

# (Digital Circuits) in short I-3 - 813

## Combinational circuits

### Code Convertors :-

- BCD to E-3
- E-3 to BCD
- 4-bit Binary to Gray code
- 4-bit Gray to Binary
- 5-4-2-1 BCD to Natural BCD
- Natural BCD to 54-2-1 BCD

### Excess-3 (self-complementary) code :-

$$\begin{array}{ccc} \text{BCD} & & \text{E-3} \\ \downarrow & & \downarrow \\ 5 = 0101 & \xrightarrow{+3} & 1000 \\ 4 = 0100 & \xleftarrow{-3} & 0111 \end{array}$$

1's

- The once compliment of given BCD to excess-3 code is 9's compliment of the given BCD in E-3 form.
- E-3 codes are used in BCD subtractor ckt design.

NOTE :- E-I code is unweighted code.

Q. When 8-bit register is used to represent a BCD number, how many combinations are unused combination 156.

4 bits      4 bits  
 $b_7 \ b_6 \ b_5 \ b_4$  prop of quantity digit-0  
 $(0-9) \ (0-9)$  prop of quantity digit-1  
.....  
.....

So Total possible = 100  
BCD i.e 0 - 99

so, unused combination  $= 2^8 - 100 = \boxed{156}$

4-Bit Binary to Grey code converter:-

$B_3 B_2 B_1 B_0 \rightarrow B \rightarrow G \rightarrow Q_3 Q_2 Q_1 Q_0$

in Next Example a 4-bit binary to 4-bit as output  
from B to G

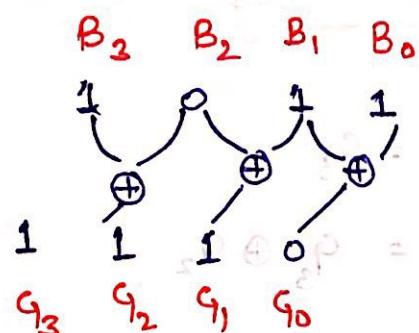
$$G_3 = B_3$$

$$G_2 = B_3 \oplus B_2$$

$$G_1 = B_2 \oplus B_1$$

$$G_0 = B_1 \oplus B_0$$

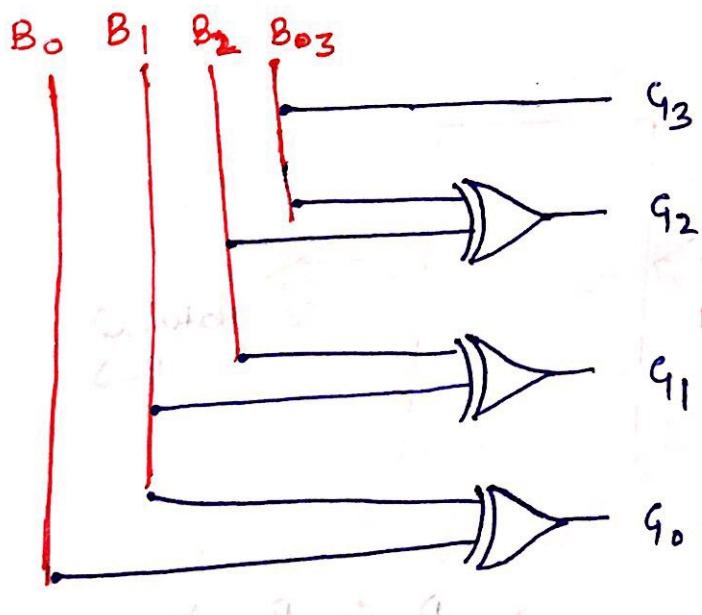
	Binary	Grey
10	1 0 1 0	1 1 1 1
11	1 0 1 1	1 1 1 0
<del>10</del>	<del>1 0 1 1</del>	<del>1 1 1 0</del>
11	1 0 1 1	1 1 1 0
12	1 1 0 0	1 0 1 0



Any two succeeding binary equivalent Gray code are differ in only 1-bit, that's why they are known as unit distance code or cyclic code or reflecting code.

NOTE:- Grey code is unweighted code.

- Grey codes are used in computer networking for error detection & correction purpose.



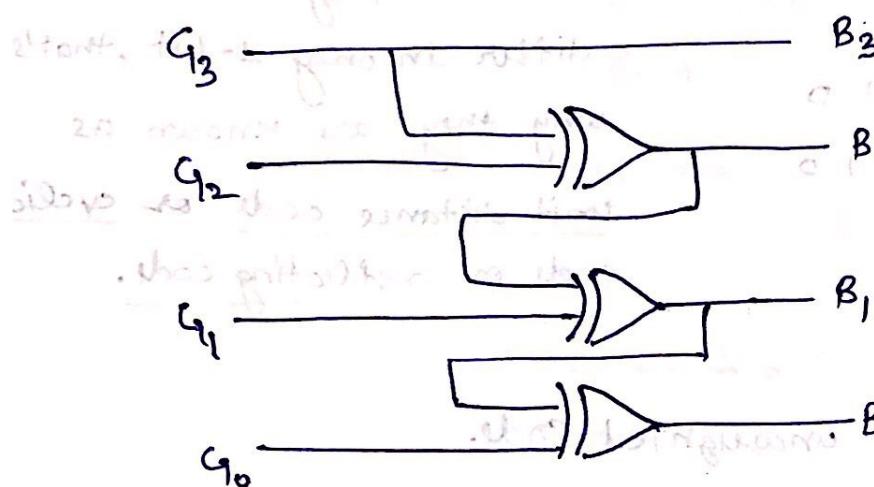
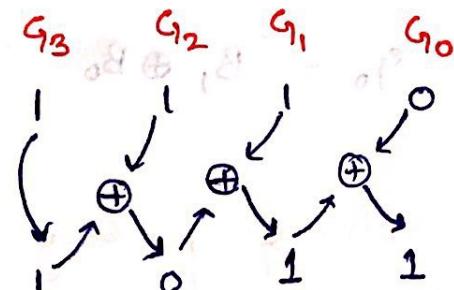
## 4-Bit Grey to Binary code converter:-

$$B_3 = G_3$$

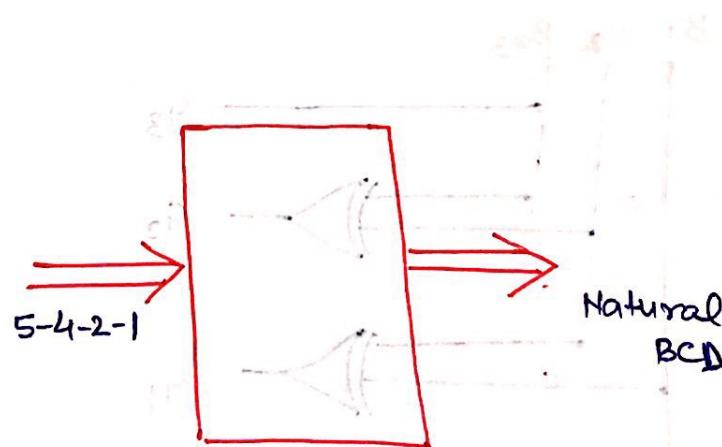
$$B_2 = G_3 \oplus G_2$$

$$B_1 = G_3 \oplus G_2 \oplus G_1$$

$$B_0 = G_3 \oplus G_2 \oplus G_1 \oplus G_0$$



## 5-4-2-1 BCD to Natural BCD :-



$$5x_3 + 4x_2 + 2x_1 + x_0 = 8D + 4C + 2B + A$$

## Truth Table :-

	$x_3$	$x_2$	$x_1$	$x_0$		D	C	B	A
0	0	0	0	0		0	0	0	0
1	1	0	0	0	1	0	0	0	1
2	2	0	0	1	0	0	0	1	0
3	3	0	0	1	1	0	0	1	1
4	4	0	1	0	0	0	1	0	0
5	5	1	0	0	0	0	1	1	0
6	6	1	0	1	0	0	1	0	0
7	7	1	0	1	0	0	1	1	1
8	8	1	0	1	1	1	0	0	0
9	9	1	0	1	1	0	1	1	0
10	10	1	1	0	0	1	0	0	1
11	11	1	1	0	0	0	0	0	0
12	12	1	1	0	0	0	0	0	1

$x_3x_2$        $x_1x_0$

$x_3x_2$	00	01	11	10
00	0	1	3	2
01	4	X	X	X
11	X	5	6	7
10	8	9	10	11

$$D(x_3, x_2, x_1, x_0) = \sum m(11, 12) + d(5, 6, 7, 13, 14, 15)$$

$$D = x_3x_2 + x_3x_1x_0$$

$x_3x_2$        $x_1x_0$

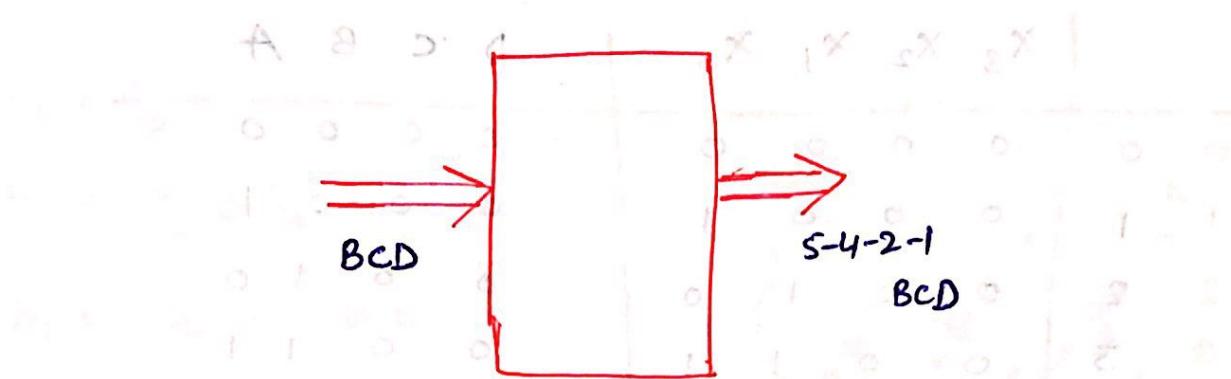
$x_3x_2$	0	1	10	$x_1$	;
0	0	1	0	X	
1	X	X	X	X	
10					
$x_3$					
				$x_0$	

$$C = \bar{x}_3\bar{x}_2 + x_3\bar{x}_2\bar{x}_1 \\ + x_3\bar{x}_2\bar{x}_0$$

$$B = \bar{x}_3x_2 + x_1\bar{x}_0$$

$$A = x_3\bar{x}_0 + \bar{x}_3x_0 = x_3 \oplus x_0$$

## Natural BCD to 5-4-2-1 code converter :-



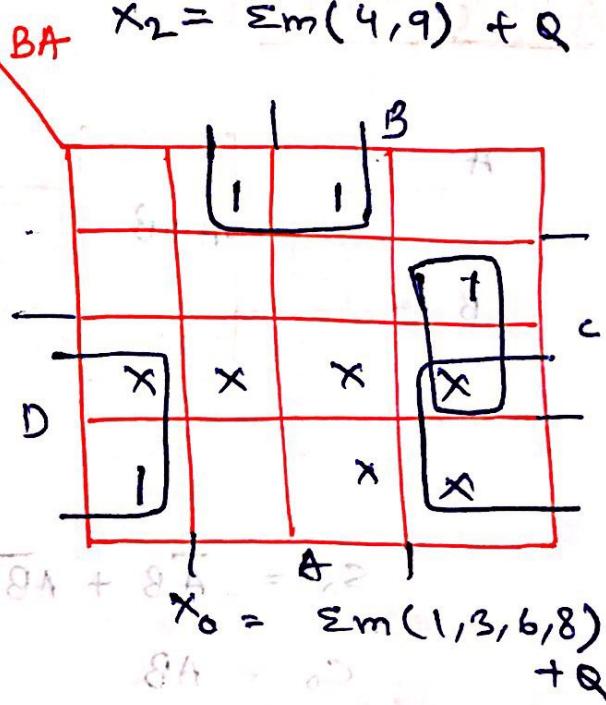
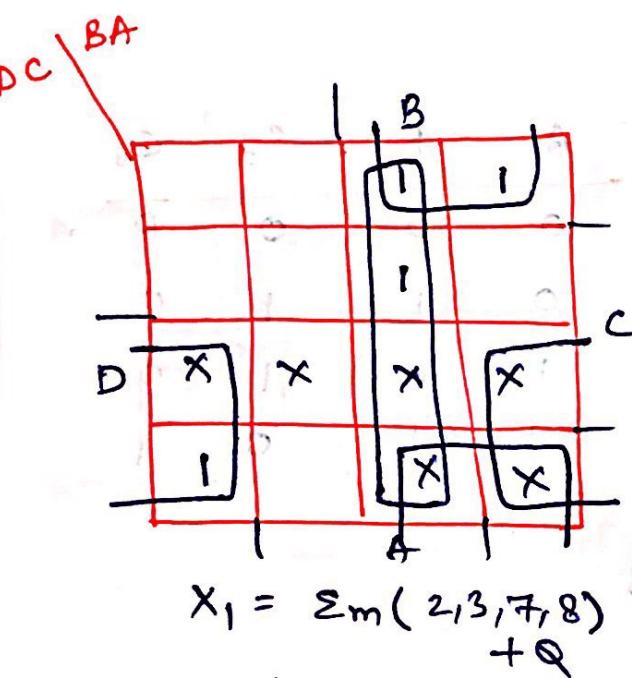
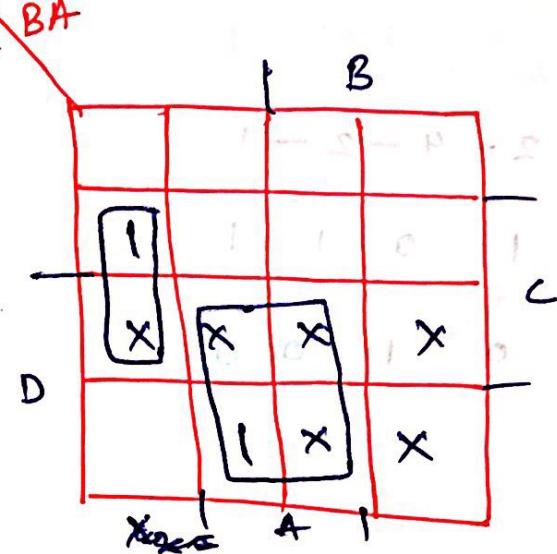
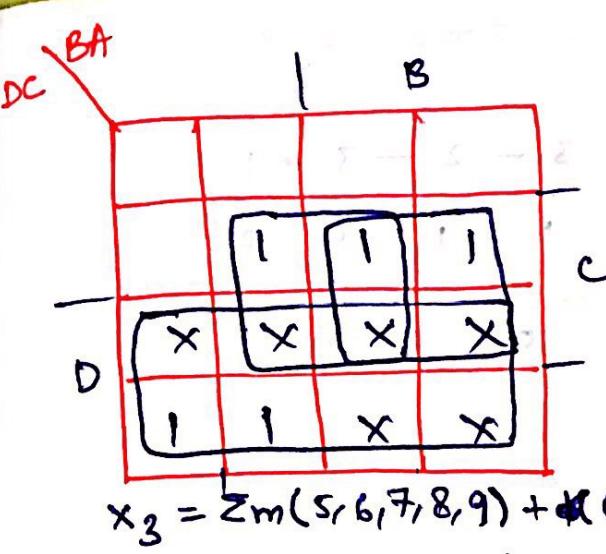
$$\begin{array}{r} 8 \ 9 \ 2 \ 1 \\ D \ C \ B \ A \end{array} \longrightarrow \begin{array}{r} 5 \ 4 \ 0 \ 2 \ 1 \\ X_3 \ X_2 \ X_1 \ X_0 \end{array}$$

$$8D + 4C + 2B + A = 5X_3 + 4X_2 + 2X_1 + X_0$$

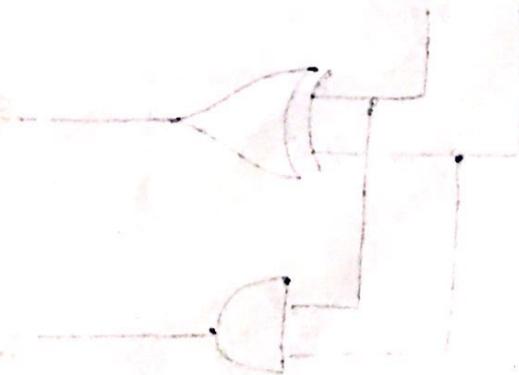
### Truth Table:-

	D	C	B	A	X <sub>3</sub>	X <sub>2</sub>	X <sub>1</sub>	X <sub>0</sub>
0	0	0	0	0	0	0	0	0
1	0	0	0	1	0	0	0	1
2	0	0	(X <sub>1</sub> ) <sup>1</sup>	0	0	0	1	0
3	0	0	1	1	0	0	1	1
4	0	1	0	0	0	1	0	0
5	0	0	X <sub>1</sub>	1	1	0	0	0
6	0	1	1	0	1	0	0	1
7	0	1	1	1	1	0	1	0
8	1	0	0	0	1	0	1	1
9	1	0	0	1	1	1	0	0

$$Q = d(10, 11, 12, 13, 14, 15)$$



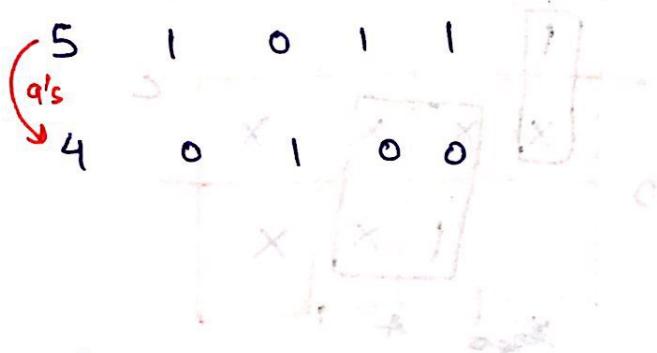
$x_3 = D + AC + BC$
$x_2 = AD + C\bar{B}\bar{A}$
$x_1 = CBA + D\bar{A} + B\bar{E}$
$x_0 = D\bar{A} + \bar{D}\bar{C}A + CBA$



NOTE:- when the sum of the weight of a BCD code equals to 9. Such BCD code is name as self complementary codes

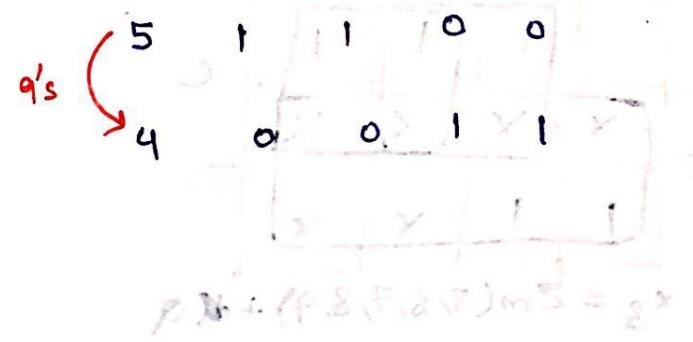
$$2+4+2+1 = 9$$

$$2 - 4 - 2 - 1$$

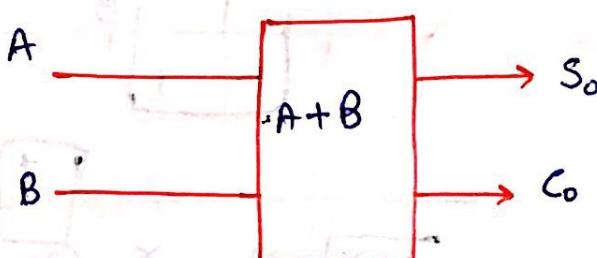


$$3+2+3+1 = 9$$

$$3 - 2 - 3 - 1$$



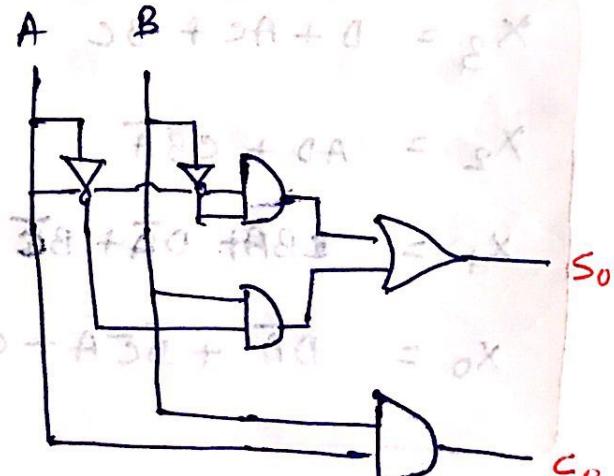
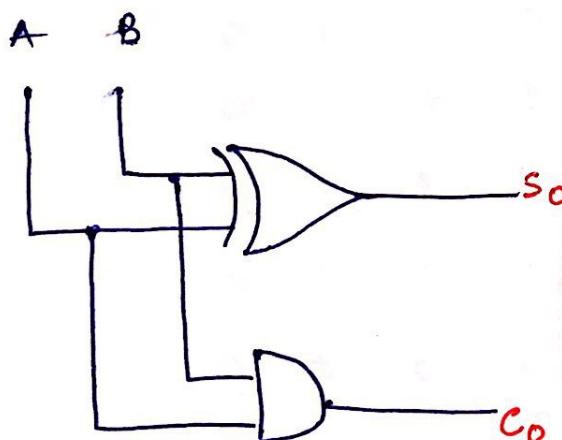
Adder (Half) :-



A	B	S <sub>0</sub>	C <sub>0</sub>
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

$$S_0 = \overline{A}B + A\overline{B} = A \oplus B$$

$$C_0 = AB$$



the C0 is to be taken off from the adder

it is same as the C0 of full adder

the feedback loop

Q. Minimum no. of NAND & NOR gates required to realize half-adder?

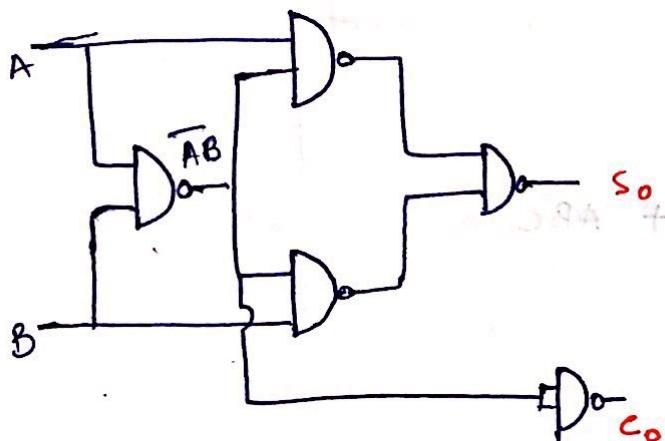
(a) 3,4

(b) 4,3

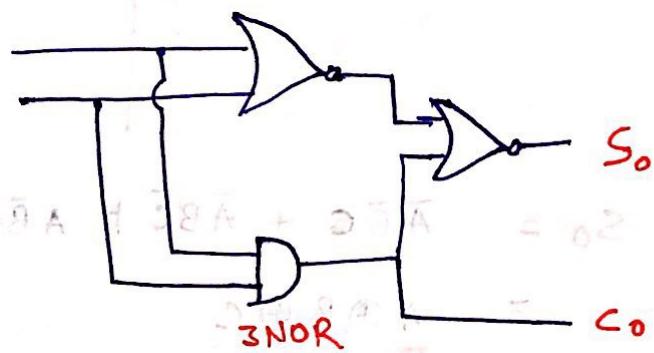
(c) 4,4,

(d) 5,5

NAND



NOR

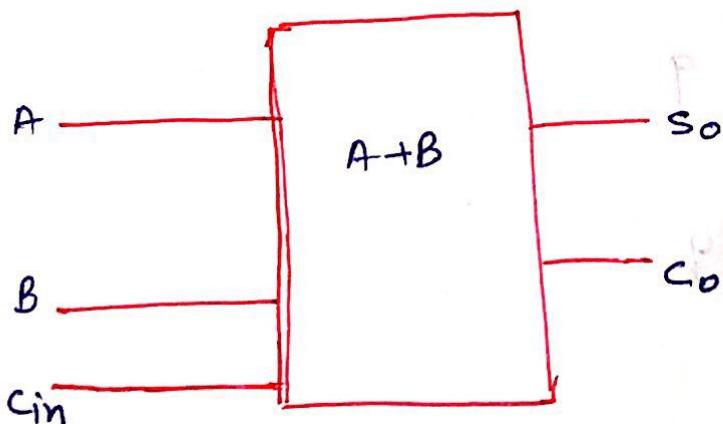


$$2\bar{S}A + \bar{S}\bar{B}A + 2\bar{B}A + \bar{S}\bar{B} = A \oplus B$$

$$\begin{aligned} &= \overline{AB} \\ &= \overline{AB + \bar{A}\bar{B}} \\ &= \overline{x+y} \end{aligned}$$

2SA & 2BA are 2's complement of A & B for addition subtraction with full adder

Full Adder :- adds 3 bits with or without borrow and generates sum and carry.



A	B	C	$S_0$	$C_0$
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

$$S_0 = \overline{A} \overline{B} C + \overline{A} B \overline{C} + A \overline{B} \overline{C} + A B C$$

$$= \underline{\underline{A \oplus B \oplus C}}$$

$$C_0 = \overline{\overline{A} \overline{B} C} + A \overline{B} \overline{C} + A B \overline{C} + A B C$$

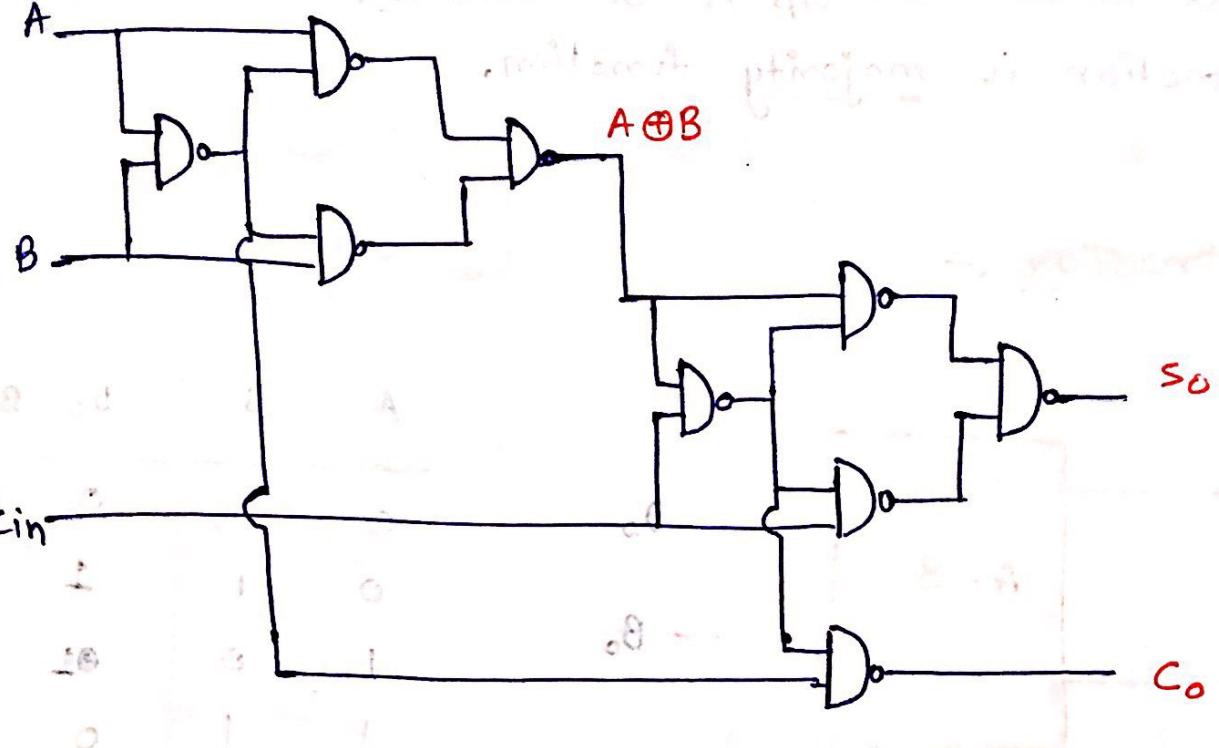
$$= \underline{\underline{AB + BC + AC}}$$

$$= \underline{\underline{C(A \oplus B) + AB}}$$

The minimum number of 2-I/p NAND & NOR gates required to realize full adder -

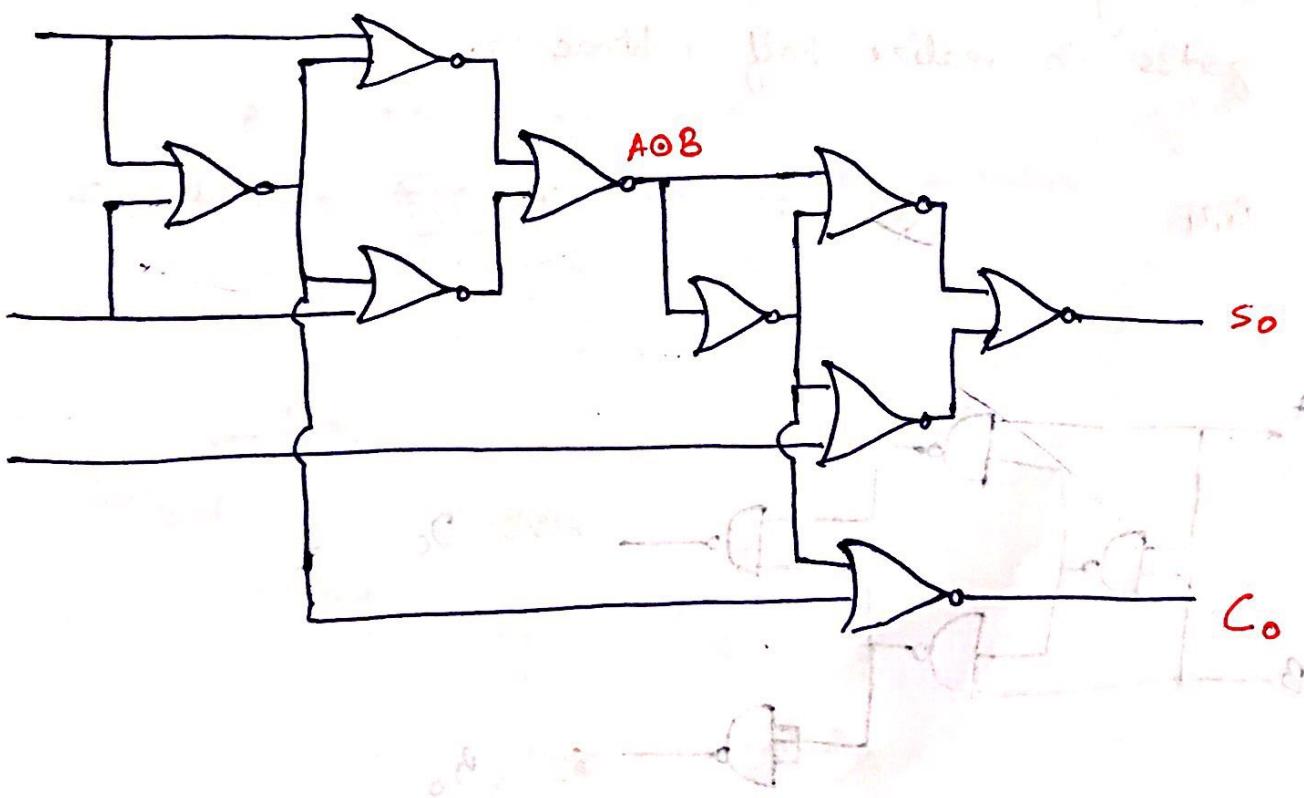
$$\boxed{\text{NAND} = 9}$$

$$\boxed{\text{NOR} = 9}$$



$$C_0 = \overline{C_{in}(A \oplus B)} + AB$$

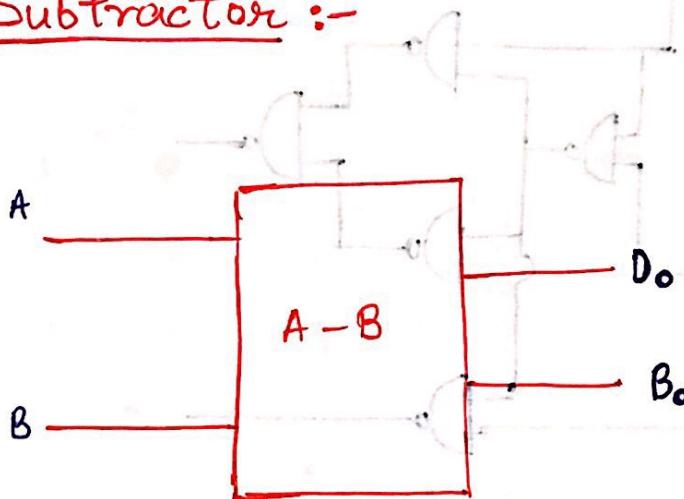
$$= \overline{AB} \cdot \overline{C_{in}(A \oplus B)}$$



- Full Adder sum o/p is odd function and carry function is majority function.

Half

Subtractor :-



A	B	D <sub>o</sub>	B <sub>o</sub>
0	0	0	0
0	1	1	1
1	0	0	1
1	1	0	0

$$\begin{aligned} D_o &= A \oplus B \\ B_o &= \overline{A}B \end{aligned}$$

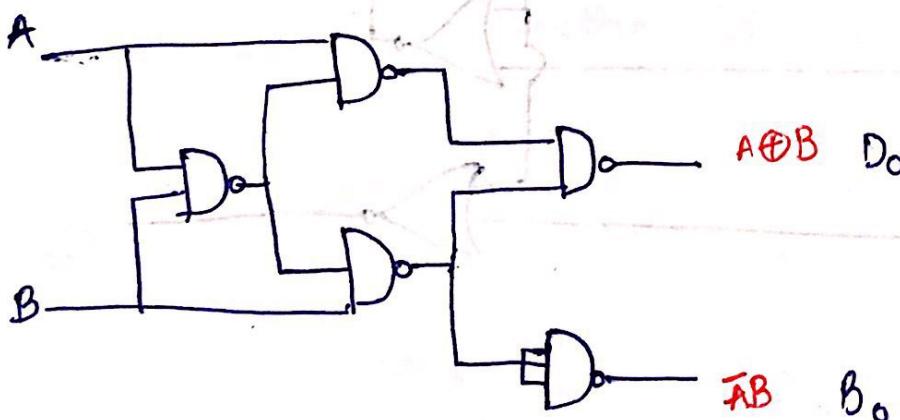
Q. Identify the minimum no. of two input NAND or NOR gates to realize half subtractor.

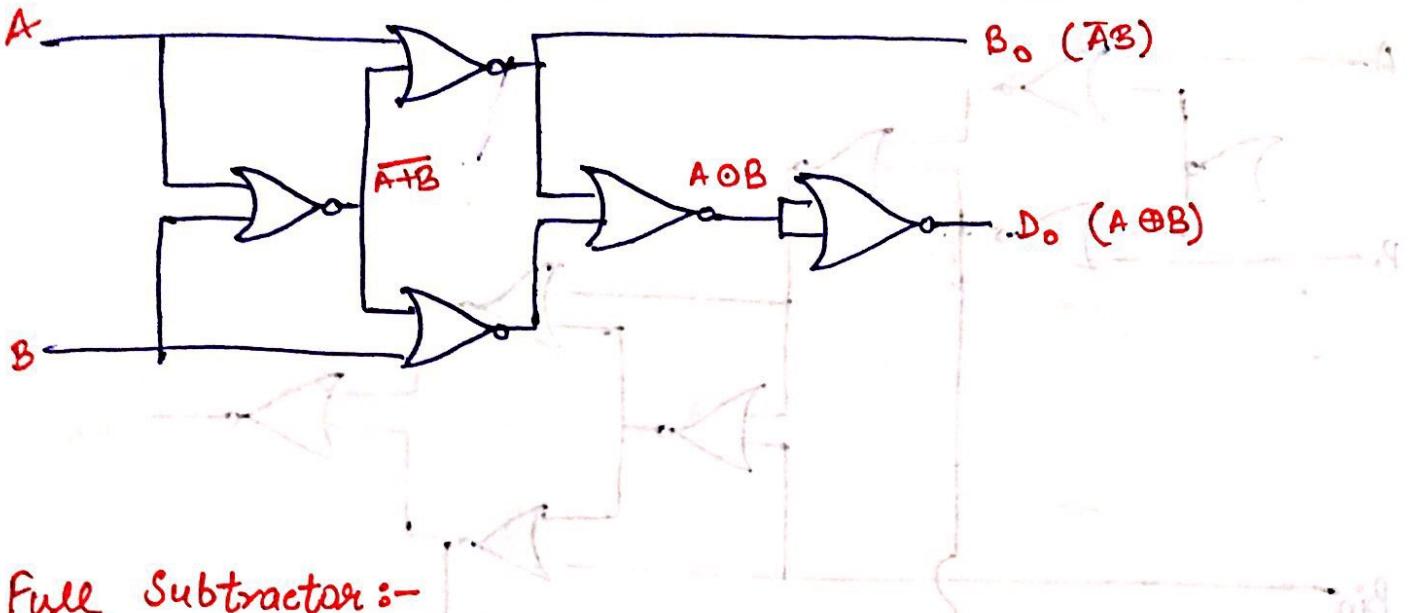
(a) 4,4

(b) 5,5

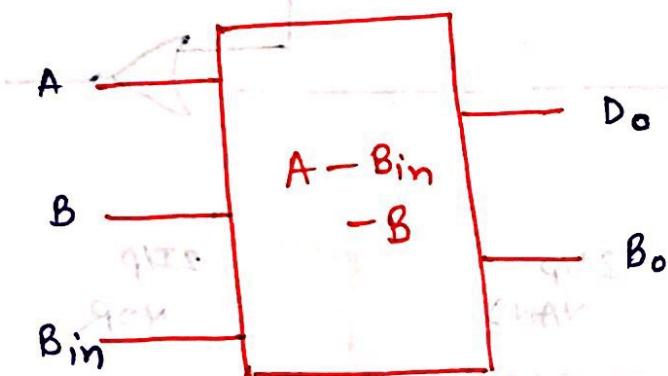
(c) 6,7

(d) 7,7





Full Subtractor :-



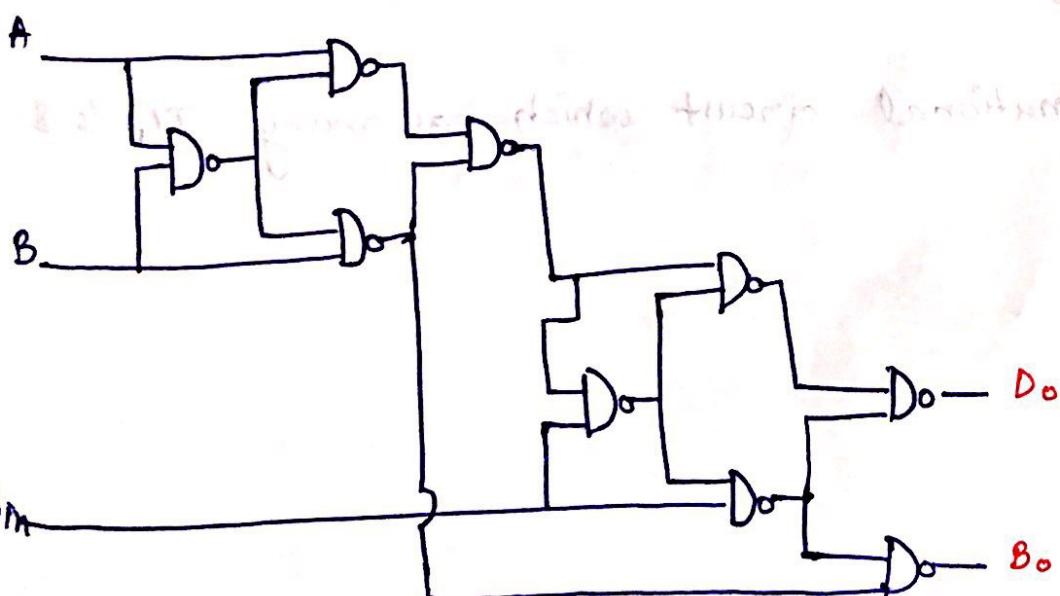
A	B	$B_{in}$	$D_0$	$B_0$
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	0
1	0	0	1	0
1	0	1	0	0
1	1	0	0	1
1	1	1	1	1

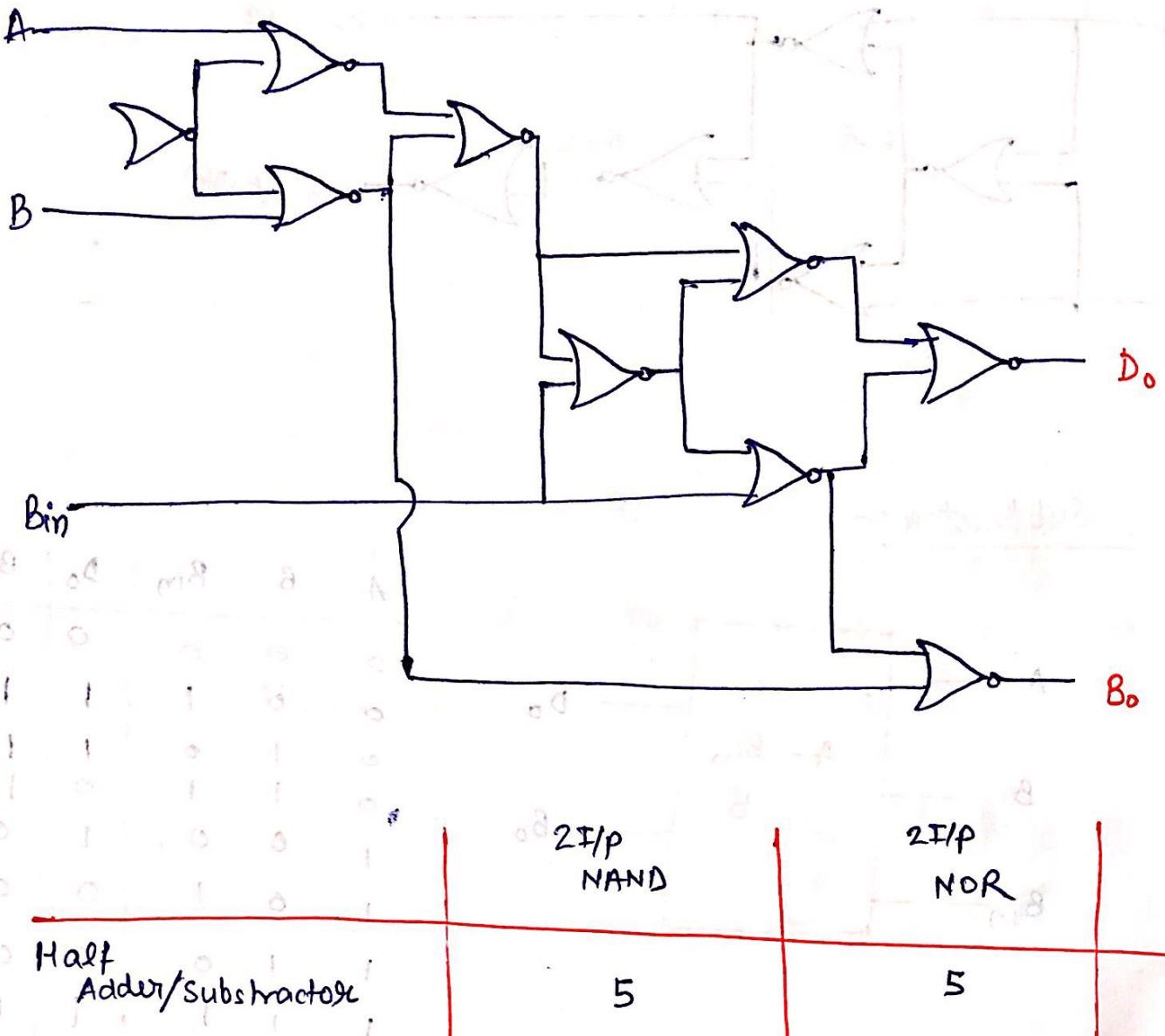
$$D_0 = A \oplus B \oplus B_{in}$$

$$B_0 = \bar{A}B + \bar{A}B_{in} + BB_{in}$$

$$B_0 = \bar{A}\bar{B}B_{in} + \bar{A}B\bar{B}_{in} + \bar{A}BB_{in} + ABB_{in}$$

$$B_0 = (\overline{A \oplus B})B_{in} + \bar{A}B$$





Half Adder/Subtractor

2I/P  
NAND

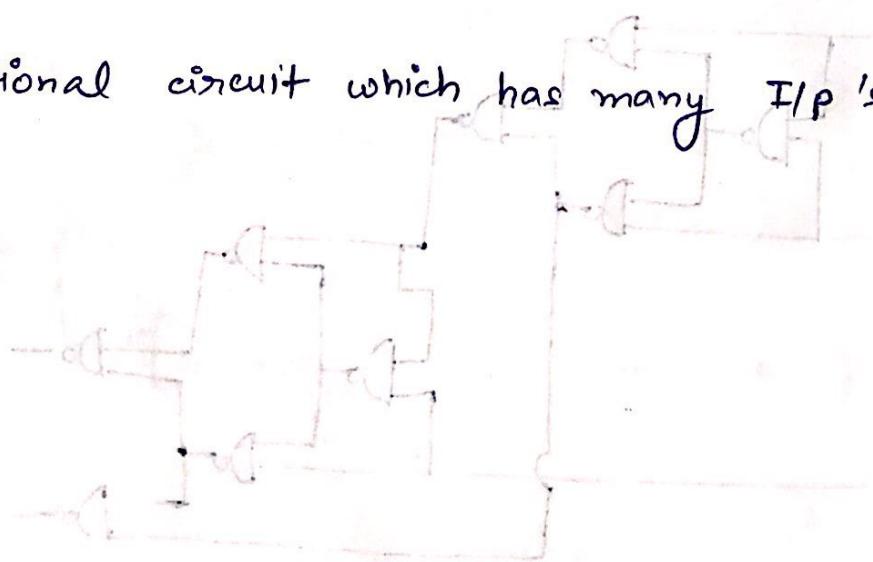
2I/P  
NOR

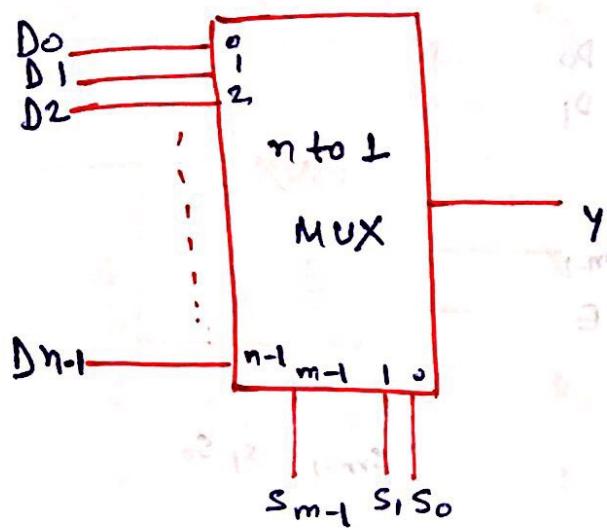
5

5

### Multiplexer :-

It is a combinational circuit which has many I/P's & single output.

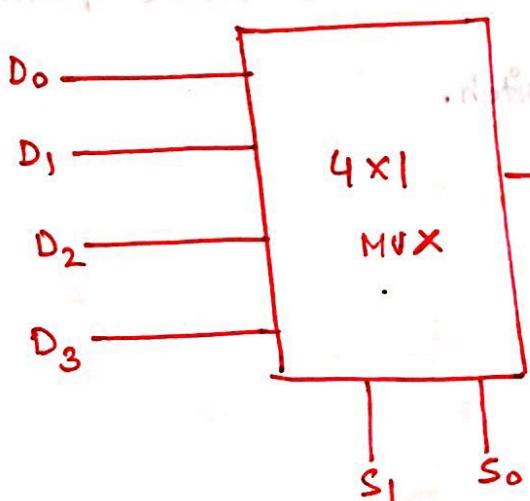




$2 \times 1$	$\boxed{S_0}$
$2:1$	$- S_0, S_1$
$4 \times 1$	$- S_0, S_1, S_2$
$8 \times 1$	$- S_0, S_1, S_2, S_3$
$16 \times 1$	$- S_0, S_1, S_2, S_3$

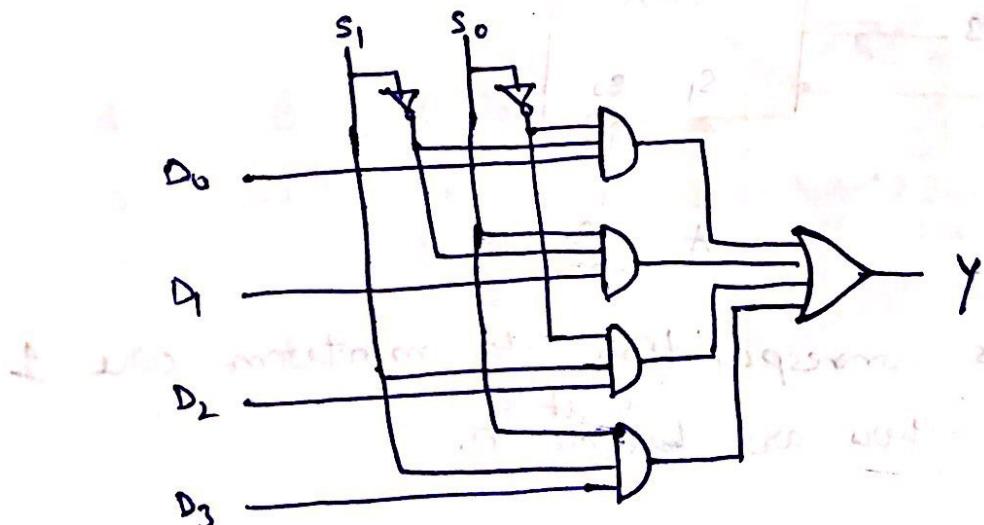
$$(\text{No. of bits} = 2^m \geq n)$$

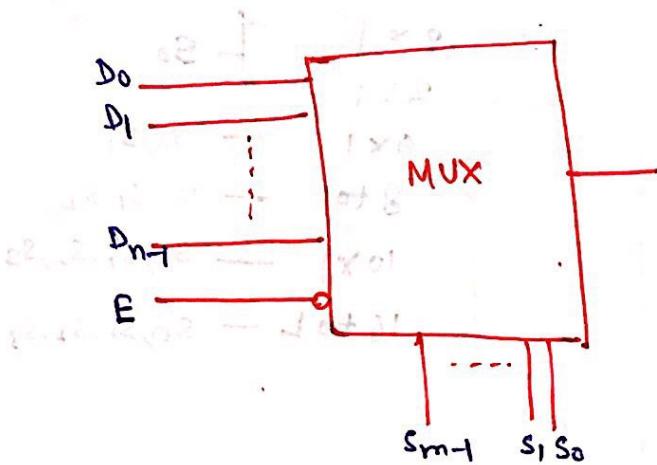
$4 \times 1$  MUX :-



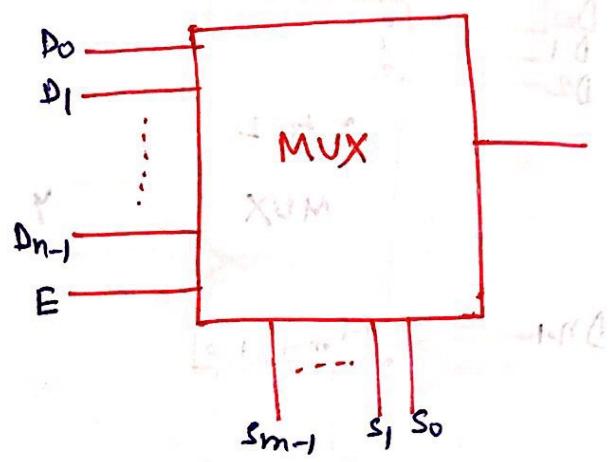
$S_{0,1}$	$S_0$	$Y$
0	0	$D_0$
0	1	$D_1$
1	0	$D_2$
1	1	$D_3$

$$y = \bar{S}_0 \bar{S}_1 D_0 + \bar{S}_0 S_1 D_1 + S_0 \bar{S}_1 D_2 + S_0 S_1 D_3$$





(Active-Low)

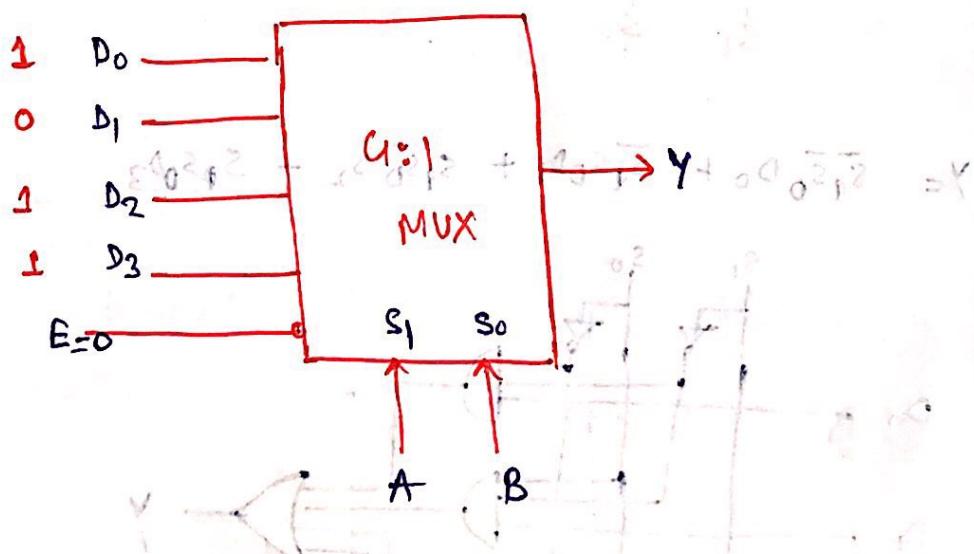


(Active-High)

### Application :-

- It is possible to convert parallel data to serial form.
- It is possible to use as TDM switch.
- Use for realizing boolean function.

$$f(A, B) = \sum m(0, 2, 3)$$



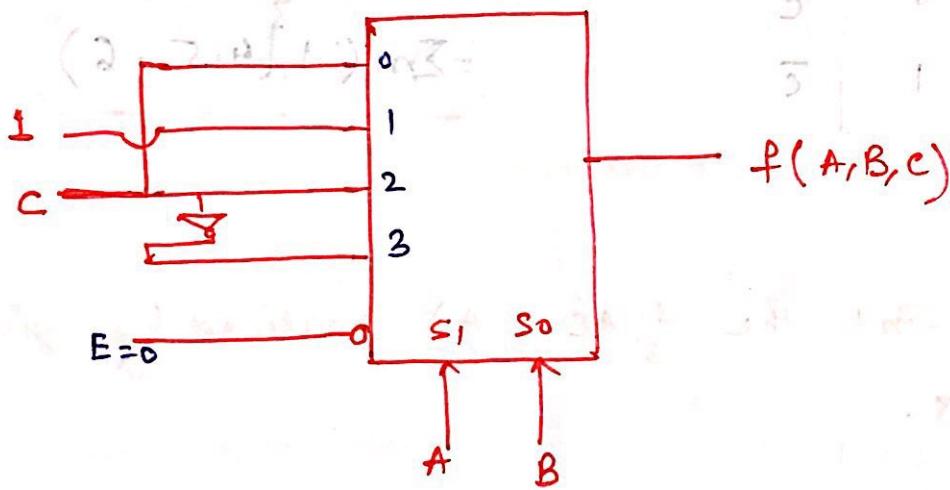
Input lines corresponding to minterm are 1 and others are set to 0.

$$S_0, \quad D_0 = D_2 = D_3 = 1$$

$$D_1 = 0$$

A S <sub>1</sub>	B S <sub>0</sub>	Y
0	0	D <sub>0</sub> (1)
0	1	D <sub>1</sub> (0)
1	0	D <sub>2</sub> (1)
1	1	D <sub>3</sub> (1)

Q. Find the function  $f(A, B, C)$  which is realized by with the given 4x1 MUX.

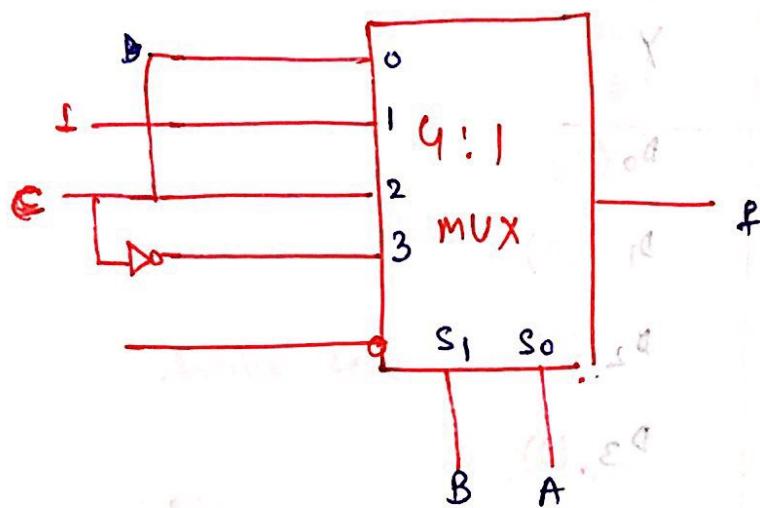


A	B	Y f(A, B, C)
0	0	D <sub>0</sub> (C)
0	1	D <sub>1</sub> (1)
1	0	D <sub>2</sub> (C)
1	1	D <sub>3</sub> (C)

$$Y = \overline{A}\overline{B}C + \overline{A}B + A\overline{B}C + AB\overline{C}$$

$$f(A, B, C) = \sum m(1, 2, 3, 5, 6)$$

Q: Find the function  $f_2(A, B, C)$  realized by the given 4:1 MUX.



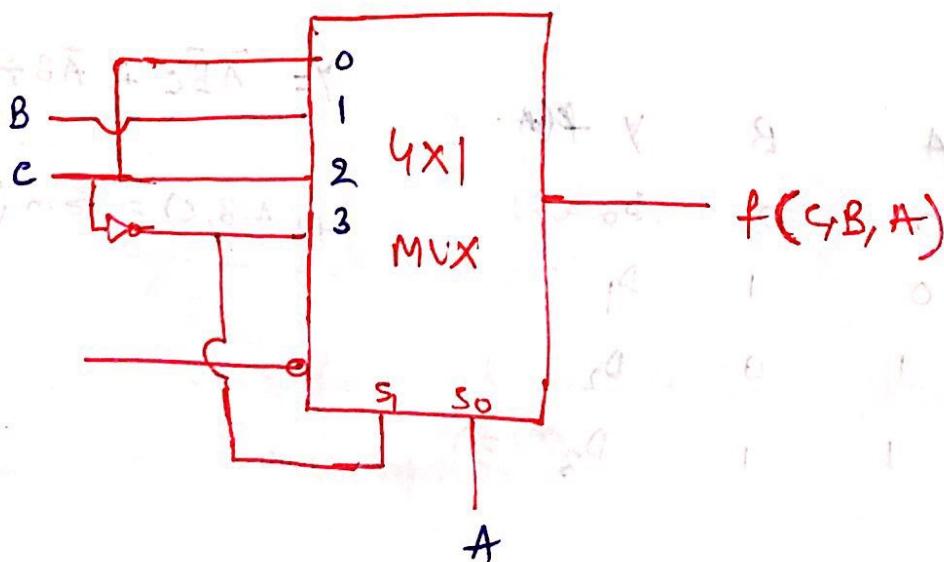
B	A	Y
S <sub>1</sub>	S <sub>0</sub>	
0	0	C
0	1	1
1	0	C
1	1	C̄

$$f(A, B, C) = \overline{AB}C + A\overline{B} + \overline{ABC}$$

$$+ ABC$$

$$(\Sigma m(1, 4, 5, 6))$$

Q: ~~Ex.~~ Find the  $f_3(AC, B, A)$  realized by given MUX.



$S_1$	$S_0$	Y
0	0	$D_0$ C
0	1	$D_1$ B
1	0	$D_2$ C
1	1	$D_3$ $\bar{C}$

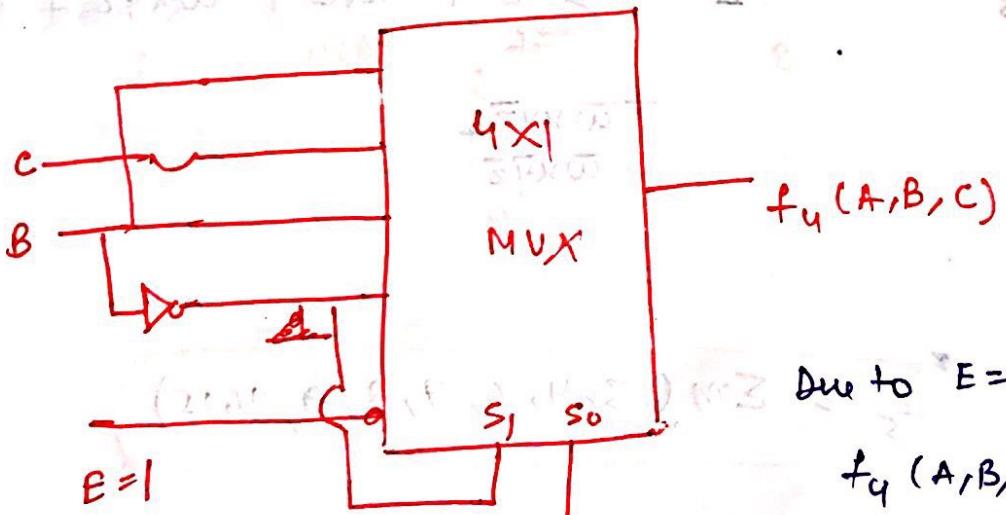
$$f = C\bar{A}C + CAB + \bar{C}\bar{A}C + \bar{C}AC$$

$$= C\bar{A} + \cancel{CAB} + 0 + \cancel{CA}$$

$$= CBA + C\bar{B}\bar{A} + CBA + \bar{C}BA + \bar{C}\bar{B}A$$

$$f(C, B, A) = \Sigma m(1, 3, 4, 6, 7)$$

Q Find the  $f_4(A, B, C)$  realized by given MUX.

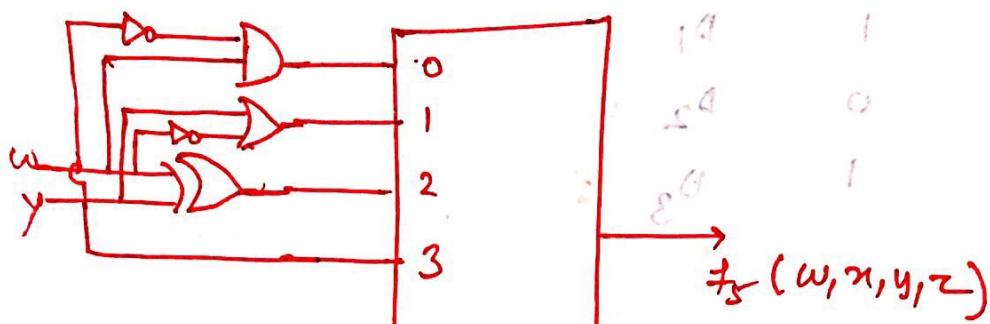


Due to  $E=1$

$$f_4(A, B, C) = 0$$

$$S_0, f_4 = \prod M(0, 1, 2, 3, 4, 5, 6, 7)$$

Q:

Find the func<sup>n</sup>  $f_5(w, x, y, z)$ 

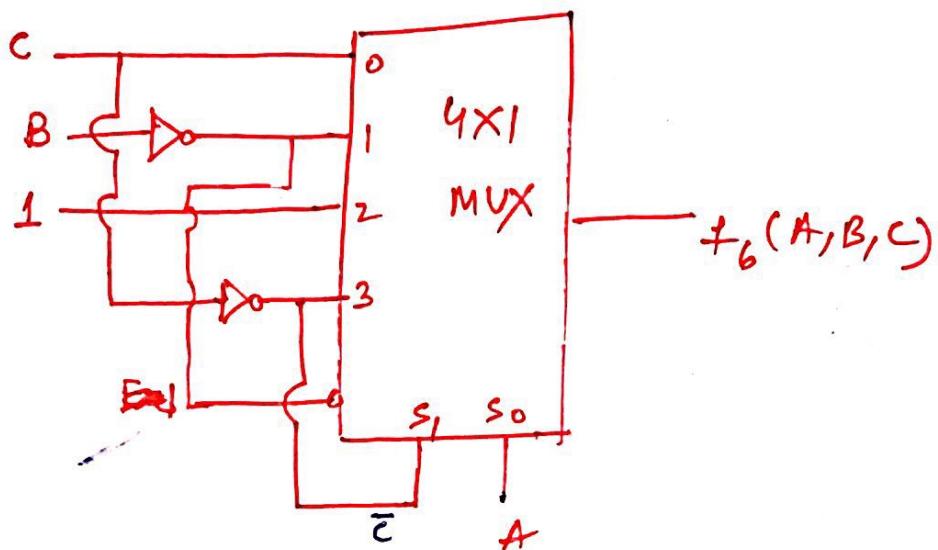
$$\begin{aligned} E = 0 & \quad \bar{A}\bar{B} + \bar{A}\bar{B} + A\bar{B} + A\bar{B} = 9 \\ & \quad \bar{A}\bar{B} + 0 + \bar{A}\bar{B} + \bar{A}\bar{B} = 9 \end{aligned}$$

$$\begin{aligned} D_0 &= w\bar{y} & A\bar{B}\bar{C} + A\bar{B}\bar{C} + A\bar{B}C + \bar{A}\bar{B}C + \bar{A}\bar{B}D = 9 \\ D_1 &= \bar{w} + y & f_5 = \bar{s}_1 \bar{s}_0 D_0 + \bar{s}_1 s_0 D_1 + s_1 \bar{s}_0 D_2 \\ D_2 &= w \oplus y & + s_1 s_0 D_3 \\ D_3 &= y & = \bar{z}\bar{x}w\bar{y} + \bar{z}x(\bar{w}+y) + z\bar{x}(w\bar{y}+\bar{w}y) \\ & & + zxy \end{aligned}$$

$$\begin{aligned} f_5 &= w\bar{x}\bar{y}\bar{z} + \bar{w}xy\bar{z} + xy\bar{z} + w\bar{x}\bar{y}z + \bar{w}\bar{x}yz \\ & & + \bar{w}xyz \\ & & + wxyz \\ & & + \bar{w}xyz \end{aligned}$$

$$f_5 = \sum m(3, 4, 6, 7, 8, 9, 14, 15)$$

Q. Find func'n  $f_6(A, B, C)$



$$\begin{aligned}
 f_6 &= [\bar{s}_1 \bar{s}_0 D_0 + \bar{s}_1 s_0 D_1 + s_1 \bar{s}_0 D_2 + s_1 s_0 D_3] \bar{B} \\
 &= [C \bar{A} C + C A \bar{B} + \bar{C} \bar{A} (1) + \bar{C} A \bar{C}] \bar{B} \\
 &= [\cancel{A B C} + \cancel{\bar{A} \bar{B} C} + \cancel{A \bar{B} C} + \cancel{\bar{A} B \bar{C}} + \cancel{\bar{A} \bar{B} \bar{C}} + \cancel{A B \bar{C}} + \cancel{A \bar{B} C}] \bar{B} \\
 &= \cancel{\bar{A} \bar{B} C} + \cancel{\bar{A} B \bar{C}} + \cancel{A \bar{B} \bar{C}} + \cancel{A \bar{B} C} \\
 &= \sum m(0, 1, 4, 5) \\
 &= \overline{\sum m(2, 3, 6)}
 \end{aligned}$$