1 Gaussian Discriminant Analysis

Log-likelihood function for GDA:

$$LL(\theta) = \sum_{i=1}^{m}[log(P(y^{(i);\phi})) + log(P(x^{(i)}/y^{(i)}; \mu_0, \mu_1, \Sigma_0, \Sigma_1))]$$

Values of parameters that maximizes the above function:

$$\phi = \sum_{i=1}^{m} \frac{1y^{(i)} = 1}{m}$$

$$\mu_0 = \frac{\sum_{i=1}^{m} 1y^{(i)} = 0x^{(i)}}{\sum_{i=1}^{m} 1y^{(i)} = 0}$$

$$\mu_1 = \frac{\sum_{i=1}^{m} 1y^{(i)} = 1x^{(i)}}{\sum_{i=1}^{m} 1y^{(i)} = 1}$$

Assuming $\Sigma_0 = \Sigma_1 = \Sigma$:

$$\Sigma = \frac{1}{m}\sum_{i=1}^{m}(x^{(i)} - \mu_{y^{(i)}})(x^{(i)} - \mu_{y^{(i)}})^T$$

In general, if we allow each target class to have its own co-variance matrix then co-variance matrix Σ_0(similarly Σ_1) is given by:

$$\Sigma_0 = \frac{\sum_{i=1}^{m} 1y^{(i)} = 0(x^{(i)} - \mu_{y^{(i)}})(x^{(i)} - \mu_{y^{(i)}})^T}{\sum_{i=1}^{m} 1y^{(i)} = 0}$$

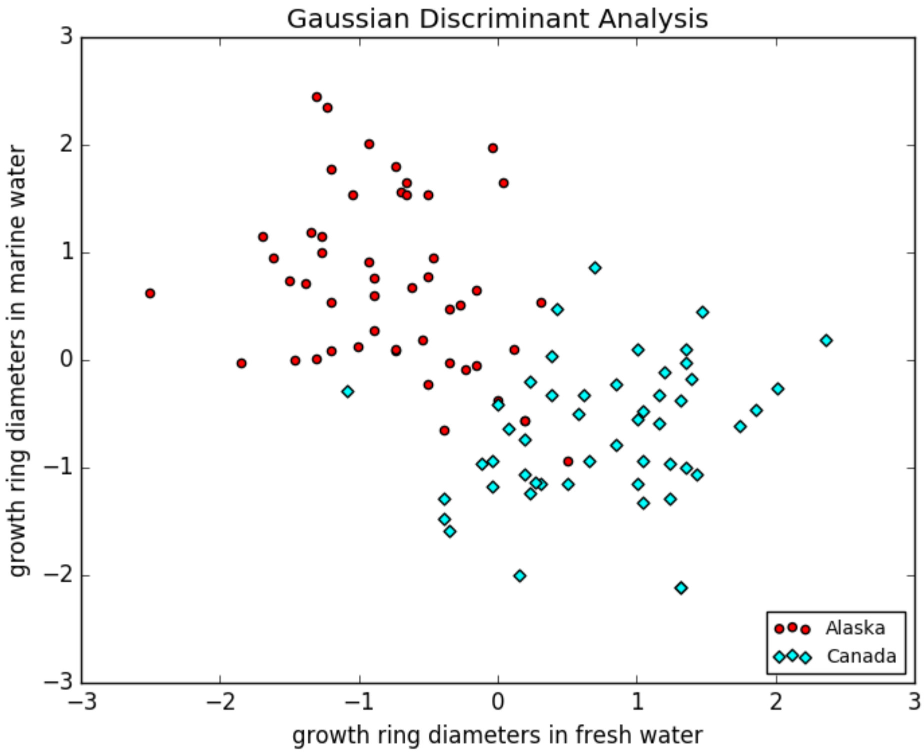
a) Values obtained are :

$$\mu_0 = \begin{bmatrix} 0.75529433 \\ -0.68509431 \end{bmatrix}$$

$$\mu_1 = \begin{bmatrix} -0.75529433 \\ 0.68509431 \end{bmatrix}$$

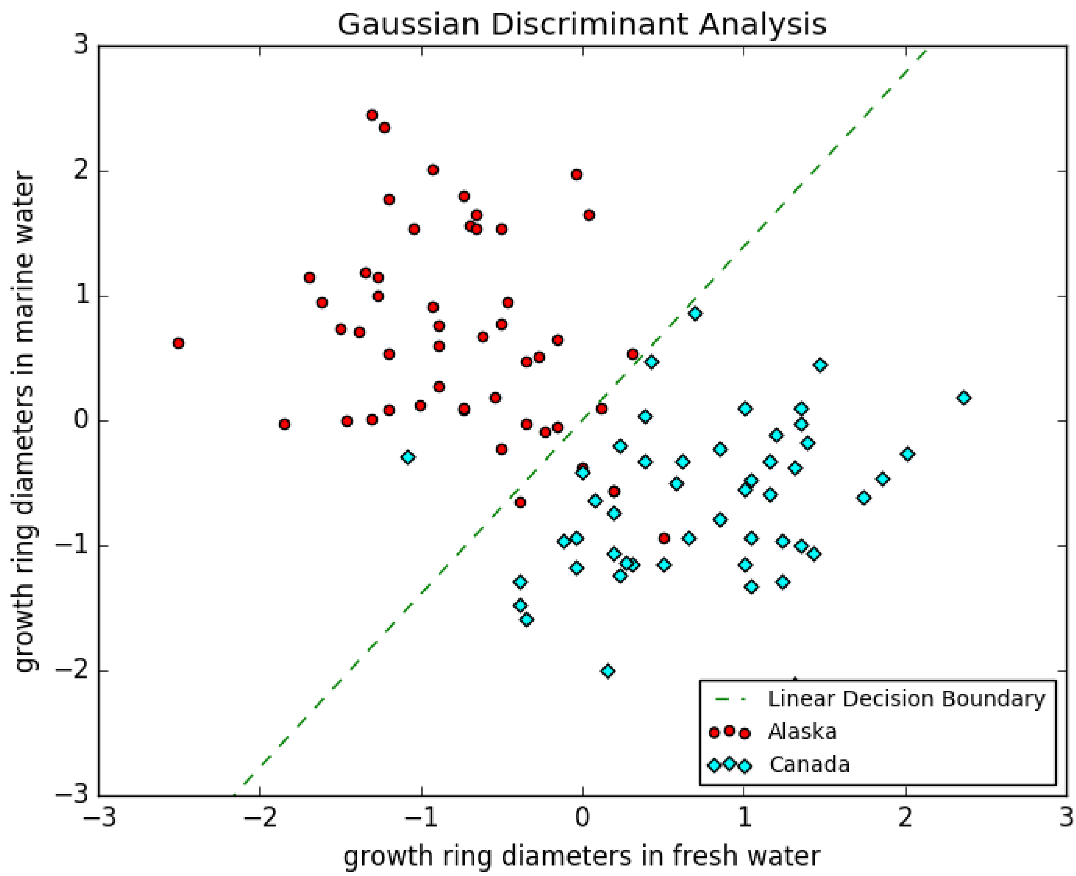
$$\Sigma = \begin{bmatrix} 0.42953048 & -0.02247228 \\ -0.02247228 & 0.53064579 \end{bmatrix}$$

b)



c) Equation for Decision Boundary:

$$X^T\Sigma^{-1}(\mu_0 - \mu_1) + (\mu_0 - \mu_1)^T(\Sigma^{-1}X) + \mu_1^T\Sigma^{-1}\mu_1 - \mu_0^T\Sigma^{-1}\mu_0 = 2 * log(\frac{\phi}{1-\phi})$$



d) Values obtained are :

$$\mu_0 = \begin{bmatrix} 0.75529433 \\ -0.68509431 \end{bmatrix}$$

$$\mu_1 = \begin{bmatrix} -0.75529433 \\ 0.68509431 \end{bmatrix}$$

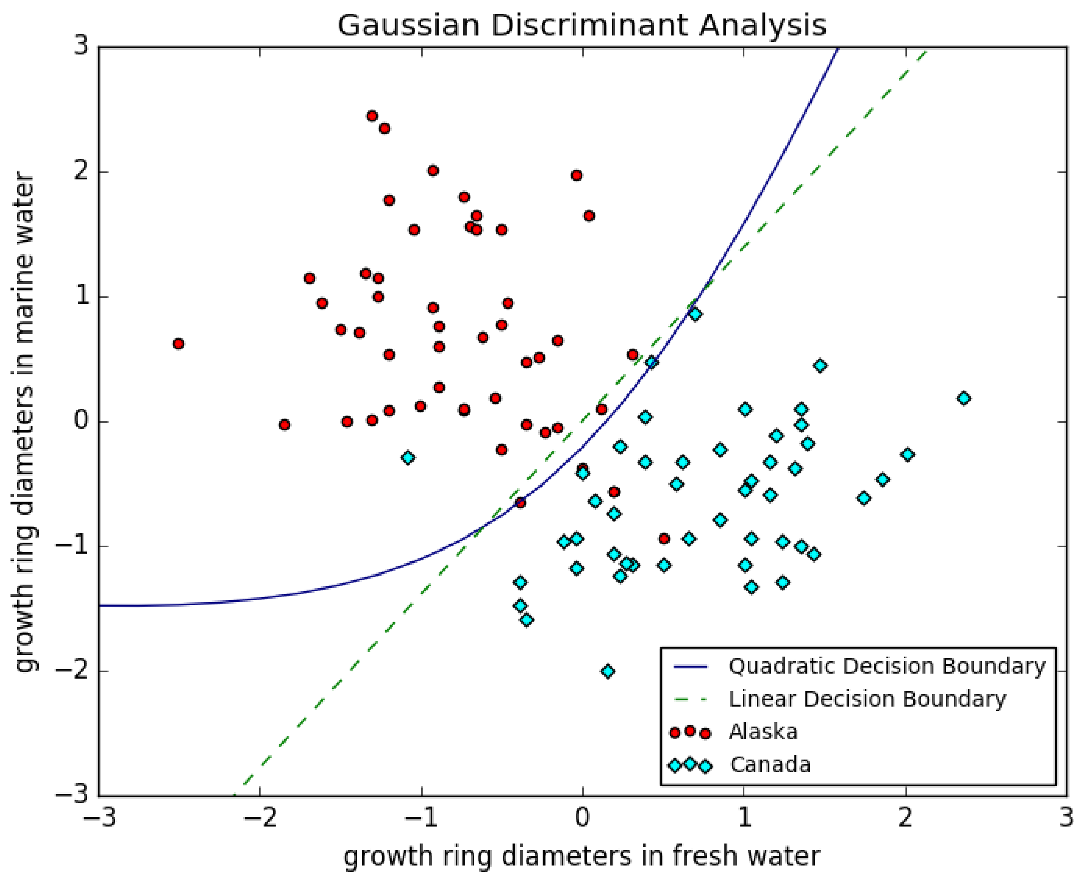
$$\Sigma_0 = \begin{bmatrix} 0.47747117 & 0.1099206 \\ 0.1099206 & 0.41355441 \end{bmatrix}$$

$$\Sigma_1 = \begin{bmatrix} 0.38158978 & -0.15486516 \\ -0.15486516 & 0.64773717 \end{bmatrix}$$

e)

Equation for Decision Boundary:

$$(X-\mu_1)^T\Sigma_1^{-1}(X-\mu_1)-(X-\mu_0)^T\Sigma_0^{-1}(X-\mu_0) = 2 * \ [\ log(\frac{\phi}{1-\phi}) - log(\frac{|\Sigma_1|^{0.5}}{|\Sigma_0|^{0.5}})\ ]$$



f) We can observe from the plot that accuracy on the training data is more with quadratic boundary than linear boundary.
For salmons having low diameter in fresh water and high diameter in marine water both boundaries predict Alaska. Similarly salmons having high diameter in fresh water and low diameter in marine water both boundaries predict Canada.
On the other hand, these boundaries differ in their behaviour if we have salmons with low diameter in fresh water as well as in marine water. Quadratic boundary in this case would predict Canada whereas linear boundary would give Alaska.

The End.