < 0$

$(0,0)$ is point of maxima

Q. $f(x,y) = x^3 + y^3 - 3xy$ has —

(a) min at $(1,-1)$ (b) max at $(1,-1)$

(c) min at $(1,1)$ ✓ (d) max at $(1,1)$

$p = 3x^2 - 3y$

$q = 3y^2 - 3x$

$r = 6x$

$t = 6y$.

$s = -3$

at $(1,-1)$ $p \neq 0$, $q = 0$
not stationary point

at $(1,1)$ $p = 0$, $q = 0$
stationary point

so, at $(1,1)$,

$$\left[\begin{array}{c} rt - s^2 > 0 \\ r > 0 \end{array}\right]$$

$r = 6$.
$t = 6$
$s = -3$

Q. $f(x,y) = x^2 + y^2 + xy + x - 4y + 5$ has

(a) min at $(2,-3)$ (b) max at $(2,-3)$

(c) min at $(-2,3)$ ✓ (d) max at $(-2,3)$

$p = 2x + y + 1$

$q = 2y + x - 4$

at $(2,-3)$ $p \neq 0$ $q \neq 0$

at $(-2,3)$ $p = 0$ $q = 0$
stationary point

$r = 2$

$t = 2$

$s = 1$

$$\left[\begin{array}{c} rt - s^2 > 0 \\ r > 0 \end{array}\right]$$

Q. $f(x,y) = 4x^2 + 6y^2 - 8x - 4y + 8$ has ___

(a) min $8/3$ (b) ✓ min $10/3$ (c) max $8/3$ (d) max $10/3$

$p = 8x - 8$

$q = 12y - 4$

$r = 8$

$t = 12$

$s = 0$

$(rt - s^2 > 0)$

$(r > 0)$

minima

Let $p = 0$ and $q = 0$

$x = 1$ $y = 1/3$

$(1, 1/3)$ is stationary point

$f(x,y)_{min} = f(1, 1/3)$

$= 4 + \frac{6}{9} - 8 - 4/3 + 8$

$= 4 + 6/9 - 12/9 = 4 - \frac{6}{9}$

$= (10/3)$

Q. $f(x,y) = \sin x + \sin y + \sin(x+y)$ has ___

(a) min $\sqrt{3}/2$ (b) max $\sqrt{3}/2$ (c) min $\frac{3\sqrt{3}}{2}$ (d) ✓ max $\frac{3\sqrt{3}}{2}$

$p = \cos x + \cos(x+y) = 0$ —(i)

$q = \cos y + \cos(x+y) = 0$ —(ii)

$r = -\sin x - \sin(x+y)$

$t = -\sin y - \sin(x+y)$

$s = -\sin(x+y)$

$\cos x - \cos y = 0$

$(x = y)$

Substitute it in (i) & (ii)

$\cos x + \cos 2x = 0$

$\cos 2x = -\cos x$

$\cos 2x = \cos(\pi - x)$

so, $2x = \pi - x$

$3x = \pi$

$\left(x = \pi/3\right) \quad \left(y = \pi/3\right)$ stationary point

$r = -\sqrt{3}$

$t = -\sqrt{3}$

$s = -\sqrt{3}/2$

$\begin{bmatrix} rt - s^2 > 0 \\ r < 0 \end{bmatrix}$ Maxima

Max-value $= \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} = \left(\frac{3\sqrt{3}}{2}\right)$

Q. $f(x,y) = x^2 + y^2 + \frac{2}{x} + \frac{2}{y}$ has —

(a) min at (0,0) (b) max at (0,0)

(c) min at (1,1) (d) max at (1,1)

$p = 2x - 2/x^2$ (1,1) is stationary point

$q = 2y - 2/y^2$

$r = 2 + \frac{4}{x^3} = 6$ $s = 0$

$t = 2 + 4/y^3 = 6$

$\begin{bmatrix} rt - s^2 > 0 \\ r > 0 \end{bmatrix}$

Q. $f(x,y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$

Find stationary point.

$p = 3x^2 + 3y^2 - 30x + 72 = 0$

$q = 6xy - 30y = 0$

solve eq q,

$6y(x-5) = 0$

$(y=0)$ put in eq p

$(x=5)$ put in eq p

$3x^2 - 30x = 0$

$3x(x-10) = 0$

$(x = 0, 10)$

$(0,0)$, $(10,0)$

$75 + 3y^2 - 150 + 72$

$(y = \pm 1)$

$(5,-1)$, $(5,1)$

⇒ Definite & Improper Integrals :-

- $$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx \qquad (a < c < b)$$

- $$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

- $$\int_a^b \frac{f(x)}{f(a)+f(b)}\,dx = \frac{b-a}{2}$$

- $$\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$$

- $$\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx \qquad \text{EVEN}$$

$$= 0 \qquad \text{ODD}$$

- $$\int_0^{2a} f(x)\,dx = 2\int_0^a f(x)\,dx \qquad \text{if } f(2a-x) = f(x)$$

$$= 0 \qquad \text{if } f(2a-x) = -f(x)$$

$$\int_0^{\pi/2} \frac{f(\sin x)}{f(\sin x)+f(\cos x)}\,dx = \int_0^{\pi/2} \frac{f(\tan x)}{f(\tan x)+f(\cot x)}\,dx = \pi/4$$

$$\int_0^{\pi/2} \sin^n x\,dx = \int_0^{\pi/2} \cos^n x\,dx = \frac{(n-1)(n-3)(n-5)\cdots}{n(n-2)(n-4)\cdots} \times K$$

$K = 1$ if n is odd

$K = \pi/2$ if n is even

$$\int_0^{\pi/2} \sin^m x \cdot \cos^n x\,dx = \frac{(m-1)(m-3)(m-5)\cdots(n-1)(n-3)\cdots}{(m+n)(m+n-2)(m+n-4)\cdots} \times K$$

$K = 1$ if either m or n or both odd

$K = \pi/2$ if both m & n are even

Indefinite Integrals :-

$\int x^n\,dx = \frac{x^{n+1}}{n+1}$

$\int 1/x\,dx = \log x$

$\int 1/x^2\,dx = -1/x$

$\int 1/x^3\,dx = \frac{-1}{2x^2}$

$\int e^{ax}\,dx = \frac{e^{ax}}{a}$

$$\int \sin x\, dx = -\cos x$$

$$\int \cos x\, dx = \sin x$$

$$\int \sec^2 x\, dx = \tan x$$

$$\int \sec x \cdot \tan x\, dx = \sec x$$

$$\int \tan x\, dx = \log(\sec x)$$

$$\int \cot x\, dx = \log(\sin x)$$

$$\int \sinh x = \cosh x$$

$$\int \cosh x = \sinh x$$

$$\int \frac{1}{\sqrt{a^2-x^2}} = \sin^{-1}(x/a)$$

$$\int \frac{1}{\sqrt{a^2+x^2}} = \sinh^{-1}(x/a)$$

$$\int \frac{1}{\sqrt{x^2-a^2}} = \cosh^{-1}(x/a)$$

$$\int \frac{1}{a^2+x^2} = \tan^{-1}(x/a)$$

$$\int \frac{1}{a^2-x^2}\, dx = \frac{1}{2a}\log\left(\frac{a+x}{a-x}\right)$$

$$\int \frac{1}{x^2-a^2}\, dx = \frac{1}{2a}\log\left(\frac{x-a}{x+a}\right)$$

$$\int \frac{f'(x)}{f(x)}\, dx = \log(f(x))$$

$$\int \frac{f'(x)}{\sqrt{f(x)}}\, dx = 2\sqrt{f(x)}$$

$$\int \frac{f'(x)}{1+(f(x))^2}\, dx = \tan^{-1}(f(x))$$

$$\int (f(x))^n f'(x)\, dx = \frac{f(x)^{n+1}}{n+1}$$

$$\int e^{f(x)} f'(x)\, dx = e^{f(x)}$$

$$\int e^x[f(x)+f'(x)]\, dx = e^x \cdot f(x)$$

$$\int_0^{\pi/2} \log(\cot x) = 0$$

$$\int_0^{\pi/2} \log(\tan x) = 0$$

- $\int f(x)\, g(x) = f(x) \int g(x)\, dx - \int \left(f'(x) \int g(x)\, dx \right) dx$

To choose $f(x)$

I — Inverse func^n

L — Log func^n

A — algorithm

T — trigno

E — Exponential

- $\int x e^x\, dx = e^x(x-1)$

- $\int x^2 e^x\, dx = e^x(x^2 - 2x + 2)$

- $\int x^3 e^x\, dx = e^x(x^3 - 3x^2 + 6x - 6)$

$$\boxed{\int f(x)\, e^x\, dx = e^x \left(f(x) - f'(x) + f''(x) - \cdots \right)}$$

- $\int x e^{ax}\, dx = e^{ax}\left[\frac{x}{a} - \frac{1}{a^2}\right]$

- $\int x^2 e^{ax}\, dx = e^{ax}\left[\frac{x^2}{a} - \frac{2x}{a^2} + \frac{2}{a^3}\right]$

- $\int \log x\, dx = x(\log x - 1)$

- $\int (\log x)^2\, dx = x\left((\log x)^2 - 2\log x + 2 \right)$

$$\int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2+b^2}[a\sin bx - b\cos bx]$$

$$\int e^{ax}\cos bx\,dx = \frac{e^{ax}}{a^2+b^2}\left[a\cos bx + b\sin bx\right]$$

Q: $$f(x) = \int_2^7 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{9-x}}\,dx = \frac{7-2}{2} = 5/2$$

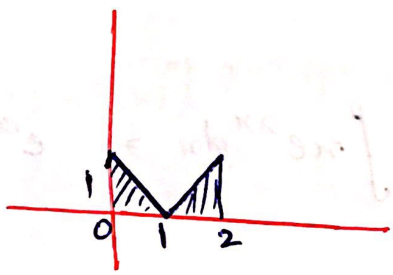
$$\boxed{\left(\int_a^b \frac{f(x)}{f(x)+f(a+b-x)}\,dx = \frac{b-a}{2}\right)}$$

Q: $$\int_0^2 |1-x|\,dx$$

$1-x=0$

$(x=1)$

$$= \int_0^1 (1-x)\,dx + \int_1^2 (x-1)\,dx$$



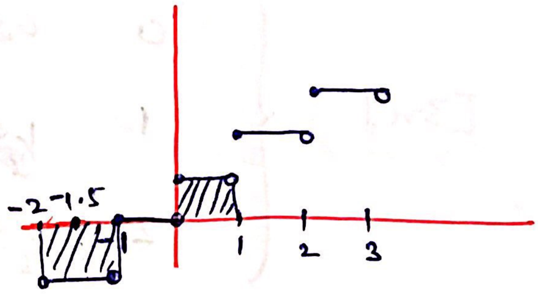
$$= 1/2 + 1/2$$

$1/2 \times 1 \times 2$

$$= (1)$$

Q. $\displaystyle\int_{-1.5}^{1} [x+1]\, dx = \left(1/2\right)$

$$x+1 \begin{cases} -1 & -1.5 \le x < -1 \\ 0 & -1 \le x < 0 \\ 1 & 0 \le x < 1 \end{cases}$$



Q. $\displaystyle\int_{1}^{2} x[x]\, dx$

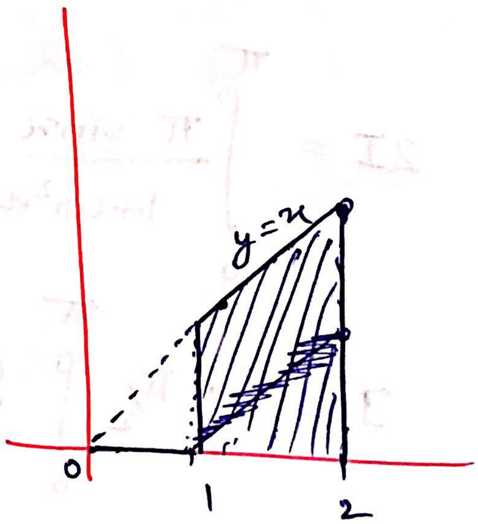
$$x[x] \begin{cases} 0 & 0 \le x < 1 \\ x & 1 \le x < 2 \\ 2x & 2 \le x < 3 \end{cases}$$

so, $\displaystyle f(x) = \int_1^2 x$

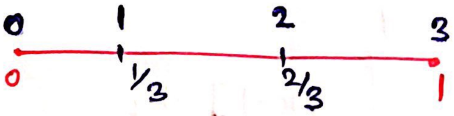
$$= \left.\frac{x^2}{2}\right]_1^2$$

$$= 2 - 1/2$$

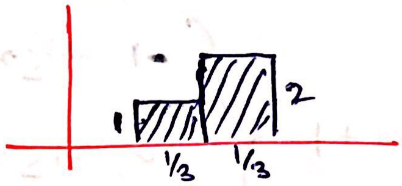
$$= \left(3/2\right)$$



Q. $\displaystyle\int_0^1 [3x]\, dx = (1)$



$$[3x] \begin{cases} 0 & 0 \le x < 1/3 \\ 1 & 1/3 \le x < 2/3 \\ 2 & 2/3 \le x < 3 \end{cases} =$$



$$= 1/3 + 2/3 = 1$$

Q. $\displaystyle\boxed{\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}\, dx = \frac{\pi^2}{4}}$

$$I = \int_0^a f(x) = \int_0^a f(a-x)$$

$$I = \int_0^{\pi} \frac{(\pi-x)\sin(\pi-x)}{1+\cos^2(\pi-x)} = \int_0^{\pi} \frac{(\pi-x)\sin x}{1+\cos^2 x}\, dx$$

$$2I = \int_0^{\pi} \frac{\pi \sin x}{1+\cos^2 x} = -\pi \int_0^{\pi} \frac{-\sin x}{1+\cos^2 x}$$

$$I = -\pi/2 \int_0^{\pi} \frac{(-\sin x)}{1+\cos^2 x} \qquad \text{apply } \tan^{-1}[f(x)]$$

$$I = \left[-\pi/2 \quad \tan^{-1}[\cos x] \right]_0^{\pi}$$

$$I = -\pi/2 \left(\tan^{-1}(-1) - \tan^{-1}(1) \right)$$

$$I = -\pi/2 \left(-\pi/4 - \pi/4 \right)$$

$$I = \frac{\pi^2}{4}$$

Q. $\int_0^{\pi/2} \log(\tan x)\, dx = 0$

$$I = \int_0^{\pi/2} \log(\tan x)\, dx$$

$$I = \int_0^{\pi/2} \log\left(\tan \left(\pi/2 - x\right)\right)$$

$$2I = \int_0^{\pi/2} \left(\log(\tan x) + \log \cot x\right) dx$$

$$2I = \int_0^{\pi/2} \log(\tan x \cdot \cot x)\, dx$$

$$2I = 0 \qquad \boxed{I = 0}$$

Q. $$\int_{-1}^{1} \frac{1}{1+x^2}\,dx = \pi/2$$

$\left(1/_{1+x^2}\right)$ is an even funcn.

$$= 2\int_{0}^{1} \frac{1}{1+x^2} = 2\left[\tan^{-1}x\right]_0^1$$

$$= 2\left(\tan^{-1}1 - \tan^{-1}0\right)$$

$$= 2 \times \pi/4$$

$$= \left(\pi/2\right)$$

Q. $$\boxed{\int_{-a}^{a} \sqrt{\frac{a-x}{a+x}}\,dx}$$

Rationalize; $$= \int_{-a}^{a} \sqrt{\frac{(a-x)(a-x)}{(a+x)(a-x)}}\,dx$$

$$= \int_{-a}^{a} \frac{a-x}{\sqrt{a^2-x^2}}\,dx$$

$$= \int_{-a}^{a} \frac{a}{\sqrt{a^2-x^2}} - \frac{x}{\sqrt{a^2-x^2}}$$

even odd

$$= 2\int_0^a \frac{a}{\sqrt{a^2-x^2}} = 2a\left[\sin^{-1} x/a\right]_0^a$$

$$= 2a\left(\sin^{-1}(\cancel{a}/\cancel{a}) - \sin^{-1}(0)\right)$$

$$= \cancel{2}a \; \pi/\cancel{2}$$

$$= (a\pi)$$

Q. $\int_0^{\pi/2} \sin^5 x \, dx$

$$= \frac{4 \times 2}{5 \times 3 \times 1} \times K$$

$K=1$ $n=5$ odd

$$= \left(\frac{8}{15}\right)$$

Q. $\int_0^{\pi/2} \cos^7 x \, dx$

$$= \frac{6 \times 4 \times 2}{7 \times 5 \times 3 \times 1} = \left(16/35\right)$$

$K=1$ n is odd

Q. $\int_0^{\pi/2} \cos^8 x \, dx$

$$= \frac{7 \times 5 \times \cancel{3} \times 1}{8 \times \underset{2}{\cancel{6}} \times 4 \times 2} \times \frac{\pi}{2} = \left(\frac{35\pi}{256}\right)$$

Q. $\int_0^1 \frac{x^6}{\sqrt{1-x^2}} \, dx$

$x = \sin\theta$

$dx = \cos\theta \, d\theta$

$$= \int_0^{\pi/2} \frac{\sin^6\theta \cdot \cancel{\cos\theta} \cdot d\theta}{\cancel{\cos\theta}}$$

$$= \frac{5 \times \cancel{3} \times 1}{\underset{2}{\cancel{6}} \times 4 \times 2} \times \frac{\pi}{2} = \left(\frac{5\pi}{32}\right)$$

Q. $\int_0^{\pi/2} \sin^{\overset{m}{4}} \cos^{\overset{n}{5}} x \, dx$

$m + n = 9$

$$= \frac{(\cancel{3} \times 1)\,(4 \times 2)}{9 \times 7 \times 5 \times \cancel{3} \times 1} = \frac{8}{315}$$

($K = 1$
as n is odd)

Q. $\int_0^{\pi/2} \sin^6 x \cos^4 x \, dx$

$m=6$
$n=4$

$(k = \pi/2)$

Both even

$$I = \frac{(5\times3\times1)(3\times1)}{10\times8\times6\times4\times2} \times \frac{\pi}{2} = \left(\frac{3\pi}{512}\right)$$

Q. $\int_0^{\pi/2} \sin^5 x \cos^3 x \, dx$

$k = 1$

$5+3 = \underline{8}$

$$I = \frac{(\cancel{4\times2})(2)}{8\times6\times\cancel{4\times2}} \times 1$$

$$I = \left(\frac{1}{24}\right)$$

Q. $\int_0^{\pi} \sin^3 x$

$$I = 2\int_0^{\pi/2} \sin^3 x$$

$\sin x = \sin(\pi - x)$

$$= 2 \times \frac{2}{3\times1} \times 1 = \left(4/3\right)$$

⇒ Improper Integral :-

An Integral $\int_a^b f(x)\,dx$ is said to be an improper integral, if

(i) f becomes infinite in the interval of integration

(ii) one or both of the limits are infinite.

Q. $\displaystyle\int_0^1 \sqrt{\frac{1+x}{1-x}}\,dx$

$$\int_0^1 \frac{\sqrt{(1+x)(1+x)}}{\sqrt{1-x^2}}\,dx = \int_0^1 \frac{1}{\sqrt{1+\sqrt{x^2}}} + -\frac{1}{2}\frac{-2x}{\sqrt{1+x^2}}\,dx$$

$$= \left[\sin^{-1}(x) + 2\sqrt{1-x^2} \times -\frac{1}{2}\right]_0^1$$

$$= \left(\frac{\pi}{2} + 1\right)$$

Q. $\int_0^3 \frac{1}{(x-1)^{2/3}}\,dx = \int_0^3 (x-1)^{-2/3}\,dx$

$$= \frac{(x-1)^{-2/3+1}}{-2/3+1}$$

$$= \frac{(x-1)^{1/3}}{1/3} = 3\left[(x-1)^{1/3}\right]_0^3$$

$(-1)^3 = -1$

$$= 3\left(\sqrt[3]{2} - \sqrt[3]{-1}\right)$$

$$= 3\left(\sqrt[3]{2} + 1\right)$$

Q. $\int_1^\infty \frac{1}{x^3}\,dx = \int_1^\infty x^{-3}\,dx$

$$= \left[\frac{x^{-2}}{-2}\right]_1^\infty = \left[-\frac{1}{2x^2}\right]_1^\infty$$

$$= -\left(\frac{1}{\infty} - \frac{1}{2}\right)$$

$$= -\left(0 - 1/2\right) = (1/2)$$

Q. $\int_{-\infty}^{0} \sinh x \, dx = [\cosh x]_{-\infty}^{0} = \left[\frac{e^x + e^{-x}}{2}\right]_{-\infty}^{0}$

$$= \frac{e^0 + e^{-0}}{2} - \frac{e^{\infty} + e^{-\infty}}{2} = \frac{1}{2} - \infty$$

$= (-\infty)$ divergent

NOTE:-

$$\boxed{\begin{aligned} \sinh x &= \frac{e^x - e^{-x}}{2} \\ \cosh x &= \frac{e^x + e^{-x}}{2} \end{aligned}}$$

Q. $\int_{-\infty}^{\infty} \frac{1}{1+x^2} dx = 2\int_{0}^{\infty} \frac{1}{1+x^2} dx$ even func^n

$$= 2 (\tan^{-1} x)_{0}^{\infty}$$

$$= 2 (\tan^{-1}\infty - \tan^{-1} 0)$$

$$= 2 (\pi/2 - 0)$$

$$= (\pi)$$

Q. $\int_0^\infty 2x e^{-x^2} dx$

$x^2 = t$

$\frac{dt}{dx} = 2x$

$dt = 2x\,dx$

$$= \frac{-1}{2} \int_0^\infty e^{-x^2} (-2x\,dx)$$

$e^{f(x)} \cdot f'(x)$

$$= \frac{-1}{2} \left[e^{-x^2}\right]_0^\infty = \left(\frac{1}{2}\right)$$

Q. $\boxed{\int_0^\infty e^{-x^2} dx = \frac{\sqrt{\pi}}{2}}$ (solve using Gamma funcn)

$$\Gamma n = \int_0^\infty e^{-x} \cdot x^{n-1} dx$$

$\Gamma(n+1) = n!$, if $n \in N$

$\Gamma(n+1) = n\Gamma n$, if $n > 0$

$\Gamma 1 = 1$

$\Gamma \tfrac{1}{2} = \sqrt{\pi}$

$$\left(I = \frac{\sqrt{\pi}}{2}\right)$$

$x^2 = t$

$\frac{dt}{dx} = 2x$

$dt = 2x\,dx$

$x = \sqrt{t}$

$\frac{dx}{dt} = \frac{1}{2\sqrt{t}}$

$dx = \frac{dt}{2\sqrt{t}}$

$$I = \int_0^\infty e^{-t} \cdot \frac{dt}{2\sqrt{t}}$$

$$= \frac{1}{2} \int_0^\infty e^{-t} \cdot t^{-1/2} dt$$

$$= \frac{1}{2} \int_0^\infty e^{-t} \cdot t^{(1/2)-1}$$

$(n = \tfrac{1}{2})$

Q.

$$\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}\,dx\,dy$$

$$I = \int_0^\infty \int_0^\infty e^{-x^2} \cdot e^{-y^2}\,dx\,dy$$

$$= \int_0^\infty e^{-x^2}\,dx \cdot \int_0^\infty e^{-y^2}\,dy$$

$$= \frac{\sqrt{\pi}}{2} \times \sqrt{\pi}/2 = \left(\frac{\pi}{4}\right)$$

Q.

$$= \int_0^\infty x^3 e^{-x}\,dx$$

$$= \int_0^\infty e^{-x} \cdot x^{4-1}\,dx$$

$$=$$

$$= 3! = 6$$

$$\int_0^\infty x^n e^{-x}\,dx = n! \quad ; \quad \int_0^\infty x^4 e^{-x}\,dx = 4!$$

Q. $\displaystyle\int_0^\infty \frac{2x}{(x^2+9)^2}\,dx$

$t = x^2+9$

$dt = 2x\,dx$

$\displaystyle\int_0^\infty \rightarrow \int_9^\infty$

$$\frac{1}{2}\int_9^\infty \frac{dt}{t^2} = \frac{1}{2}\left[\frac{t^{-1}}{-1}\right]_9^\infty = \frac{1}{2}\left[-\frac{1}{t}\right]_9^\infty$$

$$= \frac{1}{2}\left[0 + \frac{1}{9}\right]$$

$$= \left(\frac{1}{18}\right)$$

Q. $\displaystyle\int_{-\infty}^{\infty} \frac{x}{(x^2+9)^2}\,dx = 0$ f(x) is odd

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