

## = Membership test:-

$X \rightarrow Y$  FD member of FD set(F)  $x^+$  must determine Y in FD set(F).

$$F = \{ \dots \dots \} \leftarrow \xrightarrow{x \rightarrow y}$$

$$x^+ = \{ \dots \dots y \dots \} \text{ then } \underline{x \rightarrow y \in F}$$

eg:-  $\{AB \rightarrow C, BC \rightarrow D, CD \rightarrow E, E \rightarrow F\}$  Test FD's member of FD set?

(i)  $AB \rightarrow F \quad \checkmark$

$$(AB)^+ = \{ A, B, C, D, E, \textcircled{F} \}$$

(ii)  $BCD \rightarrow E \quad \checkmark$

$$(BCD)^+ = \{ B, C, D, \textcircled{E}, F \}$$

(iii)  $BC \rightarrow A \quad \times$

$$(BC)^+ = \{ B, C, D, E, F \}$$

~~(A)~~ absent

## = Equality test :-

$F \& g$  are FD sets,  $F \& g$  are equal if

(i)  $F$  covers  $g$  :- Every FD of  $g$ , must be member of  $F$   
 $F \supseteq g$

(ii)  $g$  covers  $F$  :- Every FD of  $F$ , must be member of  $g$   
 $g \supseteq F$

i.e.  $(F \supseteq g \text{ and } g \supseteq F) \rightarrow (F = g)$

### NOTE :-

$F$ covers $g$	$g$ covers $F$	Result
Yes	Yes	$g = F$
Yes	No	$F > g$
No	Yes	$g \supseteq F$
No	No	$F \& g$ not comparable

Q.  $F = \{A \rightarrow B, B \rightarrow C, C \rightarrow A\}$

$$g = \{A \rightarrow BC, B \rightarrow AC, AB \rightarrow C, BC \rightarrow A\}$$

- (a)  $F \subset g$       (b)  $\checkmark F > g$       (c)  $F = g$       (d) None

Since  $F$  covers  $g$  but  $g$  doesn't cover  $F$   
 $\{C \rightarrow A\}$

F covers g : True

F	$\supseteq$	g
$A \rightarrow B$		$A \rightarrow BC$ ✓
$B \rightarrow C$		$B \rightarrow AC$ ✓
$C \rightarrow A$		$AB \rightarrow C$ ✓
		$BC \rightarrow A$ ✓

g covers F : False

g	$\supsetneq$	F
$A \rightarrow BC$		$A \rightarrow B$ ✓
$B \rightarrow AC$		$B \rightarrow C$ ✓
$AB \rightarrow C$		$C \rightarrow A$ ✗
$BC \rightarrow A$		

Q.  $F = \{ A \rightarrow BC, BC \rightarrow AB, B \rightarrow F, D \rightarrow E \}$

$$g = \{ A \rightarrow BCDEF, BC \rightarrow ADEF, B \rightarrow F, BCDEF \rightarrow E, D \rightarrow F \}$$

$$g > F = \text{True} \quad \& \quad F > g = \text{True}$$

$$\text{so } (F = g)$$

## Properties of Decomposition :-

- (i) Loss Less join Decomposition
- (ii) Dependency Preserving Decomposition

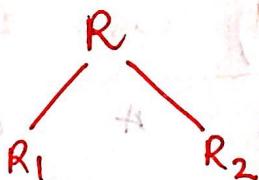
### (i) Loss Less join :-

Relational schema ' $R$ ' with instance ' $r$ ' decomposed into subrelations  $R_1, R_2, \dots, R_n$

In general  $\{R_1 \bowtie R_2 \bowtie \dots \bowtie R_n\} \geq r$

if  $\{R_1 \bowtie R_2 \bowtie \dots \bowtie R_n\} = r$  Loss Less Join

if  $\{R_1 \bowtie R_2 \bowtie \dots \bowtie R_n\} \geq r$  Lossy Join



(i)  $R_1 \bowtie R_2 = R$  possible (Loss Less)

(ii)  $R_1 \bowtie R_2 \supset R$  possible (Lossy)

(iii)  $R_1 \bowtie R_2 \subset R$  Not possible

NOTE :- In  $(R_1 \bowtie R_2) \rightarrow t$  tuples  $t \in R_1$  Joined to  $t$  tuples of  $R_2$  which have common attributes

eg:-

R	Sid	Sname	Cid
S <sub>1</sub>	A	C <sub>1</sub>	
S <sub>1</sub>	A	C <sub>2</sub>	
S <sub>2</sub>	B	C <sub>2</sub>	
S <sub>3</sub>	B	C <sub>3</sub>	

$$\boxed{\text{Sid} \rightarrow \text{Sname}} \\ \boxed{\text{CK} = \{\text{Sid}, \text{Cid}\}}$$

decomposition

R <sub>1</sub>	Sid	Sname
S <sub>1</sub>	A	
S <sub>2</sub>	B	
S <sub>3</sub>	B	

R <sub>2</sub>	Sid	Cid
S <sub>1</sub>	C <sub>1</sub>	
S <sub>1</sub>	C <sub>2</sub>	
S <sub>2</sub>	C <sub>2</sub>	
S <sub>3</sub>	C <sub>3</sub>	

Join

R<sub>1</sub> ⋈ R<sub>2</sub>

	Sid	Sname	Cid
S <sub>1</sub>	A	C <sub>1</sub>	
S <sub>1</sub>	A	C <sub>2</sub>	
S <sub>2</sub>	B		C <sub>2</sub>
S <sub>3</sub>	B		C <sub>3</sub>

Since R<sub>1</sub> ⋈ R<sub>2</sub> = R so Loss Less Join

eg:

R	sid	Sname	Cid
$s_1$	A	$c_1$	
$s_1$	A	$c_2$	
$s_2$	B	$c_2$	
$s_3$	B	$c_3$	

$sid \rightarrow Sname$   
 $CK = \{ sid \rightarrow Cid \}$

decomposition

R <sub>1</sub>	sid	Sname
$s_1$	A	
$s_2$	B	
$s_3$	B	

R <sub>2</sub>	Sname	Cid
A	$c_1$	
A	$c_2$	
B	$c_2$	
B	$c_3$	

Join

$R_1 \bowtie R_2$

R <sub>1</sub> $\bowtie$ R <sub>2</sub>	sid	Sname	Cid
$s_1$	A	$c_1$	
$s_1$	A	$c_2$	
$s_2$	B	$c_2$	
$s_2$	B	$c_3$	
$s_3$	B	$c_2$	
$s_3$	B	$c_3$	

As  $R_1 \bowtie R_2 \supset R$

so Lossy Decomposition

spurious tuples  
 causes  
 inconsistency

NOTE:-

if

Relational schema R with FD set (F) decomposed into  
sub relation  $R_1$  and  $R_2$  then

### (i) Decomposition loss less join :-

iff (i)  $R_1 \cup R_2 = R$  (attr should not be lost)

$$\begin{aligned} \text{(ii)} \quad & (R_1 \cap R_2) \rightarrow R_1 \\ & \text{common attr} \qquad \qquad \qquad \text{all attr} \\ \text{OR} \quad & (R_1 \cap R_2) \rightarrow R_2 \\ & \text{common attr} \qquad \qquad \qquad \text{all attr} \end{aligned}$$

(common attr should be CK or SK for any of relation  $R_1$  or  $R_2$ )

Q.  $R(A B C D E) = \{AB \rightarrow C, C \rightarrow D, B \rightarrow E\}$  Test given decomposition Loss less join?

(i)  $R_1(ABC) \bowtie R_2(CD) \times$

common attr = C which is not SK/CK for  $R_1$  or  $R_2$

(ii)  $R_1(ABC) \bowtie R_2(DE) \times$

No common attributes

(iii)  $R_1(\underline{ABC}) \bowtie R_2(\underline{CD}E) \times$   
 $R_1 \cap R_2 = C$  and  $C^+ = \{C, D\}$   
 C is not SK/CK for  $R_1$  or  $R_2$

(iv)  $R_1(\underline{ABC}) \bowtie R_2(\underline{BE}) \checkmark$   
 $R_1 \cap R_2 = B$  and  $B^+ = \{B, E\}$   
 so, B is CK for  $R_2$  so it is Loss Less

Q.  $R(ABCDEFGH) = \{AB \rightarrow C, BC \rightarrow A, AC \rightarrow B, B \rightarrow D, D \rightarrow EF, F \rightarrow GH\}$

decomposed into

(i)  $R_1(\underline{ABC}) \quad R_2(\underline{FGH}) \quad R_3(\underline{BD}) \quad R_4(\underline{DEF})$

$R_1 \cup R_2 \cup R_3 \cup R_4 = R$  (no attributes is missed)

if  $(R_i \cap R_j \rightarrow R_i / R_j)$  then  $R_i \bowtie R_j$  is Loss Less join

~~$R_1(\underline{ABC}) \bowtie R_3(\underline{BD}) ; B^+ = \{B, D\}, E, F, G, H\}$~~

~~so, B is SK for  $R_3$~~

~~so  $\underline{R_1 \bowtie R_3}$  is LL~~

$R_{13}(ABC)$

$$R_{13}(A \underline{B} C \underline{D}) \bowtie R_4(\underline{DEF}) \text{ & } D^+ = \{D, E, F, G, H\}$$

so,  $D$  is SK for  $R_4$

$$\text{then } R_{13} \bowtie R_4 = \boxed{R_{134}(ABCDEF)}$$

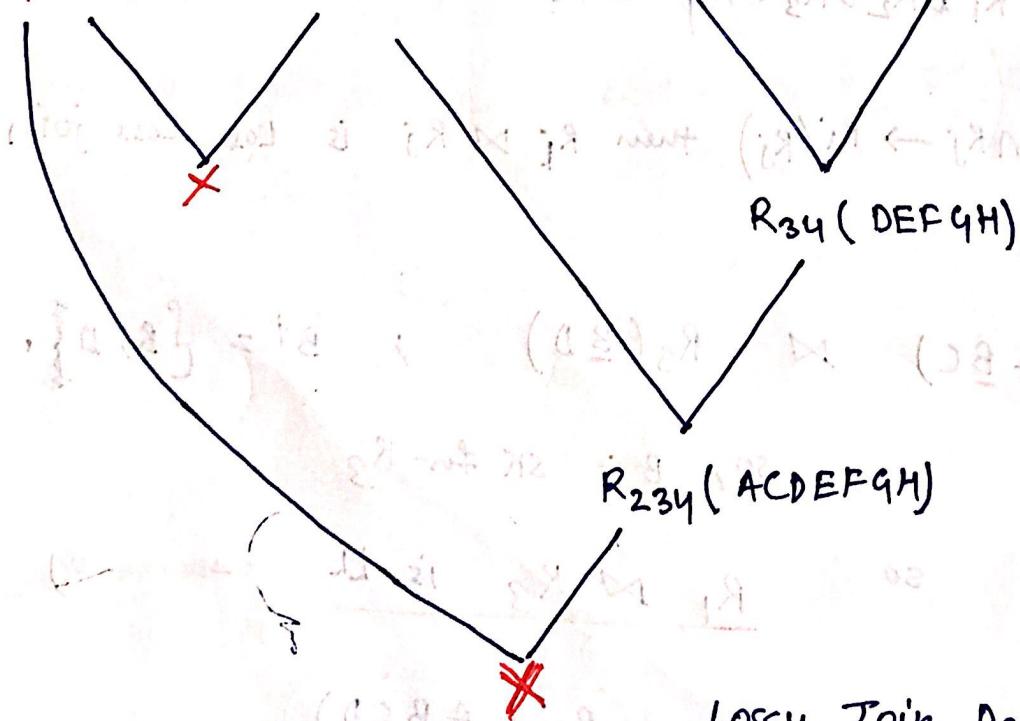
$$R_{134}(ABCDEF) \bowtie R_2(\underline{FGH}) \text{ & } F^+ = \{F, G, H\}$$

so  $F$  is SK for  $R_2$  then

$$R_{134} \bowtie R_2 = \boxed{R_{1342}(ABCDEFGH)}$$

Hence decomposition is loss less join.

$$(ii) R_1(\underline{AB}) \text{ } R_2(\underline{ACD}) \text{ } R_3(\underline{FGH}) \text{ } R_4(\underline{DEF})$$



Lossy Join Decomposition

(ii) Dependency preserving decomposition :-

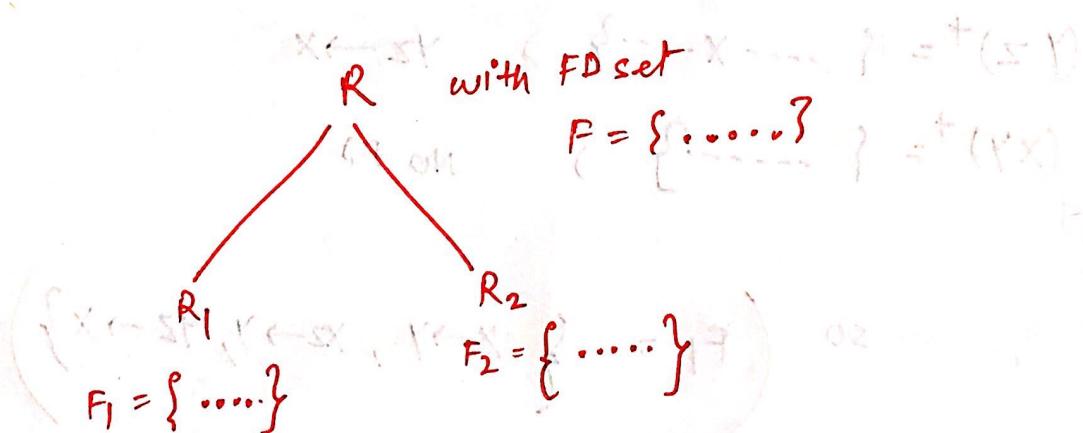
Relation schema 'R' with FD set 'F' decomposed into subrelation  $R_1, R_2, R_3, \dots, R_n$  with FD sets  $F_1, F_2, F_3, \dots, F_n$  respectively

In general,

$$\{F_1 \cup F_2 \cup \dots \cup F_n\} \subseteq F$$

If  $\{F_1 \cup F_2 \cup \dots \cup F_n\} = F$  then dependency preserving

If  $\{F_1 \cup F_2 \cup \dots \cup F_n\} \subset F$  then not dependency preserving



vi)  $\{F_1 \cup F_2\} = F$  DP of decomposition

vii)  $\{F_1 \cup F_2\} \subset F$  not DP decomposition

viii)  $\{F_1 \cup F_2\} \supset F$  (Not possible)

FD's  $F_1$  and  $F_2$  of  $R_1$  and  $R_2$  (decomposed relation)

are not given so we have to find them:

other decomposition is to find FD's of subreln

Procedure to find FD's of subreln:-

$$R(\dots) = F(\dots)$$

Assume  $R_1$  one subreln :  $R_1(x,y,z) \& F_1 = \{ \}$

check closures for each combination of  $(xyz)$  -

$$x^+ = \{ \dots y \dots \} \Rightarrow x \rightarrow y$$

$$y^+ = \{ \dots z \dots \} \Rightarrow \text{No FD}$$

$$z^+ = \{ \dots \} \Rightarrow \text{No FD}$$

$$(xz)^+ = \{ \dots y \dots \} \Rightarrow xz \rightarrow y$$

$$(yz)^+ = \{ \dots x \dots \} \Rightarrow yz \rightarrow x$$

$$(xy)^+ = \{ \dots \} \Rightarrow \text{No FD}$$

$$\text{so } (F_1 = \{ x \rightarrow y, xz \rightarrow y, yz \rightarrow x \})$$

Q)  $R(ABCDE) = \{ A \rightarrow B, B \rightarrow C, C \rightarrow D, D \rightarrow BE \}$

decomposed into :

Check for dependency preservation -

$R_1(AB)$      $R_2(BC)$      $R_3(CD)$      $R_4(DE)$

$R_1(AB)$	$R_2(BC)$	$R_3(CD)$	$R_4(DE)$
$A^+ = \{A, \underline{B}\}$ $(A \rightarrow B)$	$(B \rightarrow C)$	$(C \rightarrow D)$ $(D \rightarrow C)$	$(D \rightarrow E)$
$B^+ = \{B, C, D, E\}$ $(F_1)$	$(C \rightarrow B)$ $(F_2)$	$(F_3)$	$(F_4)$
		Non-trivial FD's	

so, every FD's of relation  $R$  is member of sub Relation

$$(F_1 \cup F_2 \cup F_3 \cup F_4) = F$$

DP decomposition

Q:  $R_F(ABCD) = \{AB \rightarrow CD, D \rightarrow A\}$  decomposed into  
 $\{ABC, AD, BCD\}$  test DP decomposition?

$R_1(ABC)$	$R_2(AD)$	$R_3(BCD)$
$(AB \rightarrow C)$	$(D \rightarrow A)$	$(BD \rightarrow C)$
$F_1$	$F_2$	$F_3$

$$F = \{ AB \rightarrow C, \underline{AB \rightarrow D}, D \rightarrow A \}$$

$$F_1 \cup F_2 \cup F_3 = \{ AB \rightarrow C, D \rightarrow A \}$$

so,  $(F_1 \cup F_2 \cup F_3 \subset F)$  Not DP decomposition

### Canonical Cover :-

- Also known as Minimal cover

Minimal cover of FD set ( $F$ ) is minimal set of FD's which are equal to  $F$  (Non-redundant FD set which is equal to  $F$ ).

$$F = \{ A \rightarrow B, B \rightarrow C, A \rightarrow C, AC \rightarrow B, B \rightarrow B \}$$

$$F_m = \{ A \rightarrow B, B \rightarrow C \}$$

- as  $A \rightarrow B$  and  $B \rightarrow C$   
 $\Rightarrow A \rightarrow C$

-  $A \rightarrow B \Rightarrow AC \rightarrow BC$   
 $\hookrightarrow AC \rightarrow B$

-  $B \rightarrow B$  (trivial)

## Procedure to find minimal cover:-

Given FD set ( $F$ ) :

- (i) Remove extraneous Attributes from each determinant [LHS of FD] of FD set ( $F$ ).

e.g:

$$\left[ \begin{array}{l} wxy \rightarrow z \quad \text{and} \quad w^+ = \{ \dots x \dots \} \\ \uparrow \\ \text{extraneous Attr} \end{array} \right]$$

i.e  $w \rightarrow x$

so,

$$\begin{aligned} \{ABC \rightarrow D, A \rightarrow B\} &= \{AC \rightarrow D, A \rightarrow B\} \\ \{PQ \rightarrow R, Q \rightarrow P\} &= \{Q \rightarrow R, Q \rightarrow P\} \end{aligned}$$

- (ii) Remove Redundant FD's from FD set ( $F$ ).

i.e  $x \rightarrow y$  FD of FD set ( $F$ ) is redundant iff

$$x \rightarrow y \text{ must be a member of } \{F - (x \rightarrow y)\}.$$

$$F = \{ A \rightarrow B, B \rightarrow C, A \not\rightarrow C \}$$

$A \rightarrow C$  is still member of  $F$  so,

$A \rightarrow C$  is redundant

Q.  $F = \{AB \rightarrow CD, BC \rightarrow DEF, CD \rightarrow F, ABD \rightarrow E\}$ , find minimal cover of FD set ( $F$ ).

$$AB \rightarrow CD$$

$$BC \rightarrow DEF$$

$$CD \rightarrow F$$

$$\cancel{ABD \rightarrow E}$$

$$AB \rightarrow D$$

Remove  
extraneous  
attr

$$AB \rightarrow CD$$

$$BC \rightarrow DEF$$

$$CD \rightarrow F$$

$$AB \rightarrow E$$

split

$$AB \rightarrow C$$

$$\times \cancel{AB \rightarrow D}$$

$$BC \rightarrow D$$

$$BC \rightarrow E$$

$$\times \cancel{BC \rightarrow F}$$

$$CD \rightarrow F$$

$$\times \cancel{AB \rightarrow E}$$

$$(AB)^+ = \{A, B, C, \textcolor{red}{D}, \textcolor{red}{E}, F\}$$

$$(BC)^+ = \{B, C, D, \textcolor{red}{F}\}$$

Minimal cover =  $\boxed{\{AB \rightarrow C, BC \rightarrow DE, CD \rightarrow F\}}$

Q.  $F = \{A \rightarrow BC, CD \rightarrow E, E \rightarrow C, D \rightarrow AH, ABH \rightarrow BD, DH \rightarrow BC\}$   
Find minimal cover

$$A \rightarrow B$$

$$A \rightarrow C$$

$$CD \rightarrow E$$

$$E \rightarrow C$$

$$D \rightarrow A$$

$$D \rightarrow H$$

$$\cancel{ABH} \rightarrow P$$

$$\cancel{ABH} \rightarrow D$$

$$\cancel{DH} \rightarrow B$$

$$\cancel{DH} \rightarrow C$$

$$AH \rightarrow B$$

$$AH \rightarrow D$$

$$D \rightarrow B$$

$$D \rightarrow C$$

removing  
extraneous  
attr

$A \rightarrow B$  $A \rightarrow C$  $CD \rightarrow E$  $E \rightarrow C$  $D \rightarrow A$  $D \rightarrow H$ 

$\times \text{ } AH \rightarrow B$        $A \rightarrow B$       so  $AH^+ = \{ \dots B \dots \}$

 $AH \rightarrow D$ 

$\times \text{ } D \rightarrow B$        $D \rightarrow A$  and  $A \rightarrow B$

$\times \text{ } D \rightarrow C$        $D \rightarrow A$  and  $A \rightarrow C$

Minimal cover =  $\{ A \rightarrow BC, CD \rightarrow E, E \rightarrow C, D \rightarrow AH \}$

Q. Find minimal cover of  $\{ A \rightarrow BC, CD \rightarrow AD \}$

 $A \rightarrow B$  $A \rightarrow C$  $A \rightarrow D$  $BC \rightarrow A$  $BC \rightarrow D$ 

no extraneous attributes

so, remove redundant FD's:-

 $\times A \rightarrow D$ 

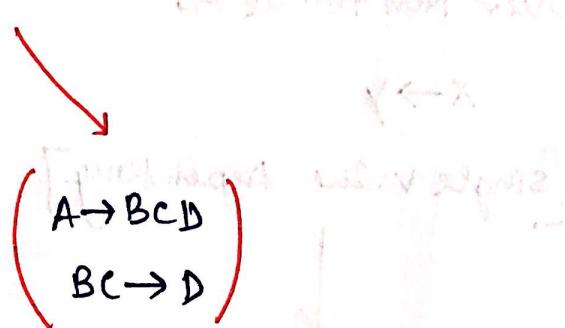
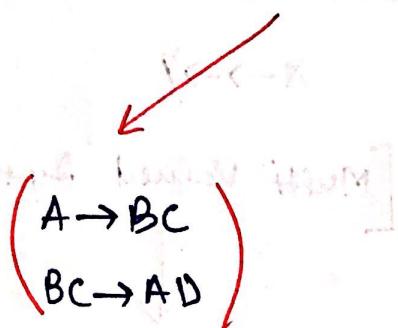
as  $A \rightarrow BC$  &  $BC \rightarrow D$

Minimal cover =  $\{ A \rightarrow BC, BC \rightarrow AD \}$

OR

$\times BC \rightarrow D =$  as  $BC \rightarrow A$  and  $A \rightarrow D$

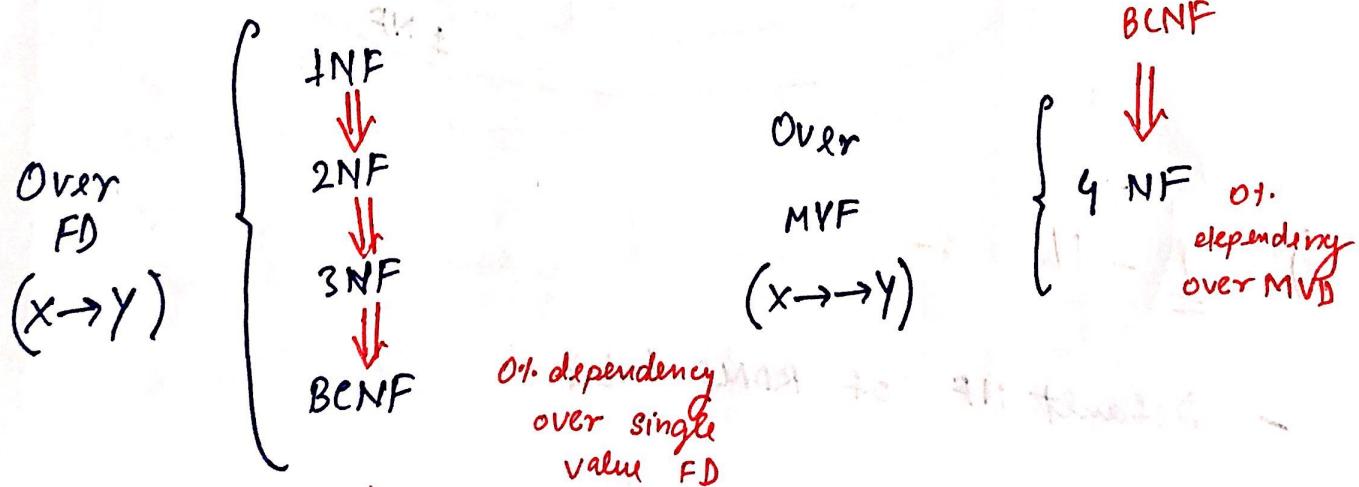
so we have two minimal covers



NOTE:- Minimal cover of FD set F may not be unique  
but all minimal covers are logically equal which  
also equal to given FD set(F).

## Normal Forms :-

Used to identify degree of redundancy and also used to eliminate redundancy.



## Redundancy in

### Relational schema

Over Non-trivial FD

$$X \rightarrow Y$$

[Single value dependency]

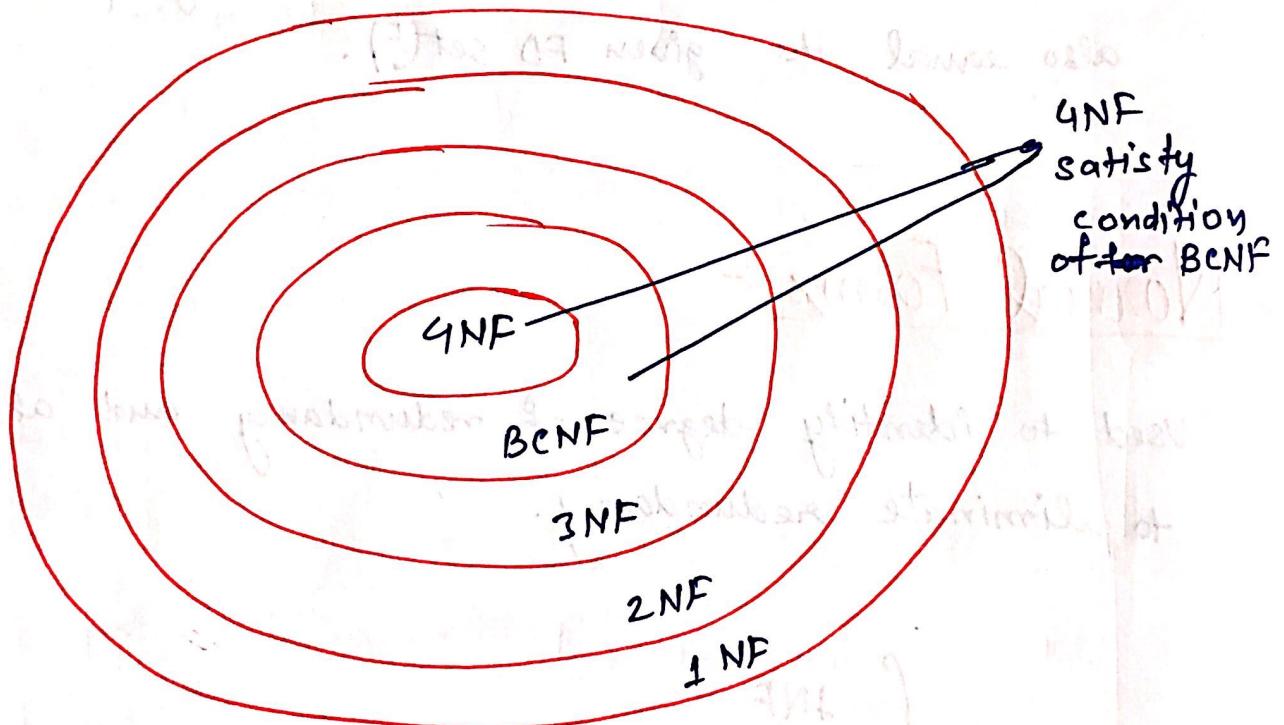
Over Non-trivial MVD

$$X \rightarrow \rightarrow Y$$

[Multi valued dependency]

To eliminate redundancy of FD's of rel<sup>n</sup> schema, schema should be decomposed into BCNF

To eliminate redundancy over MVD's, rel<sup>n</sup> schema should be decomposed into 4NF



(i) 1-NF :-

- Default NF of RDBMS table

- Relation R is in 1-NF iff no multivalued attribute in R. (Every attribute of R must be Atomic/ Single valued)

Multivalued attribute:- Set of values over attribute for some record

FD			
R	Sid	Sname	Cid
s <sub>1</sub>	A	c <sub>1</sub> /c <sub>2</sub>	
s <sub>2</sub>	B	c <sub>2</sub>	
s <sub>3</sub>	B	c <sub>3</sub>	

Multivalue attr

1NF design  
design candidate keys by using FD set

FD			
R	Sid	Sname	Cid
s <sub>1</sub>	A	c <sub>1</sub>	
s <sub>1</sub>	A	c <sub>2</sub>	
s <sub>2</sub>	B	c <sub>2</sub>	
s <sub>3</sub>	B	c <sub>3</sub>	

(CK: Sid/cid)

Multivalue attr. added to sid for CK

"MV attribute is added to PK to make CK of 1NF relation"

eg:- Students (poshivaram.com) Hi multivalued attr with pk ~

R	Sid	Sname	DOB	Cid	Phone	Email	
S <sub>1</sub>	A	1995	c <sub>1</sub> /c <sub>2</sub>	P <sub>1</sub> /P <sub>2</sub> /P <sub>3</sub>	E <sub>1</sub> /E <sub>2</sub> /E <sub>3</sub>		18 cases
S <sub>2</sub>	B	1994	c <sub>1</sub> /c <sub>3</sub>	P <sub>4</sub> /P <sub>5</sub> /P <sub>6</sub>	E <sub>4</sub> /E <sub>5</sub>		12 cases

INF design

(CK : Sid Cid Phone Email)

R	Sid	Sname	dob	Cid	Phone	Email	
1	S <sub>1</sub>	A	1995	c <sub>1</sub>	P <sub>1</sub>	E <sub>1</sub>	
2	S <sub>1</sub>	A	1995	c <sub>1</sub>	P <sub>1</sub>	E <sub>2</sub>	
3	S <sub>1</sub>	A	1995	c <sub>1</sub>	P <sub>1</sub>	E <sub>3</sub>	
4	S <sub>1</sub>	A	1995	c <sub>2</sub>	P <sub>3</sub>	E <sub>1</sub>	
5	S <sub>1</sub>	A	1995	c <sub>2</sub>	P <sub>3</sub>	E <sub>2</sub>	
6	S <sub>1</sub>	A	1995	c <sub>2</sub>	P <sub>3</sub>	E <sub>3</sub>	
7	S <sub>2</sub>	B	1994	c <sub>1</sub>	P <sub>4</sub>	E <sub>4</sub>	
8	S <sub>2</sub>	B	1994	c <sub>1</sub>	P <sub>4</sub>	E <sub>5</sub>	
9	S <sub>2</sub>	B	1994	c <sub>2</sub>	P <sub>6</sub>	E <sub>4</sub>	
10	S <sub>2</sub>	B	1994	c <sub>2</sub>	P <sub>6</sub>	E <sub>5</sub>	
11	S <sub>2</sub>	B	1994	c <sub>3</sub>	P <sub>6</sub>	E <sub>4</sub>	
12	S <sub>2</sub>	B	1994	c <sub>3</sub>	P <sub>6</sub>	E <sub>5</sub>	

18 tuples

All combination of MVattr

12 tuples

redundancy

NOTE:- - Degree of Redundancy is very high in 1NF and h

- 1NF is only default RDBMS table
- Condition to reduce or eliminate redundancy are starts from 2NF.

NOTE:-

Redundant FD's

$X \rightarrow Y$  is Non-trivial

$X$  is not superkey

Non Redundant FD's

$X \rightarrow Y$  is Trivial FD

$X$  is superkey

R <sub>1</sub>	Sid	Sname	Cid
S <sub>1</sub>	A	C <sub>1</sub>	
S <sub>1</sub>	A	C <sub>2</sub>	
S <sub>1</sub>	A	C <sub>3</sub>	
S <sub>2</sub>	B	C <sub>1</sub>	

Not CK

$$FD = \{ Sid \rightarrow Sname \}$$

Non trivial

$$CK = Sid Cid$$

R <sub>2</sub>	Sid	Sname	DOB
S <sub>1</sub>	A	D <sub>1</sub>	
S <sub>2</sub>	A	D <sub>2</sub>	
S <sub>3</sub>	B	D <sub>3</sub>	
S <sub>4</sub>	B	D <sub>3</sub>	

CK

$$FD = Sid \rightarrow Sname DOB$$

non trivial

$$CK = Sid$$

eg:

eid	ename	rating	hourlywages
101	John	5	10

$$FD = \left\{ \begin{array}{l} eid \rightarrow ename \text{ rating} \\ rating \rightarrow hourlywages \end{array} \right\}$$

Non trivial  
 $eid \rightarrow ename \text{ rating}$   
 SK

Not Redundant

Non trivial  
 $rating \rightarrow hourlywages$   
 Not SK

Redundant

so redundancy can occur

Q:  $R(ABCDEF) = \{ AB \rightarrow C, B \rightarrow D, D \rightarrow E, AE \rightarrow F, E \rightarrow A \}$

check which FD is creating redundancy.

SK  
 $AB \rightarrow C$  X

Not SK  
 $D \rightarrow E$  ✓

Not CK  
 $C \rightarrow A$  ✓

Not SK  
 $B \rightarrow D$  ✓

Not SK  
 $AE \rightarrow F$  ✓

$$CK = \{AB, BC\}$$

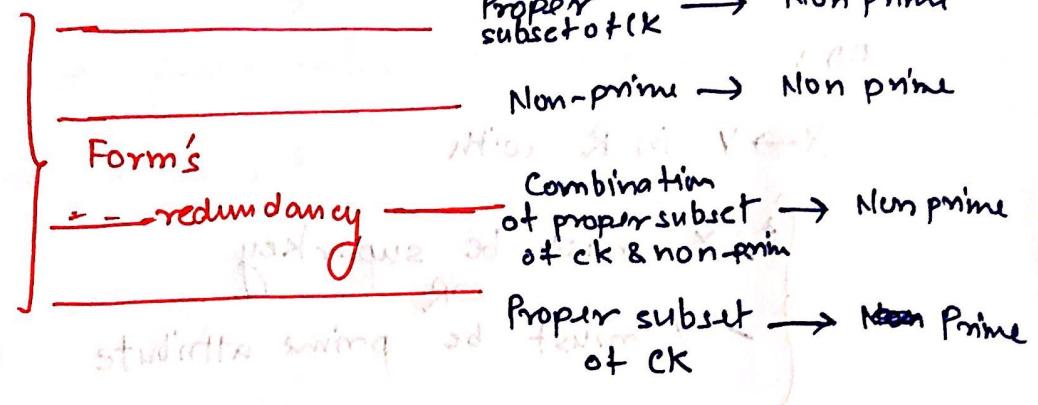
$AB \rightarrow C$

$B \rightarrow D$

$D \rightarrow E$

$AE \rightarrow F$

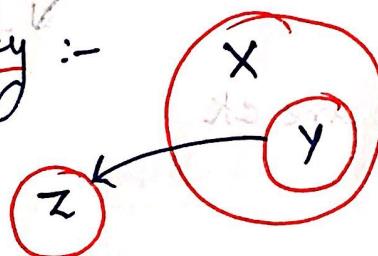
$C \rightarrow A$



## (ii) 2-NF:-

Relational schema R is 2NF iff NO partial dependency in R

Partial dependency :-



X : any CK

$Y \subset X$  (proper subset)

& Z is non prime

then  $(Y \rightarrow Z)$  is PD

(Proper subset of CK → Non-prime)

→  $\left[ \begin{matrix} (\text{Proper subset of CK})^+ = \{ \dots \text{Non prime} \dots \} \end{matrix} \right]$

### (iii) 3-NF :-

Relational schema R is in 3NF iff every non-trivial FD:

$X \rightarrow Y$  in R with

- X must be superkey
- Y must be prime attribute OR

$$\begin{matrix} X \\ \text{SK} \end{matrix} \rightarrow \begin{matrix} Y \\ \text{prime} \end{matrix}$$

e.g.  $R(ABC) = \{ AB \rightarrow C, C \rightarrow A \}$  in 3NF

→ forms redundancy

and  $\{AB, BC\}$  are CK

Proper subset of  
candidate key → Proper subset of  
some other CK

forms redundancy and allowed in 3NF

### (iv) Boyce Codd NF (BCNF) :-

Relational schema R is in BCNF iff every non-trivial FD

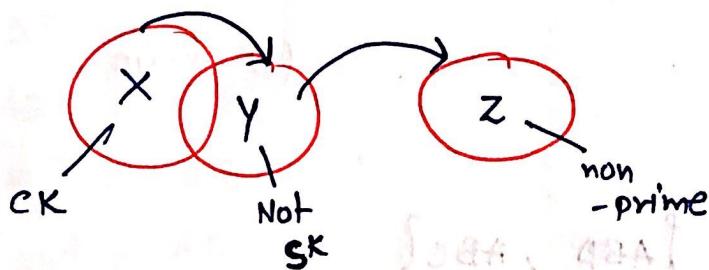
- {  $X \rightarrow Y$  with  $X$  must be superkey }

$\nvdash (x \rightarrow y)$

SK

- Non-prime Attribute transitively determined by superkey

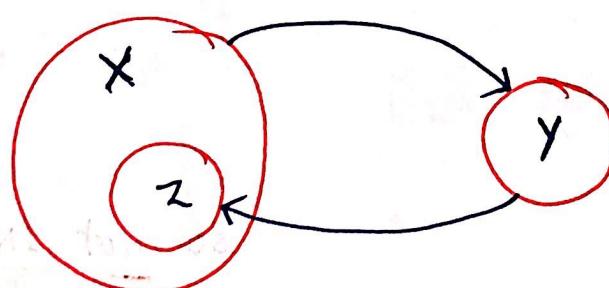
is not allowed in 3NF.



$\{x \rightarrow y \text{ & } y \rightarrow z\}$  not allowed in 3NF

- Prime Attribute transitively determined by superkey is

allowed in 3NF but not in BCNF



$\{x \rightarrow y \text{ & } y \rightarrow z\}$

allowed in 3NF

not allowed in BCNF

Q.  $R(ABCDE) = \{ ABD \rightarrow C, BC \rightarrow D, CD \rightarrow E \}$  what is the highest NF satisfied by R?

(A) 1NF

(B) 2NF

(C) 3NF

(D) BCNF

Candidate keys :  $\{ABD, ABC\}$

prime attr =  $\{A, B, C, D\}$

Non-prime attr =  $\{E\}$

$ABD \rightarrow C$

$BC \rightarrow D$

$CD \rightarrow E$

} causes redundancy

so not BCNF

in  $CD \rightarrow E$   
not prime

so not 3NF

Now for 2NF: check PD (Proper subset of CK)<sup>+</sup> = Non prime

$$\text{CK}_1 = \underline{\text{ABD}}$$

$$A^+ = A$$

$$B^+ = B$$

$$D^+ = D$$

$$AB^+ = A, B$$

$$BD^+ = B, D$$

$$AD^+ = AD$$

No PD

$$\text{CK}_2 = \underline{ABC}$$

$$BC^+ = B, C, D, E$$

non-prime

so PD exist

Thus Not 2NF

So, it must be 1NF

Q.  $R(ABCD) = \{ AB \rightarrow C, BC \rightarrow D \}$   $\text{CK} = \underline{AB}$

- check PD for CK:

$$A^+ = A$$

so, No partial dependency

$$B^+ = B$$

Hence 2NF

- Now  $BC \rightarrow D$  Not SK Not prime

so not 3NF and hence  
not BCNF

1NF	2NF	3NF	BCNF
✓	✓	✗	✗

Q.  $R(ABCDEF) = \{ AB \rightarrow C, C \rightarrow DE, E \rightarrow F, F \rightarrow A \}$

check highest Normal form.

(CK = AB, BF, BE, BC)

prime attr = {A, B, C, E, F}

non-prime = {D}

$C \rightarrow DE$   
Not SK

so not BCNF

$C \rightarrow D$   
Not SK  
Not prime

so not 3NF

$C \rightarrow D$   
proper subset  
not prime

PD so not 2NF

1NF

2NF

3NF

BCNF



Q:  $R(ABCD) = \{ AB \rightarrow C, C \rightarrow A, AC \rightarrow B \}$

$$CK = \{ CD, ABD \}$$

$$\text{so prime Attr} = \{ A, B, C, D \}$$

since all FD's determining prime attr

so they are in 3NF,

But not in BCNF as  $AB$  is not SK.

Now check PD:

as since all are prime attributes so there will be no chances for non-prime attr

$(\text{proper subset})^+ \rightarrow \text{Non-prime}$

So it is in 2NF

1NF	2NF	3NF	BCNF
✓	✓	✓	✗

Q.

$$R(ABCD) = \{ A \rightarrow B, B \rightarrow AC, C \rightarrow D \}$$

CK = {A, B}

prime attributes = {A, B}

Non prime = {C, D}

since

$$\begin{array}{c} C \rightarrow D \\ \text{Not} \quad \text{Non} \\ \text{SK} \quad \text{-prime} \end{array}$$

so it is not in 3NF

And there are no proper subset of CK so

no chances for P.D. works on ad

Hence it is in 2NF (for the reason)

1NF	2NF	3NF	BCNF
✓	✓	✗	✗

Q.  $R(ABCDEF) = \{ AB \rightarrow C, C \rightarrow D, CD \rightarrow AF, DE \rightarrow F, EF \rightarrow B \}$

$$CK = \{ AB, AEF, ADE, C \}$$

All are prime attributes so in 3NF

$DE \rightarrow F$  so not in BCNF  
 Not SK

1NF	2NF	3NF	BCNF
✓	✓	✓	✗

Q.  $R(ABCDE) = \{ A \rightarrow B, B \rightarrow C, C \rightarrow D, D \rightarrow A \}$

$$CK = \{ AE, BE, CE, DE \}$$

All are prime attributes so it follows 3NF

1NF	2NF	3NF	BCNF
✓	✓	✓	✗

$$Q. R(ABeD) = \{ AB \rightarrow e, e \rightarrow A, Ac \rightarrow D \}$$

$$CK = \{ AB, Be \}$$

$$\text{prime attr} = \{ A, B, C \}$$

$$\text{Non-prime} = \{ D \}$$

$AC \rightarrow D$   
 Not SK      non prime  
 so not 3NF

$(C \subset BC) \Rightarrow C^+ = \{ c, A, D \dots \}$  so not 2NF  
 non prime

1NF	2NF	3NF	BCNF
✓	✗	✗	✗

### NOTE:-

- Simple candidate key - CK only one attribute

- Compound candidate key - CK contains atleast 2 attributes

- If rel<sup>n</sup>R consist only simple Fk then relation R always in 2NF but may or may not in 3NF
- If rel<sup>n</sup>R with only prime attributes then relation will always in 3NF but may or may not in BCNF.
- A rel<sup>n</sup>R with <sup>no</sup> non-trivial FD's then R always in BCNF (all are non-redundant FD's)
- Rel<sup>n</sup>R with only two attributes will be in BCNF & 4NF also.

