(Boolean Algebra)

- Base of variable ($r = 2$)

variables – (A, B, C, ----Z) or (a, b, c ----, z)

operators – $\underbrace{\text{OR, AND,}}_{\text{binary}}$ $\underbrace{\text{NOT}}_{\text{unary}}$

(i) OR – {+, U, v}

$Y = A+B$ or $A \cup B$ or $A \vee B$
(2-variables)

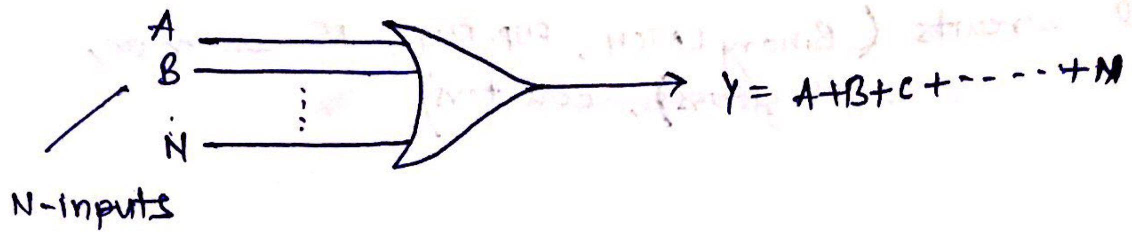
$Y = A+B+C$
(3-variables)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

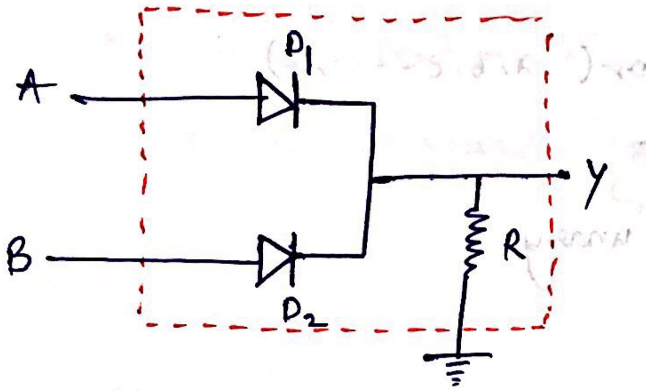
A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

(Y only 0 when all I/p's are 0)

OR - gate



Internal circuits of OR - gate :-



- Truth Table consist all possible combination (2^n) for n-inputs with corresponding output.

(ii) AND op^n : $\{\cdot, \cap, \wedge\}$

$Y = A \cdot B$

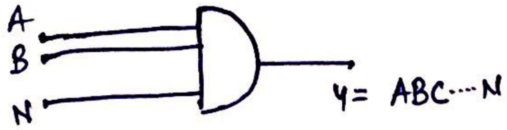
A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

$Y = A \cdot B \cdot C$

A	B	C	Y
0	0	0	0
0	0	1	0
0	0	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

(Y is 1 when all I/p are 1 otherwise 0)

AND - Gate :

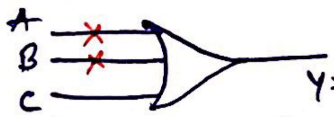


NOTE :- for Y = f(A, B)

	OR	AND
if A=0	Y = B	Y = 0
A=1	Y = 1	Y = B
B=0	Y = A	Y = 0
B=1	Y = 1	Y = A

NOTE :- Enable I/p which allows other I/p's to passe the value . .

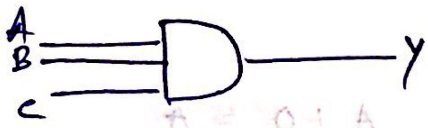
Disable I/P which blocks other I/P's.



c=1

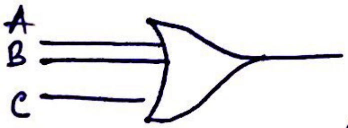
(disabled I/P)

(A and B not required)



c=0

(Disabled I/P)

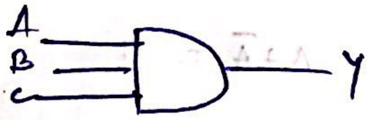


Y= A+B

(depends upon A&B)

C=0

(Enabled I/P)



C=1

(Enabled I/P)

(iii) NOT : {$-$, $'$}

$Y = \bar{A}$ or A'

A	Y
0	1
1	0

NOT gate

Boolean algebra Properties:-

1. $A + A + A + \cdots + A = A$

   $A \cdot A \cdot A \cdot \cdots \cdot A = A$

2. $A + 0 = A$

   $A \cdot 1 = A$

3. $A \cdot \bar{A} = 0$

   $A + \bar{A} = 1$

4. $A + 1 = 1$

   $A \cdot 0 = 0$

5. Principal of Duality $\begin{bmatrix} OR \leftrightarrow AND \\ 0 \leftrightarrow 1 \end{bmatrix}$

Q. find the dual of given boolean expression :

$$\bar{A} + B[C + \bar{D}(E + \bar{F})]$$

Dual $\rightarrow$ $\bar{A}\,[B + C(\bar{D} + E\bar{F})]$

NOTE:

Variable 'x' $\Rightarrow$ contains specific x or $\bar{x}$

literal 'x' $\Rightarrow$ can contain both x or $\bar{x}$ form.

---

6. $(\bar{\bar{A}}) = A$

7. $A + BC = (A+B)(A+C)$ (distributive property)

$A(B+C) = (A\cdot B) + (A\cdot C)$ dual

8. $A + \bar{A}B = A + B$

9. $\bar{A} + AB = \bar{A} + B$

10. $A + AB = \cancel{A(A+B)}\ A(1+B) = A$

Q. Identify minimum no. of logic gates to realize given expression:

$$\bar{x}\bar{y}z + xz + yz$$

(a) 0 (b) 1 (c) 2 (d) None

$\bar{x}\bar{y}z + xz + yz$

$\bar{x}\bar{y}z + (x+y)z \Rightarrow (\bar{x}\bar{y} + (x+y)')z$

$(x+\bar{y}+y)z$

$(x+1)z = z$

no logic gate required

Q. simplified form of given boolean expression:

$AB + \bar{A}B + A\bar{B}$

(a) A (b) $A+\bar{B}$ (c) $A+B$ (d) None

$= \underbrace{(A+\bar{A})}_{1}B + A\bar{B}$

$= B + A\bar{B}$

$= (B+A)\underbrace{(B+\bar{B})}_{1}$

$= (B+A)$

Q. $A\bar{B} + AB\bar{C} + A\bar{B}C\bar{D}$

(a) $\bar{A}(B+C)$ (b) $A(\bar{B}+C)$ ✓ (c) $B(\bar{A}+\bar{C})$ (d) None

$= A\bar{B} + AB\bar{C}$

$= A(\bar{B} + B\bar{C})$

$= A(\bar{B}+\bar{C})$

11. $AB + \bar{A}C + BC = AB + \bar{A}C$ [Consensus property or Redundant property]

redundant

$= AB + \bar{A}C + BC \cdot 1$

$= AB + \bar{A}C + BC(A+\bar{A})$

$= AB + \bar{A}C + ABC + \bar{A}BC$

$= AB + \bar{A}C + \bar{A}CB$

$= AB + \bar{A}C$

$(A+B)\cdot(\bar{A}+C)\cdot(B+C) = (A+B)\cdot(\bar{A}+C)$

redundant

12. $\overline{(A+B+C)} = \bar{A}\cdot\bar{B}\cdot\bar{C}$ [De-morgan's property]

$\overline{(A B \cdot C)} = \bar{A}+\bar{B}+\bar{C}$

Q: Apply De-morgan's law —

$$\overline{A} + B (C + \overline{D} (\overline{E + \overline{F}}))$$

$$A (\overline{B} + \overline{C} (D + \overline{E} \cdot F))$$

Q: if $f(A,B) = \overline{A} + B$ then

$$F (F (x+y, y), z) = ?$$

(a) $\overline{x}y + z$

(b) $x\overline{y} + z$

(c) $y + \overline{x}z$

(d) None

$$= \overline{(\overline{x+y} + y)} + z$$

$$= (\overline{\overline{x+y} + y}) + z$$

$$= ((x+y) \cdot \overline{y}) + z$$

$$= (x\overline{y} + 0) + z$$

$$= (x\overline{y} + z)$$

Q. If $\cancel{f}(x * y) = \bar{x} + y$ and $z = x * y$ then

$z * x = ?$

(a) y (b) x

(c) $\bar{x}$ (d) None

$= \cancel{(\bar{x}+y)\,x}$

$z * x$

$(x * y) * x$

$(\bar{x} + \bar{y}) * x$

$z * x = \bar{z} + \cancel{y}x$

$= \overline{(x * y)} + \cancel{y}x$

$= \overline{(\bar{x}+y)} + \cancel{y}x$

$= (x \cdot \bar{y}) + \cancel{y}x$

$= x$

Q. $v + \bar{v}w + \bar{v}\bar{w}x + \bar{v}\bar{w}\bar{x}y + \bar{v}\bar{w}\bar{x}\bar{y}z$

$\bar{v}\bar{w}\bar{x}(\underbrace{y + \bar{y}z})$

$\bar{v}\bar{w}x + \bar{v}\bar{w}\bar{x}(y+z)$

$\bar{v}w + \bar{v}\bar{w}(x+y+z)$

$= v + \bar{v}(w+x+y+z)$

$= (v+w+x+y+z)$

Q: $\bar{A}B + \bar{B}C + \bar{C}A + \underline{B\bar{C}} + B + \underline{\bar{C}}$

$$\underbrace{\bar{C}A + \bar{C}} + B$$

$$\bar{A}B + \bar{B}C + \bar{C} + B$$

$$\underline{B + \bar{B}C} + \bar{C}$$

$$(B + C) + \bar{C}$$

$$B + 1$$

$$1$$

Q: $\overline{A\bar{C} + \bar{B}C\left[AC + \overline{D(B + \bar{A}CD)}\right]}$

$$= A\bar{C} + \bar{B}C\left[\overline{AC}\,(\bar{D} + \underline{B + \bar{A}CD})\right]$$

$$= A\bar{C} + \bar{B}C\,(\cancel{\overline{AC}\cdot\bar{D} + \overline{AC}\cdot B + \overline{AC}}$$

Q: Find the values of Boolean variable A, B, C & D while solving the following Boolean eq.:

$$\bar{A} + AB = 0$$

$$A\bar{B} = AC$$

$$AB + BC + CA = C\bar{D}$$

$\bar{A}+B = 0$

$A=1$
$B=0$ $\quad C=1$ $\quad D=0$

$A\bar{B} = AC$ $\qquad 0 + 0 + 1 = 1 \cdot \bar{D}$

$\bar{B} = C$

$A \ B \ C \ D$

$$\boxed{\begin{matrix} A=1 \\ B=0 \\ C=1 \\ D=0 \end{matrix}}$$

Q. $f(x,y,z) = \bar{x}y + y\bar{z} + x\bar{y}z$ — Sum-of-Product form

(OR)

$f(x,y,z) = (\bar{A}+B)(\bar{B}+\bar{C})(A+\bar{B}+C)$ — Product-of-sum form

(AND)

~~The truth table of~~

Q. The truth table of $f(A,B,C)$ is given below —

	A	B	C	f(A,B,C)	
0	0	0	0	0	
1	0	0	1	1	–
2	0	1	0	1	–
3	0	1	1	0	
4	1	0	0	1	–
5	1	0	1	0	
6	1	1	0	1	–
7	1	1	1	0	

SOP :-

- Refer to only those combination of truth table where the function output is 1

- if the variable is having value as 0 or 1 -

$$\begin{cases} x = 0 \Rightarrow \bar{x} \\ x = 1 \Rightarrow x \end{cases}$$

- Then OR all the combination.

$$SOP = \overbrace{f_1 + f_2 + f_4 + f_6}^{\text{min-terms}}$$

$$= \underbrace{(\bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + AB\bar{C})}_{\text{canonical sop form}}$$

Minterm - It is a product term, it contains all the variable either (complementary $\bar{x}$ or un-complementary x), and the corresponding function output must be 1.

$$f(A,B,C) = \Sigma(m_1, m_2, m_4, m_6)$$

$$= \Sigma(1,2,4,6)$$

POS :-

- Refer to those combination of truth table where the function output is 0.

- if $x = 0 \rightarrow x$

  $x = 1 \Rightarrow \bar{x}$ in Maxterm

- Apply AND opⁿ on maxterms

$$POS = (A+B+C)(A+\bar{B}+\bar{C})(\bar{A}+B+\bar{C})(\bar{A}+\bar{B}+\bar{C})$$

$$= \underbrace{\pi(0,3,5,7)}_{\text{canonical POS form}}$$

$$= (A+B+C)(\bar{A}+B+\bar{C})(\bar{B}+\bar{C})$$

Maxterm - It is a sum term, it contains all the variables either (x or $\bar{x}$ form), where the corresponding function output is 0.

Derived operators :-

(i) NAND - AND → NOT $\overline{AB}$

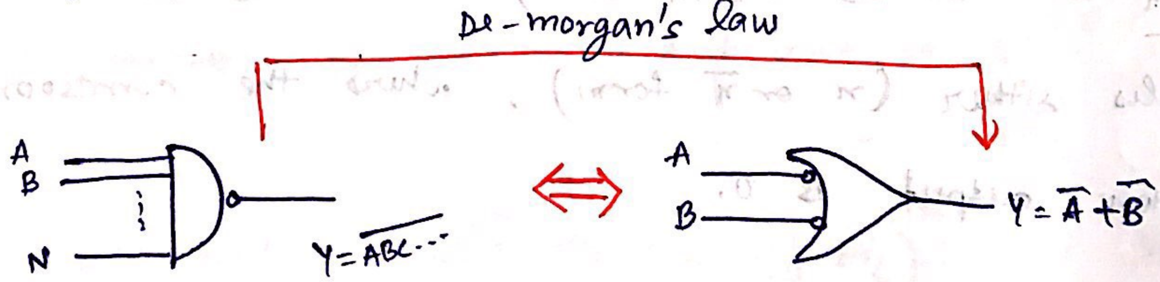
$Y = \overline{AB}$ $Y = \overline{ABC\ldots}$

A	B	$Y=\overline{AB}$
0	0	1
0	1	1
1	0	1
1	1	0

A	B	C	Y
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

The result of NAND op^n is 0 if and only all the variables are 1.

NAND gate :



Bubbled
- OR gate

if $A = 0$ then $y = 1$ ← disabled I/p $A=0$

$A = 1$ then $y = \overline{B}$ ← Enabled I/p $A=1$

(ii) NOR – OR → NOT $\overline{A+B}$

enable I/p

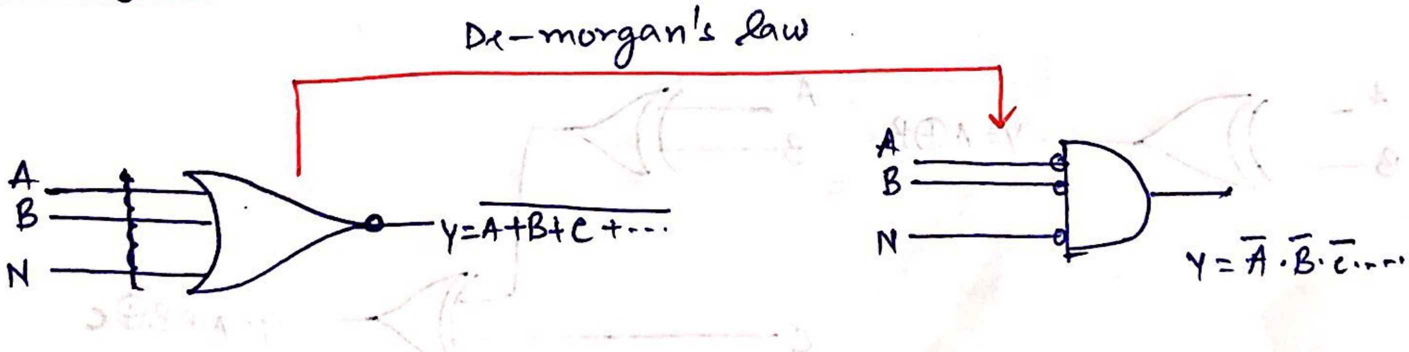
A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

disable I/p

A	B	C	Y
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

The O/p of NOR gate is zero if at-least one of the variable is 1.

NOR gate:-



if $A = 1$ then $Y = 0$ ← disabled I/p

$A = 0$ then $Y = \overline{B}$ ← enabled I/p

(iii) EX-OR (exclusive - OR)

$Y = A \oplus B$
$= \bar{A}B + A\bar{B}$

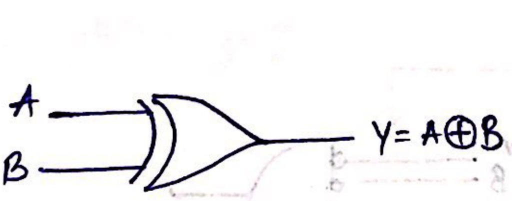
$Y = A \oplus B \oplus C$

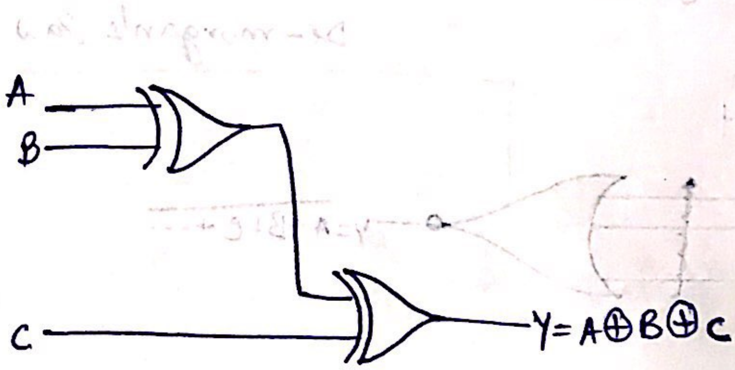
A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

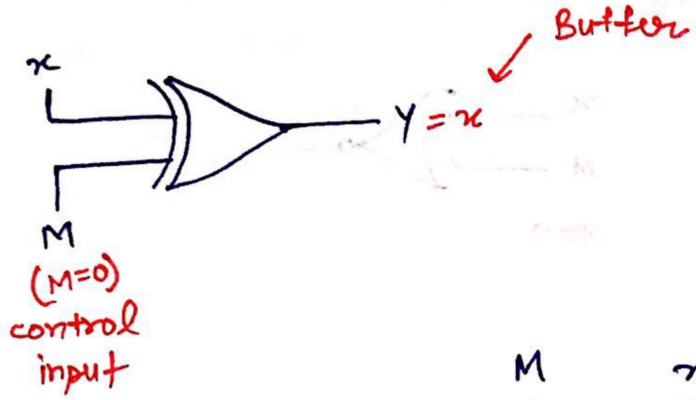
A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

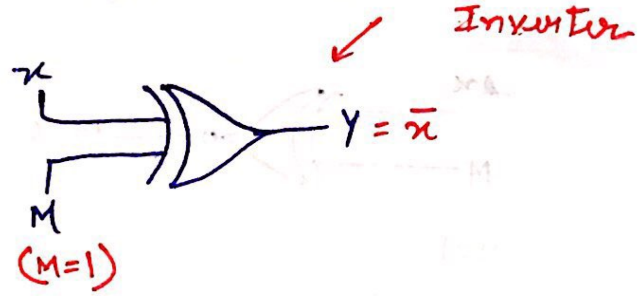
The o/p is 1 whenever odd number of variables are 1 hence it is named as <u>ODD function</u>.

XOR gate









M A	x B	Y	
0	0	0	(x)
0	1	1	
1	0	1	$(\bar{x})$
1	1	0	

NOTE: There we can't assign any input as enabled or disabled

(iv) EX – NOR (Only defined on two variables or I/p)

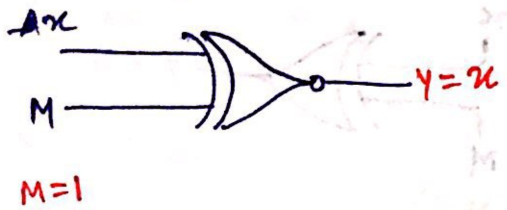
$Y = A \odot B$ or $\overline{A \oplus B}$ (EX-OR $\rightarrow$ NOT)

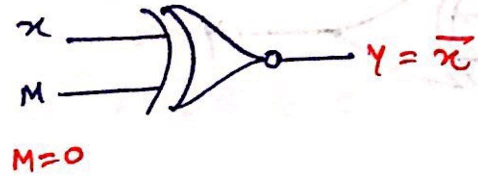
A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

$$Y = A \odot B$$

$$= \bar{A}\bar{B} + AB \quad \text{SOP}$$

$$= (A+\bar{B})(\bar{A}+B) \quad \text{POS}$$





A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

when $A = B$ then $Y = 1$

i.e X-NOR is equality checking gate

NOTE:-

XOR — inequality checking gate

XNOR — equality checking gate

Q. why more than 2-inputs are not allowed is XNOR.

$$Y = \underbrace{A \odot B}_{x} \odot C$$

$$= xc + \overline{x}\,\overline{c}$$

$$= (A \odot B)c + (A \oplus B)\overline{c}$$

$$= (\overline{A \oplus B})c + \underbrace{(A \oplus B)}_{M}\overline{c} = \overline{M}c + M\overline{c}$$

$$= A \oplus B \oplus C$$

Thus ~~XOR~~ XNOR converts to XOR for odd number of inputs.

NOTE :- For odd number of inputs XOR and XNOR behaves same.

NOTE :- Associativity are valid on OR, AND, & EX-OR, and not valid on NOR & NAND

Q. simplify given Boolean expressions:

(i) $(A \oplus B) \oplus (A \oplus \overline{B})$ $\quad 1$

(ii) $A \oplus A\overline{B} \oplus \overline{A}$ $\quad \overline{A}+B$

(i) $(A \oplus A) \oplus (B \oplus \overline{B})$ associativity

$(A \oplus A) \oplus 1$

$0 \oplus 1$ $\quad (1)$

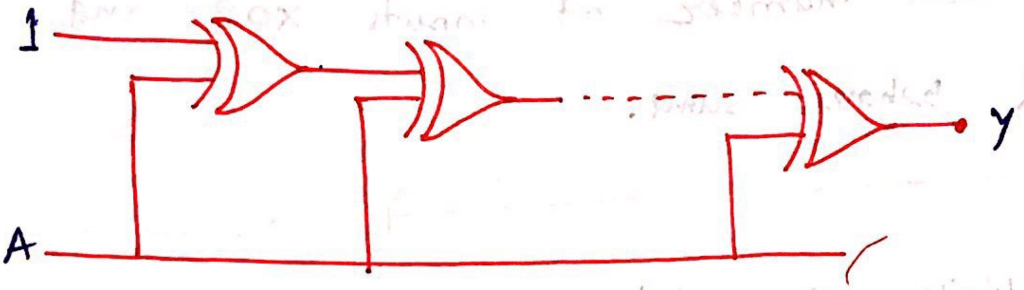
(ii) $A \oplus A\overline{B} \oplus \overline{A}$

$A \oplus \overline{A} \oplus A\overline{B}$ associativity

$1 \oplus A\overline{B}$

$0 \cdot A\overline{B} + 1\,(\overline{A}+B) = (\overline{A}+B)$

Q. In a given ckt 20 XOR gates are cascaded as shown below. The output by each -



(a) A (b) $\bar{A}$ (c) ✓ 1 (d) 0

$= 1 \oplus A \oplus A \oplus ------ \oplus A$

$= 1 \oplus 0$

$= 1$

Q. In the given Boolean express A is having XOR opn n - no. of times, it ~~n is odd~~

(i) n is odd $\rightarrow$ 0

(ii). n is even $\rightarrow$ A

$Y = A \oplus A \oplus ------$

when n is ~~odd~~ even $n = 2$

$Y = \underbrace{A \oplus A}_{0} \oplus A$

$= A$

if n is ~~even~~ odd ($n = 3$)

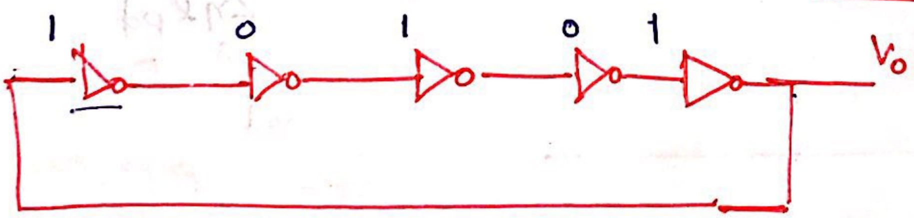
$Y = \underbrace{\underbrace{\underbrace{A \oplus A}_{0} \oplus A}_{A} \oplus A}_{(0)}$

Q: $Y = A \oplus A \oplus A \oplus \ldots$, A is repeating n - number of times.

so, for n even $Y = 0$ $(A \oplus A)$

n odd $Y = A$ $(A \oplus A \oplus A)$

Q. In the given ckt each NOT gate has propagation delay of 100ps, what is the frequency of output signal V_0 -



$1 \to 0 \to 1 \ldots$

$= \frac{1}{100 \times 5} \times 10^{12}$

$= \frac{1000}{5} \times 10^{9}$

$= 2 \times 10^{9}$

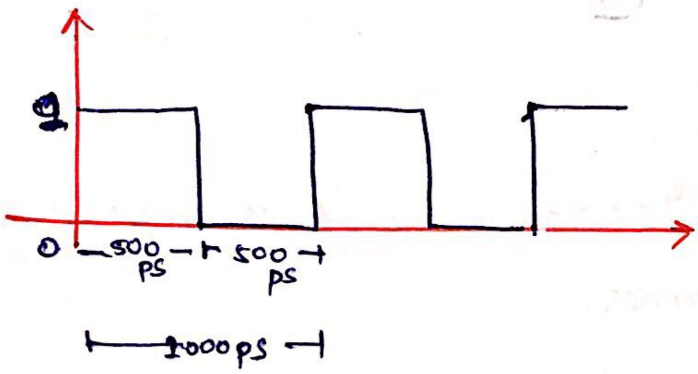
(a) 500 KHz

(b) ✓ 1 GHz

(c) 2 GHz

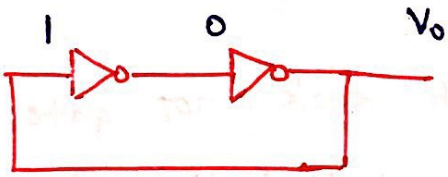
(d) None

The propagation delay is the amount of time taken by the logic gate to respond for the given input.



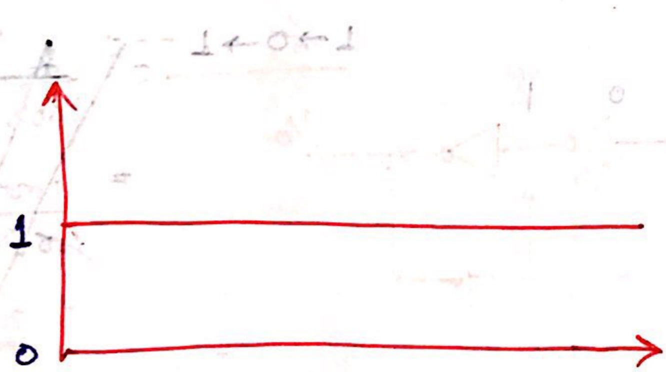
so, $f = \frac{10^{12}}{1000} = 10^9$ Hz = 1GHz

Q.



$V_0 = 1-1-1 \cdots$

no, change in waveform.



$f = \frac{1}{2n t_{pd}}$

odd

propagation delay