

## Top - down Parser

Recursive Descent Parser

(with Back-tracking)

LL(1)

(without Back-Tracking)

### (i) Recursive Descent Parser :-

S()

{ select any production of S

let ( $S \rightarrow x_1, x_2, x_3, \dots, x_k$ )

for ( i=1 ;  $i \leq k$ ; i++ )

{ if ( $x_i$  is variable)

( $x_i = \alpha$ )  $x_i(\cdot)$

else

if ( $x_i ==$  Look ahead symbol)

incr i/p pointer;

else  
error (try other possibilities)

Next production

}

Eg:-  $S \rightarrow ABC \mid DEF \mid GHI$

$A \rightarrow ab \mid gh \mid mn$

$B \rightarrow cd \mid ij \mid op$

$C \rightarrow ef \mid kl \mid qr$

$D \rightarrow d$

$g \rightarrow g$

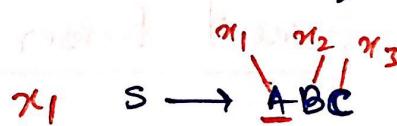
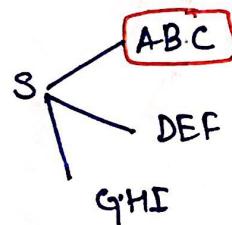
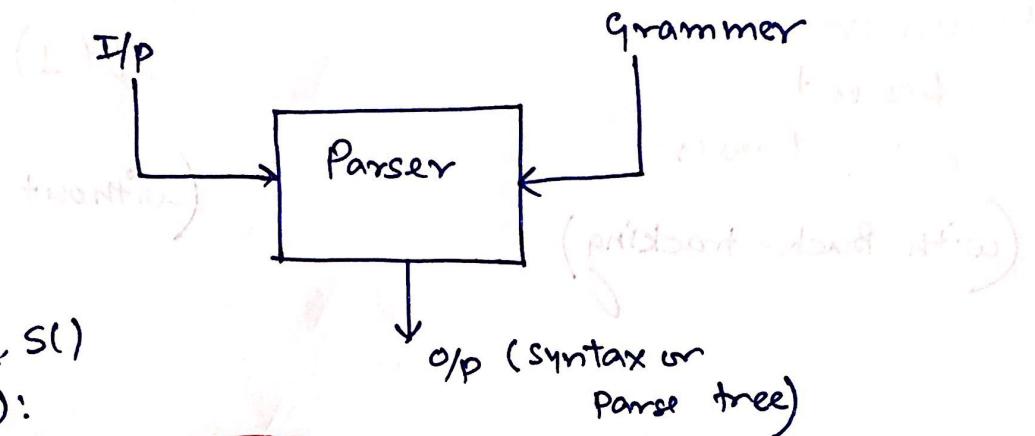
$E \rightarrow e$

$H \rightarrow h$

$F \rightarrow f$

$I \rightarrow i$

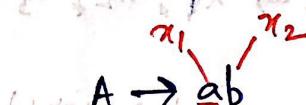
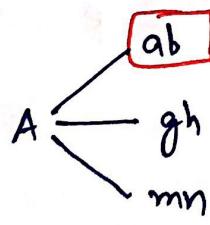
I/p:- abcdar \$



$A$  is non-terminal

PUSH , A()

A():



$x_1$  is terminal,

if ( $a == LAS$ )

a b c d q r \$  
↑  
LAS

so inc LAS;

$x_2$ : b is terminal

abcd q r \$  
↑  
LAS

& ( $b == LAS$ )

~~a b c d qr \$~~ INC LAS;

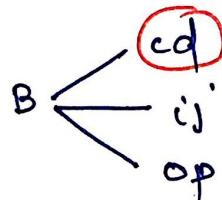
POP A();

- S() :  $x_2$  : B is non-terminal

so, B();

PUSH B()

B() :



$$B \rightarrow cd$$

$x_1$  : c is terminal

c == LAS

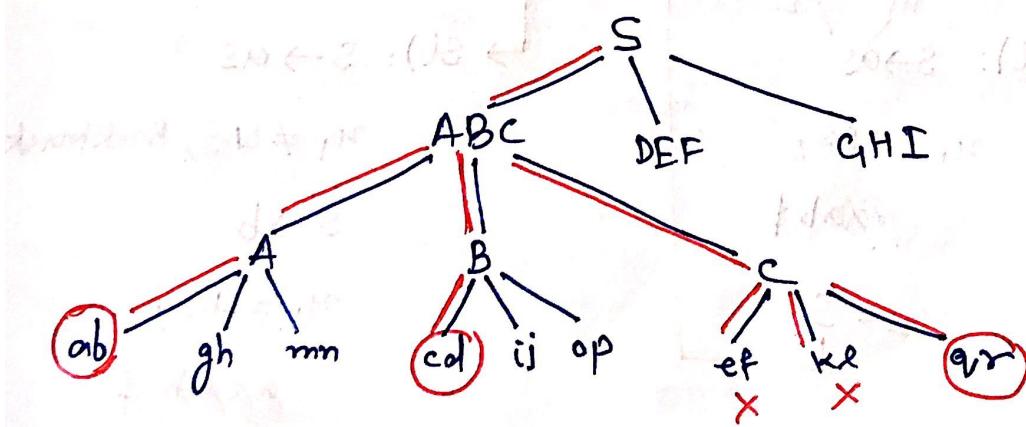
~~a b c d qr \$~~

LAS

~~a b c d qr \$~~

LAS

INC LAS;

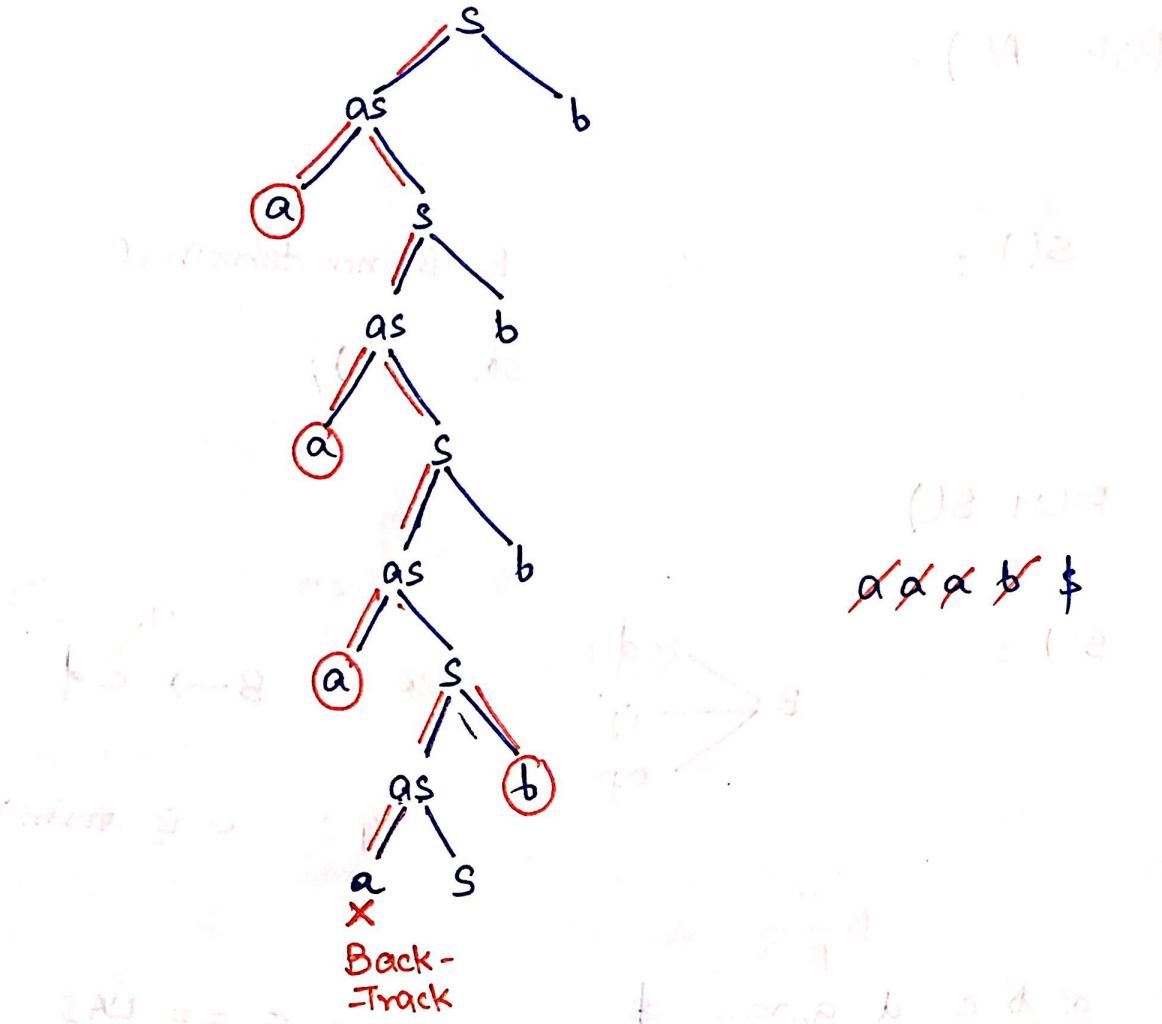


X: Error (Backtrack)

eg:-

$$S \rightarrow as \mid b$$

$$I/P:- aaab \$$$



$$S(): S \xrightarrow{x_1/x_2} as$$

$$x_1 = LAS, \\ \cancel{aaab\$}$$

$$x_2 = S$$

$$S(): S \xrightarrow{x_1/x_2} as$$

$$x_1 = LAS, \\ \cancel{aaab\$}$$

$$x_2 = S$$

$$S(): S \rightarrow as$$

$$x_1 = LAS, \\ \cancel{aaab\$}$$

$$x_2 = S$$

$$S(): S \rightarrow as$$

$x_1 \neq LAS$ , Backtrack

$$S \rightarrow b$$

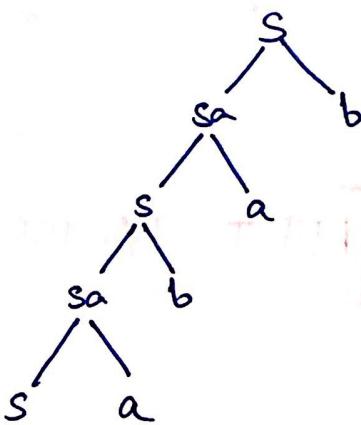
$$x_1 = LAS, \\ \cancel{aaab\$}$$

ACCEPTED

eg:

$$S \rightarrow Sa \mid b$$

I/P: baaa \$



[LEFT RECURSION]

As we can see in

$$S \rightarrow \underline{Sa}$$

as PUSH op<sup>n</sup>  
of  $S()$

first symbol in production  
is non-terminal which  
makes parser to call  
 $S()$  infinitely until  
STACK overflow.

And as we know  $S \rightarrow Sa$  is left recursion

so if left recursion is present in then  
Recursive descent parser goes into a loop.

NOTE:-

In parser STACK is used and in LA (lexical analyser) DFA is used for tokenization so,

$$(DFA + \text{STACK} = PDA)$$

so Parsing use PDA

$$S \rightarrow ABC \mid DEF \mid GHI$$

$$A \rightarrow ab \mid abc \mid abcd$$

$$B \rightarrow ef \mid gh \mid ij$$

$$C \rightarrow mkl \mid mn \mid op$$

$$D \rightarrow d$$

$$E \rightarrow c$$

$$F \rightarrow f$$

$$G \rightarrow g$$

$$H \rightarrow h$$

$$I \rightarrow i$$

## LEFT FACTORING

I/P:- abcdefghop \$

Output: abcde fghop \$

After factoring we get:  
Tree of various ABC under S  
Left factorization

S():  
• Left factorization

$$S \rightarrow ABC$$

$$S \rightarrow DEF$$

$$S \rightarrow GHI$$

A():  
 ~~$A \rightarrow ab$~~

$A \rightarrow abc$

$\circlearrowleft A \rightarrow abcd$

if this production is selected then no problem

B():  
 ~~$B \rightarrow ef$~~  X

Backtrack

As  $A \rightarrow ab$

Doesn't show any error

~~$B \rightarrow ij$~~  X

Backtrack

because it has same symbol

as  $A \rightarrow abcd$

But 'cd' can only be derived from A only so

A done its half work, due to which all other showing error.

## NOTE:-

If Grammer contain Left recursion → Recursive - Descent parser goes into  $\infty$  loop  
(can't give answer)

// if Grammer contains Left factoring → Recursive Descent parser may failed as it can show false result  
(string is derivable from grammar but parser can't)

## (ii) LL(1) Parser | Predictive parser :-

- LL(1) is without backtracking because we are using LL(1) parsing table.
- To construct LL(1) parsing table, we need two functions :

→ FIRST()

→ FOLLOW()

FIRST(): contains set of all terminals present at 1<sup>st</sup> position of every string derived by A.

e.g.  $s \rightarrow abc \mid def \mid ghi \mid \epsilon$

$$\begin{aligned} \text{FIRST}(s) &= \text{Fi}(abc), \text{Fi}(def), \text{Fi}(ghi), \text{Fi}(\epsilon) \\ &= \text{Fi}(a), \text{Fi}(d), \text{Fi}(g), \epsilon \\ &= \{a, d, g, \epsilon\} \end{aligned}$$

$\text{First}(\text{terminal}) \rightarrow \text{terminal}$

$\text{First}(\text{null}) \rightarrow \text{null } (\epsilon)$

eg:

$S \rightarrow ABC$

$A \rightarrow a|d|\epsilon$

$B \rightarrow b|g|\epsilon$

$C \rightarrow c|j|\epsilon$

$$\text{First}(S) = \text{Fi}(ABC)$$

$$= \text{Fi}(A)$$

next of A if exist

$$= \text{Fi}(a) \neq \text{Fi}(d) \Rightarrow \underline{\text{Fi}(\epsilon)}$$

$$= " , " , \text{Fi}(B)$$

$$\text{Fi}(b), \text{Fi}(g), \underline{\text{Fi}(\epsilon)}$$

next of B  
if exist

$$= \text{Fi}(c)$$

$$\text{Fi}(c), \text{Fi}(g), \underline{\text{Fi}(\epsilon)}$$

not  
next  
of C

so include

$$= \text{Fi}(a), \text{Fi}(d), \text{Fi}(b), \text{Fi}(g), \text{Fi}(c), \text{Fi}(j), \underline{\text{Fi}(\epsilon)}$$

$$= \underline{\{a, d, b, g, c, j, \epsilon\}}$$

eg:-

$$S \rightarrow aBDh$$

$$B \rightarrow bC|\epsilon$$

$$C \rightarrow cC|\epsilon$$

$$D \rightarrow FE$$

$$F \rightarrow g|\epsilon$$

$$E \rightarrow f|\epsilon$$

$$\text{First}(S) = \text{First}(aBDh)$$

$$= \text{Fi}(a)$$

$$= \{a\}$$

$x$	$\text{First}(x)$
S	a
B	b, $\epsilon$
C	c, $\epsilon$
D	g, f, $\epsilon$
F	g, $\epsilon$
E	f, $\epsilon$

$$\text{First}(B) = \text{Fi}(bC), \text{Fi}(\epsilon)$$

$$= \text{Fi}(b), \text{Fi}(\epsilon)$$

$$= \{b, \epsilon\}$$

$$\text{First}(F) = \{g, \epsilon\}$$

$$\text{First}(C) = \text{Fi}(cC), \text{Fi}(\epsilon)$$

$$= \text{Fi}(c), \text{Fi}(\epsilon)$$

$$= \{c, \epsilon\}$$

$$\text{First}(E) = \{f, \epsilon\}$$

$$\text{First}(D) = \text{Fi}(FE)$$

$$= \text{Fi}(F)$$

$$= \text{Fi}(g), \text{Fi}(\epsilon)$$

 $=$ 

$$\text{Fi}(E)$$

$$= \{g, f, \epsilon\}$$

$$\text{Fi}(f), \text{Fi}(\epsilon)$$

eg:-  $s \rightarrow aSbS \mid bSaS \mid \epsilon$

$$\begin{aligned}\text{First}(S) &= \text{Fi}(aSbS), \text{Fi}(bSaS), \text{Fi}(\epsilon) \\ &= \underline{\{a, b, \epsilon\}}\end{aligned}$$

eg:-  $s \rightarrow AaAb \mid BbBa$

$$A \rightarrow \epsilon$$

$$B \rightarrow \epsilon$$

$$\text{First}(S) = \text{Fi}(AaAb), \text{Fi}(BbBa)$$

$$\begin{array}{c} \text{Fi}(A) \quad , \quad \text{Fi}(B) \\ | \quad \quad | \\ \text{Fi}(\epsilon) \text{ next} \quad \text{Fi}(\epsilon) \text{ next} \\ | \quad \quad | \\ \text{Fi}(aAb) \quad \text{Fi}(bBa) \\ | \quad \quad | \\ \text{Fi}(a) \quad \text{Fi}(b) \\ | \quad \quad | \\ \text{Fi}(a) \quad \text{Fi}(b) \end{array}$$
$$= \{a, b\}$$

$x$	$\text{First}(x)$
$s$	$a, b$
$A$	$\epsilon$
$B$	$\epsilon$

FOLLOW(:): contain set of all terminals present at 1<sup>st</sup> place of every string immediately right of that variable.

e.g:-

$$S \rightarrow ABC$$

$$A \rightarrow DEF$$

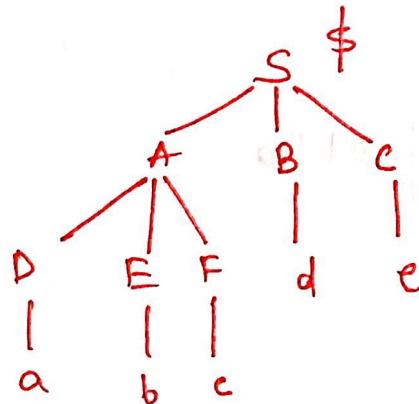
$$D \rightarrow a$$

$$E \rightarrow b$$

$$B \rightarrow d$$

$$C \rightarrow e$$

$$F \rightarrow c$$



$$\text{Follow}(D) = \text{First}(EF) \cup \text{Follow}(A) \cup \text{Follow}(C) \cup \{\$\}$$

$$= \text{First}(EF)$$

$$= b$$

$$\text{Follow}(F) = \text{Follow}(A) \cup \text{Follow}(B) \cup \{\$\}$$

$$= \text{First}(BC)$$

$$= \text{First}(B)$$

$$= d$$

$$\text{Follow}(C) = \text{Follow}(S)$$

$$= \text{First}(\$)$$

$$= \$$$

$$S \rightarrow ABC$$

c is last

var in production

Given:  $S \rightarrow ABC$

$A \rightarrow DEF$

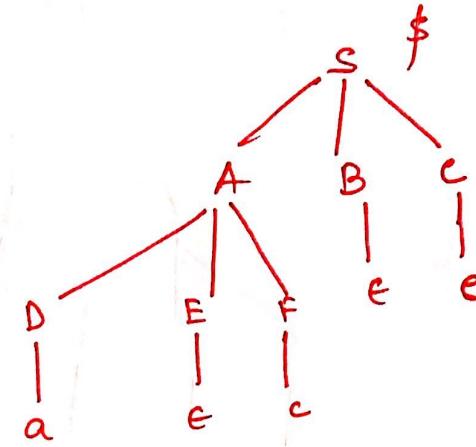
$D \rightarrow a$

$B \rightarrow e$

$E \rightarrow e$

$C \rightarrow e$

$F \rightarrow c$



$$\text{Follow}(D) = \text{First}(EF)$$

$$= \text{First}(E)$$

$\text{Fr}(e) \text{ next } \text{Fi}(F)$

$$\text{Fi}(c) = \underline{c}$$

$$\text{Follow}(F) = \text{Follow}(A)$$

$$= \text{First}(BC)$$

$$= \text{Fi}(B)$$

$\text{Fr}(e) \text{ next } \text{Fi}(C)$

$$\text{Fi}(e) = \underline{e}$$

$$\text{Follow}(C) = \text{Follow}(S)$$

$$= \text{First}(\$)$$

$$= \underline{\$}$$

e.g:-

$S \rightarrow ABC$

$A \rightarrow DEF$

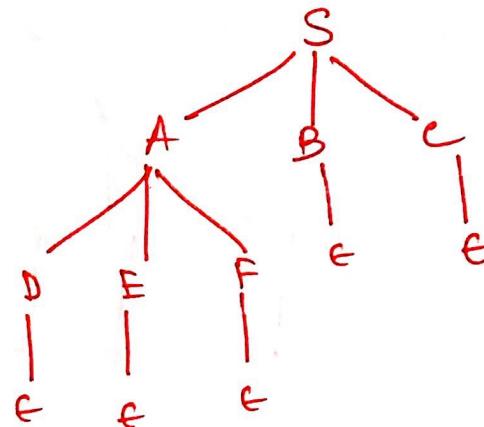
$B \rightarrow E$

$C \rightarrow E$

$D \rightarrow E$

$E \rightarrow E$

$F \rightarrow E$



$$\text{Follow}(B) = \text{First}(C)$$

$\downarrow$   
Fi( $\epsilon$ ) next

$\downarrow$   
Follow(s)

$$= \text{Fi}(\$) = (\$)$$

$$\text{Follow}(D) = \text{First}(EF)$$

$\downarrow$   
Fi(E)

$\downarrow$   
Fi(F) next

$\downarrow$   
Fi(F)

$\downarrow$   
Fi(E) next Follow(A)

$\downarrow$   
Fi(BC)

$\downarrow$   
Fi(B)

$\downarrow$   
Fi(E) next Fi(C)

$\downarrow$

$\downarrow$   
Fi(\\$)

$$\text{Follow}(S) = (\$)$$

NOTE:- "Follow can't contain  
 $\epsilon$ "

## Rules to find follow:-

Assume A is variable

i) If A is start symbol  $\text{Follow}(A) = \$$

ii)  $B \rightarrow e A(DY)$

$D \rightarrow b$

$$\text{Follow}(A) = \text{Fi}(DY)$$

$$= \text{Fi}(D) = \text{Fi}(b) = \{b\}$$

iii)  $B \rightarrow cA$  or  $B \rightarrow cAD$

$$\text{Follow}(A) = \text{Follow}(B)$$

Q.

$$E \rightarrow TE'$$

$$E' \rightarrow e \mid +TE'$$

$$T \rightarrow ET'$$

$$T' \rightarrow e \mid *FT'$$

$$F \rightarrow id \mid (E)$$

$x$	$\text{First}(x)$	$\text{Follow}(x)$
E	id, (	$\$, )$
E'	e, +	$\$, )$
T	id, (	$\$, +, )$
T'	e, *	$\$, +, )$
F	id, (	$\$, *, +, )$

$$\text{First}(E) = \text{First}(TE')$$

$$= \text{Fi}(T)$$

$$= \text{Fi}(FT')$$

$$= \text{Fi}(F)$$

$$= \text{Fi}(id), \text{Fi}((E))$$

$$= \{id, (\}$$

Q:  $s \rightarrow asbs \mid bsas \mid \epsilon$

Find First and Follow

$$\begin{aligned} \text{First}(s) &= \text{Fi}(asbs), \text{Fi}(bsas), \text{Fi}(\epsilon) \\ &= \{a, b, \epsilon\} \end{aligned}$$

$$\text{Follow}(s) = \{\$, b, a\}$$

$s$  is starting symbol = { \$, ? }

$$s \rightarrow \underline{asbs} = \{\$, b, \}\}$$

$s \rightarrow \underline{asbs}$  no use

$$s \rightarrow \underline{bsas} = \{\$, b, a\}$$

$s \rightarrow \underline{bsas}$  no use

Q:  $s \rightarrow AbAb \mid BaBa$

$$A \rightarrow \epsilon$$

$$B \rightarrow \epsilon$$

$$\begin{aligned} \text{First}(s) &= \text{Fi}(AbAb), \text{Fi}(BaBa) \\ &= \{b, a\} \end{aligned}$$

$$\text{Follow}(A) = \{b\}$$

$$s \rightarrow \underline{Ab} \underline{Ab}$$

$$\text{Follow}(B) = \{a\}$$

$$s \rightarrow \underline{Ba} \underline{Ba}$$

$\alpha$	First( $\alpha$ )	Follow( $\alpha$ )
s	{a, b}	{\$, }
A	$\epsilon$	{b}
B	$\epsilon$	{a}

Q:

$$s \rightarrow (L) | a$$

$$L \rightarrow SL'$$

$$L' \rightarrow \epsilon | ; SL'$$

$$\begin{aligned} \text{First}(S) &= \text{Fi}(L) , \quad \text{Fi}(a) \\ &= \{(, a\} \end{aligned}$$

$$\text{First}(L) = \text{first}(SL') = \text{Fi}(S)$$

$$\begin{aligned} \text{First}(L') &= \text{Fi}(\epsilon) , \quad \text{Fi}(, SL') \\ &= \{\epsilon, ;\} \end{aligned}$$

$$\text{Follow}(L) = )$$

$$\text{Follow}(L') = )$$

	First	Follow
s	(, a	\$, ;, )
L	(, a	)
L'	$\epsilon, ;$	)

Q:

$$S \rightarrow aBDh$$

$$B \rightarrow bB|\epsilon$$

$$C \rightarrow cC|\epsilon$$

$$D \rightarrow FE$$

$$E \rightarrow f|\epsilon$$

$$F \rightarrow g|\epsilon$$

c is not reachable  
(useless symbol)

$\alpha$	First	Follow
S	a	\$
B	b, $\epsilon$	f, g, h
C	c, $\epsilon$	-
D	f, g, $\epsilon$	h
E	f, $\epsilon$	h
F	g, $\epsilon$	f, h

## LL(1) - parsing table construction algo:-

For each production  $A \rightarrow \alpha$

Repeat following steps :

- ADD  $A \rightarrow \alpha$  under  $M[A, b]$   
 $\forall b \in \text{First}(\alpha)$
- if  $\text{First}(\alpha)$  contain  $\epsilon$  then add  
 $A \rightarrow \alpha$  under  $M[A, c]$ ,  $\forall c \in \text{Follow}(A)$

(Parsing table)

no. of rows  
= no. of variable

no. of columns  
= no. of terminal + 1 (\$)

eg:-  $S \rightarrow (L) \mid a$

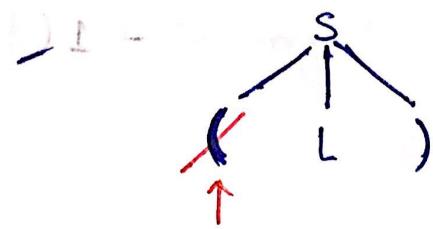
$L \rightarrow SL'$

$L' \rightarrow \epsilon \mid , SL'$

LL(1)	,	a	(	)	\$
S		$S \rightarrow a$	$S \rightarrow (L)$		
L		$L \rightarrow SL'$	$L \rightarrow SL'$		
$L'$	$L' \rightarrow , SL'$			$L' \rightarrow \epsilon$	

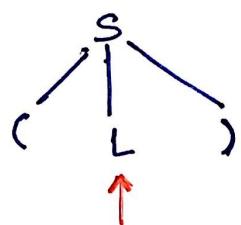
$\alpha$	First( $\alpha$ )	Follow( $\alpha$ )
S	(, a	\$, )
L	(, a	)
$L'$	$\epsilon, ,$	)

I/P:  $(a, a, a) \notin$

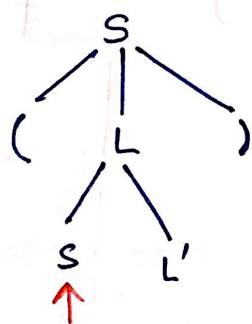


$M[S, \square]$

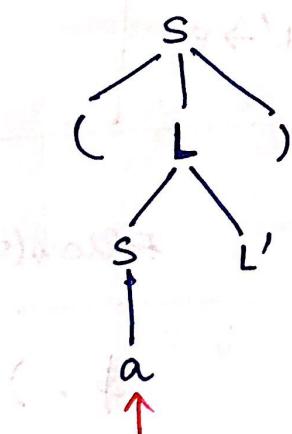
$(a, a, a) \notin$



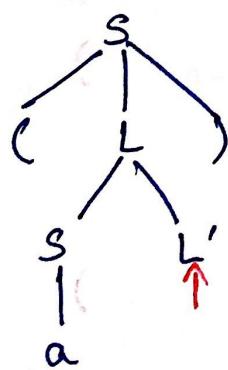
$M[L, a]$



$M[S, a]$

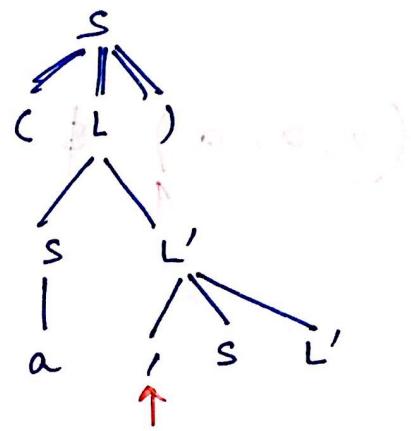


$(a, a, a) \notin$

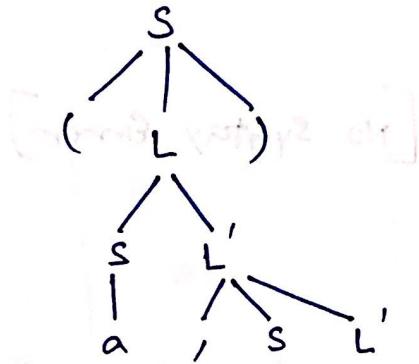


$(a, a, a) \notin$

$M[L', \square]$



$(\alpha/a, a) \uparrow \$$

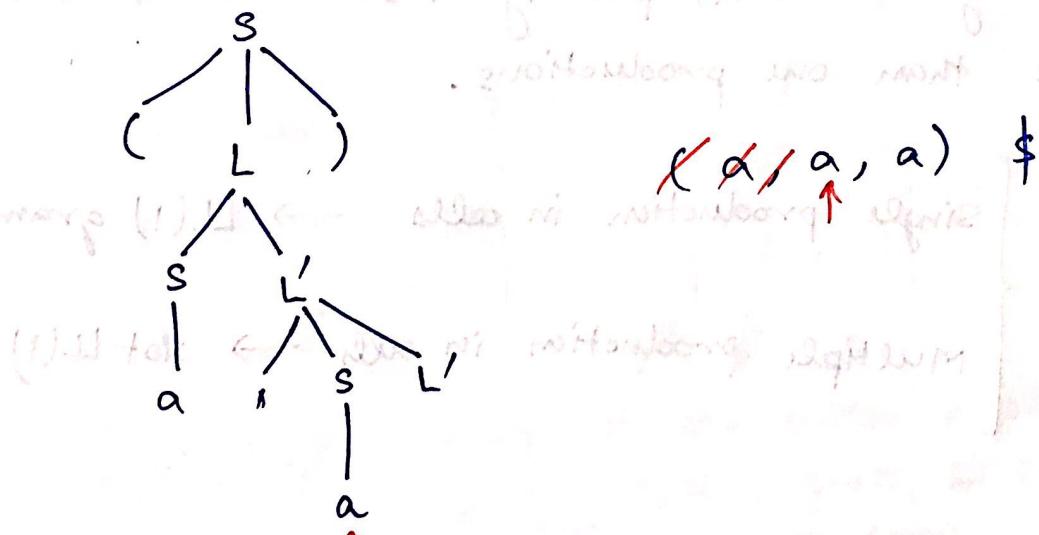


$(\alpha/a, a) \uparrow \$$

$M[S, a]$

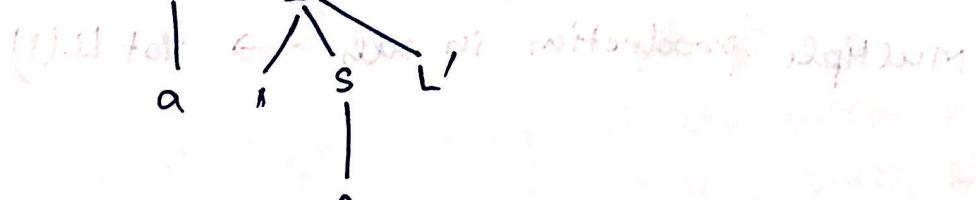
Distinct words at second and third nodes  $\rightarrow$   $a$

distinct words after popping (1) is all others are  
quitting the stack more

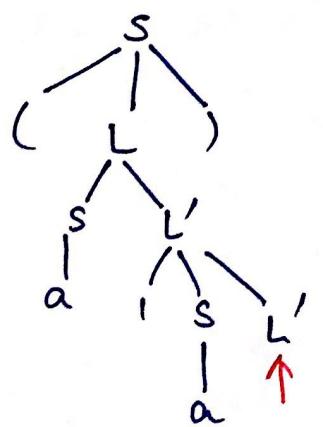


$(\alpha/a, a) \uparrow \$$

leaving (1) is all other in matching rule

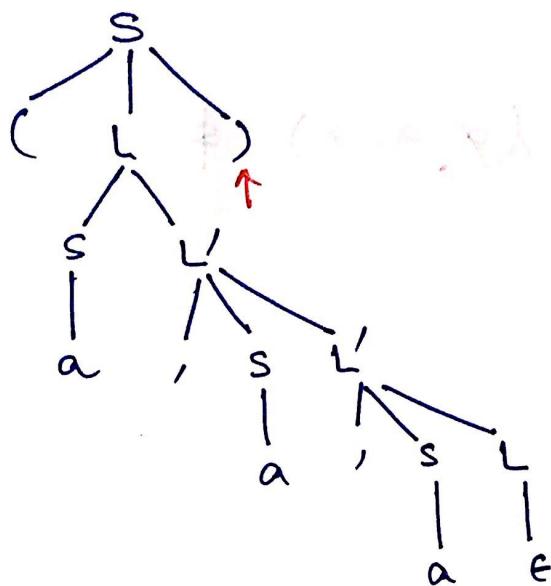


This is first step of reducing left hand side



$(\alpha/a, a) \uparrow \$$

$M[L', , ]$



$(a, a, a)$  \$

Hence string accepted

[No syntax error]

NOTE:- - Blank Entries are known as Error Entries

- no entry in LL(1) parsing table should contain more than one productions.

$\left\{ \begin{array}{l} \text{single production in cells} \rightarrow \text{LL(1) grammar} \\ \text{multiple production in cells} \rightarrow \text{Not LL(1)} \end{array} \right.$

Q: Check the following grammar is LL(1) or not

$$S \rightarrow AaAb \mid BbBa$$

$$A \rightarrow \epsilon$$

$$B \rightarrow \epsilon$$

$M[ ]$	a	b	\$
s	$s \rightarrow AaAb$	$s \rightarrow BbBa$	
A	$A \rightarrow \epsilon$	$A \rightarrow \epsilon$	
B	$B \rightarrow \epsilon$	$B \rightarrow \epsilon$	

	First()	FOLLOW()
s	a, b	\$
A	$\epsilon$	a, b
B	$\epsilon$	a, b

(Grammes is LL(1))

### NOTE:-

$s \rightarrow x_1 | x_2 \rightarrow$  it can suffer problem in which both  $s \rightarrow x_1$  &  $s \rightarrow x_2$  goes to same cell

$$x_1 \rightarrow a$$

$$x_2 \rightarrow b$$

they have only one option (choice) so they can't make any double entries

Q. Check LL(1) or not:

$$S \rightarrow aSbS \mid bSaS \mid \epsilon$$

$\text{Follow}(S)$

$$= \{a, b, \$\}$$

	a	b	\$
S	$S \rightarrow aSbS$ $S \rightarrow \epsilon$	$S \rightarrow bSaS$ $S \rightarrow \epsilon$	$S \rightarrow \epsilon$

Multiple Entries

so

Not LL(1) grammar

Q.

$$E \rightarrow TE'$$

$$E' \rightarrow \epsilon \mid *TE'$$

$$T \rightarrow FT'$$

$$T' \rightarrow \epsilon \mid +FT'$$

$$F \rightarrow id \mid (E)$$

Follow set calculation routine now for each symbol

M[]	id	+	*	\$	( )
E	$E \rightarrow TE'$				$E \rightarrow TE'$
E'			$E' \rightarrow *TE'$	$E' \rightarrow \epsilon$	$E' \rightarrow \epsilon$
T	$T \rightarrow FT'$				$T \rightarrow FT'$
T'		$T' \rightarrow +FT'$	$T' \rightarrow *$	$T' \rightarrow \epsilon$	$T' \rightarrow \epsilon$
F	$F \rightarrow id$				$F \rightarrow (E)$

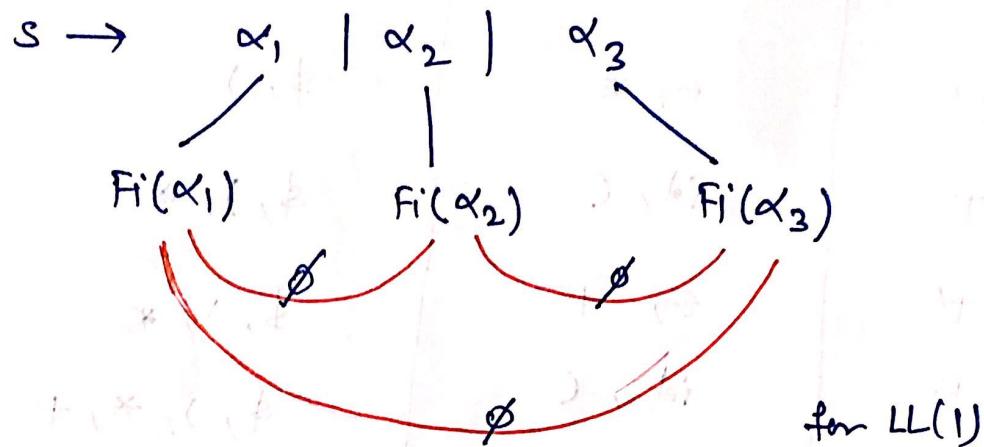
	first()	Follow()
E	id, c	\$, )
E'	$\epsilon$ , *	\$, )
T	id, c	\$, ), *, A
T'	$\epsilon$ , +	\$, ), *, *
F	id, c	\$, ), *, +

Q  $S \rightarrow aBDh$   
 $B \rightarrow bC | \epsilon$  check for LL(1)  
 $C \rightarrow cB | \epsilon$  tracing of following  $\Rightarrow$   $+ \epsilon \leftarrow$   
 $D \rightarrow FE$   
 $E \rightarrow cB | E$   
 $F \rightarrow bC | \epsilon$  (Not LL(1))

- check only for those variable which contains more than one choices. i.e

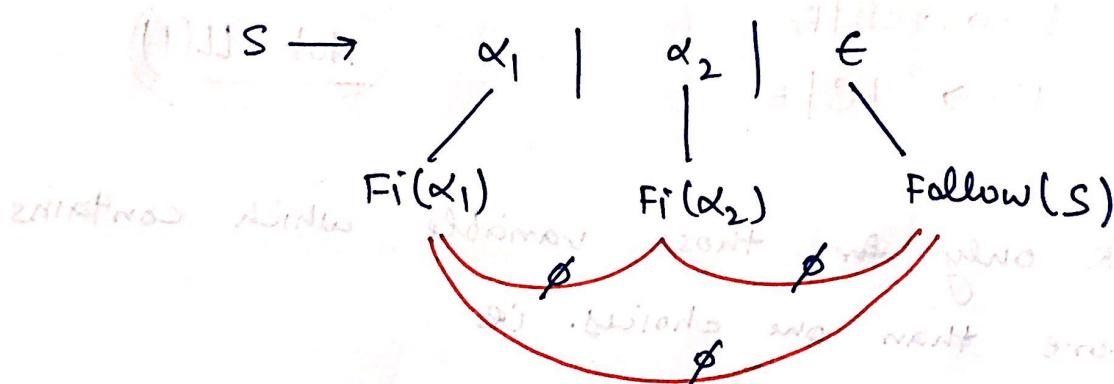
M[i]	a	b	c	d	e	f	g	h	\$
B		$B \rightarrow bC$ $B \rightarrow \epsilon$	$B \rightarrow \epsilon$		$B \rightarrow E$				
C			$C \rightarrow cB$ $C \rightarrow \epsilon$		$C \rightarrow \epsilon$				
E				$E \rightarrow cB$	$E \rightarrow h$				
F		$F \rightarrow bC$	$F \rightarrow \epsilon$		$F \rightarrow \epsilon$				

NOTE :- If productions have no null production -



Every possible pair of  $F_i()$  is disjoint i.e.  
they should not have common "first terminals".

→ If  $\epsilon$  production is present



Every possible pair should be disjoint for

LL(1)

Q. Check LL(1) or not:

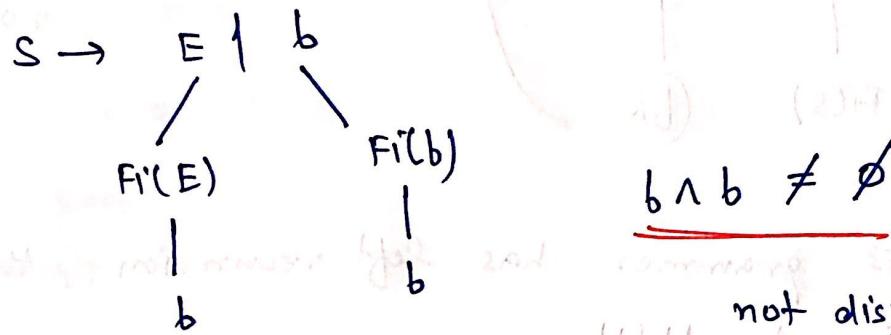
$$S \rightarrow E \mid b$$

$$E \rightarrow b$$

Since  $S$  can create multiple entries so,

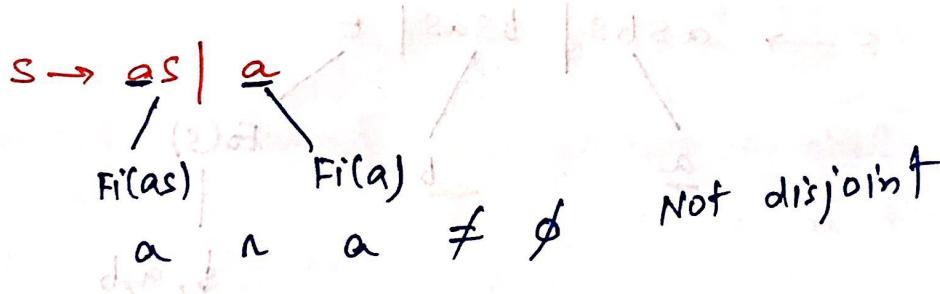
it is not standard.

Take look here



so, (Grammar is not LL(1))

Q. Check LL(1) or not:



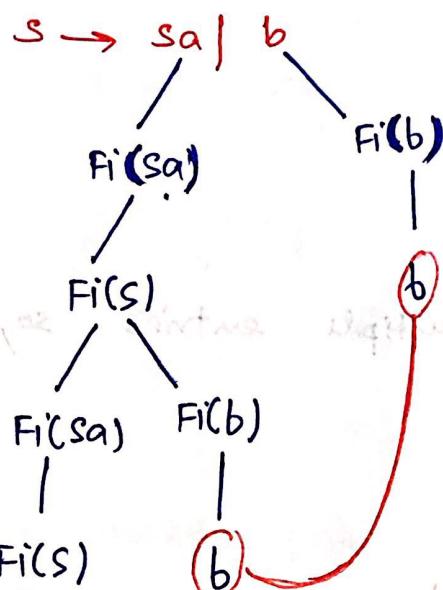
So, if grammar contains left factoring then it

cannot be LL(1)

Left factoring  $\rightarrow$  not LL(1)

Q.

Check LL(1) or not:



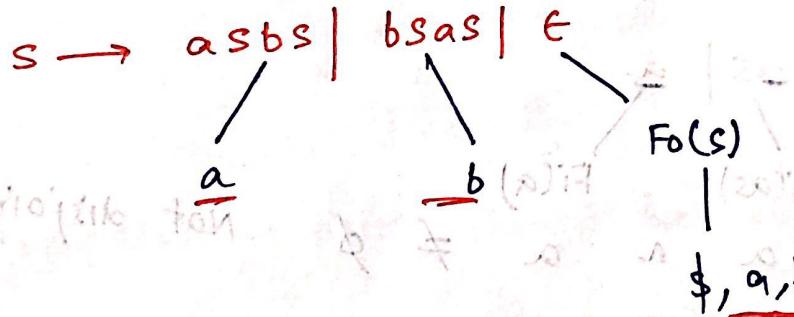
$$b \cap b = b \neq \emptyset$$

not disjoint

so, if grammar has left recursion, then grammar is not LL(1).

Left recursion  $\rightarrow$  not LL(1)

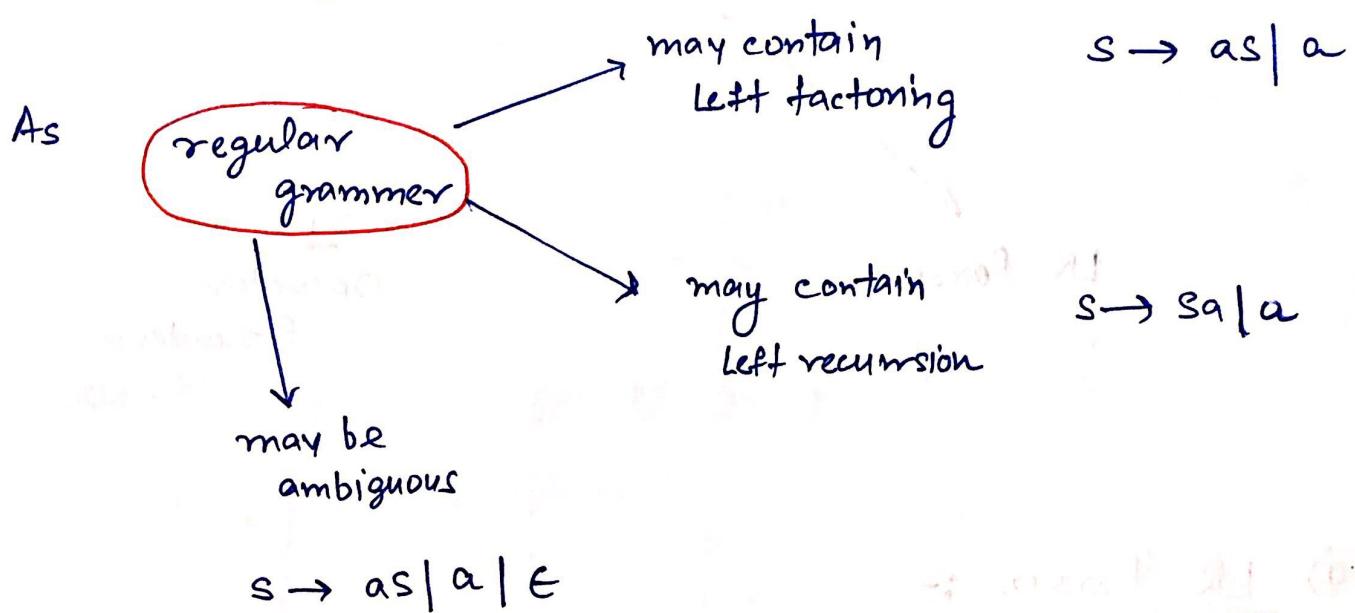
Q.



\$, a, b

Ambiguous grammar  $\rightarrow$  not LL(1)

NOTE:- Every regular grammar need not be LL(1)



Q.  $S \rightarrow ab | ac | ad$

(a) LL(1) X

(b) LL(2) ✓

LL(1)

one symbol is scanned  
at a time.

$S \rightarrow \underline{ab} | \underline{ac} | \underline{ad}$

Left factoring

not LL(1)

LL(2)

two symbol is scanned  
at a time

$S \rightarrow \underline{ab} | \underline{ac} | \underline{ad}$

LL(2)

Power:  $LL(1) \leq LL(2)$