Limitation of Boolean algebra in simplification of Boolean expression.

$$f(A,B,C) = \Sigma m(0,2,3,4,5,7)$$

$$= \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}\bar{C} + A\bar{B}C + ABC$$

$$= \bar{A}\bar{C} + \bar{A}B + A\bar{B} + AC \quad \text{— (i)}$$

The expression which is not possible to simplify any more further is called an <u>irredundant</u> or <u>irreducible expression</u>.

$$= \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}\bar{C} + A\bar{B}C + ABC$$

$$= \bar{A}\bar{C} + \bar{B}\bar{C} + BC + AC \quad \text{— (ii)}$$

$$= \bar{A}B\bar{C} + \bar{A}\bar{B}\bar{C} + \bar{A}BC + A\bar{B}\bar{C} + A\bar{B}C + ABC$$

$$= \bar{A}\bar{C} + BC + A\bar{B} \quad \text{--- (iii)}$$

exp (i), (ii) and (iii) are irredundant expression

where expression (iii) is minimal expression.

- **Minimal expression** is the expression which must contain minimum number of terms and each term with minimum no. of literals.

- The given function may be having more than one forms of an irredundant expression.

- All irredundant expression may not be minimal.

- Minimal expression may not be unique.

To overcome these problem we use some systematic approch or methods for simplification of expression:

↳ K-Map

↳ Quine Mc clusky Method

K- Map :-

K-Map is a alternative way of writing truth table.

Truth Table $\leftrightarrow$ 2^n rows

K-map $\leftrightarrow$ 2^n cells

f (A,B)

A \ B	$\overline{B}$ 0	B 1
$\overline{A}$ 0	0	1
A 1	2	3

B \ A	$\overline{A}$ 0	A 1
$\overline{B}$ 0	0	2
B 1	1	3

f (x, y,z)

x \ yz	00	01	11	10
0	0	1	3	2
1	4	5	7	6

xy \ z	0	1
00	0	1
01	2	3
11	6	7
10	4	5

z \ xy	00	01	11	10
0	0	2	6	4
1	1	3	7	5

yz \ x	0	1
00	0	4
01	1	5
11	3	7
10	2	6

$f(A, B, C, D)$

AB \ CD	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

CD \ AB	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

In the K-map if any two cell products are differ in only one-variable, such two cells are said to be adjacent cell.

$1 \rightarrow \bar{A}B$

$3 \rightarrow AB$

(cell 1 $\leftrightarrow$ cell 3)

NOTE :- In a func^n of n variable any reference cell i, the no. of adjacent cells are n.

$f(A, B) = \Sigma m(0, 2, 3)$

A \ B	$\bar{B}$ 0	B 1
$\bar{A}$ 0	[0] 1	[1]
A 1	[2] 1	[3] 1

2 cells   4 cells   8 cells .... 2^m cells
subcube

It is possible to group ~~to~~ 2 cells if and only those two cells are adjacent to each other.

whatever the variables which are common ~~in~~ on the total subcubes only those variables are present in the product representation of the subcubes.

$A \backslash B$	$\overline{B}$ 0	B 1
$\overline{A}$ 0	1	
A 1	1	ϕ

$$(Y = A + \overline{B})$$

Eg:- $f(x,y,z) = \Sigma m(0,2,3,5,6,7)$

$x \backslash yz$	$\overline{y}\overline{z}$ 00	$\overline{y}z$ 01	yz 11	$y\overline{z}$ 10
$\overline{x}$ 0	1		1	1
x 1		1	1	1

$$(f = y + \overline{x}\overline{z} + xz)$$

Steps:-

(i) Identify all independent cells

(ii) Mark all possible 2 cell subcubes such that there should not be part of any larger size subcubes

(iii) Mark all possible 4-cell subcubes such that they should not be part of any larger size subcubes.

Eg:- Write minimal SOP expression for the function:

$$f(A,B,C,D) = \Sigma m(3,5,7,8,13,14,15)$$

AB \ CD	00	01	11	10
00	0	1	3 : 1	2
01	4	5 : 1	7 : 1	6
11	12	13 : 1	15 : 1	14 : 1
10	8 : 1	9	11	10

$$Y = \underline{A\bar{B}\bar{C}\bar{D} + BD + \bar{A}CD + ABC}$$

Q. In func^n of n variables, consider a possible sub-cube 2^m cells. The subcube product is represented with how many number of literals.

Number of variables eliminated $= m$

so, variables in product of subcube $= (n-m)$

Subcube	m	elements
2 - cell	1	1
4 - cell	2	2
8 - cell	3	3
2^m - cells	m	m

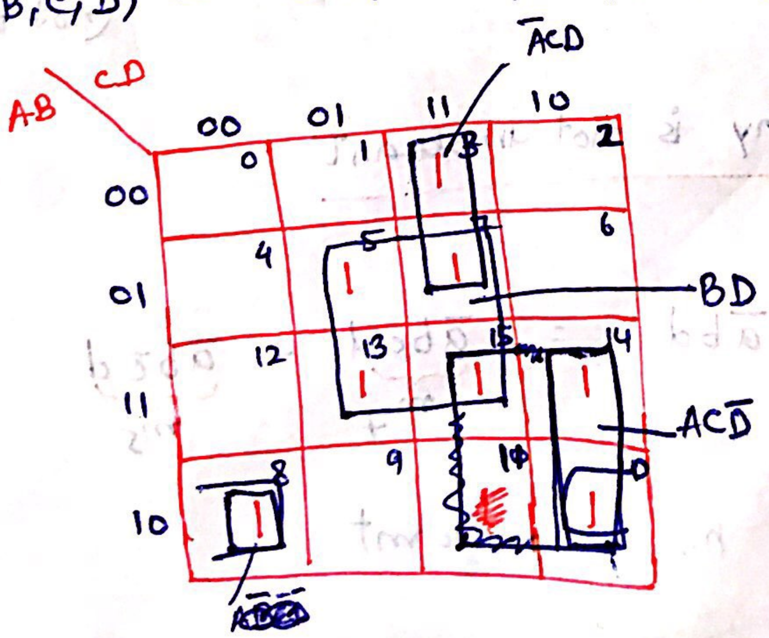
- A subcube 2^m cells is possible if and only in that subcube for every cell m no. of cells must be adjacent cells.

Implicant :-

If a product h is covered by the given func^n f, then the product h is said to be implicant for the func^n f.

$$h \rightarrow f$$

let $f(A,B,C,D) = \Sigma m(3,5,7,8,10,14,13,15)$



$\left(Y = \overline{A}\,\overline{B}\,\overline{D} + AC\overline{D} + \overline{A}CD + BD\right)$

Q. For the above funcⁿ which of the given product list are implicants.

$h_x = abc$ ✓

$h_y = \bar{a}bc$ ✗

$h_z = \bar{a}bd$ ✓

$$h_x = abc(d+\bar{d}) = \underbrace{abcd}_{m_{15}} + \underbrace{abc\bar{d}}_{m_{14}}$$

m_{15} and m_{14} are present in funct f

so, h_x is implicant

$$h_y = \bar{a}bc(d+\bar{d}) = \underbrace{\bar{a}bcd}_{m_7} + \underbrace{\bar{a}bc\bar{d}}_{m_6}$$

(not present)

h_y is not implicant

$$h_z = \bar{a}bd = \underbrace{\bar{a}bcd}_{m_7} + \underbrace{\bar{a}b\bar{c}d}_{m_5}$$

h_z is implicant

Prime implicants :-

Prime implicant is a smallest possible product term, removing any one of the literal from ~~which~~ it is not possible.

Q. $h_1 = BD$ ✓

$h_2 = \overline{A}CD$ ✓

$h_3 = A\overline{B}\overline{D}$ ✓

$h_4 = AC\overline{D}$ ✓

$h_5 = ABC$ ✓

$h_6 = B\overline{C}D$ ✗

$h_7 = ABD$ ✗

For the given func^n $f(A,B,C,D) = \Sigma m(3,5,7,8,10,13,14,15)$ from the given product list h_1 to h_7, how many products are prime implicants.

Q. For the given func^n $f(A,B,C,D)$ no. of prime implicants $\underline{\ 5\ }$.

$$f(A,B,C,D) = \Sigma m(3,5,7,8,10,13,14,15)$$

AB \ CD	00	01	11	10
00	(0)	(1)	(3) 1	(2)
01	(4)	(5) 1	(7) 1	(6)
11	(12)	(13) 1	(15) 1	(14) 1
10	(8) 1	(9)	(11)	(10) 1

- Number of prime implicants for the given number are equal to no. of all possible largest size subcubes in the K-Map.

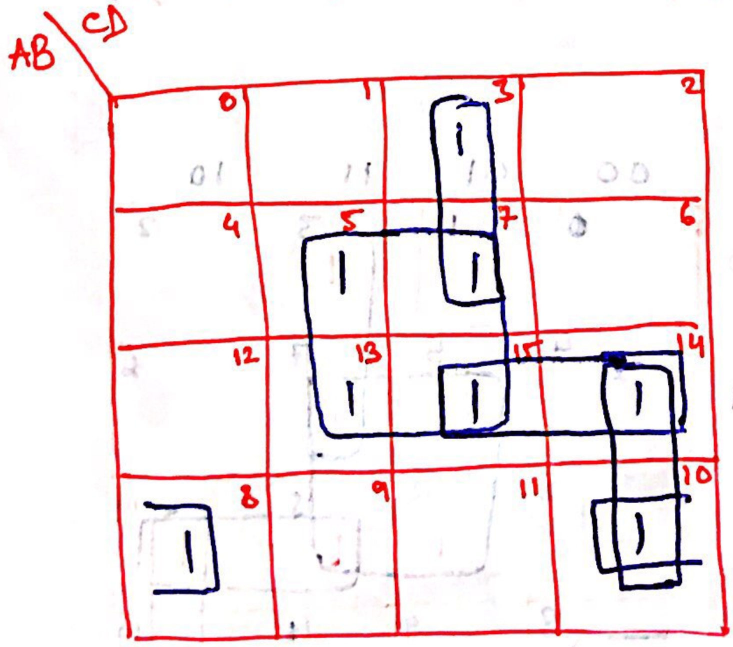
- For minimal expression, prime implicants are needed only.

Essential Prime implicants :-

It is a prime implicant, ~~which~~ it must covers atleast 1 minterm which is not covered by any other prime implicant.

Q. For the func^n $f(A,B,C,D) = \Sigma m(3,5,7,8,10,13,14,15)$

no. of Essential PI ___3___.



EPI $h_1 = BD$ (5, 7, 13, 15)

EPI $h_2 = \bar{A}CD$ (3, 7)

EPI $h_3 = A\bar{B}\bar{D}$ (8, 10)

× $h_4 = AC\bar{D}$ (10, 14)

× $h_5 = ABC$ (14, 15)

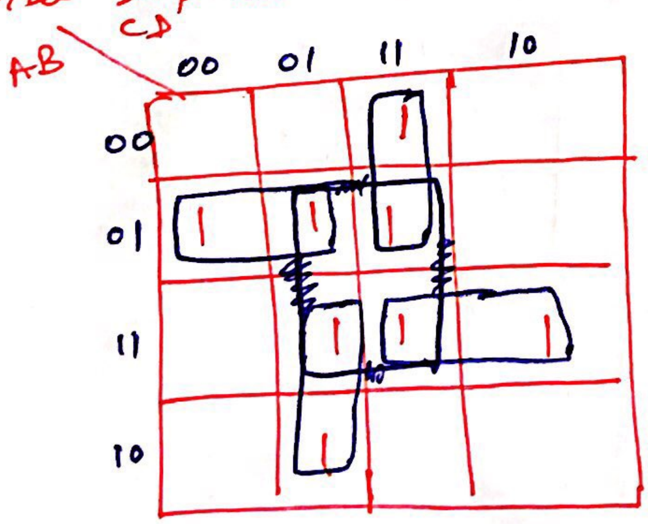
Out of 5 PI only 3 are EPI (h_1, h_2, h_3)

- No. of EPI for the given funcⁿ are equal to no. of all possible larger size subcubes in which atleast 1 cell is without overlapping.

- Essential PI must present in all possible minimal expressions.

- To cover uncovered minterms (i.e not covered by EPI's) use smallest possible PI set (i.e other than EPI's)

Q. Find number of PI, EPI and write all possible minimal expressions.



PI = 4+1 = 5

EPI = 4

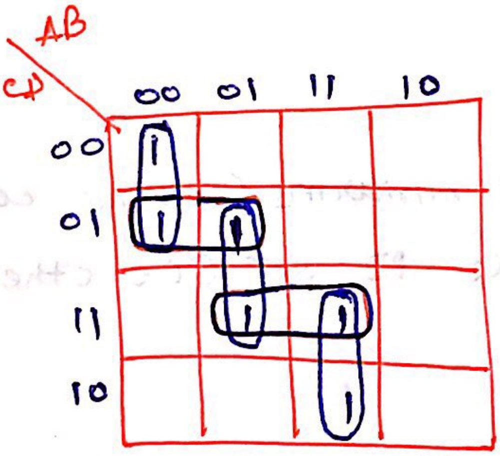
$(Y = \bar{A}B\bar{C} + \bar{A}CD + ABC + A\bar{C}D)$

Here EPI covers all the ~~p~~ minterms so there is no need to ~~usin~~ use other PI, Hence here it is a unique minimal form.

~~NOTE~~ Q. If func^n having unique minimal form, then the PI which are present in the minimal expression must be a ~~unique~~ EPI.

(a) True (b) False ✓

Q. Find the number of PI, EPI and write all possible minimal expressions:



EPI = ~~1~~ 2, PI = 3+2 = 5

EPI $h_1 = \bar{A}\bar{B}\bar{C}$

$h_2 = \bar{A}\bar{C}D$

$h_3 = \bar{A}BD$

$h_4 = BCD$

EPI $h_5 = ABC$

$$Y = \bar{A}\bar{B}\bar{C} + ABC + \bar{A}BD$$